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Quantifying the value of SHM information for bridges under flood-induced scour

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Abstract

Bridge scour is a leading cause of failure for bridges over waterways and is notoriously difficult to detect with accuracy. Dynamic Structural Health Monitoring (SHM) of bridges for scour has gained traction in recent years as monitoring systems have improved and the reliability of measurements increased. Due to the large number of bridges on typical networks and the limited financial resources within asset management agencies, decision-makers must prioritize certain structures when it comes to management in the event of flooding. The decision to install a dynamic SHM system on a bridge must be balanced by the financial benefit of doing so, as limited resources often need to be carefully rationed. In this paper, a methodology is proposed to evaluate such benefit based on the Value of Information (VoI) from Bayesian decision analysis. A case study is presented whereby a dynamic SHM system is considered to be installed on a typical bridge with the aim to support emergency management operations during flooding. The proposed methodology allows computation of the financial benefit of installing a dynamic SHM system over a certain reference period, thus accounting for multiple flood events and scenarios. The various elements of the procedure are discussed in detail.

Keywords: Scour, Bridge management, Emergency management, Value of Information, Decision making, Structural Health Monitoring, Cost-Benefit analysis

1. Introduction

The erosion of soil from around and under bridge foundations, termed scour, remains a critical issue for many bridges worldwide (Prendergast & Gavin, 2014). Scour erosion occurs due to the interaction of water flow with soil particles and is exacerbated under higher flows associated with flooding (Briaud et al., 2011). Scour reduces the load-carrying capacity of bridge foundations (Fitzgerald et al., 2019) and is considered the main cause of bridge collapse worldwide (Melville & Coleman, 2000). The primary difficulty with scour erosion lies in the challenges surrounding its detection. Scour can cause severe damage to bridges but its presence is largely invisible, as it occurs below the waterline, where the turbid nature of flowing water makes this difficult to visually identify (Maddison, 2012). Effective management of traffic over bridges during flooding is, therefore, a challenging task, as the capacity of a bridge to handle this traffic load is directly correlated to the nature of the scour condition affecting the foundation.

Decision-making for asset managers in the event of flood occurrence is not trivial. On the one hand, if a bridge is kept open during a flood, it might fail under the combination of scour and various external actions, leading to unacceptable consequences. On the other hand, implementing a bridge closure might incur significant indirect consequences related to travel detours. Intermediate actions, for example closing a bridge to heavy traffic only, might reduce the risk of structural collapse but will involve indirect, albeit minor associated consequences. The availability of information on the actual scour condition affecting a bridge foundation allows the enhancement of emergency bridge management.

Many asset agencies monitor the occurrence of scour around bridges using visual inspections, which are laborious, inefficient and, of questionable efficacy. Recent advances have sought to move away from direct human intervention towards the use of sensors and instruments capable of remotely detecting scour development (Fisher et al., 2013). Many sensors have been developed including float-out devices (Briaud et al., 2011; Hunt, 2009), radar and sound-based systems (Anderson et al., 2007), and buried rod systems (Zarafshan et al., 2012), among many others, all which aim to detect scour hole depth evolution. The common disadvantage of many of these systems is that they typically can only detect scour local to the sensors and are therefore prone to inaccuracy. Moreover, they provide no information on how the bridge structure is responding to the presence of a scour hole, which is arguably the most important aspect.

In response to the dissatisfaction associated with scour detection using remote instrumentation and visual inspections, methods that can monitor the actual performance of a scoured bridge have become popular in recent years, especially dynamic Structural Health Monitoring (SHM) systems (Prendergast et al., 2018). One such approach relies on monitoring changes in the natural frequency of a bridge structure due to the occurrence of scour, which represents a change in the structural boundary conditions and therefore results in a change in modal behaviour (Sohn et al., 2001). Frequency-based scour detection has received significant attention in the literature in recent years, and interested readers are referred to Refs. (Malekjafarian et al., 2020; Prendergast et al., 2016; Xiong et al., 2019) for more details. Traditional dynamic SHM systems can be expensive, therefore tools are required to guide decision-makers in deciding whether or not it is financially beneficial to install them.

The Value of Information (VoI) from Bayesian decision analysis can be used to compute the benefit of installing a permanent dynamic SHM system for scour detection. The VoI can be defined as the expected reduction in management cost related to informed decision-making, that is when the decision is aided by the new information (Khan et al., 2021; Straub, 2014; Thöns & Faber, 2014). The theoretical framework of the VoI in the context of management of scoured bridges has been presented in Ref. (Giordano et al., 2020). Nevertheless, several practical aspects relating to the implementation of the framework of the VoI have not yet been addressed.

This paper further develops the framework presented in Ref. (Giordano et al., 2020), and presents a viable methodology to evaluate the benefit of installing a permanent dynamic SHM system on a bridge prone to scour. In addition, the theoretical framework is extended to include multiple floods during the life-cycle of the structure. A case study is presented to demonstrate the proposed methodology and to guide decision-makers in how to apply the procedure. This paper contains a number of novel contributions: (1) a reliability-based definition of scour-induced damage states affecting a bridge is proposed; (2) a more involved definition of a flood hazard exploiting the properties of the Peaks over Threshold series model (Kottegoda & Rosso, 2009) is incorporated; (3) the output from a SHM system is directly modelled using Monte Carlo simulations by considering the dynamic response of a two-span integral bridge subjected to a moving load, with inherent uncertainty incorporated in the load and bridge structural properties; (4) intermediate traffic restrictions are considered in the case study application.

The objective of the decision problem tackled in this paper is the management of scoured bridges during a flood event. The decision-maker must decide the type of traffic restriction (none, partial, total) to impose on a scoured bridge after the beginning of a flood event. Each of these three alternative actions entails different consequences and leads to an expected cost that depends on the uncertain state of the bridge. Information about the condition of the bridge during a flood can reduce the uncertainty thereby improving the choice of the optimal management action (leave the bridge open, reduce the traffic, close the bridge). The VoI is used to quantify the benefit of such information.

The paper is organized as follows. Section 2 introduces the theoretical background of the VoI, which is subsequently detailed in Section 3, where the steps needed to quantify the VoI are described for the specific decision problem analysed. Section 4 presents a case study of a two-span integral road bridge under scour. Section 5 is devoted to the conclusions of the paper.

2. Theoretical background of the Value of Information

In general terms, the VoI quantifies the gain associated with the acquisition of new information in a given decision-making context (Raiffa & Schlaifer, 1961). To quantify the VoI, it is necessary to compare two situations, namely the situation in which decision-making is carried out using the available knowledge, and the situation where decision-making is supported by new information (Thöns et al., 2018). The decision analysis that is carried out using the available knowledge is referred to as Prior analysis. In this case, the decision-maker selects the action $\hat{A}$ that maximises their expected utility, u_1, as follows:

$$\hat{A} = \arg\max_n E[u(A_n)] \tag{1}$$

$$u_1 = E[u(\hat{A})] = \sum_{l=1}^{L} E[u(\hat{A})|DS_l]P(DS_l) \tag{2}$$

where A_n $(n = 1, \dots, N)$ is one of N alternative actions; DS_l $(l = 1, \dots, L)$ is one of L possible states of the system; $P(DS_l)$ is the prior probability assigned to DS_l; and $E\left[u(\hat{A})|DS_l\right]$ is the expected utility of action $\hat{A}$ when the state of the system is DS_l.

The availability of new information on the state of the system allows updating of the prior probabilities $P(DS_l)$, according to Bayes' Theorem, which reads:

$$P\left(DS_l\middle|O_j\right) = \frac{P\left(O_j|DS_l\right)P(DS_l)}{P\left(O_j\right)} = \frac{P\left(O_j|DS_l\right)P(DS_l)}{\sum_{l=1}^{L} P\left(O_j|DS_l\right)P(DS_l)} \tag{3}$$

where O_j $(j = 1, \dots, J)$ is one of J possible outcomes provided by a data collection strategy; $P\left(O_j|DS_l\right)$ is the probability of obtaining the outcome O_j when the state of the system is DS_l, which is referred to as a likelihood function; $P\left(DS_l\middle|O_j\right)$ is the posterior probability of DS_l, i.e. the updated probability that the system is in damage state DS_l given the outcome O_j; and $P\left(O_j\right)$ is the probability of observing O_j. When the outcome O_j is available, the decision-maker can perform a Posterior analysis, that is they can select the action $\breve{A}_{O_j}$ that maximises the expected utility $E\left[u\left(\breve{A}_{O_j}\right)|O_j\right]$ as follows:

$$\breve{A}_{O_j} = \breve{A}\left(O_j\right) = \arg\max_n E\left[u(A_n)\middle|O_j\right] \tag{4}$$

$$E\left[u\left(\breve{A}_{O_j}\right)|O_j\right] = \sum_{l=1}^{L} E\left[u\left(\breve{A}_{O_j}\right)|DS_l\right] P\left(DS_l\middle|O_j\right) \tag{5}$$

When the VoI analysis is performed to decide if it is worth acquiring new information with a certain data acquisition strategy, this information (outcome O_j) is not yet available. In this case, the decision-maker must compute the expected utility associated with all possible outcomes. This is termed Pre-Posterior analysis and comprises two steps. Firstly, the Posterior analysis is performed for each possible outcome O_j. This implies the identification of the optimal action $\breve{A}_{O_j}$ and the evaluation of the corresponding expected utility $E\left[u\left(\breve{A}_{O_j}\right)|O_j\right]$. Secondly, the weighted sum of the expected utilities of the optimal actions is computed according to Eq. (6):

$$u_0 = \sum_{j=1}^{J} E\left[u\left(\breve{A}_{O_j}\right)|O_j\right] P\left(O_j\right) \tag{6}$$

where $P(O_j)$ is computed according to Eq. (3). The VoI represents the difference between u_0 (Pre-Posterior analysis), and the expected utility u_1 (Prior analysis):

$$\text{VoI} = u_0 - u_1 \tag{7}$$

The general framework of the VoI to compute the benefit of SHM data is adapted to the case of emergency management of scoured bridges in Ref. (Giordano et al., 2020). In this context, the elements of the VoI analysis are specified as follows. A_n ($n = 1, \dots, N$) is one of the N emergency management actions, such as "close the bridge", "issue traffic restrictions", and "leave the bridge open". DS_l ($l = 1, \dots, L$) is one of the L damage states of the bridge, linked to a specific scour depth affecting the structure. Accordingly, the l-th prior probability $P(DS_l)$ represents the probability of a bridge being in a given damage state during a flood event. In general, the prior probabilities $P(DS_l)$ are conditional on the intensity measure characterizing the severity of the flood, herein described as the water flow rate Q, i.e., $P(DS_l|Q)$. The decision maker is assumed to be risk neutral (Verzobio et al., 2021), i.e., the utility function u is assumed equal to minus cost. Therefore, the action which maximises the utility is the one associated with the minimum expected cost. In this case, the VoI can be understood as the expected reduction in emergency management costs resulting from the use (in the future) of the information from a SHM system. The expected utility $E[u(A_n)|DS_l]$ of the action A_n when the system is in damage state DS_l, depends on the cost of the consequence of the action A_n. In the case considered here, the bridge can fail or can survive following the choice of the action A_n. The relevant expected utility, used in Equations (2) and (5), is thus computed (in absolute value) as follows:

$$E[u(A_n)|DS_l] = c_F(A_n)P(F|A_n, DS_l) + c_{\bar{F}}(A_n)[1 - P(F|A_n, DS_l)] \tag{8}$$

where $c_F(A_n)$ and $c_{\bar{F}}(A_n)$ are the cost of bridge failure and survival, respectively, which depend on the action A_n; $P(F|A_n, DS_l)$ is the failure probability, conditional on both the action A_n and damage state DS_l. The failure probability accounts for the fact that, after the selection of the management action, the structure damaged by the flood might (or not) fail under operational loads (e.g., traffic).

The probabilities $P(O_j|DS_l)$ are retrieved by the so-called likelihood functions, which represent the Probability Density Functions (PDF) of the information (SHM output O_j) when the damage state of the bridge is DS_l. The VoI is conditional on the intensity of the flood, i.e., $\mathrm{VoI}(Q)$ (since the prior probabilities are conditional on Q). The unconditional VoI is obtained as follows:

$$\mathrm{VoI} = \int_Q \mathrm{VoI}(Q) f(Q) dQ \tag{9}$$

where $f(Q)$ is the probability density function (PDF) of the flow. The VoI computed according to Eq. (9) is related to a single flood and thus to a single emergency management decision (whose intensity is not known in advance).

Furthermore, a decision-maker could be interested in the benefit of installing an SHM system over a reference period, such as the reference life of the structure or the reference life of the SHM system. Both of these aspects can be taken into account by the Life-Cycle VoI, $\mathrm{VoI_{LC}}$, (Giordano et al., 2020; Zonta et al., 2014), which is defined as follows:

$$\mathrm{VoI_{LC}} = \sum_{i=1}^{T_{LC}} \lambda \frac{\mathrm{VoI}}{(r+1)^i} \tag{10}$$

where T_{LC} is the reference period (in years) considered in the analysis, λ is the expected number of floods above a certain threshold in one year, and r is the discount rate. The $\mathrm{VoI_{LC}}$ should be compared with the total cost of the SHM system over the reference period (including installation cost and management cost) to determine the cost-benefit of installing the SHM system itself.

3. Methodology

The application of the theoretical framework of the VoI described in the previous section entails several steps, namely:

1. Definition of the damage states DS_l affecting a given structure;
2. Computation of the probabilities of each damage state $P(DS_l)$;
3. Computation of the failure probability $P(F|A_n, DS_l)$ in the different damage states for the considered decision alternatives;
4. Bayesian updating of prior probabilities of damage states using the monitoring information modelled through the likelihood functions $P(O_j|DS_l)$;

5. Estimation of management costs;
6. Computation of the VoI over the life-cycle accounting for the exposure at the site of the structure.

In the following subsections, each of these six steps is described with reference to the emergency situation created by the flood-induced scouring of a bridge.

3.1. Definition of damage states

Scouring of bridge foundations reduces the capacity of the structure leading to the development of certain damage states, which are related to the scour depth attained during the flood. The continuous spectrum of possible damage states, each relating to a certain scour depth, is discretized into three groups corresponding to “Insignificant damage”, DS_1, “Moderate damage”, DS_2, and “Heavy damage”, DS_3. Following the approach proposed in Ref. (Sharma et al., 2017), these damage states are not defined in terms of the physical damage but in terms of its effect on the reliability of the bridge through different reliability index values. Three threshold values of the reliability index define the considered damage states: reliability index β_0 of the original system, acceptable reliability index β_{acc}, and minimum tolerable reliability index β_{tol}. The reliability index β is directly linked to the probability of failure $P(F)$ of the bridge in virtue of its definition: $P(F) = \Phi(-\beta)$, where Φ is the standard normal cumulative function. The threshold β_0 relates to the probability of failure under normal situations (no scour, and full traffic load). The reliability thresholds β_{acc} and β_{tol} must be selected by the decision-maker. The bridge is in DS_1 for $\beta_{acc} \leq \beta < \beta_0$, in DS_2 for $\beta_{tol} \leq \beta < \beta_{acc}$, and in DS_3 for $\beta < \beta_{tol}$. When the bridge is in DS_1, its reliability decreases but remains above a certain acceptable level. The reliability of the bridge in DS_2 is below the acceptable threshold but still above the minimum tolerable threshold. The reliability of the bridge in DS_3 is below the tolerable threshold.

In the case of scoured bridges, the failure probability $P(F|y_s)$, which depends on the attained scour depth y_s, can be computed as follows:

$$P(F|y_s) = P[C(y_s) - D \leq 0] \tag{11}$$

where $C(y_s)$ is the total capacity of the bridge as a function of y_s, and D is the demand on the bridge. Both demand and capacity must be represented by suitable distribution models. Once a reliability index is fixed, the scour depth that leads to the corresponding failure probability can be identified by solving an inverse reliability problem. These scour depths constitute the scour thresholds $y_s = th_l$ ($l = 1, \ldots, L$) that define the L damage states.

Scour can lead to several different failure modes such as axial failure, adverse settlement, pile buckling, deck unseating, among others (Hughes et al., 2007; Prendergast et al., 2018). The failure mode considered in this paper relates to the loss of axial capacity of pier foundations due to scour. It is assumed that scour occurrence reduces the available pile capacity, which eventually leads to foundation failure (Malekjafarian et al., 2020). It should be noted that any failure mechanism can be considered with the present framework so long as an appropriate demand model can be obtained, the purpose of using axial capacity failure is to demonstrate the approach developed.

The axial capacity can be calculated considering the available pile group shaft and base resistance, as determined using the API design codes (API, 2007), for example. The unit shaft, f, and base resistance, q_b, are calculated as in Eq. (12) and Eq. (13), respectively, where K is the ratio of horizontal to vertical normal effective stress, $\sigma'_v(y_s)$ is the vertical effective stress (kPa) as a function of scour, δ is the interface friction angle between the soil and the pile, and N_q is the dimensionless bearing capacity factor.

$$f = K\sigma'_v(y_s)\,tan\,\delta \tag{12}$$

$$q_b(y_s) = \sigma'_v(y_s)N_q \tag{13}$$

The average shaft resistance is determined using the vertical effective stress calculated at the mid-depth of the pile, where depth refers to the penetration depth of the pile (excluding the portion lost due to scour). This is converted to shaft capacity $C_s(y_s)$ by multiplying by the available shaft shear area of the pile group $A_s(y_s)$, which depends on the scour depth y_s, using Eq. (14) where N_{pile} is the number of piles in the group. Similarly, the base bearing capacity $C_b(y_s)$, which also depends on the scour depth as it relates to the effective stress at the base, is determined by multiplying the base resistances by the cross-sectional area of the piles in the group, A_b, see Eq. (15).

$$C_s(y_S) = N_{pile} f A_s(y_s) \tag{14}$$

$$C_b(y_S) = N_{pile} q_b(y_s) A_b \tag{15}$$

The total capacity $C(y_s)$ is determined as the sum of the available shaft and base capacities under a given scour scenario, as follows:

$$C(y_s) = C_s(y_s) + C_b(y_s) \tag{16}$$

The demand consists of the external actions acting on the pier foundation and includes the dead load of the bridge and vehicular loading.

3.2 Probability of a damage state

When a monitoring system or other measurements are not available, the decision-maker must rely on theoretical models, or on their own engineering judgement, to estimate the probabilities of the different damage states. These are called 'prior' probabilities since they are estimated before the collection of measurements. Herein, it is assumed that the decision-maker uses the methodology described in Ref. (Arneson et al., 2012; Jones & Sheppard, 2000) to compute the scour depth associated with a flood of intensity Q for complex pier foundations. The total scour depth y_s is obtained as the sum of three terms related to the components of a complex pier, namely pier stem, pile cap, and pile group, as follows:

$$y_s = y_{s,pier} + y_{s,pc} + y_{s,pg} \tag{17}$$

where $y_{s,pier}$ is the scour component for the pier stem in the flow according to Eq. (18); $y_{s,pc}$ is the scour component for the pile cap in the flow according to Eq. (19); and $y_{s,pg}$ is the scour component for the piles exposed to the flow according to Eq. (20). Figure 1 shows the different components of a generic complex pier.

$$\frac{y_{s,pier}}{y_1} = K_{h,pier}\left[2.0 K_1 K_2 K_3 \left(\frac{a_{pier}}{y_1}\right)^{0.65} \left(\frac{V_1}{\sqrt{g y_1}}\right)^{0.43}\right] \tag{18}$$

$$\frac{y_{s,pc}}{y_2} = 2.0 K_1 K_2 K_3 K_w \left(\frac{a^*_{pc}}{y_2}\right)^{0.65} \left(\frac{V_2}{\sqrt{g y_2}}\right)^{0.43} \tag{19}$$

$$\frac{y_{s,pg}}{y_3} = K_{hpg}\left[2.0K_1K_3\left(\frac{a_{pg}^*}{y_3}\right)^{0.65}\left(\frac{V_3}{\sqrt{gy_3}}\right)^{0.43}\right] \tag{20}$$

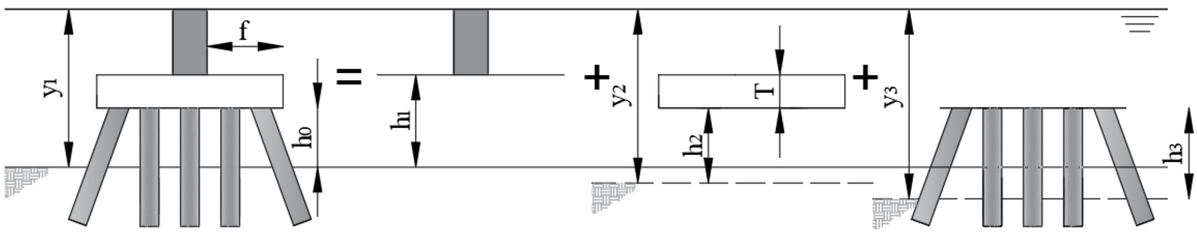

Figure 1 Definition of scour components for a complex pier foundation (modified from Arneson et al., 2012)

The variables used in the computation are defined in Table 1. In particular, y_1 is the original flow depth (i.e., without scour), y_2 is the adjusted flow depth for pile cap computations (i.e., the increased flow depth accounting for the amount of scour due to the pier stem), and y_3 is the adjusted flow depth for pile group computations (i.e., the increased flow depth accounting for the amount of scour due to both the pier stem and the pile cap). The effect of modelling uncertainty associated with scour modelling is considered by multiplying the right side of Eq. (17) by a model correction factor λ_{y_s}, $y_s = \lambda_{y_s}\left(y_{s,pier} + y_{s,pc} + y_{s,pg}\right)$.

Hypothesizing a rectangular channel of width B, the terms y_1 and V_1 can be computed as follows (Highways Agency, 2012):

$$y_1 = \left(\frac{Qn}{Bs^{0.5}}\right)^{3/5} \tag{21}$$

$$V_1 = \frac{Q}{By_1} \tag{22}$$

where Q is the flow rate; n is the Manning's coefficient; s is the slope of the channel.

The introduction of random variables in the formulation allows obtaining a distribution of scour depth. The probabilities of the different damage states are obtained as follows:

$$\begin{aligned} &P(DS_l|Q) = P[\{y_s \geq th_l\} \cap \{y_s < th_{l+1}\}] && for\ l \neq L \\ &P(DS_l|Q) = P(y_s \geq th_l) && for\ l = L \end{aligned} \tag{23}$$

where th_l ($l = 1, \dots, L$) are the scour thresholds that define the L damage states.

Table 1 Variables used in scour depth computation

Variable	Description	Variable	Description
f	distance between the front edge of the pile cap and the pier	K_2	correction coefficient for angle of attack of flow
T	pile cap thickness	K_3	correction coefficient for bed conditions
S	spacing between piles (pile centre to centre)	a_{pier}	pier width
g	acceleration due to gravity	a_{pc}	full pile cap width
h_0	height of the pile cap above the riverbed before scour	a	single pile width
h_1	$= h_0 + T$	m	number of pile rows in flow direction
h_2	$= h_0 + y_{s,pier}/2$	a_{proj}	sum of non-overlapping projected widths of piles
h_3	$= h_0 + y_{s,pier}/2 + y_{s,pc}/2$	a^*_{pc}	pile cap equivalent width, (function of a_{pc}, T/y_2, h_2/y_2)
y_1	flow depth before scour	a^*_{pg}	$= a_{proj} K_{sp} K_m$
y_2	$= y_1 + y_{s,pier}/2$	$K_{h,pier}$	suspended pier scour ratio, (function of h_1/a_{pier}, f/a_{pier})
y_3	$= y_1 + y_{s,pier}/2 + y_{s,pc}/2$	K_w	wide pier correction factor
V_1	mean velocity of flow upstream of pier	K_{hpg}	pile group height adjustment factor, (function of h_3/y_3)
V_2	$= V_1(y_1/y_2)$	K_{sp}	coefficient for pile spacing, (function of a_{proj}/a, S/a)
V_3	$= V_1(y_1/y_3)$	K_m	coefficient for number of aligned rows, (function of m, S/a)
K_1	correction coefficient for pier nose shape		

3.3. Computation of the failure probability

After the selection of an emergency management action, a structure might fail due to external actions that relate to its damage state. This aspect is taken into account by the failure probability $P(F|A_n, DS_l)$, see Eq. (8).

In Section 3.1, it is shown that foundation capacity decreases with increasing scour depth, see Eq. (11). A lower capacity implies an increased failure probability. Nevertheless, the failure probability can also be influenced by the choice of the decision-maker, for example, the decision made with respect to an implemented emergency action directly determines the subsequent traffic demand on the structure. Therefore, when considering the effect of different decisions, Eq. (11) can be rewritten as follows:

$$P(F|A_n, y_s) = P[C(y_s) - D(A_n) \leq 0] \tag{24}$$

where $P(F|A_n, y_s)$ is the probability of failure conditional on the action A_n and the scour depth y_s, and $D(A_n)$ is the demand on the bridge, which depends on the action A_n. The probability $P(F|A_n, DS_l)$ is obtained by means of Eq. (24) by setting $y_s = th_l$.

In case the selected action A_n is "leave the bridge open", it can be assumed that the traffic remains the same as under normal operating conditions. More refined analyses can be performed, whereby the interaction between the characteristics of traffic and the flood magnitude is modelled (Pregnolato et al., 2017).

Traffic limitations, such as closing the bridge to trucks, result in lower demand and therefore a lower probability of failure for the same scour depth with respect to the previous situation. As for the closure of the bridge, it is assumed that the only cause of the collapse is the exceedance of the axial capacity of pier foundations under scour due to vertical loads.

3.4 Bayesian updating of the probability of damage states

The mathematical formulation of the VoI has been presented in Section 2. In particular, Eq. (3) describes the updating of the prior probability of the different damage states according to the Bayes' Theorem. This updating is made possible by the output of an SHM system, which is modelled by means of likelihood functions $P(O_j|DS_l)$ representing the probability of observing a certain output O_j when the structure is in damage state DS_l. In general, for existing structures it is impossible to directly observe such probabilities before the installation of an SHM system because this would require intentionally damaging a structure to observe the response. For this reason, the likelihood functions must be estimated using models (Giordano & Limongelli, 2020; Pozzi et al., 2010).

The focus of this paper is on vibration-based SHM, which exploits the global dynamic response of structures as a means of damage detection (Quqa et al., 2021). Natural frequencies are used as a damage-sensitive feature in the present case due to the ease associated with measuring this parameter and the simplicity of its meaning. Since natural frequencies are continuous parameters, the likelihood functions are modelled as PDFs, which describe the probability of observing a given value of a continuous random variable in correspondence to a certain scour depth (corresponding to the threshold values which define the damage states). It should be noted that frequencies by themselves are only a proxy for scour depth and it is inherently assumed that an asset manager would make use of a reference finite-element model with which to map the frequencies measured onto the scour depths, as per any model-based (and not data-driven) SHM technique.

The procedure for generating these distributions is outlined herein. Monte-Carlo simulations of the frequency content of the dynamic response of a bridge excited by a crossing vehicle are undertaken. The simulations make use of a numerical model of the bridge under scour and can be repeated for each damage state. Various sources of uncertainty should be accounted for in the analysis, namely uncertain structural properties, unknown excitation and environmental parameters, noise in the data acquisition and transmission process, and numerical errors in data processing, among others. The procedure involves developing a given model bridge with desired structural and geotechnical properties, which are sampled from distributions of representative parameters. The properties of the bridge are considered as random variables. The bridge is traversed by a moving load whose magnitude and velocity are sampled from distributions of these parameters. When the random vehicle crosses the bridge, the acceleration signal (structural response) can be measured at a suitable location (e.g. at the top of a pier) and analysed for its frequency content (Prendergast et al., 2017). The natural frequencies can be obtained using a Fourier transform applied to the "measured" acceleration.

3.5. Estimation of management costs

The VoI has been defined in Section 2 as the difference between the expected utility from Pre-Posterior analysis and the expected utility from Prior analysis. To compute such expected utilities, the costs of bridge failure and survival must be estimated for each action A_n (see Equation 8). Each action leads to different expected direct and indirect costs in the case of failure or survival in relation to the damage state of the bridge. Direct costs are related to damage and losses resulting directly from the failure of the bridge, whereas indirect costs are generated by the loss of functionality of the bridge (Imam &

Chryssanthopoulos, 2012). In Ref. (NASEM, 2007), the cost of bridge failure is obtained as the sum of direct costs; namely rebuilding costs C_{RB} and costs of life loss C_{LL}; and indirect costs, namely running costs C_{RN} and costs related to time loss C_{TL}, which are computed as follows:

$$C_{RB} = C_1 W L \tag{25}$$

$$C_{LL} = C_6 X \tag{26}$$

$$C_{RN} = \left[C_2 \left(1 - \frac{T}{100}\right) + C_3 \frac{T}{100}\right] D_L A d \tag{27}$$

$$C_{TL} = \left[C_4 O \left(1 - \frac{T}{100}\right) + C_5 \frac{T}{100}\right] \frac{D_L A d}{S} \tag{28}$$

where C_1 is the unit rebuilding cost (€/m^2); W is the width of the bridge (m); L is the length of the bridge (m); C_2 is the cost of running cars (€/km), e.g. cost of fuel and wear; C_3 is the cost of running trucks (€/km); D_L is the detour length (km); A is the average daily traffic (vehicles/day); d is the duration of the detour based on A (days); T is the average daily truck traffic, (% of A); C_4 is the value of time per adult car passenger (€/h); O is the average occupancy rate for cars; C_5 is the value of time for trucks (€/h); S is the average detour speed (km/h); C_6 is the cost for each life lost (€/n); X is the number of life losses (n).

Table 2 shows which costs are considered in the estimation of the cost of bridge failure and survival depending on the selected action. In case the selected action is “leave the bridge open”, the failure of the bridge involves rebuilding costs, running costs, costs related to time loss, and costs of life loss. The duration of detours corresponds to the time needed to replace the bridge (in the order of months or years). The survival of the bridge results in no costs. If traffic limitations are issued, namely closing the bridge to trucks, the costs related to the failure of the bridge are the same with respect to the previous situation under the assumption that the expected number of fatalities does not change since truck traffic is only a small percentage of the total traffic crossing the bridge (however the probabilities of failure will be different). In this case, indirect costs are also generated due to the need to divert truck traffic. Since cars are still allowed to cross the bridge, the indirect costs are computed considering $C_2 = C_4 = 0$ in Equations (27) and (28). The duration of traffic limitations relates to the emergency phase (in the order of days or weeks).

Table 2 Cost of bridge failure and survival depending on the selected action

Costs	**Actions**		
	$A_n = Open$	$A_n = Limit$	$A_n = Close$
$\boldsymbol{c_F(A_n)}$	$C_{RB} + C_{RN} + C_{TL} + C_{LL}$ (d = Replacement duration)	$C_{RB} + C_{RN} + C_{TL} + C_{LL}$ (d = Replacement duration)	$C_{RB} + C_{RN} + C_{TL}$ (d = Replacement duration)
$\boldsymbol{c_{\overline{F}}(A_n)}$	0	$C_{RN} + C_{TL}$ (d = Traffic limitation duration, $C_2 = C_4 = 0$)	$C_{RN} + C_{TL}$ (d = Traffic limitation duration)

3.6 Computation of the VoI over the life-cycle

The computation of the VoI and of the Life cycle VoI requires the analysis of flood frequency of occurrence, as defined in Section 2. The Peaks over Threshold (POT) series model (Kottegoda & Rosso, 2009) is particularly suitable to represent multiple floods, where a flood is defined as a river discharge event exceeding a certain flow threshold, Q_0. The POT model consists of two components, namely (1) a probabilistic model of the distribution of the annual number of floods, and (2) a probability distribution of the flood magnitude. The number of flood events in one year n_f is assumed to follow a Poisson distribution, whose probability mass function reads

$$P(n = n_f) = \frac{e^{-\lambda}\lambda^{n_f}}{n_f!}, n_f = 0, 1, 2, \ldots \tag{29}$$

where $P(n = n_f)$ is the probability of having n_f floods in one year, and λ is the expected number of floods above the threshold in one year, which appears in Eq. (10). The use of the Poisson distribution implies that floods are considered to be statistically independent in time. Moreover, the developing scour condition resulting from a given flood is assumed independent of previous scour occurrences as it is assumed that a scour hole is re-filled when a flood ends. This assumption is made due to the lack of available data to quantify the effects of subsequent scour developments in the available design formulae. The flood magnitude is described by an Exponential distribution having the following PDF:

$$f(Q) = \nu \exp[-\nu(Q - Q_0)] \tag{30}$$

where Q is the flow rate.

The effect of climate change might modify the flood intensity and frequency over time and can be included in the VoI formulation. Specifically, climate change effects can be modelled as a gradual modification of the parameters that describe the flow intensity and frequency. Interested readers can refer to Ref. (Giordano et al., 2020) for additional details on this issue.

4. Case study – Two-span integral bridge under scour

The methodology described in the previous section is applied to a case study of a two-span integral bridge herein. The focus is on demonstrating how to apply the proposed methodology to compute the VoI from SHM, so a generic reference bridge from the literature is used (Prendergast et al., 2016). It should be noted that integral-type bridges do not represent bridges that typically fail under scour, however, any bridge can be used with the present framework and the purpose of using the current integral bridge model is that it has been used in previous research to trial several vibration-based scour detection approaches. In the case study, it is shown how to compute or estimate the variables needed in the evaluation of the VoI for a specific case without any claim for the generality of the results. The life cycle VoI is computed over a period of 20 years, which is considered as the expected life of a permanent SHM system.

The bridge model comprises two 25 m long deck spans founded on a 6 m long central pier system with two leaves, and nine 6 m high flexible-type cast-in-sleeve abutments, which are founded on piles of varying length. Each leaf of the central pier system is founded on four piles, and each set of nine columns are founded on 10 piles, with pile caps. The deck has a cross-sectional area of 9.516 m^2, and a second moment of area of 2.9487 m^4. The abutment columns have a grouped cross-sectional area of 1.7671 m^2 and second moment of area 0.0276 m^4. The central pier system has cross-sectional area 7.22 m^2, and second moment of area 1.137 m^4. The central pier piles have grouped cross-sectional area of 3.53 m^2 and second moment of area 0.124 m^4, and the abutment piles have grouped cross-sectional area 2.83 m^2 and second moment of area 0.064 m^4. The density of concrete is taken as 2400 kg/m^3 for all elements in the model. The elastic modulus and soil stiffness for each element is considered as a variable in the subsequent analyses, see Table 7 for properties. A schematic of the structure with the main dimensions is shown in Figure 2.

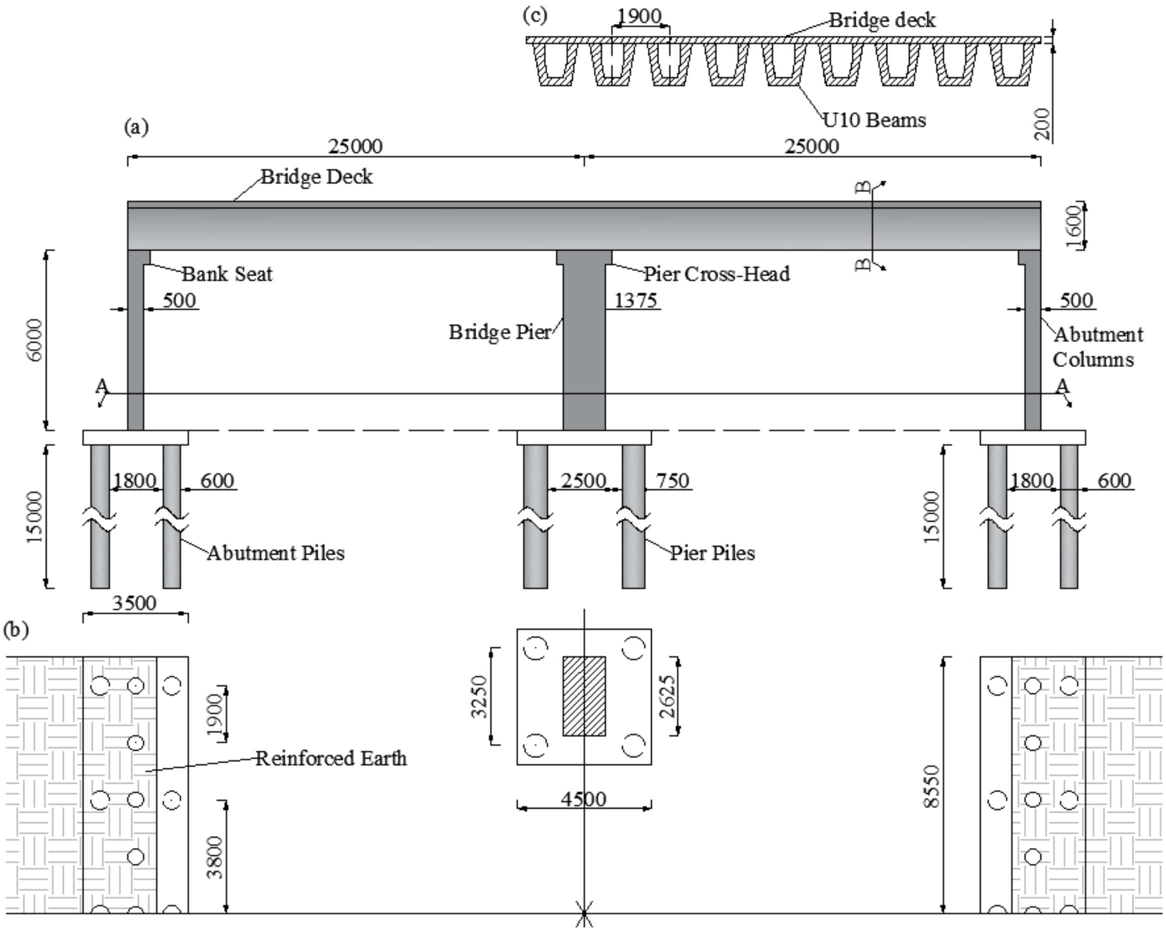

Figure 2 Schematic of bridge model used in the case study (modified after Prendergast et al., 2016).

The reason for considering an integral bridge in the present study is two-fold, (i) they are an increasingly popular form of construction as they do not require thermal expansion joints, reducing maintenance requirements (Springman et al., 1996), and (ii) the presence of moment connections between the deck and supporting piers/abutments enables longitudinal (traffic-direction) movement under vehicle actions, which is potentially beneficial for detecting scour-related stiffness losses at foundations (Malekjafarian et al., 2020; Prendergast et al., 2016). Any bridge model can be supplemented in the present framework and the current model is used as a means to demonstrate the approach only.

The bridge model is further described in Section 4.1 and is used to develop distributions of the output frequency of vibration resulting from scour affecting the central pier foundation of the bridge. For the purpose of the analyses, the abutments are considered as fully protected against scour. To incorporate uncertainty, the bridge elastic modulus and soil properties are considered as probabilistic distributions, see Table 7.

4.1. Numerical modelling of bridge

The bridge, originally developed in Ref. (Prendergast et al., 2016), is mathematically modelled using the stiffness matrix method employing 2D Euler-Bernoulli frame elements, each with six-degrees-of-freedom (DOFs). Grouped geometrical and material properties are used to model the deck, pier, abutments, and piles, and into-the-page and torsional behaviour is omitted – see section 4 for primary dimensions and properties. The dynamic response of the bridge can be obtained by solving the dynamic equation of motion, shown in Eq. (31).

$$\mathbf{M_B}\ddot{\mathbf{x}} + \mathbf{C_B}\dot{\mathbf{x}} + \mathbf{K_B}\mathbf{x} = \mathbf{F} \tag{31}$$

where $\mathbf{M_B}$, $\mathbf{C_B}$, and $\mathbf{K_B}$ are the mass, damping, and stiffness matrices of the bridge model; $\ddot{\mathbf{x}}$, $\dot{\mathbf{x}}$, and $\mathbf{x}$ are the acceleration, velocity, and displacement of each DOF at each time step; $\mathbf{F}$ is a vector describing the external action on the bridge, in this case, a moving load, which traverses the bridge simulating the actions of single vehicles. The load is applied to the relevant degrees of freedom by distribution as forces and moments to elemental nodes using Hermitian shape functions.

Soil-structure interaction is incorporated in the model using Winkler springs, whereby discrete springs are attached along the pile elements to represent the stiffness contribution from the soil. Distributed lateral and vertical springs model the horizontal and shear resistances respectively. Due to the small amplitude vibrations expected in the horizontal and vertical direction under vehicular loading, small-strain criteria are assumed and the spring stiffnesses are characterised by the initial stiffness of the load-displacement behaviour from the American Petroleum Institute (API) design code (API, 2007). Hysteretic damping is assumed negligible due to the small-strain oscillations under this loading, and radiation damping is ignored as it is assumed wave propagation away from the foundations is more rapid than the frequencies associated with applied loading and the structural response. Spring stiffnesses corresponding to medium dense sand are assumed.

Scour is modelled by the iterative removal of springs from the model at the pier foundation, corresponding to a loss in soil-structure contact, and increasing free length of pile structural elements. While several researchers, e.g. (Chortis et al., 2020; Li et al., 2020; Lin et al., 2010), have noted that the removal of overburden due to the formation of a scour hole leads to a reduction in strength and stiffness of the remaining soil, this has been ignored in the present study due to the lack of quantitative research on how it influences the shearing behaviour at the pile-soil interface in the vertical direction.

4.2. Application of the methodology for the computation of VoI in example bridge

Damage states

The first step in the computation of the VoI is in the definition of the damage states of the bridge due to scour. Following Ref. (Sharma et al., 2017), the reliability thresholds for the damage states are fixed to $\beta_{acc} = 2.5$ and $\beta_{tol} = 1.5$, corresponding to the failure probability $P_{acc}(F) = 6.2 \cdot 10^{-3}$ and $P_{tol}(F) = 6.7 \cdot 10^{-2}$, respectively. The probability of failure under normal traffic conditions is computed according to Eq. (11), by comparing the capacity and demand for an assumed axial failure mechanism at the foundation. The capacity is computed following Equations (12)-(16) using the random and deterministic parameters shown in Table 3. It should be noted that in the case of integral bridges, ultimate failure is dependent on the resistance and stiffness of many elements, not just the foundation alone. This is because these are hyperstatic structures, so strictly speaking the capacity equation does not encapsulate this. A thorough analysis should consider the contributions from various elements of the structure to the foundation capacity and assess the different failure modes. In the present paper, this is simplified for the purpose of demonstrating the framework. Concerning the model correction factor, (Ghosn et al., 2003) provide it for the case of simple foundations. In this study, it is assumed that the same level of uncertainty can be considered for the case of complex piers.

Table 3 Parameters used to obtain the capacity distribution

Parameter	Mean	CoV	Distribution	Reference
Angle of internal friction, ϕ	35°	0.086	Lognormal	Assumed
Angle of interface friction, δ	0.8 ϕ	-	Deterministic	(API, 2007)
Bulk unit weight	20 kN/m^3	0.1	Lognormal	Assumed
Number of piles in the group, N_{pile}				
Ratio of horizontal to vertical normal effective stress	1	-	Deterministic	(API, 2007)
Cross-sectional area of the piles, A_b	3.53 m^2	-	Deterministic	(Prendergast et al., 2016)
Bearing capacity factor, N_q	20	-	Deterministic	(API, 2007)

The soil is considered as medium dense sand with inherent uncertainty (API, 2007). The distribution of the traffic maximum load considered in the computation relates to a reference period of 2 weeks (it is therefore assumed that 2-weeks maxima are independent). In Ref. (Fib, 2016), the characteristic traffic load X_k provided in the Eurocode (EN 1991-2 Eurocode 1: Actions on Structures - Part 2: Traffic Loads on Bridges, 2003) is linked to the distribution of annual maxima of road traffic load, which is modelled as a Gumbel distribution with mean $0.7X_k$ and CoV 7.5%. It can be verified numerically that a Lognormal distribution with mean $0.53X_k$ and CoV 15% corresponds to the annual distribution provided in Ref. (Fib, 2016), meaning that the maximum of 26 independent and identically distributed 2-weeks-variables is roughly a Gumbel distribution with mean $0.7X_k$ and CoV 7.5%. The model error for traffic load is considered in addition (Fib, 2016). The random variables used to compute the demand are shown in Table 4.

Table 4 Parameters used to obtain the demand distribution

Parameter	Mean	CoV	Distribution	Reference
Dead load	7100 kN	0.05	Lognormal	(Prendergast et al., 2016)
Traffic load	$0.53X_k$ kN	0.15	Lognormal	Computed

Eq. (11) is solved using Monte-Carlo simulations for each scour damage condition, where the capacity distribution reflects the scour condition. A total of 100,000 iterations have been carried out for each scour depth value. The surface plot in Figure 3 reports the values of the failure probability computed with respect to increasing pile lengths in the model varying from 10 m to 15 m, under scour depths ranging from 0 m to 6 m (in 0.1m increments). Shorter pile lengths lead to higher failure probability for a given scour depth as a result of the lower available shaft shear area contributing to the vertical foundation capacity.

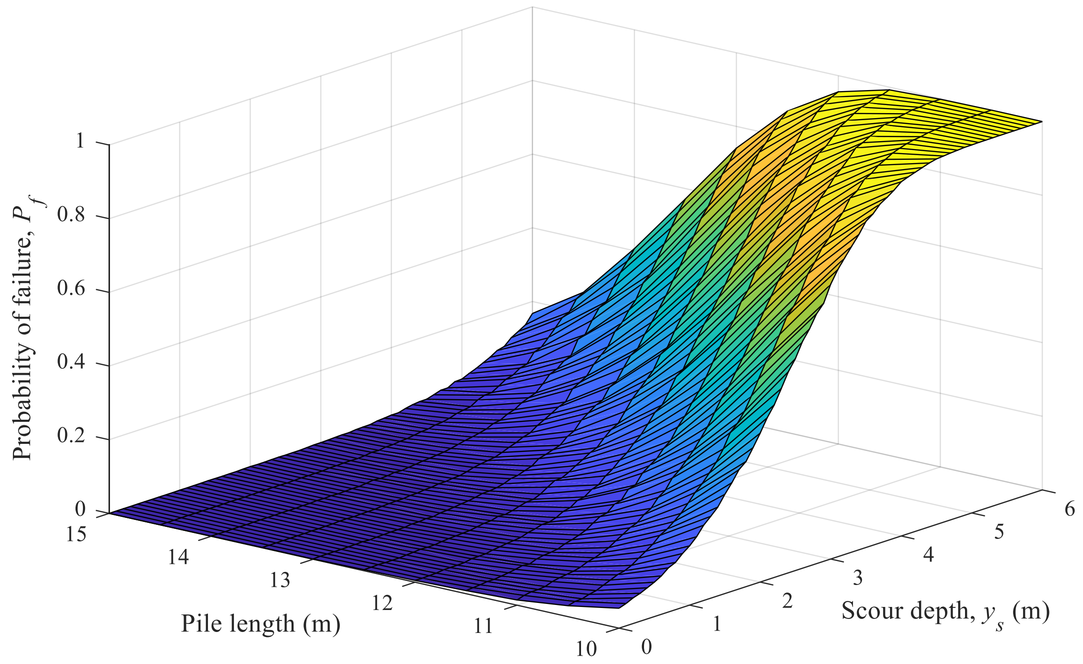

Figure 3 Failure probability of the bridge as a function of foundation pile length and scour depth.

For each value of the pile length, the surface plot in Figure 3 shows the variation of the failure probability with the scour depth. This enables the definition of the threshold values th_l of the scour depth corresponding to the reliability thresholds β_{acc} and β_{tol}. In Figure 4, the failure probability for the case of a bridge model with 14 m long piles at the central foundation is reported, which can be understood as a section cut through Figure 3. Figure 4(a) shows the probability of failure for the three cases of full demand, reduced traffic, and no traffic in light of a decision to implement traffic restrictions. Figure 4(b) shows a zoomed-in portion of Figure 4(a) to highlight DS_1. The damage state thresholds corresponding to the two reliability thresholds are shown on each plot. It can be seen that the bridge will be in DS_1 ($\beta_{acc} \leq \beta < \beta_0$) for scour depths ranging from 0 m to 2.2 m; in DS_2 ($\beta_{tol} \leq \beta < \beta_{acc}$) for scour ranging from 2.2 m to 4.2 m; and in DS_3 ($\beta < \beta_{tol}$) for scour exceeding approximately

4.2 m (for full demand). These values of scour depth will be assumed herein as thresholds of the three damage states: $th_1 = 0m$, $th_2 = 2.2m$, $th_3 = 4.2m$.

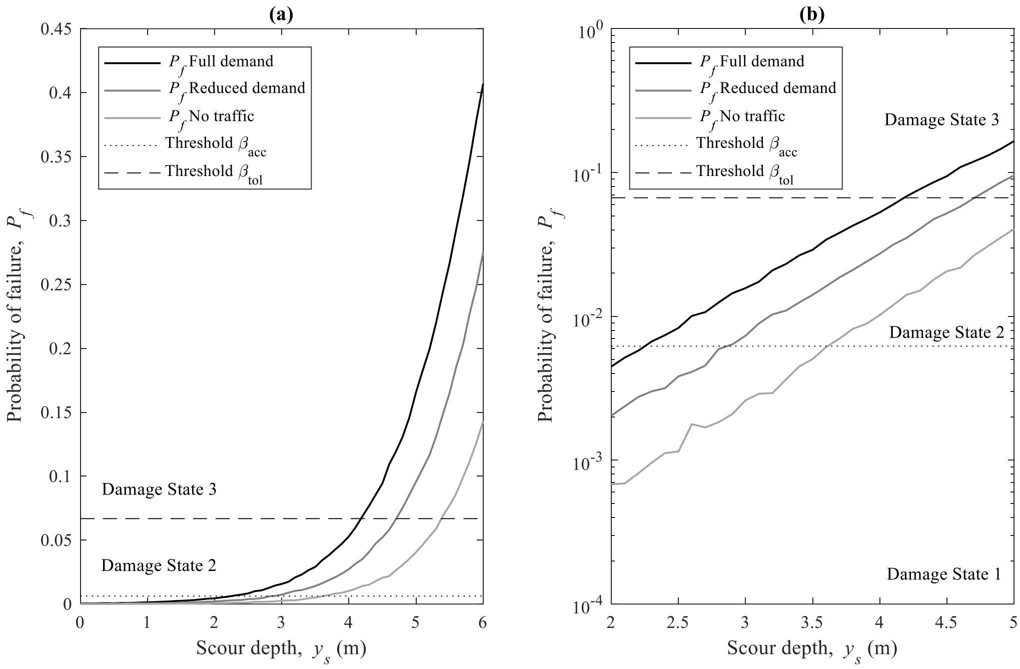

Figure 4 Failure probability of the bridge as a function of scour depth for pile length of 14 m, (a) P_f for full demand and reduced traffic, (b) zoomed in P_f in (a) showing DS_1 region

Prior and failure probabilities

The definition of the scour depth thresholds for each damage state enables obtaining the following elements of the VoI analysis; (1) the prior probabilities of damage states according to Section 3.2, (2) the failure probability related to different emergency management actions as discussed in Section 3.3, and (3) the likelihood functions for modelling the SHM output in the different damage states using the procedure described in Section 3.4.

Table 5 shows the list of input variables used in the probabilistic determination of scour depth for a given flow rate and therefore those used to compute the prior probabilities of damage states according to Section 3.2. The prior probabilities $P(DS_l|Q)$ are obtained by solving Eq. (23) by means of Monte Carlo simulations, using 100,000 iterations to estimate for each scour depth.

Table 5 Input variables for probabilistic scour depth analysis

Variable	Symbol	Unit	Distribution	Mean	CoV	Reference
Correction factor for pier nose shape	K_1	-	Det.	1	-	-
Correction factor for angle of attack of flow	K_2	-	Det.	1	-	-
Coefficients for bed conditions	K_3	-	Uniform	1.2	0.048	(Johnson & Dock, 1998)
Pier width	a_{pier}	m	Det.	1.375	-	-
Distance between front edge of pile cap or footing and pier	f	m	Det.	0.9375	-	-
Height of the pile cap above bed	h_0	m	Det.	0	-	-
Thickness of the pile cap exposed to the flow	T	m	Det.	0.5	-	-
Pile cap width	a_{pc}	m	Det.	4.5	-	-
Single pile width	a	m	Det.	0.75	-	-
Pile spacing	s	m	Det.	3.25	-	-
Number of row in the flow direction		-	-	2	-	-
River width	B	m	Lognormal	50	0.05	Assumed
Slope of the channel	s	-	Lognormal	0.002	0.05	Assumed
Manning's coefficient	n	-	Lognormal	0.035	0.28	(Davis & Burnham, 1987)
Model correction factor	λ_{y_s}	-	Normal	0.55	0.52	(Ghosn et al., 2003)

The failure probabilities $P(F|y_s, A_n)$ corresponding to the identified scour depth thresholds th_l, i.e., $y_s = th_l$ $(l = 1, 2, 3)$, and emergency management actions A_n are computed according to Eq. (24) as discussed in Section 3.3. The traffic load for the different management actions must be computed since it determines the overall vertical demand $D(A_n)$ on the central pier. When $A_n =$ "leave the bridge open", the failure probability corresponds to the reliability thresholds defined in Section 3.1 since the traffic level is supposed to be the same as under normal operation. When $A_n =$ "reduce the traffic", the characteristic traffic load is computed omitting (for the sake of simplicity) the tandem two-axle systems prescribed by Eurocode (EN 1991-2, 2003) to represent trucks. When A_n = "close the bridge", the probability of failure is computed considering the dead load only. Table 6 shows the failure probability for different actions and damage states.

Table 6 Failure probability for different actions and damage states

	$A_n = Open$	$A_n = Limit$	$A_n = Close$
DS_1	$3.9 \cdot 10^{-4}$	$2 \cdot 10^{-4}$	$2 \cdot 10^{-5}$
DS_2	$6.2 \cdot 10^{-3}$	$2.8 \cdot 10^{-3}$	$8 \cdot 10^{-4}$
DS_3	$6.7 \cdot 10^{-2}$	$3.5 \cdot 10^{-2}$	$1.4 \cdot 10^{-2}$

Likelihood functions

According to Section 3.4, the numerical bridge model, described in Section 4.1, is used to generate the likelihood functions whereby the properties of the bridge are considered as random variables, to incorporate uncertainty (see Table 7 for variables). In this analysis, the deck, three supports (pier and two abutments), and the pile groups are all considered as independent systems with given properties for each run on the model, i.e., Table 7 only shows the parameters of the distribution attributed to the concrete elastic modulus, but each element is attributed a different value for this parameter for each model run. The procedure involves generating a given model bridge (by matrix assembly (Prendergast et al., 2016)) with properties sampled from distributions for the various components, which is traversed by a moving load (vehicle) with given magnitude and velocity.

Table 7 Parameters used in bridge model

Parameter	**Mean**	**CoV**	**Distribution**	**References**
Concrete Elastic modulus, E	3.5×10^{10} N/m^2	0.15	Lognormal	(JCSS, 2000)
Lateral soil stiffness, k_l	4.17×10^6 N/m to 1.25×10^8 N/m over 15 m pile depth	0.10	Lognormal	(API, 2007)
Vertical soil stiffness, k_v	7.79×10^6 N/m to 2.34×10^8 N/m over 15 m pile depth	0.10	Lognormal	(API, 2007)

The velocity of the moving vehicle is considered as a lognormal distribution with mean 60 km/hr and CoV = 0.33, and the vehicle weight is considered as a lognormal distribution with mean 150 kN and CoV = 0.17. These values were adopted as a reasonable estimate of the mean speed and mass of vehicles travelling across the bridge. A given vehicle crosses the bridge and the resulting acceleration signal is calculated at a node near the top of the pier by solving Eq. (31), and this is analysed for its frequency content. A signal corresponding to 50 seconds of free vibration after the vehicle departs is used to calculate the natural frequency as this removes the influence of vehicle-related frequencies from the output spectra, which are known to cause issues with signal quality (Prendergast et al., 2017). It should be mentioned that the bridge can also be monitored for its frequency content under in-service traffic loads, as has been undertaken by other authors (Farrar et al., 1999), and there is no requirement for the bridge to be free from traffic aside for the convenience of removing traffic-related frequencies from the output spectra. The value of the first natural frequency is obtained by applying a Fourier transform to the calculated nodal acceleration. To make the signal more realistic, noise is added using the approach in Ref. (Lyons, 2011) with a Signal-to-Noise Ratio (SNR) of 20. A Monte-Carlo type simulation is performed where 1000 vehicle crossings are considered for each of the three damage states, shown in section 4.1. A lognormal distribution is fitted to the data for each damage state to represent the probability of measuring a certain frequency value for a given damage state due to scour, see Figure 5.

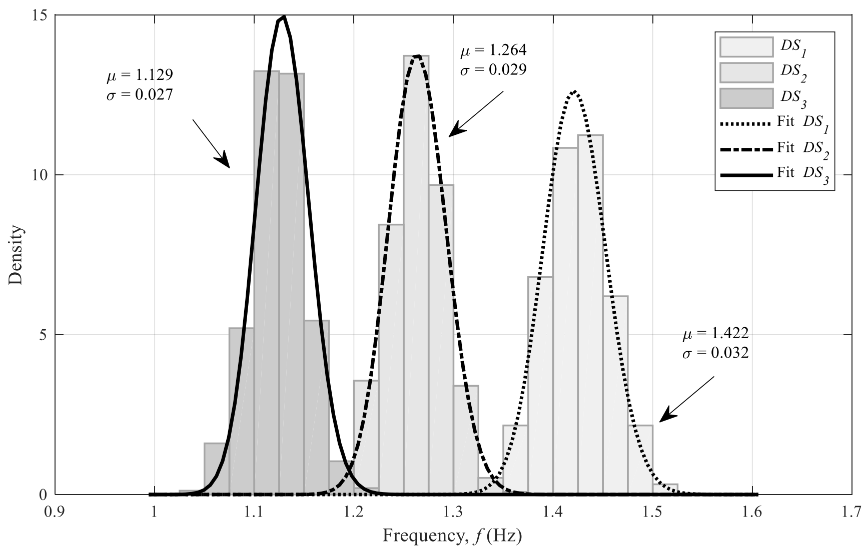

Figure 5 Empirical likelihood functions for the three considered limit damage states

Cost analysis and flood hazard

The values assumed for the cost analysis are listed in Table 8. The costs found in the references have been adjusted accounting for currency revaluations in 2020. In general, such values strongly depend on the specific structure and country. Herein, indicative values referring to Italy have been chosen to carry out the analysis. The values related to the traffic conditions have been assumed. The time for the replacement of the bridge is assumed equal to 1 year whereas the duration of traffic limitation is considered as 10 days. The last assumption is not meant to describe the actual duration of the emergency phase but rather the time needed to organize a more detailed investigation to collect information able to support the decision about interventions to recover the full bridge functionality. Regarding the modelling of the probability distribution of the flood, the following parameters are employed: $Q_0 = 500 m^3/s$, $\nu = 0.0033 (m^3/s)^{-1}$. The scale parameter can be obtained as $\nu = 1/(\bar{Q} - Q_0)$, where $\bar{Q}$ is the average value of the recorded flow during floods, assumed as $800 m^3/s$.

Table 8 Value assumed for the VoI analysis

Variable	Description	Value	Ref.
C_1	Unit rebuilding cost	2000 €/m^2	(Maffei & Boccacini, 2002)
L	Length of the bridge	50 m	(L.J. Prendergast et al., 2016)
W	Width of the bridge	17.1 m	(Prendergast et al., 2016)
C_2	Cost of running cars	0.50 €/km	(Maibach et al., 2006)
C_3	Cost of running trucks	1.10 €/km	(Maibach et al., 2006)
C_4	Value of time per car passengers	5.7 €/h	(Fiorello & Pasti, 2003)
C_5	Value of time for trucks	38 €/h	(Fiorello & Pasti, 2003)
C_6	Cost for each life lost	1,670,000 €/person	(MIT, 2015)
A	Average daily traffic	800	-
D	Detour length	10 km	-
T	Average Daily Truck Traffic (% of A)	20%	-
d	Duration of the detour	14 days (traffic limitation) – 365 days (bridge failure)	-
O	Average occupancy rate for cars	2	-
S	Average detour speed	50 km/h	-
X	Number of life losses	2/4	-

4.3. Analysis and results

The VoI is computed according to the procedure in Section 2 using the variables described in the previous section. Figure 6(a) shows the results of the Prior analysis, that is the expected cost of the three emergency management actions as a function of the river flow considering 2 fatalities. The expected cost of the three actions increases with the increasing flow since the probability of being in the severe damage states increases. For values of the flow below about 800 m^3/s, the optimal action is to "leave the bridge open". Between flows of 800 and 1700 m^3/s, to "issue traffic limitation" becomes the optimal choice. For values of the river flow higher than 1700 m^3/s, the optimal action is "close the bridge".

The expected cost of the optimal action is presented in Figure 6(b) (dashed-dotted line). In this figure, also shown is the expected cost from Pre-Posterior analysis (solid line), computed by means of Eq. (6). The difference between these two functions is the VoI as a function of the flow, which is displayed in Figure 6(c).

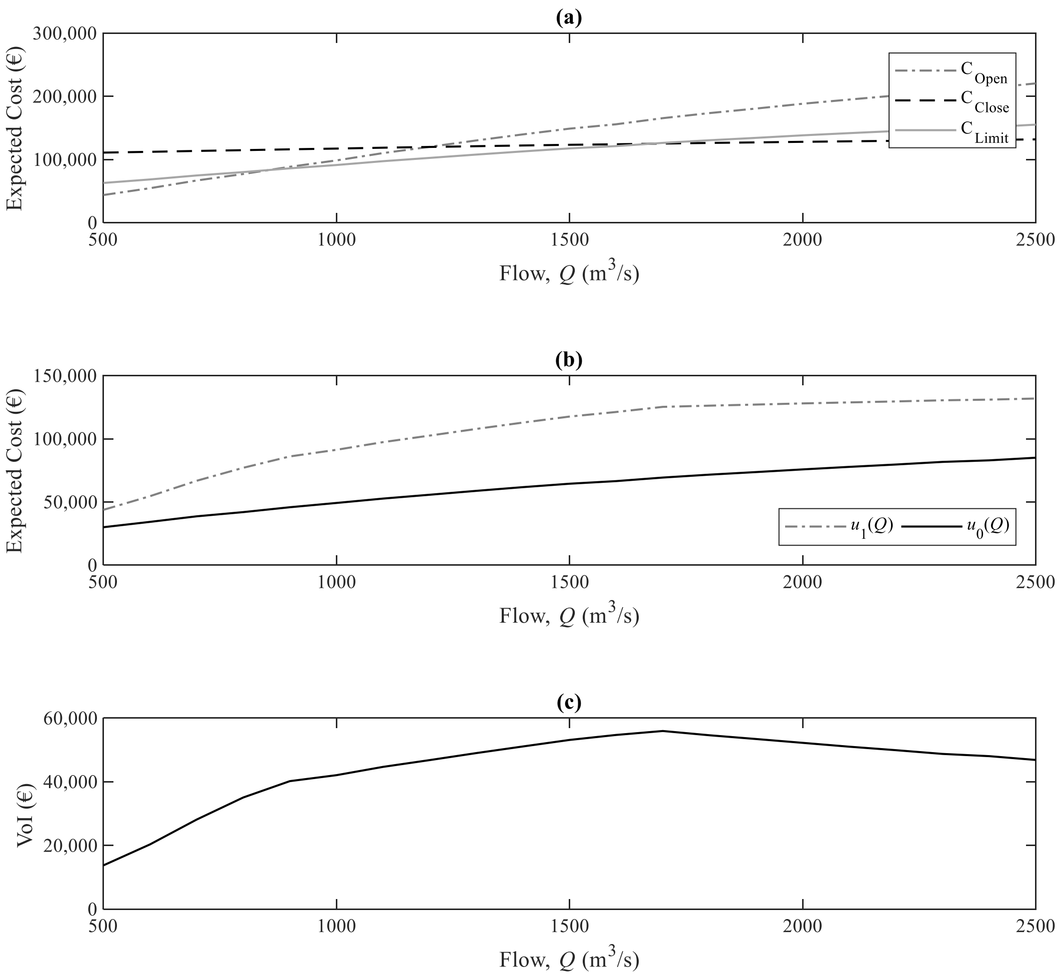

Figure 6 Expected costs and VoI considering 2 fatalities, (a) results of Prior analysis, (b) expected cost of the optimal action from Prior and Pre-Posterior analysis, (c) VoI.

Figure 7 displays the same type of analysis however 4 fatalities have been considered in this case. It is observed that the expected costs of the three actions are higher for the same value of the flow with respect to the results shown in Figure 6 since the failure of the bridge leads to higher direct consequences in this case. The actions “issue traffic limitation” and “close the bridge” become the optimal choices for correspondingly lower values of the flow. Both in Figure 6(c) and Figure 7(c), it can be noted that the VoI line exhibits a change of slope when there is a change of the optimal action according to prior analysis. This is due to the different dependency of the expected costs of the actions on the intensity of the flood.

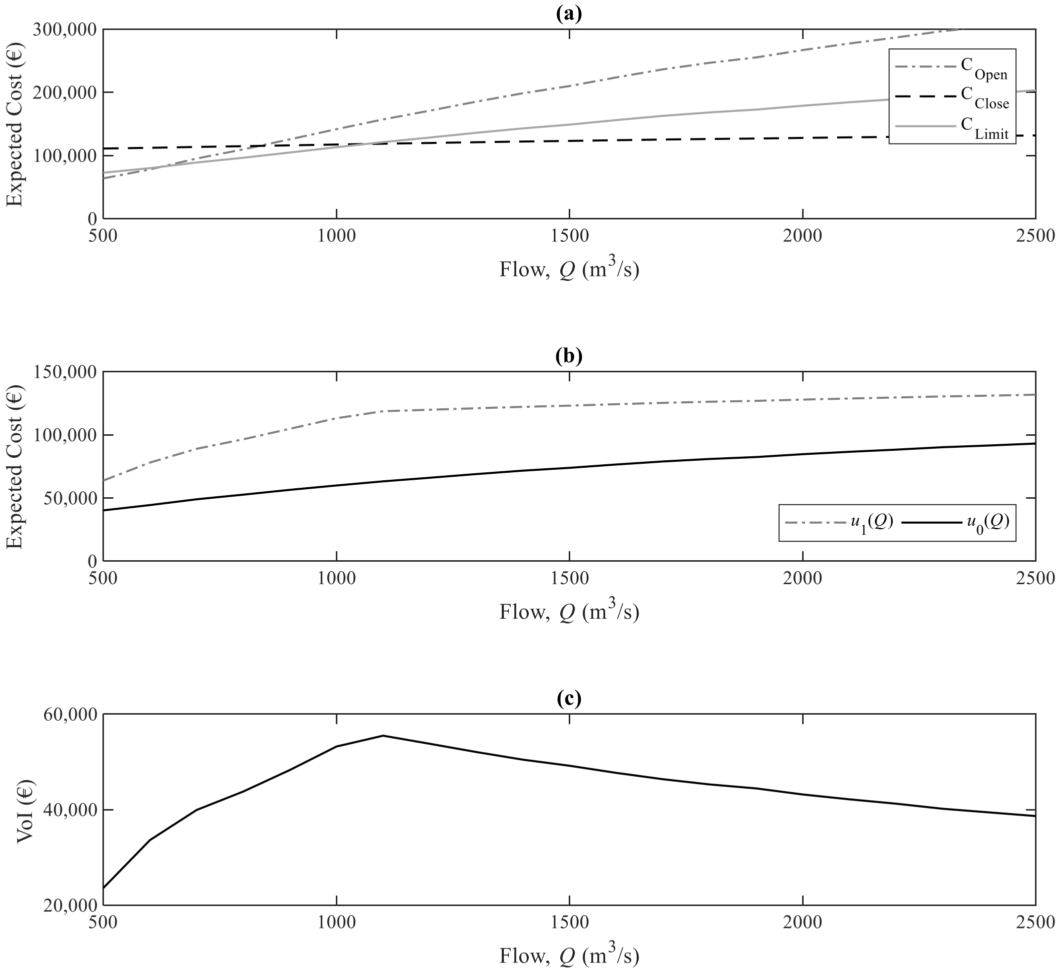

Figure 7 Expected costs and VoI considering 4 fatalities, (a) results of Prior analysis (b) expected cost of the optimal actions from Prior and Pre-Posterior analysis, (c) VoI.

Figure 8 shows the results of the VoI analysis considering 2 fatalities and no model error in the computation of the scour depth. In this case, the VoI goes to zero for flow values higher than 1200 m^3/s.

When the model error is not considered, the estimation of the scour depth is affected by small uncertainty thereby the prior probabilities that the bridge is in DS_3 is close to unity for high values of the flow. In this situation, the information from the SHM system has no value since it does not significantly modify the prior knowledge of the decision-maker. Conversely, when the model error is considered, the prior probability that the bridge is in DS_3 increases slower with increasing flow. In this situation, the SHM information is valuable even for high values of the flow since it gives insight on the actual structural conditions and is able to modify the selection of the optimal actions.

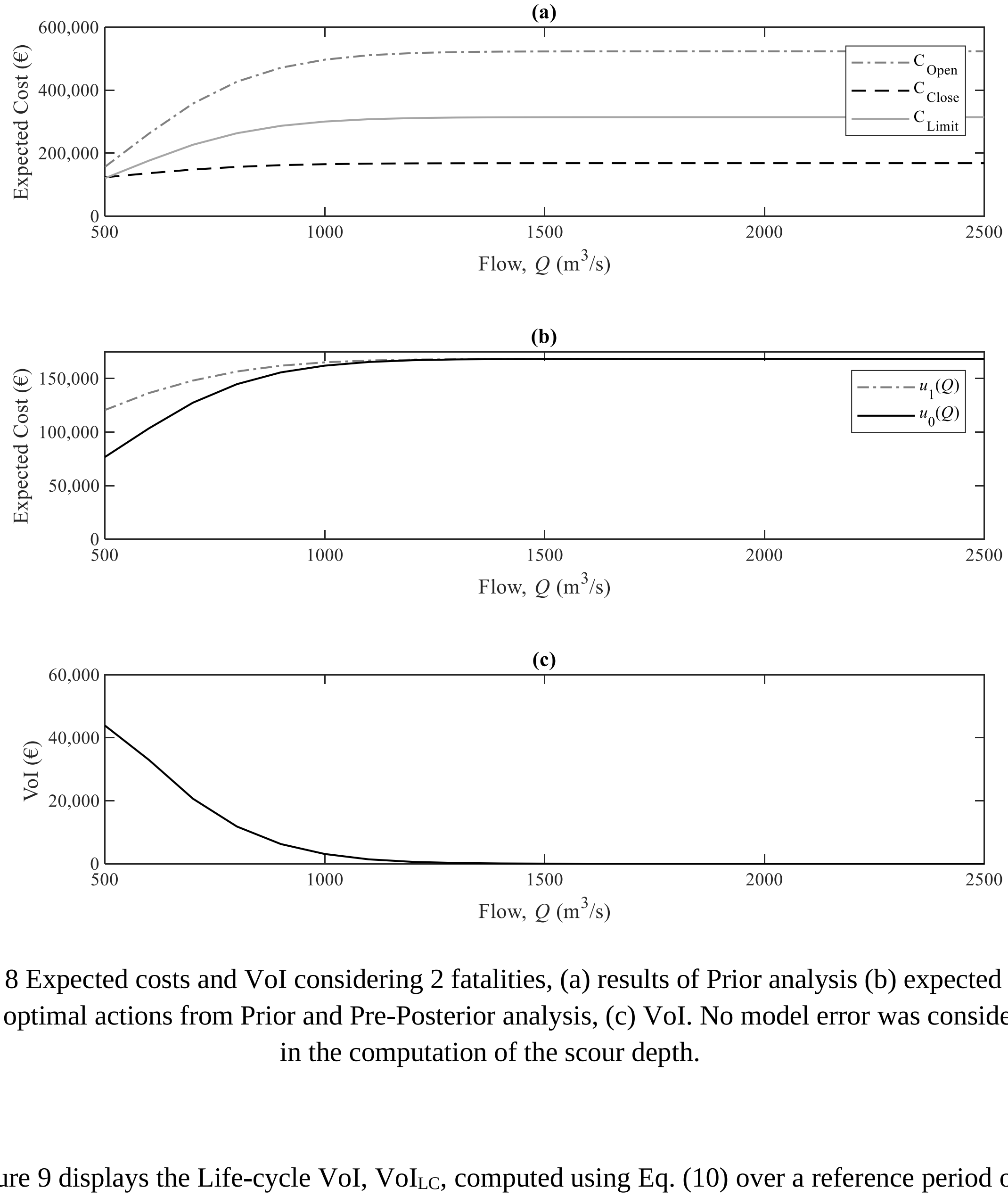

Figure 8 Expected costs and VoI considering 2 fatalities, (a) results of Prior analysis (b) expected cost of the optimal actions from Prior and Pre-Posterior analysis, (c) VoI. No model error was considered in the computation of the scour depth.

Figure 9 displays the Life-cycle VoI, VoI_{LC}, computed using Eq. (10) over a reference period of 20 years considering 2 fatalities with, (i) the model correction factor for scour evaluation, and (ii) no model correction factor. The discount rate is 0.01. The VoI_{LC} in Figure 9 is expressed as a function of the expected number of floods per year, λ. It can be observed that the VoI_{LC} increases linearly for increasing numbers of floods. For this case study, the VoI_{LC} is considerable (in the order of millions of Euros). The VoI_{LC} constitutes the maximum cost that the decision-maker should spend on the SHM system over its life cycle.

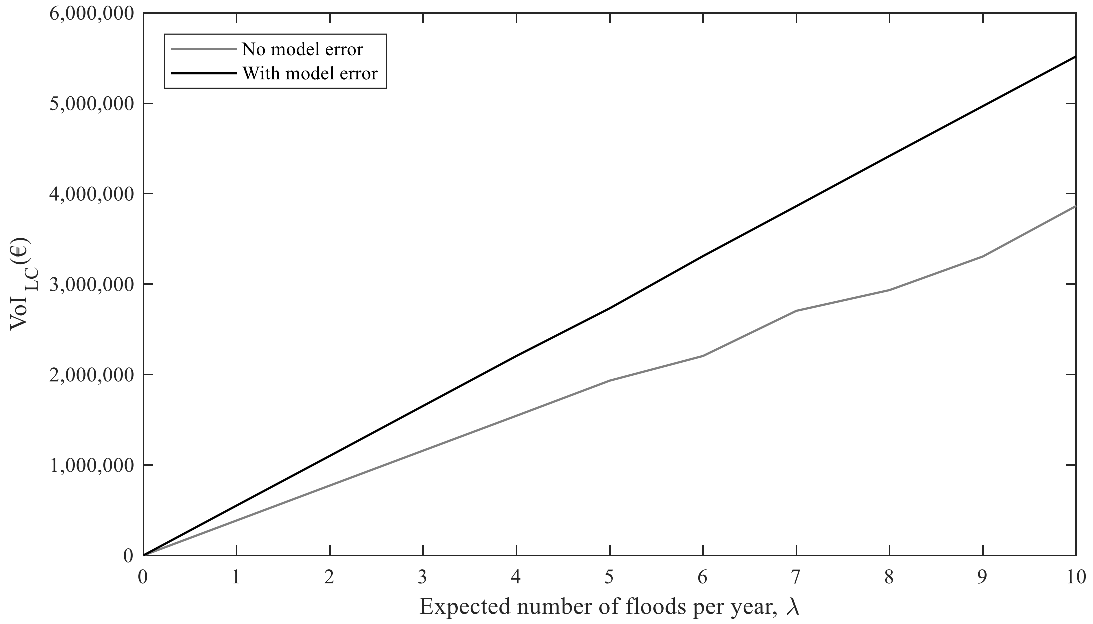

Figure 9 VoI_{LC} as a function of the expected number of floods per year λ.

5. Conclusion

In this paper, a framework for calculating the financial benefit of SHM information applied to the case of structural damage due to scour erosion of a bridge pier is presented. The benefit is evaluated using the VoI from Bayesian decision theory. The VoI should be compared with the cost of the SHM system to decide if its installation is financially worthwhile. The decision problem supported by the dynamic SHM information includes keeping the bridge open, limiting traffic, and closing the bridge, upon the occurrence of a flood.

The various steps of the VoI analysis are presented such as: the definition of scour-related damage states affecting the structure and their probabilities of occurrence, the computation of the probability of failure of the structure associated with each damage state and each decision alternative, the updating of these probabilities when new information from a health monitoring scheme is available, and the computation of the VoI over the life cycle as a function of the flood hazard.

In the case study, three decision alternatives are considered in the event of a flood; keep the bridge open, implement traffic restrictions, and close the bridge. Prior probabilities of a bridge being within a certain damage state (scour condition) are calculated using hydraulic scour equations that estimate scour depth based on bridge geometry and flow data. The probabilities of failure associated with each damage state are computed using Monte-Carlo simulations whereby the bridge capacity is considered as the vertical capacity of the piled foundations, and the demand is considered as that imposed on the structure by the structural loads and traffic. The prior probabilities of each damage state are updated using Bayes' Theorem to consider natural frequency of the bridge obtained from a health monitoring system. The likelihood functions are obtained by modelling a two-span integral bridge affected by scour and simulating vehicles crossing the bridge to generate structural accelerations, which are analysed for their associated frequency content for a given scour condition affecting the foundation of the bridge. The Life-cycle VoI is computed accounting for different flood scenarios and the number of floods during a reference period corresponding to the expected life of the dynamic SHM system.

For the cases considered, the expected costs increase as a function of flow rate with each of the three decisions (keep open, limit traffic, close the bridge) resulting in a minimum cost for certain flow rates. The VoI can be understood as the gain provided by the inclusion of additional information in the decision problem. It is shown that the VoI is strongly influenced by the magnitude of the estimated failure costs. Information relevant to any of the system components (the bridge, the exposure to floods, and the users of the bridge) can enhance the decision-making process. In this paper, only the VoI related to the information provided by a dynamic monitoring system is investigated.

The example case is specific to this paper and the relevant data should be tailored for each case, should the method be applied to other structures. Assumptions made in the present paper are outlined for the benefit of readers to facilitate alternative applications. This paper should be of interest to asset management agencies tasked with making decisions related to monitoring scour critical bridges on a network using emerging and available dynamic SHM systems. Future work will enhance the approach by incorporating multiple failure modes for bridges, the use of alternative metrics from dynamic SHM systems (such as mode shapes), and alternative scour management strategies.

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