A Petri net approach to assess the effects of railway maintenance on track availability

Author 1

- Claudia Fecarotti, Assistant Professor
- Department of Industrial Engineering & Innovation Sciences, Eindhoven University of Technology, Eindhoven, Netherlands
- ORCID number

Author 2

- John Andrews, Professor
- Department of Mechanical Engineering, Faculty of Engineering, The University of Nottingham, Nottingham, UK
- ORCID number

Corresponding author: Claudia Fecarotti, e-mail: c.fecarotti@tue.nl

Abstract

The railway infrastructure includes a portfolio of assets which are subjected to degradation and failure processes due to both usage and aging. As a consequence of degradation and failures, speed restrictions and line closures may be imposed to control the risk of derailment. Such actions have a direct impact on service as they lead to delays and journey cancellations. Maintenance is implemented to control the state of the assets. Different maintenance strategies determine different asset conditions and performance profiles, and consequently a different impact on service. This paper presents a simulation tool based on Petri nets, which combines degradation and maintenance processes to predict the future track geometry conditions, including the probability of those failure modes leading to speed restrictions and line closures. Such a model is a valuable feature of an effective infrastructure asset management system which intends to support cost-effective informed decisions on railway maintenance.

Keywords chosen from ICE Publishing list

Railway Systems; Maintenance & Inspection; Mathematical Modelling.

List of notation

This is an example created from parts of other articles, it is not designed to be read for sense.

T_i transition node in the Petri net

P_i place node in the Petri net

σ_{op} threshold of standard deviation for track vertical alignment inducing opportunistic maintenance

σ_{rm} threshold of standard deviation for track vertical alignment inducing routine maintenance

σ_{sr} threshold of standard deviation for track vertical alignment inducing speed restriction

σ_{lc} threshold of standard deviation for track vertical alignment inducing line closure

$\sigma_{g,op}$ threshold of standard deviation for track gauge inducing opportunistic maintenance

$\sigma_{g,rm}$ threshold of standard deviation for track gauge inducing routine maintenance

$\sigma_{g,er}$ threshold of standard deviation for track gauge inducing emergency maintenance

$\sigma_{g,ir}$ threshold of standard deviation for track gauge inducing immediate maintenance

β,η parameters of Weibull distribution, shape parameter and scale parameter respectively

1. Introduction

The ultimate aim of a railway system is the transport of people and goods at the required level of service and safety. The railway comprises a variety of heterogeneous assets which are subject to degradation and failures due to usage and aging. As a consequence of degradation and failures, speed restrictions and line closures may be imposed to control the risk of derailment. Such actions have a direct impact on service as they lead to delays and journey cancellations. To control the state of the assets maintenance strategies must be developed which specify the inspection and intervention activities to be performed, and the rules for their implementation. Different strategies will determine different asset conditions and performance profiles, and consequently a different impact on service. The capability to evaluate such an impact is paramount for a cost-effective planning of maintenance interventions. In (Burkhalter, Martani, & Adey, 2018) the impact of speed restrictions or line closures being imposed is considered when computing the costs and benefits of intervention plans. Here the probabilities of an object requiring a speed restriction or a line closure is computed by means of fault tree analysis (an object being either a component, such as a bridge or a switch, or a track section) and used to evaluate the risk related to a given intervention program. In (Moreu, Spencer Jr., Foutch, & Scola, 2017) the authors develop a framework to prioritise maintenance interventions on railroad bridge networks. The operational costs associated to the probability that the bridges will exceed the “service limit states” depending on the maintenance decision are minimised.

This paper presents a modelling approach based on the Petri net method, which combines the degradation, failure and intervention processes to predict the future track geometry conditions, including the probability of those failure modes leading to speed restrictions and line closures. The approach enables the asset response to a variety of potential maintenance strategies to be investigated and predicted. The model is state-based, where states which are relevant from a maintenance perspective are explicitly modelled. Of those states, the ones which correspond to the imposition of speed restrictions and line closures are particularly relevant for their impact on service. Due to the stochastic nature of the modelled processes, stochastic simulation via the Monte Carlo method is the most appropriate approach to analyse and solve the model. Statistics are collected on the probability that the asset is in any of the modelled states and on the number of interventions performed during a given time horizon. With such models, a wide range of maintenance strategies can be analysed. The resulting probability, number and duration of speed restrictions and line closures are an indicative measure of the effects that a given maintenance strategy will have on service. Such statistics can be then used directly, or within an optimisation procedure, to support the planning and development of maintenance strategies to achieve given service performance targets.

1.1 Track geometry degradation and maintenance

Track geometry, both vertical and horizontal, strongly affects the quality and safety of the ride. Track geometry is periodically inspected by running Track Recording Vehicles along the network. The vehicle measures the location of the rails and provides the variations of the rails vertical and horizontal position, gauge, twist and cyclic top over 1/8th mile section. Measurements, particularly the ones related to the vertical alignment, are then used to categorise the track in terms of quality bands, to plan maintenance and, if necessary, to take actions such as speed restrictions and line closure to control the risk of derailment. The track's components responsible for keeping the required track geometry are the ballast, sleepers and fastenings. Specifically, while vertical alignment defects are mainly attributable to degradation of the ballast, gauge, horizontal alignment, cross-level and twist defects are mainly due to degradation and failure of sleepers and fastenings. Gauge widening is the effect of multiple sleepers/fastenings failures. To maintain those components means to keep track geometry to acceptable levels. Although the degradation and maintenance processes affecting these components are very different, many dependencies arise due to common processes such as inspection, opportunistic maintenance and combined renewal. However, most of the models presented in the literature address the modelling of degradation and maintenance of each component individually.

In order to represent different degradation states, the transitions between these states and the restorative actions provided by the maintenance processes, the model must accommodate a state-based approach. Due to the variability of the times for degradation and maintenance to occur it should also be stochastic. The main approaches utilised in the literature to model degradation and maintenance are Monte Carlo simulation methods using either statistical models or state-based models to describe the asset degradation and the effects of maintenance activities. As an example of state-based approaches, Markov models have been developed in Meier Hirmer et al. (2009), Bai et al. (2013) and Prescott and Andrews (2013a) to represent track geometry degradation and its maintenance processes. Markov-based models are stochastic models capable of describing dynamic systems for which future states depend only on the current state. The history of what has occurred in the past is irrelevant to future behaviour. The size of a Markov model can increase considerably with the number of components to consider. An alternative modelling technique to the Markov approach is the Petri net (PN) method. PNs are a formalisms to model discrete event dynamic systems with concurrencies and dependencies (Murata, 1989; David and Alla, 2010). Andrews (2012) presents a PN to model ballast degradation and maintenance for a 1/8th of mile section of track. The author first analyses track geometry data from inspection and maintenance records so to evaluate the distribution of times to degrade from/to different states of interest from a maintenance perspective. These distributions are then used to sample the times to degrade of the ballast. A similar model is presented by Prescott and Andrews, (2013b) who develop a PN combining degradation, inspection, maintenance and renewal processes for ballast on a railway network with different regions each one supervised by a regional maintenance engineer. Andrews et al. (2014) apply the previously developed PN model to predict the state of the track geometry over any specified asset management strategy. In addition the model is capable of computing the expected whole life costs. In (Lake et al., 2000a,b) the authors develop a simulation model to predict the distribution of groups of failed timber sleepers in a track section for different renewal strategies. The sleepers' lifetime is assumed to follow a 2-parameter Weibull distribution, and the renewal strategies are based on the minimum number of adjacent sleepers failed and the frequency of intervention. The same model is applied in (Yun and Ferreira, 2003) to a track with 1000 sleepers. Zhao et al. (2007) evaluate the reliability of a segment of sleepers modelled as a k-out-of-n system, where the lifetime of the sleepers is assimilated to a 2-parameter Weibull distribution.

The aforementioned models focus on individual components of the track system, either the ballast or the sleepers. Therefore, dependencies induced by common processes such as inspection, opportunistic maintenance and combined renewal of multiple components and adjacent track sections would require a framework that combines the individual models into one. A modular representation is presented in this paper, where independent modules for modelling the degradation of the vertical alignment and gauge are then combined to predict track geometry response to maintenance. The models include a representation of the degraded states and corresponding actions that have a direct impact on service. These are conditions that, according to the stakeholders' policies, require the imposition of speed restrictions or line closures.

2. Track geometry model

The track geometry model consists of two modules which represent vertical geometry and gauge degradation respectively, and the corresponding maintenance actions that can be performed. Variations in vertical alignment is mainly due to ballast degradation, while sleepers and fastenings failures are mainly responsible for gauge spreading. The modelling approach adopted is based on the PN method. In the following, a brief introduction on PN is given to enable a better understanding of the proposed model.

2.1 The Petri net method.

A PN (Murata, 1989; David and Alla 2010) is a bi-partite graph with nodes called places and transitions. Places are represented as circles while transitions are represented as rectangles. Places model possible states for a component/system, while transitions model events that cause the system state to change. *Input* and *output* arcs connect places to transitions and vice versa, and are represented by arrows. Tokens are held in places; the number of tokens in a place P_i is called *marking* of P_i and is indicated by m_{Pi}. The number and distribution of tokens across the PN, called marking of the PN, represent the system state at a given time. Transitions are responsible for "consuming" tokens from the input places, and "producing" tokens into the output places thus determining a change in the marking. This is referred to as "*firing"* of the transition and corresponds to the occurrence of the event modelled by the transition. The number of tokens consumed and produced depends on the multiplicity of the input and output arcs respectively. An additional type of arc called *inhibitor* is often used to forbid the firing of a transitions under given conditions. Inhibitor arcs are as arcs with a circle end rather than an arrow end. The rules according to which transitions fire are as follows:

- First, a transition is enabled to fire if (1) the number of tokens in each input place is at least equal to the multiplicity of the corresponding input arc, and (2) the places connected by an inhibitor arc contain a number of tokens lower than the multiplicity of the inhibitor arc.
- When firing, the transition "consumes" a number of tokens from the input places equal to the input arcs multiplicity, and "produce" a number of tokens into the output places equal to the output arcs multiplicity.

Figure 1 gives an example of a simple PN where transition T1 has two input places P1 and P3, one place connected by inhibitor arc, P2, and two output places P4 and P5.

Examples of marking which do not enable T1 are given in Figure 1-a and 1-b, while firing is enabled for marking in Figure 1-c.

FIGURE 1 HERE

Figure 1 PN with different possible markings.

Transitions can be immediate or timed. Timed transitions, once enabled, will only fire when a given firing time interval has elapsed. Figure 2 shows a PN with timed transition T1 and associated firing time interval Δt before firing (2-a) at a given time t, and after firing (2-b) at time $t+\Delta t$.

FIGURE 2 HERE.

Figure 2 PN with timed transition, before (a) and after (b) firing.

The firing time interval can be either deterministic or stochastic. Events or processes affected by randomness, such as the degradation or failure of a component, are modelled by stochastic transitions whose firing time intervals are sampled from appropriate stochastic distributions. Multiple distributions can be associated to the same transition, and the appropriate one is selected based on the marking of specific places called *conditional places.* In a PN representation, a transition is linked to its conditional place by a dotted arrow. This feature is useful to represent events whose distribution of times of occurrence changes depending on some given condition. The mode in which a transition fires, referred to as *firing mode*, can be standard or non-standard. According to a standard firing mode, a transition consumes and produces tokens only according to the multiplicity of the input and output arcs. If a non-standard firing mode is associated to a transition, then the new marking is evaluated according to a given firing mode function. An explanatory example of a PN where a transition has a non-standard firing mode is given in Figure 3. The PN consists of one transition T1, with one input place, P1, three output places, P2 to P4, and one conditional place, P5.

FIGURE 3 HERE

Figure 3 Petri net with a transition having multiple firing time functions and a non-standard firing mode.

T1 is a timed stochastic transition whose firing time *t* can be sampled from one out of two Weibull distributions $W_1(\beta_1, \eta_1)$ and $W_1(\beta_1, \eta_1)$ depending on the marking of conditional place P5, m_{P5}. The new marking resulting from firing of T1 is also random. Each output place has a given probability α_i of receiving a token after firing of T1. When T1 fires, it will sample a random number *p* from a uniform distribution between 0 and 1. Depending on the value *p*, a token will be added to one out of the three output places P2 to P4. Such type of transition is called *routing probabilistic* transition. It can be used to represent maintenance actions whose effectiveness is uncertain. The symbols used to represent the different types of transitions are showed in Figure 4.

FIGURE 4 HERE.

Figure 4 Symbols used for different types of transitions.

2.2 Model for vertical alignment: ballast

The degradation of the ballast is modelled as a phased process. The conditions of the ballast are implied by the values of the standard deviation (SD) of the vertical alignment provided by the Track Recording Vehicles. The SD of the vertical alignment is therefore considered the indicator of the ballast conditions. Once degraded conditions are revealed by inspection, then the appropriate maintenance action is scheduled and performed at the required time. The model therefore includes ballast deterioration, inspection, routine and opportunistic maintenance, and emergency repair.

Degradation process. Figure 5 represents the degradation process.

FIGURE 5 HERE

Figure 5 PN representing ballast degradation.

Degradation is modelled as a phased process where a number of discrete states which are relevant from a maintenance perspective are considered. These states are represented by places P1 to P7. Each state represents a severity degradation level and is characterised by a threshold value for the SD of the vertical alignment. If the corresponding SD value is reached, then the state is entered. A different urgency of intervention is associate to each degradation level. Three approaches to maintenance are modelled, namely routine, opportunistic and emergency (or corrective) maintenance. Routine maintenance is a scheduled intervention often planned weeks or months ahead of execution. Opportunistic maintenance means that once a routine intervention is going to be carried out on a given section, adjacent sections which are in a condition close enough to require routine maintenance will be also included in the intervention. Finally, an emergency intervention is carried out when inspection reveals a degraded condition which could potentially cause a derailment. In such circumstances a speed restriction or even a line closure is imposed immediately, and intervention is performed as soon as possible. Place P1 indicates new conditions (or following renewal). Place P2 represents a state where opportunistic maintenance is possible. The corresponding threshold of the SD is σ_{op}. Place P3 models a state requiring routine maintenance to be scheduled and performed within a given period of time. The SD threshold is σ_{rm}. Place P5 and P6 represent two levels of degradation such that a speed restriction or line closure respectively must be imposed to control the risk of derailment while an emergency intervention is scheduled. These are very undesirable states that, if revealed, cause a disruption to the railway service, whereas if not detected could constitute potentially hazardous situations. P5 and P6 are entered when the threshold values σ_{sr} or σ_{lc} are exceeded respectively. It is possible that, if the inspection process reveals that the track is in a state which will soon reach σ_{sr}, an emergency repair might be carried out to avoid reaching the undesirable state requiring speed restriction. This state is represented by place P4 with threshold σ_{crit}. After maintenance, track geometry is never restored to as good as new conditions. Place P7 is used here to indicate the best possible state achievable following repair. The time to degrade from one state to the next depends on the value chosen for each SD threshold and is ruled by stochastic transitions T1 to T6. A set of firing time distributions is associated to each of these transitions. The distribution of times to degrade from one state to the next depends on the value of these thresholds. Therefore, depending on the SD threshold, the appropriate distribution is selected for each of transitions T1 to T6. The SD thresholds triggering a speed restriction or a line closure for a given track category are usually fixed for safety reasons. The thresholds for opportunistic σ_{op} and routine maintenance σ_{rm} instead can be varied to investigate the effects of more or less conservative approaches to condition-based maintenance on the track long-term behaviour. Different values of σ_{op} determine different distributions of times to degrade associated to transitions T1, T2 and T6, while different values of σ_{rm} determine different distributions associated to transitions T2 and T3 (or T4 if the critical state coincides with the state requiring a speed restriction). In order to automate the selection of different values for σ_{op} and σ_{rm}, and the appropriate distributions for the corresponding transitions, places P15 and P16 are introduced (Figure 6). If a correspondence is established between their marking and given values of σ_{op} and σ_{rm}, then P15 and P16 can be used as conditional places for transitions T1 to T4, and T6. For example, if two potential values are considered for σ_{op}, then two distributions are associated to each of transitions T1, T2 and T6. Depending on the marking of P15, the appropriate distribution will be selected between the available two for each of the above transitions.

FIGURE 6 HERE

Figure 6 PN accounting for different SD thresholds triggering opportunistic and routine maintenance.

Inspection process. The periodic inspection process is represented by loop P19-T18-P20-T17-P19 in Figure 7.

FIGURE 7 HERE

Figure 7 PN describing ballast degradation and inspection.

When inspection is not performed, place P20 is marked while place P19 is empty; such marking will enable transition T17 that will fire after the specified time interval $\vartheta 1$. Transition T17 is a timed deterministic transition and the time interval $\vartheta 1$ depends on the marking of place P18 which defines the inspection strategy. By firing, T17 will remove the token from place P20 and add a token in place P19 indicating that the track is now under inspection and degraded states, if any, can be revealed. Places P8 to P12 represent the revealed states corresponding to each possible degraded condition.

Intervention processes. Only once a degraded condition has been revealed, maintenance can be scheduled and carried out with different urgency depending on the level of degradation detected. The PN in Figure 8 includes the intervention activities that can be performed on the ballast to restore geometry conditions.

FIGURE 8 HERE

Figure 8 PN describing ballast degradation, inspection and maintenance processes.

These are represented by transitions T12 to T16. Specifically, T12 to T14 indicate the imposition of a speed restriction or line closure, and the scheduling and execution of an emergency intervention. T15 and T16 represent the scheduling and execution of a routine intervention. In order to account for the randomness in the effectiveness of tamping, the output state after firing of T16 (execution of routine tamping) is randomly selected among places P2, P3, P4 and P7. Transition T16 will therefore add a token to one of places P2, P3, P4 and P7, each with a given probability. Since the effectiveness of tamping strongly depends on the maintenance history, this probability changes with the number of tamping intervention performed, and thus depends on the marking of place P14. The latter is simply used to count the number of tamping that have been performed. In case of an emergency intervention which is often a manual tamping, a good state (place P7) is usually restored. It is worth specifying that the model explicitly represents speed restrictions and line closures due to unplanned maintenance, namely emergency interventions that are triggered when track geometry has degraded above a given limit. At its current stage the model does not explicitly account for the section closures to carry out routine maintenance, nor the speed restrictions that are often imposed after a renewal as these are considered as ‘planned’.

Since the ballast degradation rates increases with the number of tamping interventions previously performed, the marking of place P14 will affect the parameters of the distributions associated to transitions T1 to T6. The time for ballast renewal depends on the renewal strategy adopted. Renewal can be based on age or maintenance history. In the first case, the ballast is renewed after a fixed number of years, and transition T35 is used, with a deterministic firing time equal to the ballast lifetime. If renewal is based on the past maintenance, then the ballast is renewed as soon as a maximum number of tamping interventions are performed. Transition T37 is used, which is enabled as soon as the marking of place P14 reaches the threshold $N_{tamp,max}$. A third renewal strategy can be considered, according to which ballast is renewed as soon as the sleepers in the same sections are recommended for renewal. Transition T36 is used in this case; it provides a link between the ballast PN module and the PN module for sleepers presented in the next section. Place P37 is a conditional place for the renewal transitions T35, T36 and T37. Depending on its marking, one of the three renewal options can be selected. For example, if marking of P37 is 1, then transition T35 is enabled, namely ballast renewal is based on ballast age. If marking of P37 is 2, then transition T37 is enabled, namely ballast renewal is based on past maintenance. Finally, if marking of P37 is 3, then transition T36 is enabled, namely the ballast is renewed when all sleepers (and consequently fastenings as well) within the 1/8th mile section are scheduled for renewal. The conditions for sleepers/fastenings renewal are described in the following section.

2.3 Model for track gauge: sleepers and fastenings.

This model represents the effects of sleepers and fastenings failures on gauge widening. Individual ineffective sleepers or elements of the fastening system do not have a direct effect on gauge widening. It is only when a number of elements in a given length of track are ineffective that the gauge will actually spread to a level that will eventually lead to a potential derailment risk. Inspection by Track Recording Vehicles only reveals group of failed components that have already caused the gauge to spread, while detection of isolated failed elements relies on visual inspection. Possible interventions are replacement of clips and/or rail pads, and spot re-sleepering.

Degradation process. The different degraded states in the PN, correspond to different levels of gauge widening requiring intervention with different levels of urgency. Each level corresponds to a given number of ineffective elements within a certain length of track (Figure 9). Failure dependencies between sleepers and fastenings are not currently accounted for in the model.

FIGURE 9 HERE

Figure 9 PN for gauge degradation.

The considered section (1/8th of mile) is therefore divided into clusters of consecutive sleepers/fastenings. If the assumption is that a failed sleeper has the same effect on gauge widening as a pair of failed fastenings, then the cluster size is equivalent to the number of consecutive sleepers and/or fastenings whose failure will determine a line closure. Here, the clusters' size corresponds to a single track length containing 10 consecutive sleepers and 20 fastenings (2 per sleeper). As 1/8th of a mile single track section contains typically 300 sleepers (and corresponding 600 fastenings), it follows that each 1/8th of a mile section contains 30 clusters. The model currently assumes that gauge widening is due to a failure of multiple elements within the same cluster, but does not account for the situation when elements at the edges of two adjacent clusters fail. Coloured tokens are used to represent each cluster which is defined by the following attributes: an ID to uniquely identify the cluster, the cluster's size (as defined previously), the number of working sleepers $N_{sl,s}$ and fastening components $N_{f,s}$, the number of ineffective sleepers $N_{sl,i}$ and fastening components $N_{f,i}$, the total number of ineffective elements $N_{tot,i}$. The IDs are given to each cluster in order, e.g. 1,2,3… so that it is possible to identify adjacent clusters. The number of working components in each cluster can decrease over time, while sleepers and/or elements of the fastenings become ineffective. The value of each token is therefore updated every time a component fails, by decreasing the number of working elements and increasing the number of failed elements. In order to avoid confusion with the standard tokens, the coloured tokens defined above are referred to as token-cluster. Five levels of gauge widening have been considered and are represented by places P21 to P25. Thresholds g_{op}, g_{rm}, g_{er} and g_{ir} are associated to places to P22 to P25 and correspond to gauge conditions requiring opportunistic, routine, emergency and immediate interventions. P21 corresponds to no gauge widening. Degradation from one state to the next is caused by failure of components in the same cluster. When a specified number of components within the same cluster have failed, then the corresponding token-cluster is moved to the next degraded state through the transition.

Inspection process. The inspection process, depicted in Figure 10, has the same features as described above for the ballast module, except that here also visual inspection can be considered (loop P32-T31-P33-T32).

FIGURE 10 HERE

Figure 10 PN for gauge degradation and inspection.

Upon inspection the current gauge level is revealed. This is represented by one or more among transitions T23 to T26 firing and adding a token-cluster to the corresponding output place (P26 to P29).

Intervention process. Once the gauge level has been revealed, then maintenance is scheduled and performed, this being modelled by means of transitions T27 to T30 as shown in Figure 11.

FIGURE 11

Figure 11 PN for gauge degradation, inspection and maintenance.

Transitions T28 to T30 represent routine, emergency and immediate interventions respectively. T27 models the opportunistic maintenance which is only possible if a routine intervention is already planned to replace an adjacent group of sleepers/fastenings. Places P30 and P31 are used to simply keep track of the number of sleepers and fastenings replaced respectively. When an intervention is scheduled, only failed components are replaced. This means that functioning fastenings holding a failed sleepers are not replaced along with the sleeper; this however is not always the case in reality. Two renewal policies are considered, either based on age or conditions. Transition T33 represent age-based renewal and it fires when the sleepers lifetime is reached. T34 models a condition-based renewal. This transitions 'checks' the marking of places P26 to P29. Specifically, places P26 to P29 will (potentially) contain one or more token-cluster. For each of these places, the number of failed sleepers contained in each cluster is counted. If the overall number of ineffective sleepers is above a given threshold, then renewal is recommended. Place P35 is a conditional place for both transitions T33 and T34, and its marking determines the renewal strategy to be selected. For example, marking of P35 equal to 1 corresponds to time based renewal, thus transition T33 is enabled. If marking of P35 is 2, then the selected renewal strategy is based on conditions, thus transition T35 is enabled. After renewal (firing of either T33 or T34) the overall state of the system is reset to new. When the section is recommended for renewal, place P36 will receive a token. This place is used to link the gauge module to the ballast module when combined renewal is considered.

2.4 Modules assembly

The PN models for vertical alignment and gauge described above, can be combined into one model by considering the dependencies resulting from the inspection and the renewal processes. The resulting model is depicted in Figure 12, where place P36 in the gauge module is input to transition T36 in the ballast module. T36 represents the event of a combined ballast and sleepers/fastenings renewal driven by sleepers. Indeed, a necessary condition for T36 to fire is that place P36 is marked. This circumstance occurs when sleepers (and consequently also fastenings) are recommended for renewal. This can be due to either sleepers' age, in which case transition T33 fires and adds a token to place P36, or sleepers' conditions, in which case transition T34 fires and adds a token to place P36.

FIGURE 12 HERE

Figure 1 PN combining ballast and gauge models.

2.5 Model analysis

The PN models presented in this work contain several non-conventional features that cannot be accommodated by software commercially available for the construction and execution of PN models. Therefore, a bespoke programme have been developed in a C++ environment, that accounts for the additional features introduced in the models. The behaviour of the track system over time is intrinsically stochastic, thus simulation via the Monte Carlo method is the most suitable analysis technique. The Monte Carlo method consists of running a number of simulations duplicating the system behaviour. This process can be seen as a statistical experiment where each simulation is one observation of the system. This approach requires the knowledge of the distributions of times of occurrence of all the significant events which determine the evolution of the system state over time (transitions). For each stochastic transition, the firing time is sampled from the associated stochastic distribution.

Here, 2-parameter Weibull distributions are associated to stochastic transitions representing components degradation or failure, while lognormal distribution is generally used for the distribution of times to schedule and perform maintenance. The 2-parameter Weibull cumulative distribution function is given by Equation 1.

$$F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}} \qquad (1)$$

where η is the scale parameter and β is the shape parameter. The scale parameter is the time at which 63.2% of the population failed (or degraded to a given state). The shape parameter is indicative of the rate of degradation. Values of β >1 indicates that the degradation rate increases with time; this is typical of components subject to wear and ageing. A value of β = 1 instead is typical of components exhibiting a constant degradation (or failure) rate. In this last case the Weibull distribution becomes a negative-exponential distribution and the scale parameter represents the mean time to failure of the component.

From the cumulative distribution the firing time is evaluated by first generating a random number *X* uniformly distributed between 0 and 1, and then equating it to the cumulative probability as in Equation 2,

$$F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}} = X \qquad (2)$$

from which the time is obtained as in Equation 3

$$t = \eta[-lnX]^{\frac{1}{\beta}}. \qquad (3)$$

Each simulation represents a lifecycle of the track section. During each simulation, the marking of all places and the firing of all transitions of interest in the PN is monitored. This enables the following statistics to be evaluated:

- Number of routine maintenance actions,
- Number/duration/probability of speed restrictions and line closures,
- Probability of being in any of the possible states (good/requiring routine maintenance/requiring emergency intervention),
- Average time of section renewal.

3. Model application

The effects of a range of different maintenance policies on track geometry have been investigated through simulation of the PN model for a number of combinations of the maintenance parameters. Table 1 contains the value of the parameters related to the inspection frequency ϑ_i, the time to perform routine maintenance and emergency intervention (mean μ_i and variance s_i^2 of the corresponding lognormal distributions) for both vertical alignment and gauge.

Table 1. Maintenance parameters.

Inspection (days)	Θ_1=15		Θ_2=30		Θ_3=120	
Routine (days)	$\mu_{rm,1}$=20	$s_{rm,1}^2$=5	$\mu_{rm,2}$=45	$s_{rm,2}^2$=10	$\mu_{rm,3}$=60	$s_{rm,3}^2$=10
Emergency (days)	$\mu_{em,1}$=1	$s_{em,1}^2$=0.25	$\mu_{em,2}$=3	$s_{em,2}^2$=1	$\mu_{em,3}$=7	$s_{em,3}^2$=2

Table 2 below specifies the thresholds on the SD of the vertical alignment triggering opportunistic (σ_{op}), routine (σ_{rm}), emergency maintenance with speed restriction (σ_{sr}) and emergency maintenance with line closure (σ_{lc}). For this numerical application it has been assumed that the thresholds for critical state is σ_{cr}= σ_{sr} (in simple terms, the next degraded state after the one requiring routine maintenance, is the state requiring a speed restriction. Transition T3 is therefore immediate.)

Table 2. Thresholds of SD for vertical alignment and corresponding parameters of Weibull distributions associated to transitions.

		σ_{op}	σ_{rm}	σ_{sr}	σ_{lc}
SD thresholds	(1)	3.5	4.25	4.5	5
	(2)	3.5	3.75	4.5	5
Transitions		T1 and T6	T2	T4	T5
Weibull parameters	(1)	β=1.4, η=1000	β=1.55, η=300	β=1.6, η=400	β=1.7, η=300
	(2)	β=1.4, η=1000	β=1.45, η=500	β=1.6, η=500	β=1.7, η=300

In this application two SD levels for routine maintenance σ_{rm} have been considered, namely 4.25 and 3.75, while the other thresholds are left unchanged. Two approaches have been adopted for selecting the SD thresholds. The first pushes the threshold for routine maintenance closer to the limit triggering a speed restriction. The second approach is more conservative, as it establishes a lower threshold for routine interventions. To each pair of consecutive thresholds is associated a distribution of times to degrade from one threshold level to the next. It is assumed that these times are distributed according to a 2-parameter Weibull. Because two different values of σ_{rm} have been considered, this means that two different sets of parameters (β, η) will be associated to transitions T2 (from place P2 to place P3) and T3 (from place P3 to place P4) one for each value of σ_{rm}. The Weibull parameters for transitions T1 to T6 are also given in Table 2.

Concerning the gauge, in this application it is assumed that the number of consecutive failed elements (sleepers and/or fastenings) that trigger a line closure and a speed restriction is 10 and 8 sleepers and/or pairs of fastenings respectively. From the definition of a cluster size as given in section 2.3 follows that each cluster contains 10 sleepers and 20 fastenings. While the thresholds on the number of consecutive failed elements for speed restriction and line closure are kept unchanged, the number of failed elements triggering a routine intervention, $N_{f,min}$, is varied. Specifically, three different $N_{f,min}$ values have been analysed, i.e. 2, 4 and 5. An opportunistic intervention is possible if one or more ($<N_{f,min}$) components fail. Speed restrictions and line closures are followed by emergency and immediate interventions respectively. It is assumed here that the lifetimes of the individual sleeper and fastenings follow a 2-parameter Weibull distribution with parameters given in Table 3.

Table 3. Weibull parameters for sleepers and fastenings lifetime.

Concrete sleepers	β=1.4, η=9125
Fastenings	β=1.2, η=3650

The combinations of the maintenance parameters in Tables 1 with the two thresholds used for σ_{rm} (Table 2) result in 54 strategies for maintaining the vertical alignment (C1 to C54); these are detailed in Table 4. The parameters in Table 1, combined with the three values of $N_{f,min}$ determine 81 strategies for gauge maintenance (S1 to S81) as shown in Table 5. It is assumed that the ballast is renewed as soon as five out of eight unit sections (1/8th mile) every mile of track has been tamped 15 times ($N_{max,tamp}$=15). Sleepers are renewed as soon as they reach their lifetime which, in this example is assumed to be 35 years. It is also assumed that ballast, sleepers and fastening all start from new conditions.

Table 4 Strategies for maintaining the vertical alignment.

	σ_{rm1}=4.25							σ_{rm2}=3.75					
ID	**Θ**	μ_{rm}	s_{rm}^2	μ_{em}	s_{em}^2	$N_{max,tamp}$	**ID**	**Θ**	μ_{rm}	s_{rm}^2	μ_{em}	s_{em}^2	$N_{max,tamp}$
C1	15	20	5	1	0.25	15	C28	15	20	5	1	0.25	15
C2	15	45	10	1	0.25	15	C29	15	45	10	1	0.25	15
C3	15	60	10	1	0.25	15	C30	15	60	10	1	0.25	15
C4	15	20	5	3	1	15	C31	15	20	5	3	1	15
C5	15	45	10	3	1	15	C32	15	45	10	3	1	15
C6	15	60	10	3	1	15	C33	15	60	10	3	1	15
C7	15	20	5	7	2	15	C34	15	20	5	7	2	15
C8	15	45	10	7	2	15	C35	15	45	10	7	2	15
C9	15	60	10	7	2	15	C36	15	60	10	7	2	15
C10 to C18	30	Same as C1 to C9					C37 to C45	30	Same as C28 to C36				
C19 to C27	120	Same as C1 to C9					C46 to C54	120	Same as C28 to C36				

Table 5 Strategies for maintaining the gauge.

	Θ =15				
ID	$N_{f,min}$	μ_{rm}	s_{rm}^2	μ_{em}	s_{em}^2
S1	2	20	5	1	0.25
S2	4	20	5	1	0.25
S3	5	20	5	1	0.25
S4	2	45	10	1	0.25
S5	4	45	10	1	0.25
S6	5	45	10	1	0.25
S7	2	60	10	1	0.25
S8	4	60	10	1	0.25
S9	5	60	10	1	0.25
S10 to S18	Same as S1 to S9			3	1
S19 to S27	Same as S1 to S9			7	2
	Θ =30				
ID	$N_{f,min}$	μ_{rm}	s_{rm}^2	μ_{em}	s_{em}^2
S28 to S54	Same as S1 to 27				
	Θ =120				
ID	$N_{f,min}$	μ_{rm}	s_{rm}^2	μ_{em}	s_{em}^2
S55 to S81	Same as S1 to 27				

3.1 Results

Convergence of results is reached after 500 simulations. In the following, simulation results showing the effects on both track gauge and vertical alignment are presented and discussed. Figures are given per mile of track, under the assumption of homogeneous characteristics, and provide average values over the entire simulated time. The simulated time varies as it depends on when renewal is required (a simulation is stopped when the section is recommended for renewal). The computational time required to simulate all considered strategies, each 500 times is about 10 minutes. Figures 13 to 17 show the probability of being in good conditions (Figure 13), of a speed restriction being imposed (Figure 14), the average number of routine interventions (Figure 15) and opportunistic interventions (Figure 16), and the renewal times (Figure 17) for each maintenance strategy. The combination of the maintenance parameters corresponding to each strategy is also specified in each figure.

FIGURES 13, 14, 15, 16, 17 HERE.

Figure 2 Probability of good state (ballast)

Figure 3 Probability of speed restriction imposed due to ballast degradation.

Figure 4 Number of routine interventions on ballast.

Figure 5 Number of opportunistic interventions on ballast.

Figure 6 Ballast renewal times in days.

Results show that the parameter with a major influence on the asset performance is the threshold σ_{rm} triggering routine maintenance, followed by the inspection frequency and the mean time to perform routine maintenance. The probability of being in good conditions is generally higher for lower thresholds σ_{rm}, and decreases with increasing mean time to perform routine maintenance, while the probability of a speed restriction shows a complete opposite trend. This is because the longer it takes to perform routine maintenance, and the higher it is the likelihood that conditions will deteriorate further to a level requiring a speed restriction. Such condition, however, is only discovered through inspection; clearly the longer the inspection period, the less the probability of actually revealing such a state. The number of routine interventions is higher for lower thresholds $\sigma_{rm,2}$, meaning also higher intervention costs. The renewal times are mainly affected by the threshold σ_{rm}; higher σ_{rm} values determine higher renewal times. This is mainly due to the fact that the ballast degradation rate increases with the number of tamps performed, which is higher for lower thresholds σ_{rm}, as also shown in Figure 13. The renewal times obtained for $\sigma_{rm,1}$ are in the range between 12490 and 13575 days, namely between 34,23 and 37,2 years, and always below 13000 days (35,6 years) for inspection period of 15 and 30 days. For the more conservative threshold $\sigma_{rm,2}$ the renewal times lay between 9275 and 10225 days (25,4-28 years), and always below 9750 days (26,7 years) for inspection period up to 30 days. The higher threshold $\sigma_{rm,1}$, if combined with a more frequent inspection and a quicker response to the need for maintenance, enable longer ballast lifetime to be achieved. Given that the sleepers average lifetime is approximately 35 years, this also allows for ballast and sleepers renewal to be combined without any loss of sleepers useful life, or considerably frequent need for ballast tamping.

Figures 18, 19 and 20 represents the probability of being in a state requiring routine maintenance, emergency intervention with a speed restriction and immediate intervention with line closure respectively, resulting from the implementation of each strategy.

FIGURES 18, 19, 20 HERE

Figure 7 Probability of being in a state where gauge requires routine maintenance.

Figure 8 Probability of speed restrictions due to gauge degradation.

Figure 20 Probability of line closure due to gauge degradation.

Results are clustered into three main groups based on the minimum number of components $N_{f,min}$ triggering a routine intervention. Parameter $N_{f,min}$ appears to play a fundamental role in determining the gauge response. The probability of requiring a routine intervention decreases with $N_{f,min}$ while the probability of requiring an emergency intervention given that a speed restriction or a line closure are imposed, increases. This trend can be explained by observing that the higher the threshold triggering routine maintenance, the higher is the chance that additional components, either sleepers or fastenings, will fail before a routine intervention is performed, thus causing the gauge to spread to a level requiring a speed restriction. Second to $N_{f,min}$, the inspection frequency affect the gauge response, with its influence being more evident for the probability of a line closure, especially for higher values of $N_{f,min}$. This is because the worst the conditions, the faster the degradation.

Figure 21 shows the total number of sleepers replaced, while the number of fastenings replaced is given in Figure 22. Figure 23 depicts the number of grouped interventions involving sleepers and fastenings within multiple adjacent sections. Grouped interventions means that if a routine intervention is carried out for the elements within a cluster to restore the correct gauge, then ineffective sleepers and fastenings are also replaced within adjacent clusters where routine intervention has been scheduled for a later date, or opportunistic intervention is suitable. This allows taking advantage of the track possession. As expected, the lower the threshold $N_{f,min}$, the higher the number of sleepers replacement.

FIGURE 21 HERE

Figure 21 Total number of sleepers replaced.

FIGURE 22 HERE

Figure 9 Total number of fastenings replaced.

FIGURE 23 HERE

Figure 10 Total number of grouped interventions (multiple sleepers and fastenings replacement).

A different behaviour is instead observed for grouped interventions. The highest number of grouped interventions is always obtained for $N_{f,min}$ = 4 and inspection frequencies of 15 and 30 days, regardless of the other maintenance parameters. Indeed, it will take longer for a section to enter a state requiring routine maintenance if $N_{f,min}$ = 4 than if $N_{f,min}$ = 2. This means that, before a section will need a routine intervention, it will be more likely that sleepers and fastenings in other locations along the line will have also failed if $N_{f,min}$ = 4 than if $N_{f,min}$ = 2. However, if the threshold triggering routine maintenance is pushed closer to the one triggering an emergency interventions , as for $N_{f,min}$ = 5, then it is more likely that the section currently scheduled for a routine maintenance will degrade further to a state requiring an emergency action before a failure occur in any other location along the line. This observation is also supported by the fact that the probability of being in a state requiring a speed restriction and emergency intervention takes the higher value when $N_{f,min}$ = 5 as shown in Figure 16. If inspection intervals increase to 120 days, then it will take longer for the need for routine maintenance to be revealed. Therefore, when $N_{f,min}$ = 4, the section is more likely to degrade to a state requiring a speed restriction before inspection is performed. On the other hand, when $N_{f,min}$ = 2, longer inspection intervals means that failures in other locations along the line might occur before inspection is performed.

4. Conclusions

In this paper a simulation tool based on Petri nets has been presented, which models track geometry degradation and the corresponding maintenance actions that can be performed. The model accounts for vertical alignment variations and gauge spreading due to ballast and sleepers/fastening failures respectively. The model enables the track geometry conditions, probability of failure modes leading to speed restrictions and line closures, and the number of interventions performed during a given time horizon to be predicted for a wide range of maintenance strategies. Along with the probability of speed restrictions and line closures being imposed, also the average number and duration of such restrictive measures can be recorded during simulations. These provide indirect indication of the impact that different maintenance strategies will have on service if implemented. Although the proposed model does not directly quantify the delays and corresponding costs, it enables a comparison between different strategies to be drawn, based on the number of interventions (routine, opportunistic and emergency) and the unavailability of the track due to unplanned speed restrictions and line closures. Clearly, an actual evaluation of more detailed service performance measures which account for the actual delays and/or journey cancellations, will require the use of specific software that model the interactions between train services and infrastructure failures and maintenance such as OpenTrack, RailSys and TRAIL. The results obtained from the proposed model can be used within the aforementioned software to generate disruption scenarios that are directly linked to a given maintenance strategy. The model can also be used to gain insight into the potential effects of new maintenance strategies on the asset performance thus partly compensating for the lack of real data whose collection would require years if not decades.

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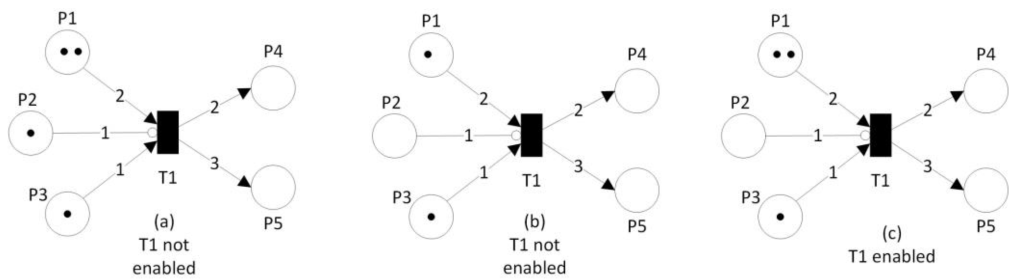
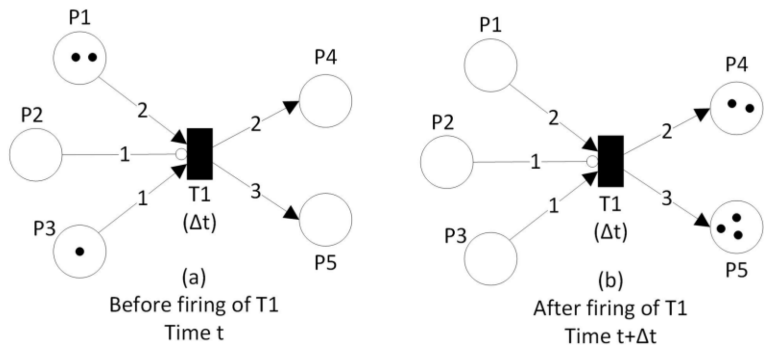
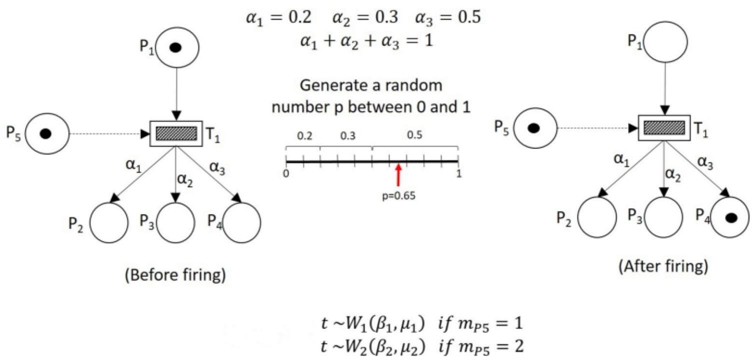
OpenTrack. OpenTrack Railway Technology Ltd. Gubelstr. 28, CH 8050 Zurich, Switzerland.

RailSys. Rail Management Consultants GmbH. Luetzerodestrasse 10, 30161 Hannover, Germany.

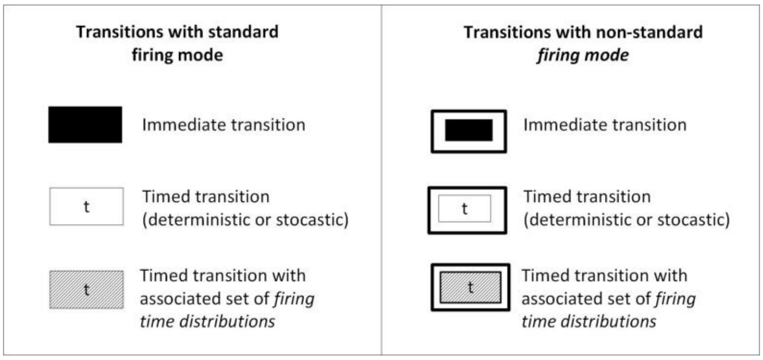
FIGURES



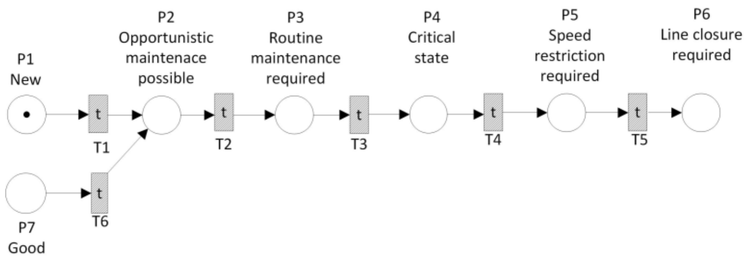




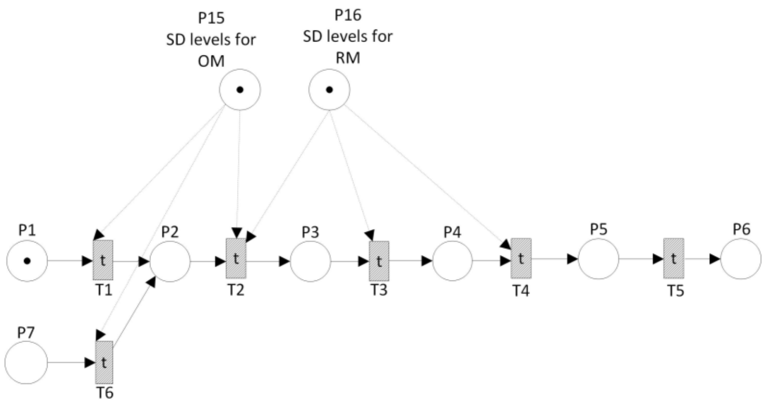
$\alpha_1 = 0.2 \quad \alpha_2 = 0.3 \quad \alpha_3 = 0.5$
$\alpha_1 + \alpha_2 + \alpha_3 = 1$
Generate a random number p between 0 and 1
p=0.65
(Before firing)
(After firing)
$t \sim W_1(\beta_1, \mu_1) \quad if\ m_{P5} = 1$
$t \sim W_2(\beta_2, \mu_2) \quad if\ m_{P5} = 2$



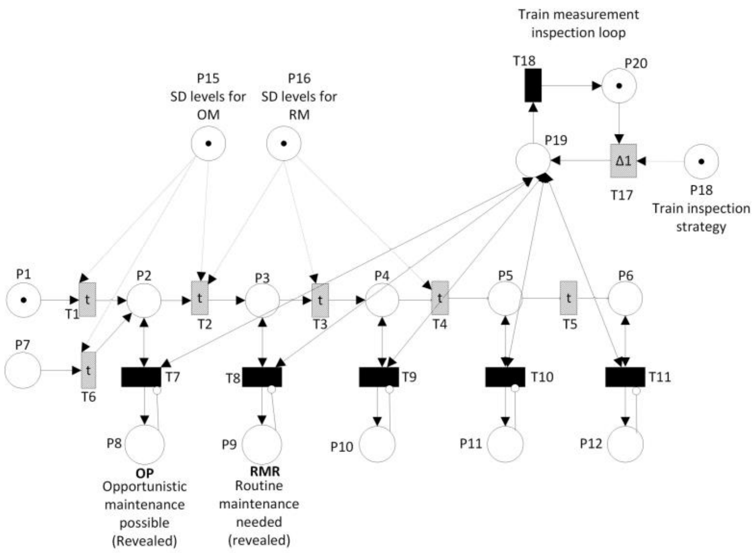
Transitions with standard firing mode
Immediate transition
Timed transition (deterministic or stocastic)
Timed transition with associated set of *firing time distributions*
Transitions with non-standard *firing mode*
Immediate transition
Timed transition (deterministic or stocastic)
Timed transition with associated set of *firing time distributions*



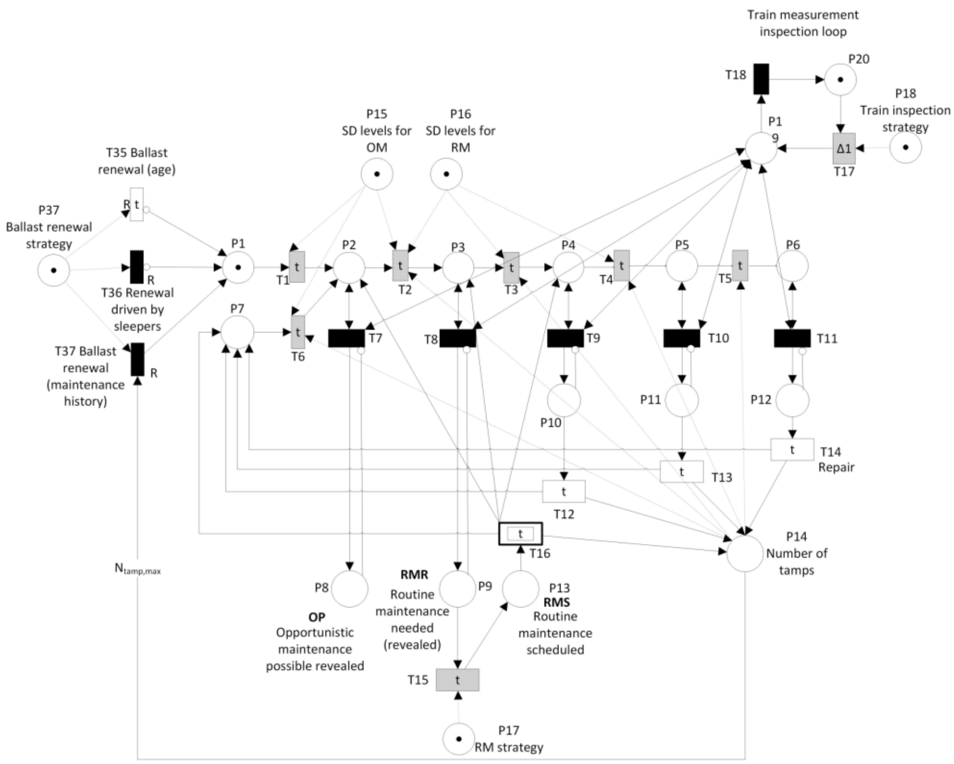
P1 New
P2 Opportunistic maintenace possible
P3 Routine maintenance required
P4 Critical state
P5 Speed restriction required
P6 Line closure required
P7 Good
T1
T2
T3
T4
T5
T6



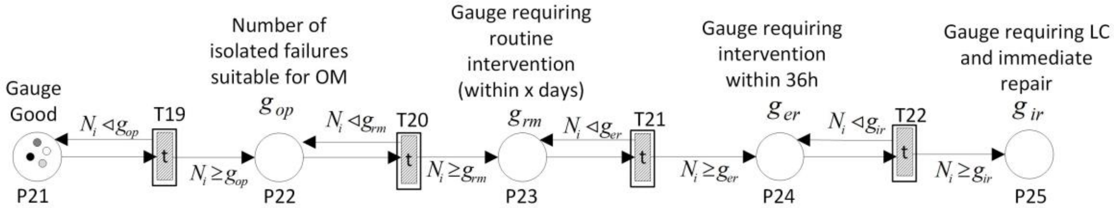
P15 SD levels for OM
P16 SD levels for RM
P1
P2
P3
P4
P5
P6
P7
T1
T2
T3
T4
T5
T6



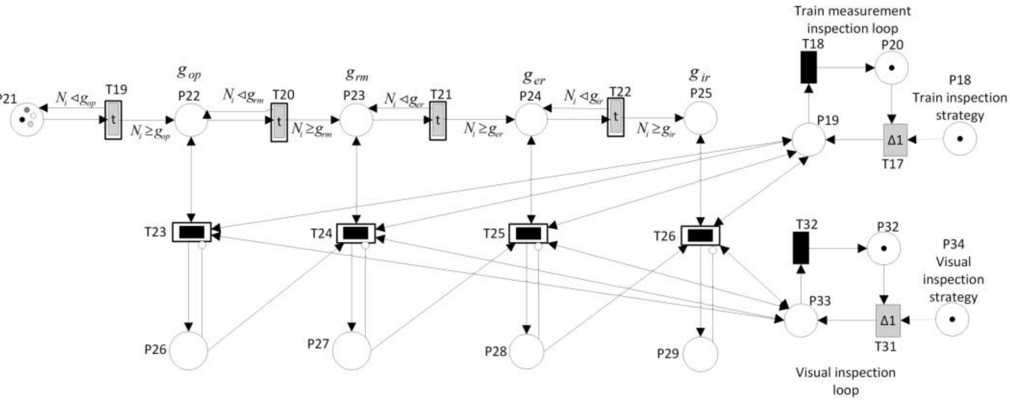
Train measurement inspection loop
T18
P20
P15 SD levels for OM
P16 SD levels for RM
P19
Δ1
T17
P18 Train inspection strategy
P1
P2
P3
P4
P5
P6
T1
T2
T3
T4
T5
P7
T6
T7
T8
T9
T10
T11
P8
P9
P10
P11
P12
OP Opportunistic maintenance possible (Revealed)
RMR Routine maintenance needed (revealed)

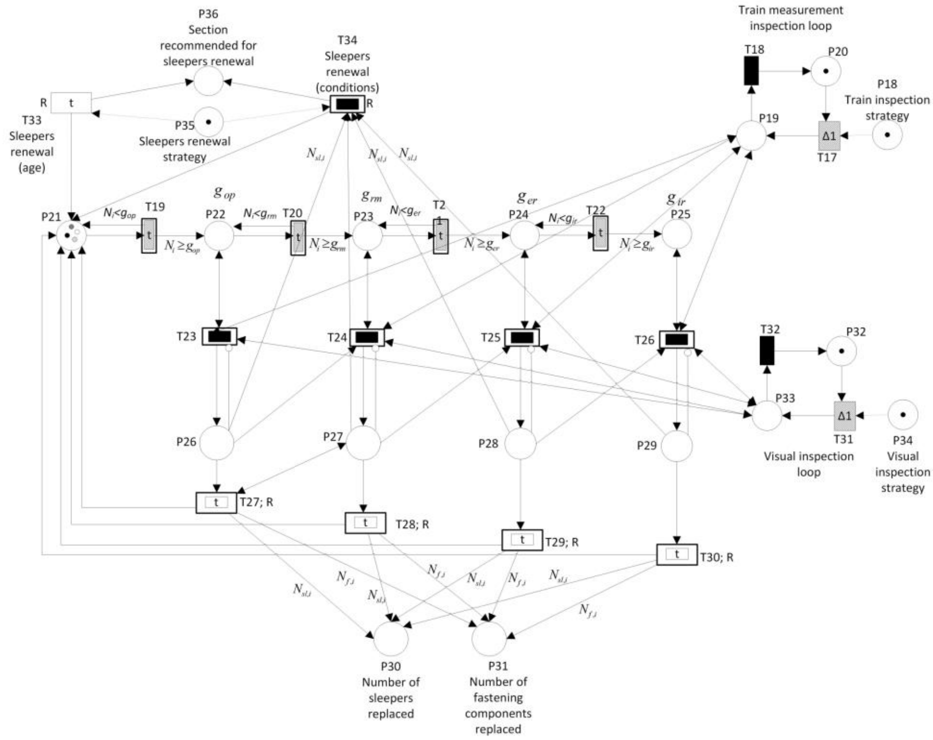


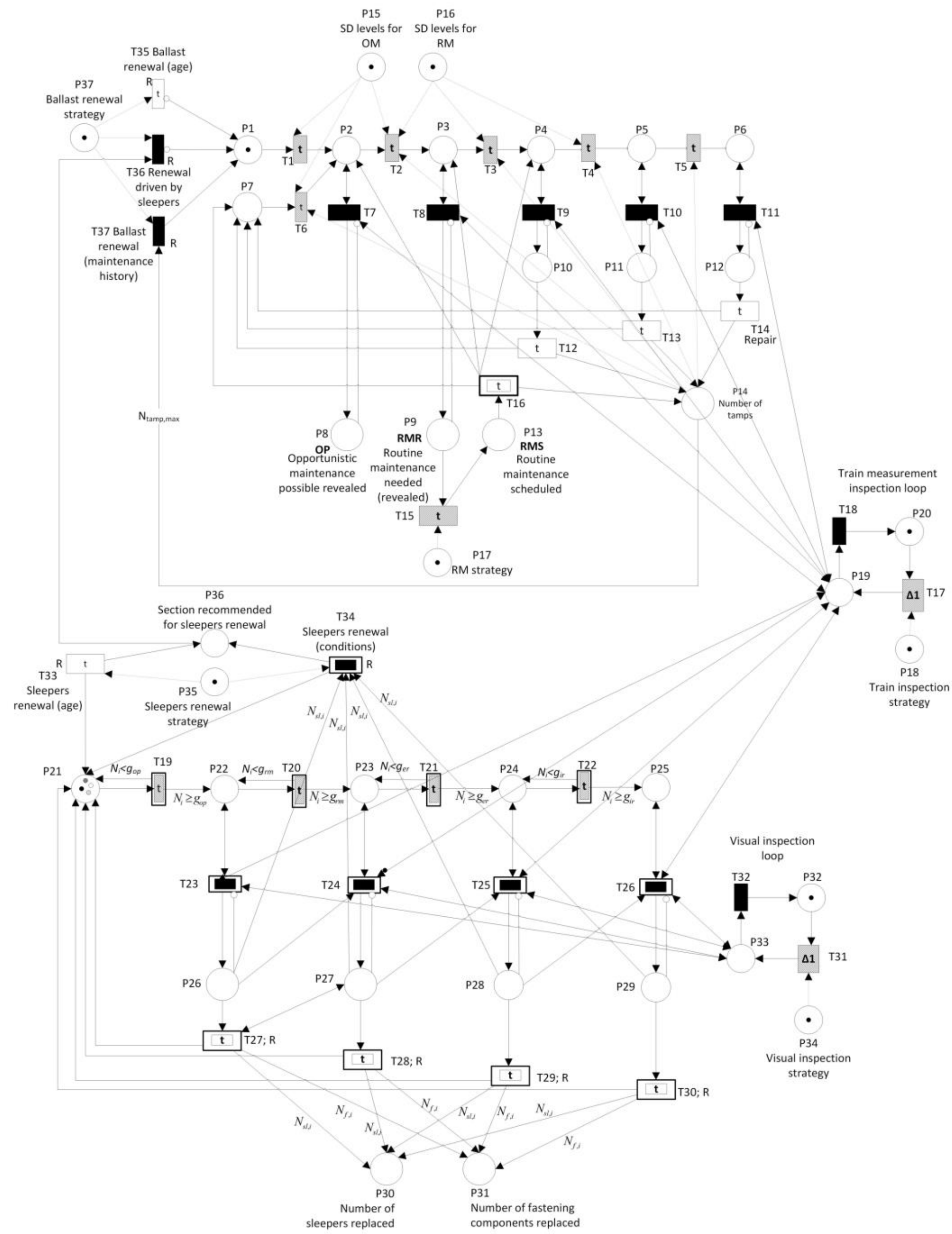
Train measurement inspection loop
T18
P20
P18 Train inspection strategy
P19
Δ1
T17
P15 SD levels for OM
P16 SD levels for RM
T35 Ballast renewal (age)
P37 Ballast renewal strategy
T36 Renewal driven by sleepers
T37 Ballast renewal (maintenance history)
P1
P2
P3
P4
P5
P6
T1
T2
T3
T4
T5
P7
T6
T7
T8
T9
T10
T11
P10
P11
P12
T14 Repair
T13
T12
T16
P14 Number of tamps
$N_{tamp,max}$
P8
OP Opportunistic maintenance possible revealed
RMR Routine maintenance needed (revealed)
P9
P13 RMS Routine maintenance scheduled
T15
P17 RM strategy

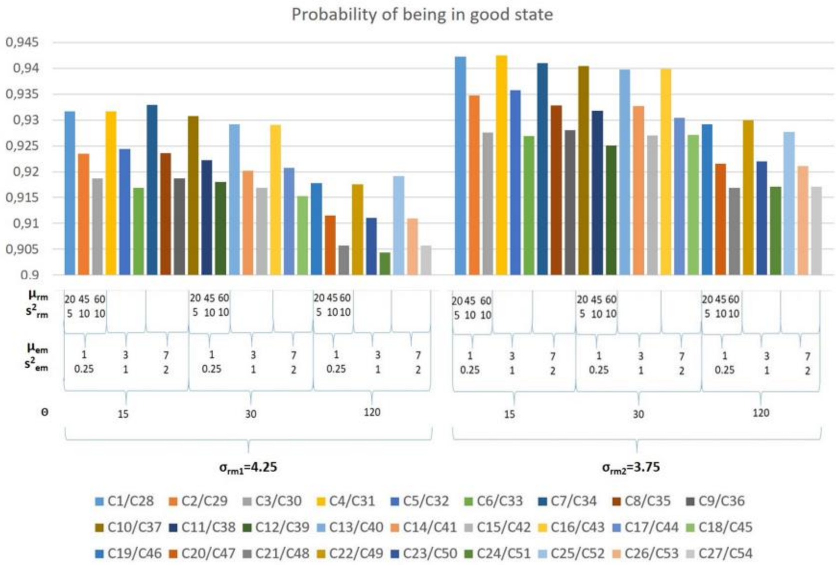


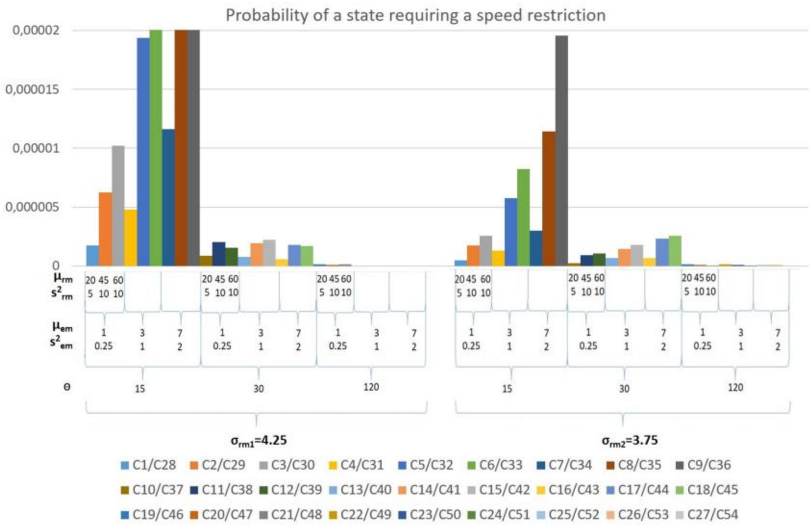
Gauge Good
P21
T19
$N_i < g_{op}$
$N_i \geq g_{op}$
Number of isolated failures suitable for OM
g_{op}
P22
T20
$N_i < g_{rm}$
$N_i \geq g_{rm}$
Gauge requiring routine intervention (within x days)
g_{rm}
P23
T21
$N_i < g_{er}$
$N_i \geq g_{er}$
Gauge requiring intervention within 36h
g_{er}
P24
T22
$N_i < g_{ir}$
$N_i \geq g_{ir}$
Gauge requiring LC and immediate repair
g_{ir}
P25

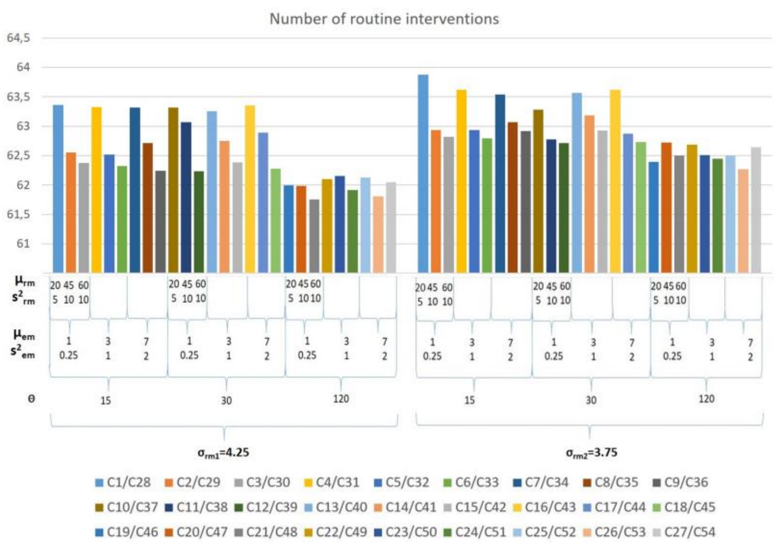

Train measurement inspection loop
T18
P20
P18 Train inspection strategy
P21
T19
P22
T20
P23
T21
P24
T22
P25
P19
T17
T23
T24
T25
T26
T32
P32
P34 Visual inspection strategy
P33
T31
P26
P27
P28
P29
Visual inspection loop

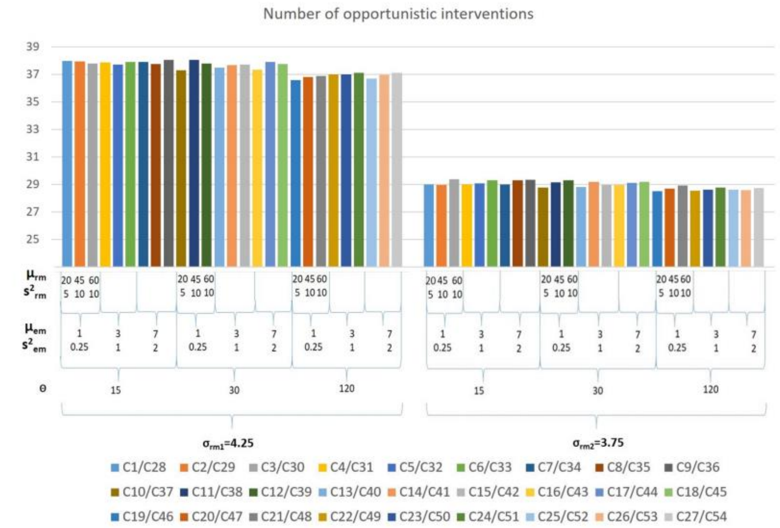

P36 Section recommended for sleepers renewal
T34 Sleepers renewal (conditions)
Train measurement inspection loop
T18
P20
P18 Train inspection strategy
T33 Sleepers renewal (age)
P35 Sleepers renewal strategy
P19
T17
P21
T19
P22
T20
P23
P24
T22
P25
T23
T24
T25
T26
T32
P32
P33
T31
P34 Visual inspection strategy
P26
P27
P28
P29
Visual inspection loop
T27; R
T28; R
T29; R
T30; R
P30 Number of sleepers replaced
P31 Number of fastening components replaced

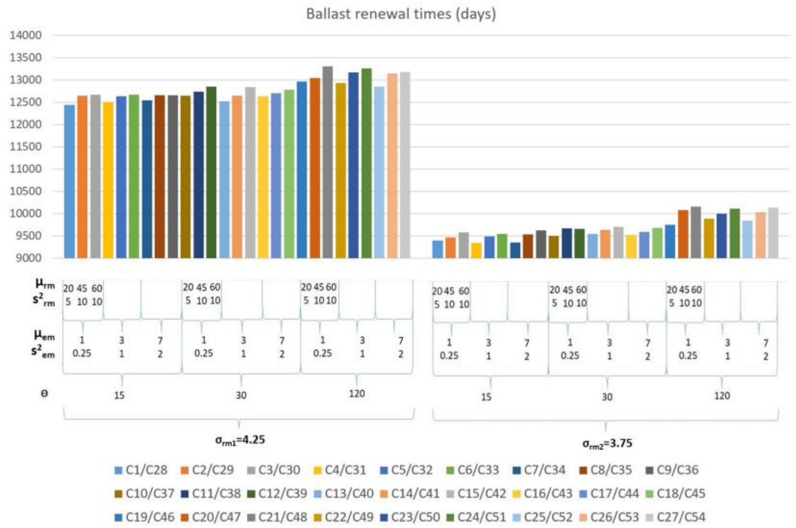

P15
SD levels for OM
P16
SD levels for RM
T35 Ballast renewal (age)
P37
Ballast renewal strategy
R
P1
P2
P3
P4
P5
P6
T1
T2
T3
T4
T5
T36 Renewal driven by sleepers
P7
T6
T7
T8
T9
T10
T11
T37 Ballast renewal (maintenance history)
P10
P11
P12
T12
T13
T14
Repair
T16
P14
Number of tamps
N tamp,max
P8
OP
Opportunistic maintenance possible revealed
P9
RMR
Routine maintenance needed (revealed)
P13
RMS
Routine maintenance scheduled
T15
P17
RM strategy
Train measurement inspection loop
T18
P20
P19
T17
Δ1
P18
Train inspection strategy
P36
Section recommended for sleepers renewal
T34
Sleepers renewal (conditions)
T33
Sleepers renewal (age)
P35
Sleepers renewal strategy
P21
T19
P22
T20
P23
T21
P24
T22
P25
Visual inspection loop
T32
P32
T23
T24
T25
T26
P33
T31
P26
P27
P28
P29
P34
Visual inspection strategy
T27; R
T28; R
T29; R
T30; R
P30
Number of sleepers replaced
P31
Number of fastening components replaced

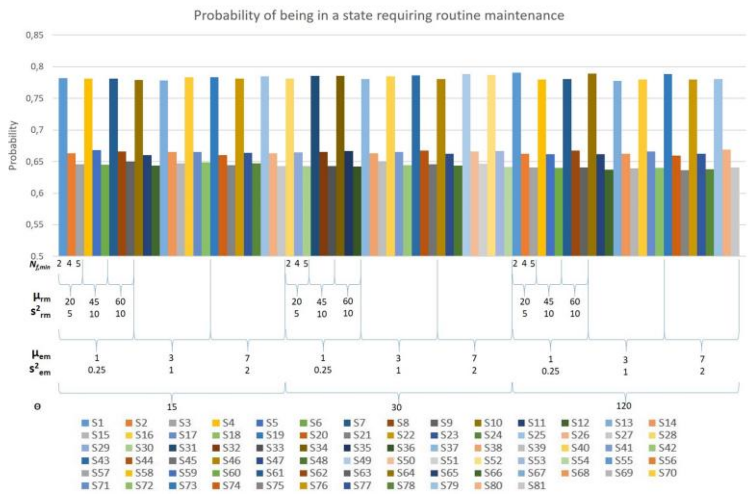

Probability of being in good state
μ_{rm} 20 45 60
s^2_{rm} 5 10 10
μ_{em} 1 3 7
s^2_{em} 0.25 1 2
θ 15 30 120
σ_{rm1}=4.25
σ_{rm2}=3.75
C1/C28 C2/C29 C3/C30 C4/C31 C5/C32 C6/C33 C7/C34 C8/C35 C9/C36
C10/C37 C11/C38 C12/C39 C13/C40 C14/C41 C15/C42 C16/C43 C17/C44 C18/C45
C19/C46 C20/C47 C21/C48 C22/C49 C23/C50 C24/C51 C25/C52 C26/C53 C27/C54

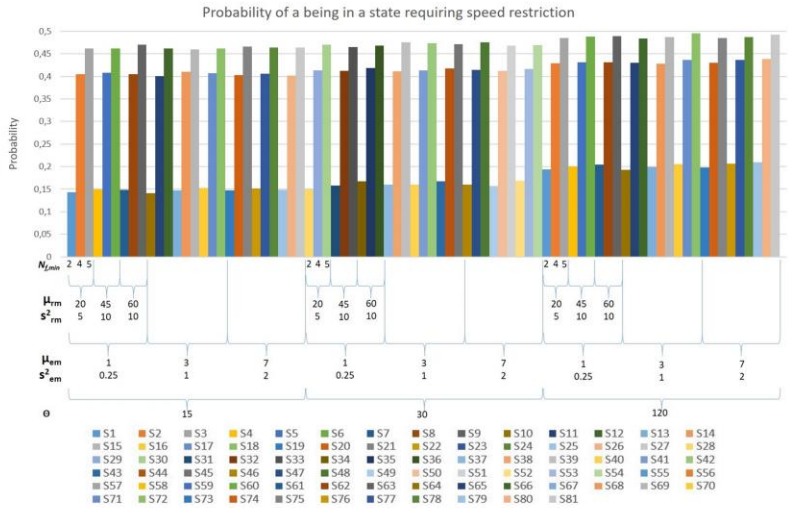

Probability of a state requiring a speed restriction
μ_{rm} 20 45 60
s^2_{rm} 5 10 10
μ_{em} 1 3 7
s^2_{em} 0.25 1 2
θ 15 30 120
σ_{rm1}=4.25
σ_{rm2}=3.75
C1/C28 C2/C29 C3/C30 C4/C31 C5/C32 C6/C33 C7/C34 C8/C35 C9/C36
C10/C37 C11/C38 C12/C39 C13/C40 C14/C41 C15/C42 C16/C43 C17/C44 C18/C45
C19/C46 C20/C47 C21/C48 C22/C49 C23/C50 C24/C51 C25/C52 C26/C53 C27/C54

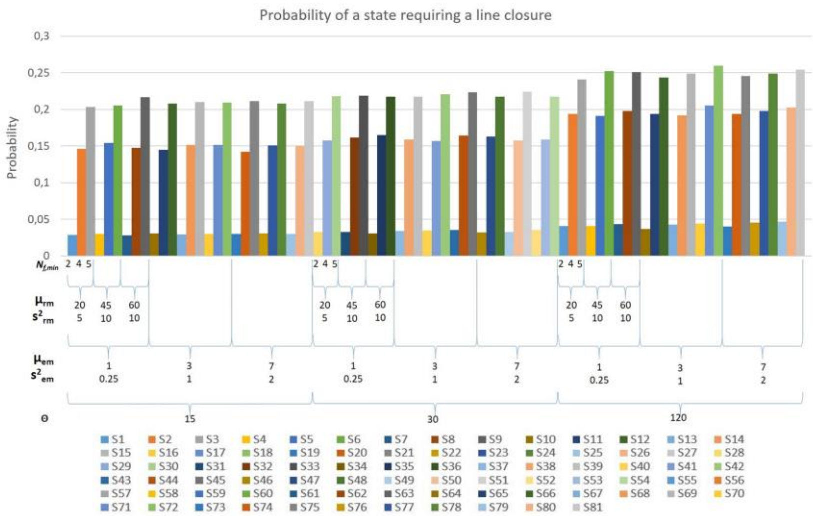

Number of routine interventions
μ_{rm} 20 45 60
s^2_{rm} 5 10 10
μ_{em} 1 3 7
s^2_{em} 0.25 1 2
θ 15 30 120
σ_{rm1}=4.25
σ_{rm2}=3.75
C1/C28 C2/C29 C3/C30 C4/C31 C5/C32 C6/C33 C7/C34 C8/C35 C9/C36
C10/C37 C11/C38 C12/C39 C13/C40 C14/C41 C15/C42 C16/C43 C17/C44 C18/C45
C19/C46 C20/C47 C21/C48 C22/C49 C23/C50 C24/C51 C25/C52 C26/C53 C27/C54

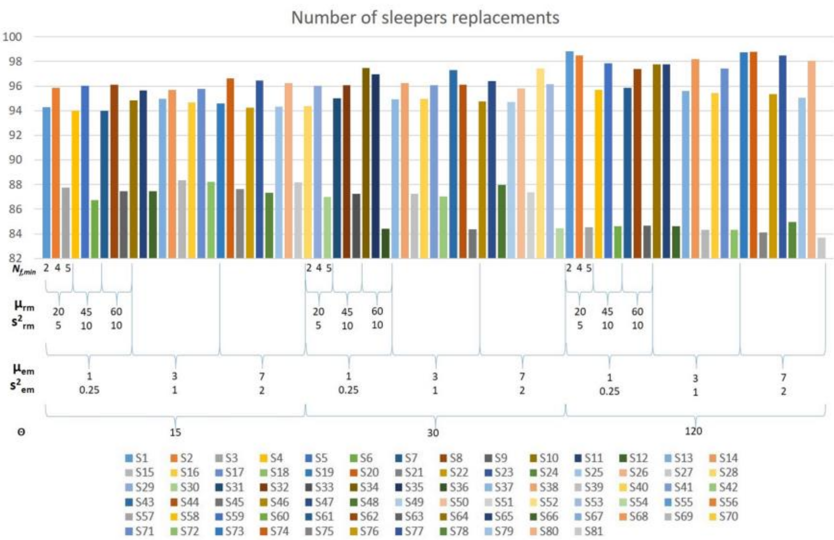

Number of opportunistic interventions
39
37
35
33
31
29
27
25
μrm
s²rm
μem
s²em
θ
15
30
120
σrm1=4.25
σrm2=3.75
C1/C28 C2/C29 C3/C30 C4/C31 C5/C32 C6/C33 C7/C34 C8/C35 C9/C36
C10/C37 C11/C38 C12/C39 C13/C40 C14/C41 C15/C42 C16/C43 C17/C44 C18/C45
C19/C46 C20/C47 C21/C48 C22/C49 C23/C50 C24/C51 C25/C52 C26/C53 C27/C54

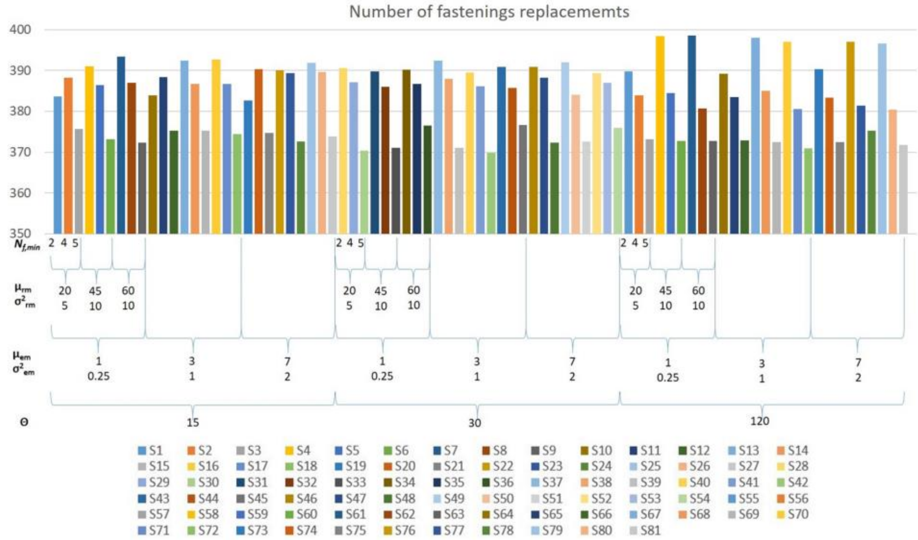

Ballast renewal times (days)
14000
13500
13000
12500
12000
11500
11000
10500
10000
9500
9000
μrm
s²rm
μem
s²em
θ
15
30
120
σrm1=4.25
σrm2=3.75
C1/C28 C2/C29 C3/C30 C4/C31 C5/C32 C6/C33 C7/C34 C8/C35 C9/C36
C10/C37 C11/C38 C12/C39 C13/C40 C14/C41 C15/C42 C16/C43 C17/C44 C18/C45
C19/C46 C20/C47 C21/C48 C22/C49 C23/C50 C24/C51 C25/C52 C26/C53 C27/C54

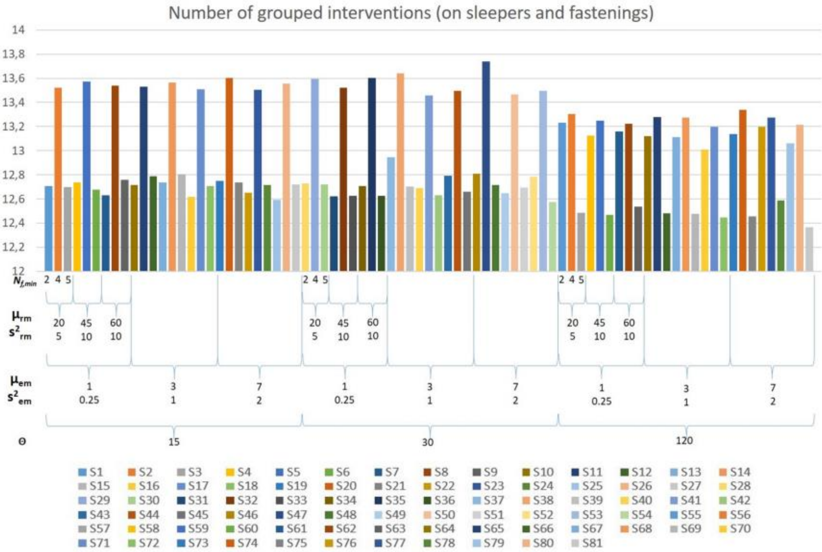

Probability of being in a state requiring routine maintenance
Probability
0,85
0,8
0,75
0,7
0,65
0,6
0,55
0,5
μrm
s²rm
μem
s²em
θ
15
30
120


Probability of a being in a state requiring speed restriction
Probability
$N_{f,min}$
μ_{rm}
s^2_{rm}
μ_{em}
s^2_{em}
θ
15
30
120


Probability of a state requiring a line closure
Probability
$N_{f,min}$
μ_{rm}
s^2_{rm}
μ_{em}
s^2_{em}
θ
15
30
120


Number of sleepers replacements
$N_{f,min}$
μ_{rm}
s^2_{rm}
μ_{em}
s^2_{em}
θ
15
30
120


Number of fastenings replacememts
400
390
380
370
360
350
2 4 5
20 5
45 10
60 10
1 0.25
3 1
7 2
Θ
15
30
120


Number of grouped interventions (on sleepers and fastenings)
14
13,8
13,6
13,4
13,2
13
12,8
12,6
12,4
12,2
12
2 4 5
20 5
45 10
60 10
1 0.25
3 1
7 2
Θ
15
30
120