A mathematical programming model to select maintenance strategies in railway networks

Claudia Fecarotti[a,*], John Andrews[b], Raffaele Pesenti[c]

[a]*Department of Industrial Engineering & Innovation Science, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands*
[b]*Department of Mechanical Engineering, The University of Nottingham, University Park, NG72DR, Nottingham, UK*
[c]*Department of Management, University Ca'Foscari Venezia, Cannaregio 873, 30121 Venezia, Italy*

Abstract

This paper presents a nonlinear integer programming model to support the selection of maintenance strategies to implement on different segments of a railway network. Strategies are selected which collectively minimise the impact of sections conditions on service, given network availability and budget constraints. Different metrics related to the network topology, sections' availability, service frequency, performance requirements and maintenance costs, are combined into a quantitative approach with a holistic view. The main contribution is to provide a simple yet effective modelling approach and solution method which are suitable for large networks and make use of standard solvers. Both an *ad hoc* heuristic solution and relaxation methods are developed, the latter enabling the quality of the heuristic solution to be estimated. The availability of railway lines is computed by exploiting the analogy with series-parallel networks. By varying the model parameters, a scenario analysis is performed to give insight into the influence of the system parameters on the selection of strategies, thus enabling more informed decisions. For its simple structure, the model is versatile to address similar problems arising in the maintenance of other types of networks, such as road and bridges networks, when deciding on the strategic allocation of maintenance efforts.

Keywords: `Maintenance Optimisation`, `Railway Networks`, `Mathematical Programming`, `Availability`

1. Introduction

Infrastructure asset maintenance for transportation networks is a complex decision making process involving the allocation of limited maintenance resources to achieve a trade-off between costs and service performance. Railway networks are an example of large scale transportation networks consisting of a variety of heterogeneous assets spatially distributed and interdependent. The network topology coupled with the operational use of the infrastructure are such that a failure of an individual section might affect the availability of an entire line.

Maintenance planning is addressed at strategic, tactical and operational level. Strategic maintenance planning involves first, at asset level (e.g. track, signalling and telecoms, bridges, earthworks, electrical power), the development of maintenance strategies which define fixed rules to maintain the assets. These

[*]Claudia Fecarotti
Email address: `c.fecarotti@tue.nl` (Claudia Fecarotti)

rules include type of inspection and its frequency, the waiting time to maintenance, the thresholds on time, age, usage and/or conditions which trigger maintenance actions with different levels of urgency. Then, at network level, strategies must be chosen for different network segments. Maintenance strategies as defined above, form the basis for the planning of maintenance activities and intervention programs defined at tactical and operational planning levels, which govern the maintenance operations.

This paper introduces an optimisation model based on mathematical programming to support the selection of the maintenance strategies to implement on different regions of a railway network. The resulting combination of strategies enables to obtain the best value from the assets from a network perspective. Its intended use is subsequent to the development of the actual strategies, which are here given as input to the model. The model is performance-oriented, it assumes that decisions are aimed at minimising the expected number of trains per time unit affected by a speed restriction, while availability targets for different railway lines have to be met and budget constraints have to be respected. The main contribution of this paper is to present a simple yet effective modelling approach and solution method which are suitable for large networks and make use of standard solvers, thus making its use in practice more attractive. From a modelling perspective, the value of the proposed model is to provide a quantitative approach which combines different metrics related to the network topology, sections availability, service frequency, performance requirements and maintenance costs, in a holistic view with a clear and simple structure rather than a black box. As the approach is quantitative in nature, a wide study of the sensitivity of the solution with respect to the model parameters is performed. The value of this study is to give insight on the relative and combined contribution of different factors on the selection of strategies. These factors include track redundancy, centrality of sections, service frequency, budget and lines availability targets. The final contribution of this paper is to have developed both a heuristic solution approach and an *ad hoc* relaxation method, thus enabling the quality of the heuristic solution to be estimated. The model addresses a real problem in practice faced by infrastructure owners/managers related to the strategic maintenance planning of their networked infrastructures. Although the model is developed and demonstrated here for a railway network, its simple structure makes it versatile and suitable to address similar problems arising in the maintenance of other types of networks when deciding on the strategic allocation of maintenance efforts. Examples are road and bridges networks, where long-term maintenance strategies must be choosen for individual or groups of road segments or bridges respectively.

1.1. Literature review

Literature proposes different optimisation models to support maintenance planning for different infrastructures and planning levels. A recent overview of the literature on asset management and maintenance of multi-unit systems is given in [1, 2], and previously in [3, 4]. The survey in [1] distinguishes between contributions dealing with multi-component and multi-asset systems, focusing more specifically on the latter. While authors usually refer to multi-component systems to describe a single asset (e.g. a machine or a bridge) and its constituent components, multi-asset systems are systems of multiple assets otherwise connected. The present paper focuses on this last category.

In the infrastructure sector, maintenance planning of systems of assets mainly focuses on the optimisation of sequential decisions. Typically these decisions concern the selection of one or more interventions out of a given set based on the current state of the assets. Example of interventions are rehabilitation, renovation, routine maintenance, or do nothing. Decisions are often constrained by limited budgets and capacity targets, while the objective is to optimise either maintenance costs and/or some measure of the system performance. The authors in [5] address the problem of planning maintenance for a system of heterogeneous assets undergoing stochastic deterioration over a finite time horizon. They develop a two-stage bottom-up approach. First, the optimal maintenance activities are determined for each asset among a set of feasible interventions. Then, a system-level optimisation selects the best combination of interventions given budget constraints. All assets in the system are independent except for the shared budget. A similar approach has been used in [6] with application to a hypothetical railway system. A number of assets are associated to each link in the network, each with a set of available maintenance activities. For each asset the best activity is obtained by solving a Markov decision problem via dynamic programming. At system-level the budget constraint includes the cost reduction achieved through opportunistic maintenance of adjacent assets. A threshold on the minimum capacity to be guaranteed between an origin and a destination node is also imposed. In [7], the authors develop a model with the objective of determining the optimal set of maintenance interventions for a system of bridge decks. Again, they suggest a two-step approach where they use the optimal cost of maintenance and replacement for each bridge resulting from a facility-level optimisation problem previously defined in [8]. Then, at system level, they select the optimal combination of maintenance and replacement activities which minimise the probability of system failure subject to budget constraints. A two-stage bottom-up optimisation approach is proposed in [9] for timely maintenance planning in heterogeneous systems. The specific maintenance action and time of execution are optimised per component based on a system perspective. The approach is demonstrated with application to a simple immaginary railway network. In a more recent contribution ([10]) the budget allocation and maintenance planning problem for multi-asset systems is addressed again as a finite-horizon Markov decision process for each individual asset. Then the system level optimisation is formulated as an integer programming problem which minimise the total expected maintenance cost of all assets under a budget constraint formulated for each decision period. The shared budget is the only interaction considered across the facilities. The problem is nonlinear, and a standard linearization method based on auxiliary binary variables is adopted. Through Lagrangian relaxation of the budget constraint, the system-level optimization can be decomposed into multiple Markov decision problems, one for each asset, and a lower bound to the primal problem is obtained. This decomposition approach is only possible because the shared budget is the only dependency among facilities. A linear optimisation model is presented in [11] to determine optimal intervention programs consisting of the interventions to be performed each year over a finite time horizon for a system of assets. The above mentioned contributions all focus on sequential decision making and formulate the problem as a Markov decision process, mostly solved via dynamic programming. The sequence of interventions to be executed for each asset per time interval (e.g. each year) during a finite time horizon is obtained. All models assume knowledge of the state of the asset following intervention.

In [12] and [13] the authors address a similar problem as the above cited contributions but with a different modelling approach. They aim at determining optimal intervention programs for railway networks with multiple assets, where interventions are selected among a given set under budget and structural constraints. They adopt a network flow model approach resulting in a mixed integer linear program which is solved via the simplex and branch and bound methods. Interventions are selected based on their cost, duration and reduction of failure risk. The approach is extended in [14] which focuses particularly on the estimation of intervention costs and the effects of intervention programs on service.

Other maintenance planning problems for multi-asset systems appearing in the literature deal with the prioritisation of assets interventions, the grouping of works and the scheduling of maintenance activities. In [15] a linear integer programming model is proposed for the scheduling of preventive small routine maintenance activities and major jobs on a single railway link. The dynamic grouping and scheduling of maintenance activities for multi-component systems is addressed in [16, 17]. Jobs are grouped together to minimise possession costs based on given maintenance frequency and which routine jobs can be combined. The optimal grouping of components is also addressed in [18] in the context of preventive maintenance of multi-component systems via a multi-level preventive decision-making model. A maintenance grouping approach for a series system with availability constraints is proposed in [19], where the maintenance planning is updated in a dynamic context. A multi-objective optimisation model is presented in [20] for scheduling renewal works of ballast, rails and sleepers while seeking a trade-off between life-cycle costs and the track availability. The problem is solved via a genetic algorithm. The optimisation of maintenance schedule for railway power supply system is addressed in [21, 22] where the authors develop a biobjective optimisation model which considers the trade-off between reliability and maintenance costs. A biobjective optimisation model is also presented in [23] to optimise planned maintenance and renewal activities for track geometry. The model determines whether a track section is tamped or renewed in a given trimester or not, and what level of speed restriction is imposed. The contributions reviewed above address maintenance planning problems at a tactical and/or operational level.

At a strategic level instead, a typical problem is to determine long-term maintenance policies defining the fixed rules for inspection and maintenance such as inspection frequency and condition (or alternatively age or usage) thresholds triggering interventions. In [24] a non-homogeneous Markov model solved via a numerical procedure is used to determine the probability of rail cracks, and it is combined with a Genetic Algorithm to find rail inspection intervals and waiting time for maintenance which minimise both costs and the probability of rail cracks. In [25] the authors consider a series-parallel systems of components of different types. They use a Markov model to describe the degradation and on-condition maintenance processes, and recour to Monte Carlo simulation to obtain the average system availability and the probability of being under maintenance at any given time t. A genetic algorithm is then used to find the maintenance thresholds per component type, which simultaneously optimise profit from system operation and system availability. In [26] a life-cycle cost model is proposed to investigate tamping and track renewal strategies to minimise ballast life-cycle costs. In particular, an iterative search algorithm is used to determine the best strategy among a set of alternatives including fixed intervals or fixed condition's thresholds triggering a tamping intervention. A similar decision problem is addressed in [27] for multi-component systems, where the optimal frequency of scheduled visits and the preventive maintenance threshold for a component within a series system are sought. Here renewal theory is used to evaluate the long-term average maintenance costs, and an iterative procedure is used to find the near optimal values of visits' frequency and maintenance thresholds that minimise the long-term average cost. The above contributions look at the development of maintenance strategies by optimising maintenance parameters such as inspection intervals and maintenance thresholds simultaneously for multiple components within a system. However they assume that the "structure"of a maintenance strategy is the same for all component types, and the only difference is in the values taken by the maintenance parameters.

This paper can be framed among those works aiming at the selection of long-term maintenance strategies for multi-asset systems with a networked structure. However, unlike the above contributions, it presents an approach that allows for fundamentally different maintenance strategies to be compared based on their costs and resulting availability and level of performance of the assets. Indeed in reality, the assets comprising an infrastructure system are quite diverse and complex, and this reflects into equally diverse and complex maintenance strategies. From here the choice to separate the problem of developing individual strategies, from the one of selecting the best strategies to be used in combination for a network of assets and allocate the maintenance budget accordingly. This paper focuses on the latter problem, thus contributing to the wider literature of portfolio decision analsyis which aims at supporting decisions for the selection of projects and the allocation of resources [28]. Recent contributions to portfolio decision analysis in infrastructure maintenance are [29, 30]. In [29], a sequential portfolio selection approach on a multi-period horizon is presented to identify the optimal risk-based maintenance portfolios for gas networks. The maintenance portfolio consists of a set of pipe segments which will undergo complete replacement. The problem of allocating prognostic and health management capabilities in power transmission networks to maximise the network global reliability efficiency under budget constraints is addressed in [30].

In the present paper, given a portfolio of potential maintenance strategies and a network of assets, one wants to select the strategies that collectively enable a trade-off between maintenance costs, system availability and service impact. This corresponds to allocate maintenance efforts among different sections of the network. The problem is formulated here for a railway network and is modelled via mathematical programming. A nonlinear combinatorial optimisation problem is presented, where the nonlinearities are due to the availability constraints imposed on railway lines which account for the network topology. It further develops the work in [31] by refining an *ad hoc* linearisation approach to approximate the optimal solution from above, and by presenting a relaxation method to quantify the quality of the solution and therefore the performance of the proposed heuristic. The model takes into account economic, functional and operational dependencies between different sections of the network by considering a shared limited budget, and by making use of reliability network theory to model the contribution of individual track sections to the availability of railway lines along which service run. The model requires the knowledge of the effects of each strategy on the long-term behavior of the assets. Input to the model can be given by state-based models as suggested in [31], which combine degradation and maintenance processes to predict the long-term assets response to maintenance strategies. Petri nets [32] can be used to this aim, as they can assess the probabilities of different states of interest, including section closures and speed restrictions, as well as the average work volumes (see e.g. [33, 34, 35, 36] for railway tracks and bridges). Alternative methods which model the assets as multi-state systems (e.g. [37, 38, 24]) can serve the same purpose, as well as historical data.

The rest of the paper is organised as follows. After providing the problem description and problem statement in section 2, the model is presented in section 3, where the approach to model the unavailability of railway lines based on the theory of reliability networks is described. Then section 4 provides the solution approach, including both the method developed to find an approximate solution and a lower bound to estimate the quality of the approximate solution. Finally the proposed modelling and solution methods are tested through a wide scenario analysis on an illustrative example considering a real portion of the UK railway network.

2. Problem description

The problem addressed in this paper is to optimise the selection of long-term maintenance strategies to implement on different segments of a given network. The aim is to obtain the best value from the assets from a network perspective based on a trade-off between maintenance costs, network availability requirements and delivered level of service. Indeed, the infrastructure owner wants to maintain its portfolio of assets to ultimately deliver the required network availability and service levels (e.g. permissible speed) [39], and he is bounded to do so with a limited budget. In railway practice, long-term maintenance strategies are normally developed per asset type (e.g. track, signalling and telecoms, bridges, earthworks, electrical power) [39, 40, 41]. Different strategies result in different long-term costs and conditions profiles, and lifecycle costs analysis tools are used to determine such profiles per strategy and per asset. The assets' conditions in turn have an impact on the proportion of time the section they insist on is closed or subject to a speed restriction. Here it is assumed that a set of maintenance strategies is given, and that the effect of each strategy on the unavailability and speed reduction of a section is known. These values can be obtained from multiple models available in the literature among which are [33, 34, 35, 36, 37, 38, 24], as well as from historical data. Railway services run along railway *lines* for which a minimum threshold of availability must be achieved to ensure the use of the infrastructure as agreed with the train operating companies (TOCs). When a speed restriction is imposed on a track section, then trains are delayed. Strategic planning looks at medium/long-term planning horizons for which no details are available on trains timetable and maintenance execution schedule. It is therefore not possible at this level of planning to obtain more precise measures of train delays. For this reason, here it is assumed that the proportion of time that a section is subject to a speed restriction combined with the train frequency, is representative of the impact of a chosen strategy on service delay per section. The superposition of the effects for all sections is then considered at network level. The logic behind the assignment of strategies to segments of the network is based on the network segmentation approach used in practice for strategic planning purposes. The UK railway network is split into 19 *Strategic Routes*, which in turn are partitioned into *Strategic Route Sections* (SRSs) [42], each including multiple tracks and stations. A SRS is a section of the network characterised by broadly homogeneous traffic and infrastructure type, thus the same maintenance strategy applies to an entire SRS. The *lines* along which railway services are provided, consist of one or more SRSs.

2.1. Modelling the unavailability of railway lines

In this paper railway lines are assimilated to series-parallel systems of the type shown in Figure 1. It

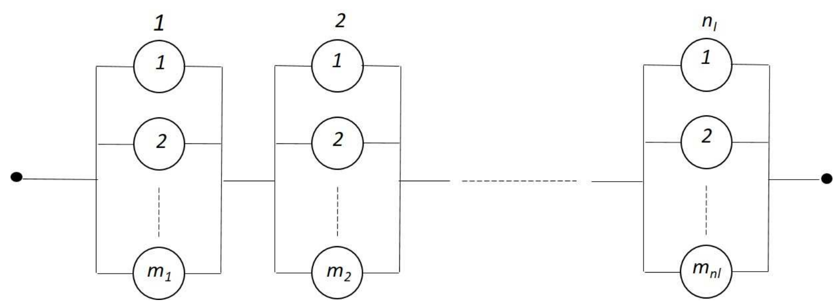


Figure 1: A generic series-parallel system

consists of a set of subsystems $I := \{i \mid i = 1, 2, \ldots, n_I\}$ connected in series. Each subsystem consists of a set of components $P_i := \{j \mid j = 1, 2, \ldots, m_i\}$ connected in parallel. The unavailability of a series-parallel system $Q_{Series-Parallel}$ is

$$Q_{Series-Parallel} = 1 - \prod_{i \in I} (1 - \prod_{j \in P_i} q_j), \tag{1}$$

where q_j is the unavailability of the generic component j. A railway line is modelled as a sequence of portions of SRSs connected in series. Each portion of SRS included in a line is in turn composed by a number of track sections connected as a series-parallel system, where each component is a continuous section of track between two consecutive stations/junctions. An example of such a representation is given in Figure 2, which depicts a railway line including two SRSs, x and y. This representation reflects the topology of a railway line as a sequence of single, double (or more) track sections connecting stations and junctions.

By exploiting the analogy with series-parallel systems, the unavailability of a railway line, Q_l, can be written as

$$Q_l = 1 - \prod_{r \in R} [1 - \delta_{lr} \prod_{i \in I_r} (1 - \prod_{j \in P_i} p_j)] \tag{2}$$

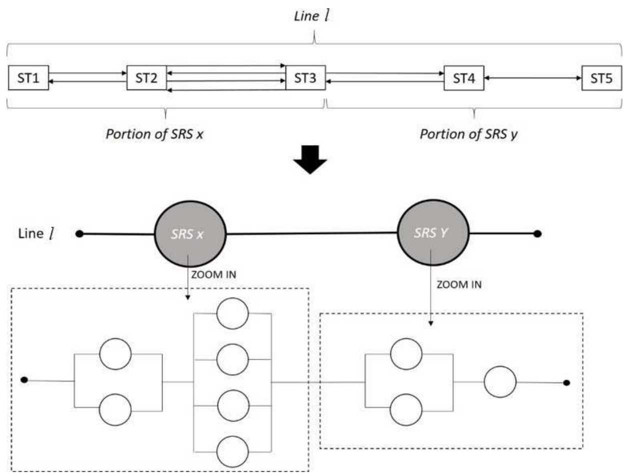


Figure 2: Schematic representation of a railway line as a series of SRSs

where δ_{lr} is a Boolean parameter which takes value 1 if line l includes SRS r, and 0 otherwise. Index $i \in I_r$ indicates the generic section between two consecutive stations/junctions within SRS r. Section i may be single track, or include more than one parallel track. It can be therefore assimilated to a parallel system with components $j \in P_i$, where j is the generic track with unavailability p_j.

2.2. Problem statement

We represent a Strategic Route as a set of SRSs, $R := \{r : r = 1, 2, ..., n_R\}$. We define $L := \{l : l = 1, 2, ..., n_L\}$ as the set of the n_L served railways lines, where $l \subseteq R$, for all $l \in L$. We also define $S := \{s : s = 1, 2, ..., n_S\}$ as the set of the n_S (maintenance) strategies available for each SRS. The same strategy s applies to an entire SRS r. Each element of S corresponds to a different combination of individual asset type sub-strategies. In addition, we use the following notation:

- x is a Boolean vector of decision variables whose components x_{sr} indicate whether strategy s is implemented on SRS r, namely

$$x_{sr} = \begin{cases} 1 & \text{if strategy } s \text{ is selected for SRS } r \\ 0 & \text{otherwise;} \end{cases} \tag{3}$$

- δ_{lr} is a Boolean parameter describing whether railway line $l \in L$ includes SRS $r \in R$, namely

$$\delta_{lr} = \begin{cases} 1 & \text{if } r \in l \\ 0 & \text{otherwise;} \end{cases} \tag{4}$$

- d_{sr} is the proportion of time that SRS $r \in R$ is subject to a speed restriction following implementation of strategy $s \in S$;

- c_{sr} is the cost per unit of time of strategy $s \in S$ implemented on SRS $r \in R$;

- f_r is the nominal frequency of trains travelling on a SRS $r \in R$;
- $Q_l(x)$ is a function $2^{R\times S} \rightarrow [0,1]$ which maps the strategies implemented on the SRSs of a line $l \in L$ into the unavailability of line l. Given that the same maintenance strategy is implemented throughout a SRS, equation 2 can be rewritten as

$$Q_l(x) = 1 - \prod_{r\in R}\{1 - \delta_{lr}\sum_{s\in S} x_{sr}[1 - \prod_{i\in I_r}(1 - \prod_{j\in P_i} p_{js})]\}, \tag{5}$$

where p_{js} is the unavailability of track section j if strategy s is implemented;
- Q_l^* is the maximum tolerable unavailability of line $l \in L$;
- b is the available budget per unit of time.

We are now ready to formally state the problem of interest.

Problem 1 (RMSP - Railway Maintenance Strategies Problem).
Consider a Strategic Route consisting of a set R of Strategic Route Sections (SRSs), a set S of strategies available for each SRS and a budget b. Choose a strategy $s \in S$ for each SRS $r \in R$ so that the total expected number of delayed trains per unit of time is minimized, the total cost of the chosen strategies does not exceed the budget b and their implementation keeps the unavailability of each line l below the acceptable threshold Q_l^*.

3. Model formulation

The problem is formulated as follows:

$$\min z(x) = \sum_{r\in R}\sum_{s\in S} d_{sr} f_r x_{sr} \quad s.t. \tag{6}$$

$$\sum_{s\in S} x_{sr} = 1 \qquad \forall \;\; r \in R, \tag{7}$$

$$\sum_{r\in R}\sum_{s\in S} c_{sr} x_{sr} \leq b, \tag{8}$$

$$Q_l(x) = 1 - \prod_{r\in R}\{1 - \delta_{lr}\sum_{s\in S} x_{sr}[1 - \prod_{i\in I_r}(1 - \prod_{j\in P_i} p_{js})]\} \leq Q_l^* \qquad \forall l \in L, \tag{9}$$

$$x_{sr} \in \{0,1\} \qquad \forall \;\; r \in R, \quad s \in S. \tag{10}$$

The objective function $z(x)$ represents the long-run expected number of trains affected by a speed restriction per unit time. Each term $d_{sr} f_r x_{sr}$ is a measure of the contribution of each SRS to the overall service disruption. The train frequency is used to weight each SRS proportionally to the normalised number of trains travelling on the SRS. The frequency also implicitly weights each SRS based on its centrality, namely its role in serving more than one line. This is because sections serving multiple lines usually carry higher train frequencies. The set of constraints (7) indicates that only one strategy can be selected for each SRS. Constraint (8) adds a bound on the overall costs according to the available budget. The set of constraints (9), one for each line l, puts a threshold on the unavailability of each line. Here, equation 5 is used to espress the unavailability of each line l. Constraints (9) are strongly nonlinear, which makes the problem a nonlinear integer otpimisation problem.

4. Solution approach

To solve the nonlinear problem we propose an *ad hoc* linearisation procedure to find an approximate solution. We then formulate a relaxed counterpart to the nonlinear problem to determine a lower bound to the optimal solution. The upper and lower bounds thus obtained are used to calculate an upper bound to the percentage error, $\varepsilon^+\%$, which is used to measure the level of suboptimality of the approximate solution.

4.1. Linearisation: upper bound solution

For the problem at hand the unavailability of a railway line as given in equation (5) is strongly non-linear. However, two assumptions are made: (i) the occurrences of sections' closure are random variables independently distributed, and (ii) the corresponding probability is small. In such circumstances it is acceptable to approximate the unavailability of a series system with an upper bound given by its *Rare Event Approximation* [43]. This is a first order approximation according to which the unavailability of a series system can be approximated with the sum of the unavailability of the individual components. It follows that the unavailability of a railway line can be simplified as:

$$Q_l(x) \leq \widetilde{Q_l}(x) = \sum_{r\in R} \delta_{lr}\{\sum_{s\in S} x_{sr}[1 - \prod_{i\in I_r}(1 - \prod_{j\in P_i} p_{js})]\}. \tag{11}$$

If the left-hand side of Constraint (9) is replaced with its *Rare Event Approximation* (11), the original nonlinear problem (6) is transformed into a linear integer programming model. This linearised model will be called from now on LM and its formal definition is

$$(LM) \qquad \min_x z_{LM}(x) := \sum_{r\in R}\sum_{s\in S} d_{sr} f_r x_{sr} \quad s.t. \tag{12}$$

$$\sum_{s\in S} x_{sr} = 1 \qquad \forall \ \ r \in R, \tag{13}$$

$$\sum_{r\in R}\sum_{s\in S} c_{sr} x_{sr} \leq b, \tag{14}$$

$$\widetilde{Q_l}(x) \leq Q_l^* \qquad \forall l \in L, \tag{15}$$

$$x_{sr} \in \{0, 1\} \qquad \forall \ \ r \in R; \quad s \in S. \tag{16}$$

The optimal solution x^*_{LM} and optimal value of the objective function z^*_{LM} for problem LM are:

$$x^*_{LM} := \arg\min_x z_{LM}(x)$$

$$z^*_{LM} := \min_x z_{LM}(x).$$

Since the linear approximation $\widetilde{Q_l}(x)$ is an upper bound to the non linear $Q_l(x)$, namely $Q_l(x) \leq \widetilde{Q_l}(x)$, any solution to the linearised problem LM will also be feasible for the original nonlinear problem. Solving LM therefore provides a suboptimal solution for the original nonlinear problem.

4.2. Associated relaxed problem: lower bound solution

Standard relaxation methods include relaxation by elimination of a subset of constraints, Lagrangian and continuous relaxation [44]. However, given some of the properties of the linearised problem AL, it is possible to build a relaxed problem with a much simpler approach than the standard relaxation methods mentioned above. This is explained in the following. The error induced by replacing $Q_l(x)$ with its linear form $\widetilde{Q_l}(x)$ is $\widetilde{Q_l}(x) - Q_l(x) = E_{Q_l}(x)$. The error $E_{Q_l}(x)$ becomes smaller, and therefore the approximation is more accurate, as the terms p_{js} decrease. Given the set of potential strategies S, it is therefore possible to identify a lower bound $E_{Q_l}\mid_{x_{min}}$ and an upper bound $E_{Q_l}\mid_{x_{max}}$ to the error incurred in the approximation of the constraint for each railway line. $E_{Q_l}\mid_{x_{min}}$ is obtained when the strategy providing the lowest value of section unavailability p_{js} is implemented to all the SRSs. It is calculated from Eq. 5 by using the lowest value of p_{js} for all sections j. $E_{Q_l}\mid_{x_{max}}$ is obtained if the strategy with the highest value of p_{js}) is implemented to all SRSs. Hence the error depends on the current value of the decision variables and is bounded as follows

$$E_{Q_l}\mid_{x_{min}} < E_{Q_l}(x) < E_{Q_l}\mid_{x_{max}}, \tag{17}$$

where x_{min} and x_{max} are the values of the decision variable vector if the best and worst strategies are selected respectively for all SRSs.

If Constraints (9) are relaxed by adding $E_{Q_l}\mid_{x_{max}}$ to the right-hand side, the solution to the relaxed problem thus obtained is a lower bound not only to the linearised problem but also to the original non-linear problem. Let us name such relaxed problem RM, formally defined as

$$(RM) \qquad \min_x z_{RAL}(x) := \sum_{r\in R}\sum_{s\in S} d_{sr} f_r x_{sr} \quad s.t. \tag{18}$$

$$\sum_{s\in S} x_{sr} = 1 \qquad \forall \;\; r \in R, \tag{19}$$

$$\sum_{r\in R}\sum_{s\in S} c_{sr} x_{sr} \leq b, \tag{20}$$

$$\widetilde{Q_l}(x) \leq Q_l^* + E_{Q_l}\mid_{x_{max}} \qquad \forall l \in L, \tag{21}$$

$$x_{sr} \in \{0, 1\} \qquad \forall \;\; r \in R, \quad s \in S. \tag{22}$$

The optimal solution x^*_{RM} and optimal value of the objective function z^*_{RM} for problem RM are:

$$x^*_{RM} := \arg\min_x z_{RM}(x)$$

$$z^*_{RM} := \min_x z_{RM}(x).$$

Once a solution to the linearised problem LM and the relaxed problem RM are obtained, the upper bound to the percentage error, $\varepsilon^{+}\%$, can be calculated as

$$\varepsilon^{+}\% = \frac{z^*_{LM} - z^*_{RM}}{z^*_{RM}}\%. \tag{23}$$

$\varepsilon^{+}\%$ is the maximum error one can commit wrt the optimal solution, by taking the approximate solution resulting from the linearized problem.

5. Numerical study: application to the East Midlands Strategic Route

The optimisation approach is applied here to select the best combination of maintenance strategies for a set of SRSs comprising one of the UK Strategic Routes, the East Midlands (EM) Route. Details of the EM route and its SRSs can be found in [42]. A simplified graphical illustration of part of the EM route showing seven SRSs is given in Figure 3.

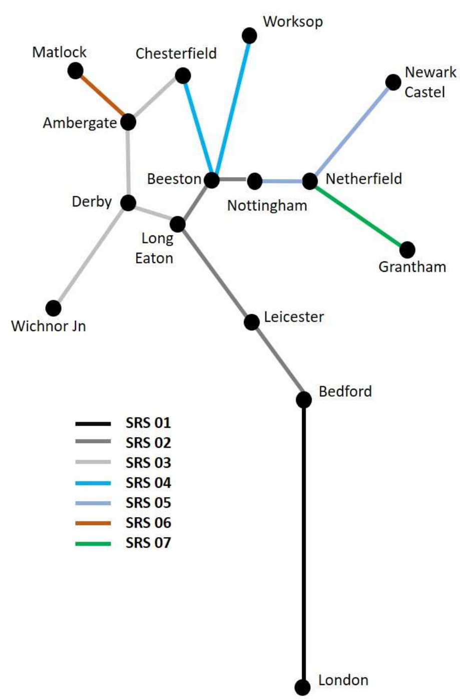


Figure 3: Simplified map of part of the East Midlands Route including seven of its eleven SRSs (11.01 to 11.07) based on [42].

The set of SRSs considered in this example are listed in Table 1 along with the train frequency.

Railway services running along the EM Route which have been considered here are listed in Table 2 along with the service type (Long distance high speed -LDHS, interurban and local) and lists the SRSs included within each service.

The series-parallel representation of each railway line is given in Figure 4. The circles represent sections of track between two consecutive stations. For example, line 1 includes SRS 01 and SRS 02. SRS 01 consists of four parallel tracks, each divided into 14 sections connected in series. Connection

Table 1: SRSs and trains frequency.

SRS	Trains per hour
01 London St. Pancras-Bedford	20
02 Bedford-Nottingham	8
03 Wichnor Jn/Long Eaton-Chesterfield	8
04 Chesterfield-Nottingham	4
05 Nottingham-Newark Castle	1
06 Matlock-Ambergate	1
07 Netherfield-Grantham	2

Table 2: Service types and SRSs included within each railway line.

Service name	Service type	SRSs
London St.Pancras to Nottingham	LDHS	{01, 02}
London St.Pancras to Sheffield(via Derby)	LDHS	{01, 02, 03}
Norwich to Liverpool	Interurban	{02, 04, 07}
Nottingham to Leeds	Interurban	{02, 04}
Newark Castle-Nottingham-Derby-Matlock	local	{02, 03, 05, 06}

between the parallel lines is at the two ends. SRS 02 comprises: a portion of four parallel tracks, each with 9 sections in series, a portion of three parallel tracks and a portion of double track, each with 9 sections in series. Three potential maintenance strategies have been considered, $S = \{s1, s2, s3\}$. Table 3 provides the cost c_{sr} and the probability of a speed restriction d_{sr} when strategy s is implemented to SRS r. The values of unavailability p_{js} corresponding to each strategy are defined per track section j (as explained in subsection 2.1). For this numerical analysis it is assumed that for all track sections j, the unavailability p_{js} takes values 0.01, 0.001 and 0.0001 for strategy $s1$, $s2$ and $s3$ respectively.

Table 3: Model parameters.

SRS	c_{1r}	c_{2r}	c_{3r}	d_{1r}	d_{2r}	d_{3r}
01	70	80	95	0.05	0.005	0.0005
02	70	80	95	0.05	0.005	0.0005
03	50	70	85	0.05	0.005	0.0005
04	45	65	80	0.05	0.005	0.0005
05	40	60	70	0.05	0.005	0.0005
06	40	60	70	0.05	0.005	0.0005
07	40	60	70	0.05	0.005	0.0005

The maximum error $E_{Q_l}|_{X_{max}}$ in the evaluation of the unavailability of each railway line induced by linearisation is given in Table 4 for each railway line.

The error is very small, with the highest value achieved for line l_5. This is due to the fact that in line l_5, each section connected in series consists of a single or a double track (see Figure 4), while the other lines all have two, three and four parallel tracks, thus making each term in the series smaller. For each railway service, different availability requirements can be considered depending on the performance targets set by the infrastructure manager. Similarly, different levels of budget can be investigated. A scenario analysis study has been conducted by varying the value of the available budget b and the

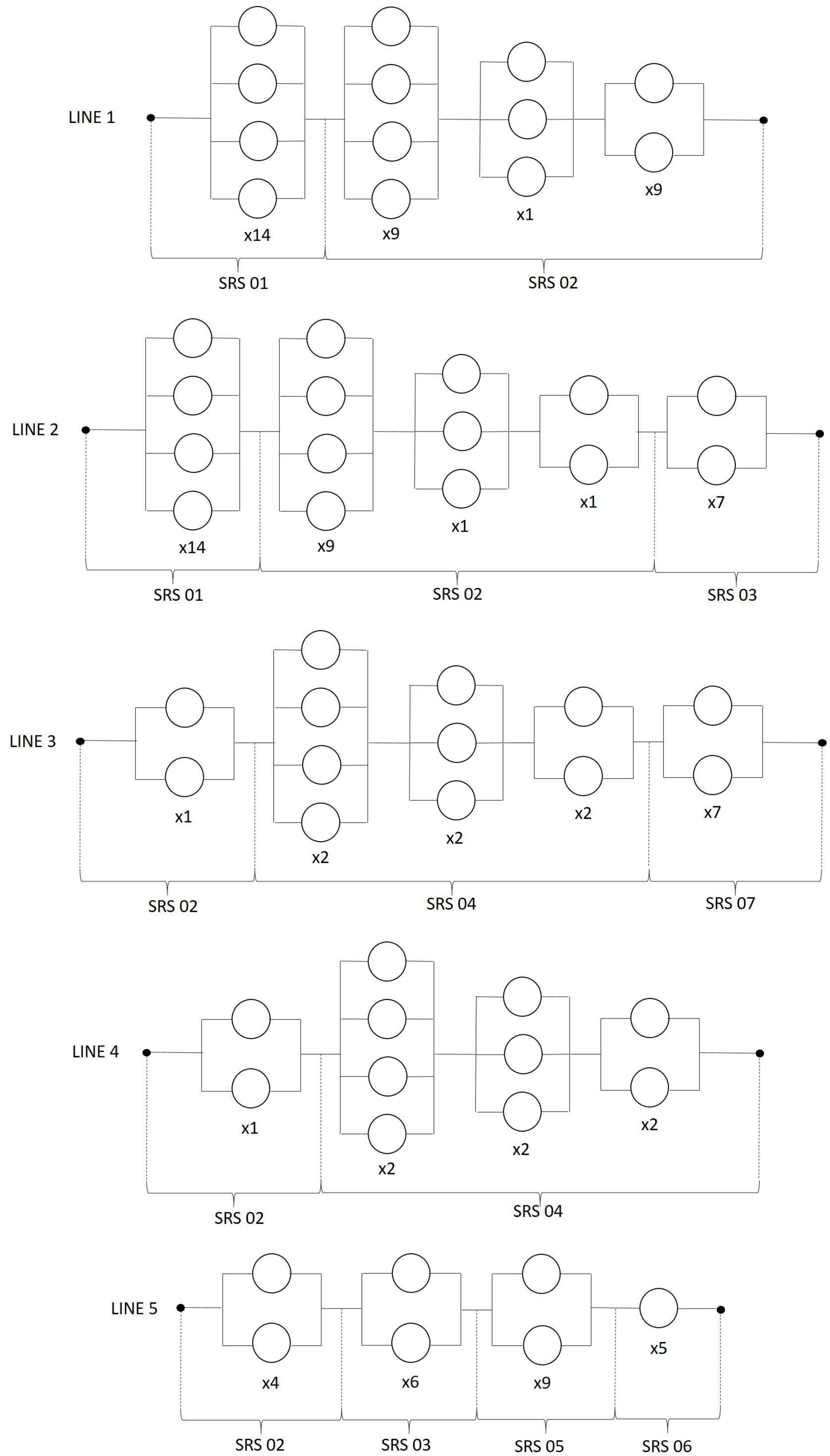


Figure 4: Railway lines as series parallel systems

threshold values on the unavailability of railway lines Q_l^*. For each scenario, problems LM and RM are solved to provide an upper and a lower bound to the global optimum of the original nonlinear problem

Table 4: Maximum error $E_{Q_l}\,|_{x_{max}}$ for each line.

Line	$E_{Q_l}\,\vert_{X_{max}}$
l_1	$E_{Q_{l_1}}\,\vert_{x_{max}} = 1.2610 \times 10^{-10}$
l_2	$E_{Q_{l_2}}\,\vert_{x_{max}} = 7.0854 \times 10^{-8}$
l_3	$E_{Q_{l_3}}\,\vert_{x_{max}} = 2.3153 \times 10^{-7}$
l_4	$E_{Q_{l_4}}\,\vert_{x_{max}} = 2.0201 \times 10^{-8}$
l_5	$E_{Q_{l_5}}\,\vert_{x_{max}} = 9.4174 \times 10^{-5}$

respectively. The estimation of the optimality gap $\varepsilon^{+}\%$ is also calculated by applying equation (23). The optimisation models have been implemented in Matlab R2018b and a standard solver based on the Branch and Bound algorithm has been used. The computational time to solve 90 istances (both LM and RM for 45 scenarios) was 2.768 seconds, run on a Intel Core i5-8350U processor, CPU 1.70GHz 1.90GHz, 8GB RAM, 64-bit Operating System.

5.1. Scenario analysis $SA1$

For this first scenario analysis $SA1$, it has been assumed that different lines have different thresholds on their unavailability, thus reflecting different criticality values. Three sets of scenarios have been analised, for each set the thresholds on the availability of the railway lines are given in Table 5. For each

Table 5: Unavailability thresholds for each line

Set	Q_1^*	Q_2^*	Q_3^*	Q_4^*	Q_5^*
$SA1_1$	0.001	0.008	0.01	0.05	0.01
$SA1_2$	0.0001	0.0008	0.001	0.005	0.008
$SA1_3$	0.00001	0.00008	0.0001	0.0005	0.0008

set, the following values of the available budget b have been tested:

$$b = \{350, 375, 400, 425, 450, 475, 500, 525, 550, 575, 600, 625, 650, 675, 700\}.$$

The number of scenarios analysed in $SA1_1$ is therefore $3 \times 15 = 45$. Tables 6, 7, 8 show the results obtained by solving problem LM for sets of scenarios $SA1_1$, $SA1_2$ and $SA1_3$ respectively. The tables detail the strategy selected for each SRS and the corresponding value of the objective function z^*_{LM}. The otimal value z^*_{RM} obtained by solving the corresponding relaxed problem RM and the upper bound to the percentage error, $\epsilon_+\%$ calculated using Equation (23) are also given.

No feasible solution to problem LM can be found for budgets $b = 350$ and $b = 375$ as the strategies that are affordable do not ensure the required level of availability for each railway line. The increased budget $b = 400$ is enough to find a feasible solution. Such solution consists of strategy $s1$ selected for all

Table 6: Results for set $SA1_1$.

Set $SA1_1$: $Q_1^* = 0.001$, $Q_2^* = 0.008$, $Q_3^* = 0.01$, $Q_4^* = 0.05$, $Q_5^* = 0.01$														
	Budget b													
SRS	350	375	400	425	450	475	500	525	550	575	600	625	650	675	700
01	-	-	s2	s2	s3	s3	s3	s3	s3	s3	s3	s3	s3	s3	s3
02	-	-	s1	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3	s3	s3
03	-	-	s1	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3	s3	s3
04	-	-	s1	s1	s2	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3
05	-	-	s1	s1	s1	s1	s1	s2	s2	s3	s3	s3	s3	s3	s3
06	-	-	s2	s2	s2	s2	s2	s2	s2	s3	s3	s3	s3	s3	s3
07	-	-	s1	s1	s1	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3
z^*_{LM}	-	-	1.255	0.535	0.265	0.175	0.103	0.058	0.031	0.022	0.022	0.022	0.022	0.022	0.022
z^*_{RM}	-	-	1.255	0.535	0.265	0.175	0.103	0.058	0.031	0.022	0.022	0.022	0.022	0.022	0.022
$\epsilon^+\%$	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 7: Results for set $SA1_2$.

Set $SA1_2$: $Q_1^* = 0.0001$, $Q_2^* = 0.0008$, $Q_3^* = 0.001$, $Q_4^* = 0.005$, $Q_5^* = 0.001$														
	Budget b													
SRS	350	375	400	425	450	475	500	525	550	575	600	625	650	675	700
01	-	-	s1	s2	s3	s3	s3	s3	s3	s3	s3	s3	s3	s3	s3
02	-	-	s2	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3	s3	s3
03	-	-	s1	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3	s3	s3
04	-	-	s1	s2	s2	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3
05	-	-	s1	s1	s1	s1	s1	s2	s2	s3	s3	s3	s3	s3	s3
06	-	-	s2	s2	s2	s2	s2	s2	s2	s3	s3	s3	s3	s3	s3
07	-	-	s1	s1	s1	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3
z^*_{LM}	-	-	1.795	0.535	0.265	0.175	0.103	0.058	0.031	0.022	0.022	0.022	0.022	0.022	0.022
z^*_{RM}	-	-	1.795	0.535	0.265	0.175	0.103	0.058	0.031	0.022	0.022	0.022	0.022	0.022	0.022
$\epsilon^+\%$	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0

SRSs except SRS 06 and 01 for which a better strategy $s2$ is selected. SRS 01 belongs to the line with the most restrictive availability requirement, which justify the selection of a better strategy. Even though SRS 06 belongs to the line with the less restrictive availability requirement, it is given priority over other sections. The explanation is that, since SRS 06 is single track, a less performing strategy than $s2$ would not enable to meet the availability threshold. This solution is also the cheapest combination of strategy that enables to achieve the required levels of availability for all lines. For any further increment of the available budget, better strategies are selected first for the SRSs with higher train frequency. These are in fact the SRSs with a higher impact on the objective function.

For set of scenarios $SA1_2$, the availability thresholds are decreased for all lines. Results are given in table 7. As before, no feasible solution exist until budget is increased to $b = 400$, when the optimal

Table 8: Results for set $SA1_3$.

Set $SA1_3$: $Q_1^* = 0.00001$, $Q_2^* = 0.00008$, $Q_3^* = 0.0001$, $Q_4^* = 0.0005$, $Q_5^* = 0.0001$															
	Budget b														
SRS	350	375	400	425	450	475	500	525	550	575	600	625	650	675	700
01	-	-	-	-	-	s1	s3	s3	s3	s3	s3	s3	s3	s3	s3
02	-	-	-	-	-	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3
03	-	-	-	-	-	s2	s2	s3	s3	s3	s3	s3	s3	s3	s3
04	-	-	-	-	-	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3
05	-	-	-	-	-	s2	s2	s2	s2	s3	s3	s3	s3	s3	s3
06	-	-	-	-	-	s3	s3	s3	s3	s3	s3	s3	s3	s3	s3
07	-	-	-	-	-	s2	s2	s3	s2	s3	s3	s3	s3	s3	s3
z_{LM}^*	-	-	-	-	-	1.1155	1.1225	0.0805	0.0355	0.022	0.022	0.022	0.022	0.022	0.022
z_{RM}^*	-	-	-	-	-	1.1155	1.1225	0.0805	0.0355	0.022	0.022	0.022	0.022	0.022	0.022
$\epsilon^+\%$	-	-	-	-	-	0	0	0	0	0	0	0	0	0	0

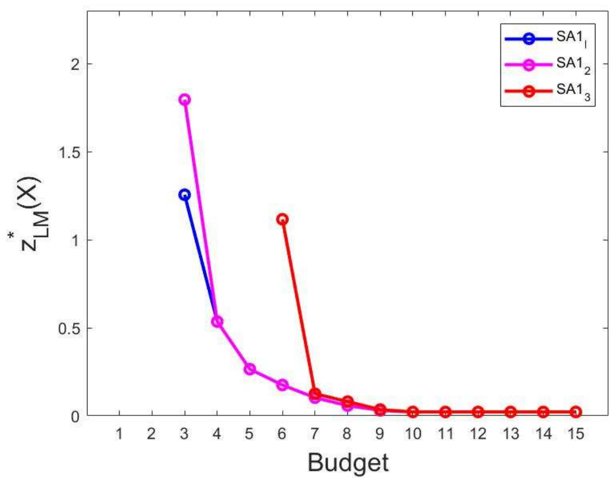


Figure 5: Expected number of delayed trains due to speed restrictions per unit time for scenario analysis $SA1$.

solution consists of the cheapest strategy ($s1$) for all SRSs except SRSs 02 and 06. While SRS 01 only belongs to line l_1, SRS 02 belongs to all lines which now have to meet more restrictive availability targets with respect to scenarios $SA1_2$. For further increases of the budget, better strategies are selected first for the SRSs with higher train frequency.

Finally, results for scenarios $SA1_3$ are given in table 8. None of the strategies affordable with a budget up to $b = 450$ constitute a feasible solutions. Strategy ($s3$) is needed for single tracked SRS 06, and at least strategy $s2$ for SRSs 02, 03 and 05 to meet the unavailability threshold on line l_5. The approximate solutions obtained by solving problem LM and the lower bound given by relaxed problem RM coincide for all scenarios. This means that the linearised problem LM yields the global optimum for the considered scenarios. This can be explained by the fact that the unavailability of all lines is of a bigger order than the corresponding error $E_{Q_{l_i}}|_{max}$ (see table 4).

The analysis of the three sets of scenarios has helped identify the factors affecting the solution, and their importance. Such factors are: (i) the level of redundancy of the SRSs, (ii) the centrality of the SRSs, namely if a SRSs belongs to one or more lines, (iii) the required level of availability, and (iv) the train train frequency. The level of redundancy and the centrality of the SRSs are indicative of the influence of the network topology. The first three factors are the features to look at first as they determine whether the affordable solutions are actually feasible for the availability targets. The more restrictive these availability targets are, the more important are the level of redundancy and centrality of each SRS. The fourth factor affecting the solution is the train frequency. As long as both budget and availability constraints are satisfied, the selection of the strategies is only based on the effect on the objective function. Better strategies are applied first to SRSs with higher train frequency. If two or more SRSs have the same train frequency, then the order in which these SRSs are attributed the best strategy does not matter as they would result in equivalent optimal solutions.

5.2. Scenario analysis $SA2$

In scenario analysis $SA1$ different lines had different availability thresholds. It is therefore difficult to evaluate the order in which the first three important factors (redundancy, centrality and availability targets) influence the solution, and the impact of the features indicative of the network topology regardless of different requirements set for each line. A second scenario analysis has been carried out here, $SA2$, where all lines have the same availability target, so that no line has priority over the others. Values of lines unavailability considered are $Q^* = \{0.08, 0.008, 0.0008\}$, while 15 different budget values are considered as in scenario analysis $SA1$. Three sets of scenarios result from the combinations of the above values, one set for each value of threshold Q^*. Each set contains 15 scenarios, one for each budget value from $b = 350$ to $b = 700$. Tables 9, 10, 11 show the results obtained by solving problem LM for values of Q^* equal to 0.08, 0.008 and 0.008 respectively. The tables detail the strategy selected for each SRS, the corresponding value of the objective function z^*_{LM}. The optimal value z^*_{RM} and the upper bound to percentage error, $\epsilon_+\%$ are also given.

For budget value $b = 350$ no feasible solution to problem LM exists given the available strategies, regardless of threshold Q^*. As it can be observed from Table 9, when the budget is increased to $b = 375$ an optimal solution is found for $Q_* = 0.08$ only, and it consists of strategy $s1$ for all SRSs; $Q^* = 0.08$ is permissive enough that the cheapest strategy is sufficient for all lines to meet the availability target. For any further increment of the budget, the optimal solutions are selected based on their impact on the objective function only and better strategies are selected first for the SRSs with higher train frequency.

When the threshold Q^* is decreased to 0.008, strategy $s1$ selected for all SRSs no longer provides a feasible solution. Budget $b = 375$ is not enough to select a better strategy, hence no feasible solution exists that meets both budget and line availability constraints. If the budget is increased to 400, strategy $s2$ can be selected for $SRS06$, which combined with strategy $s1$ for all other SRSs provides a feasible

Table 9: Results for set $SA2_1$.

Set $SA2_1$: $Q_1^* = Q_2^* = Q_3^* = Q_4^* = Q_5^* = 0.08$															
	Budget b														
SRS	350	375	400	425	450	475	500	525	550	575	600	625	650	675	700
01	-	s1	s2	s2	s3	s3	s3	s3	s3	s3	s3	s3	s3	s3	s3
02	-	s1	s2	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3	s3	s3
03	-	s1	s2	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3	s3	s3
04	-	s1	s1	s2	s2	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3
05	-	s1	s1	s1	s1	s2	s1	s2	s2	s3	s3	s3	s3	s3	s3
06	-	s1	s1	s1	s1	s1	s2	s2	s2	s3	s3	s3	s3	s3	s3
07	-	s1	s1	s1	s2	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3
z^*_{LM}	-	2.2	0.58	0.4	0.22	0.175	0.103	0.058	0.031	0.022	0.022	0.022	0.022	0.022	0.022
z^*_{RM}	-	2.2	0.58	0.4	0.22	0.175	0.103	0.058	0.031	0.022	0.022	0.022	0.022	0.022	0.022
$\epsilon^+\%$	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 10: Results for set $SA2_2$.

Set $SA2_2$: $Q_1^* = Q_2^* = Q_3^* = Q_4^* = Q_5^* = 0.008$															
	Budget b														
SRS	350	375	400	425	450	475	500	525	550	575	600	625	650	675	700
01	-	-	s2	s2	s3	s3	s3	s3	s3	s3	s3	s3	s3	s3	s3
02	-	-	s1	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3	s3	s3
03	-	-	s1	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3	s3	s3
04	-	-	s1	s1	s2	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3
05	-	-	s1	s1	s1	s1	s1	s2	s2	s3	s3	s3	s3	s3	s3
06	-	-	s2	s2	s2	s2	s2	s2	s2	s3	s3	s3	s3	s3	s3
07	-	-	s1	s1	s1	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3
z^*_{LM}	-	-	1.255	0.535	0.265	0.175	0.103	0.058	0.031	0.022	0.022	0.022	0.022	0.022	0.022
z^*_{RM}	-	-	1.255	0.535	0.265	0.175	0.103	0.058	0.031	0.022	0.022	0.022	0.022	0.022	0.022
$\epsilon^+\%$	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0

solution. However, the budget is enough to select $s2$ also for $SRS01$, thus producing a solution which is not only feasible but also optimal. For further increases of the budget, better strategies are selected first for SRSs with higher train frequencies. It is worth noticing that for $SRS06$ strategy $s1$ is never selected. Furthermore, if solutions obtained for $b = 475, Q^* = 0.08$ in table 9 and $b = 475, Q^* = 0.008$ in table 10 are compared, one can see that the only difference is that when the threshold is made more restrictive, strategy $s2$ is selected for $SRS06$ rather than $SRS05$. One could deduct that line l_5 can only reach the availability target if at least startegy $s2$ is selected for $SRS06$ as the latter is single track.

Table 11 shows that no feasible solutions exist for budget $b = 350$ and $b = 450$ when the threshold is $Q^* = 0.0008$ as at least strategy $s3$ for the single tracked SRS 06, and $s2$ for most of the others SRSs are needed to meet the availability targets. A first feasible solution is found for $b = 475$, which

Table 11: Results for set $SA2_3$.

Set $SA2_3$: $Q_1^* = Q_2^* = Q_3^* = Q_4^* = Q_5^* = 0.0008$															
	Budget b														
SRS	350	375	400	425	450	475	500	525	550	575	600	625	650	675	700
01	-	-	-	-	-	s2	s3	s3	s3	s3	s3	s3	s3	s3	s3
02	-	-	-	-	-	s2	s2	s3	s3	s3	s3	s3	s3	s3	s3
03	-	-	-	-	-	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3
04	-	-	-	-	-	s2	s2	s2	s3	s3	s3	s3	s3	s3	s3
05	-	-	-	-	-	s3	s2	s2	s2	s3	s3	s3	s3	s3	s3
06	-	-	-	-	-	s3	s3	s3	s3	s3	s3	s3	s3	s3	s3
07	-	-	-	-	-	s1	s2	s3	s2	s3	s3	s3	s3	s3	s3
z^*_{LM}	-	-	-	-	-	0.301	0.1255	0.0805	0.0355	0.022	0.022	0.022	0.022	0.022	0.022
z^*_{RM}	-	-	-	-	-	0.301	0.1255	0.0805	0.0355	0.022	0.022	0.022	0.022	0.022	0.022
$\epsilon^+\%$	-	-	0	0	0	0	0	0	0	0	0	0	0	0	0

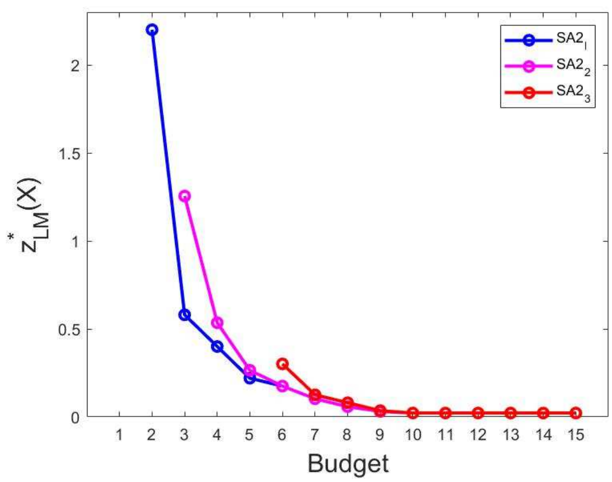


Figure 6: Expected number of delayed trains due to speed restrictions per unit time for scenario analysis $SA2$.

enables strategy $s3$ to be selected for SRS06 and double tracked SRS 05, and strategy $s2$ for SRSs 02 and 03 so that line 5 achieves the required availability target. Strategy $s2$ is also selected for the other SRSs, except 07, to satisfy the availability requirements. Higher budgets lead to better strategies to be chosen for higher frequency sections first so to yield better values of the objective function. The approximate solutions obtained by solving problem LM and the lower bound given by relaxed problem RM coincide for all scenarios. This means that the linearised problem LM yields the global optimum for the considered scenarios. This can be explained by the fact that the unavailability of all lines is of a bigger order than the corresponding error $E_{Q_{l_i}}|_{max}$ (see table 4). In Figure 6 the optimal values of the objective function obtained in scenario analysis $SA2$ are compared.

From Figure 6 it is possible to deduce the following:

- for fixed Q^* the optimal number of trains affected by a speed restriction decreases for increasing values of the available budget;

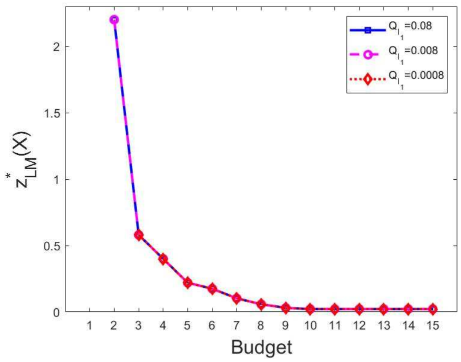
- for fixed budget, more restrictive availability thresholds lead to higher numbers of trains affected by a speed restriction.

This last result might seem contradictory but it is in fact not. An explanation to this behaviour can be found by looking at the network topology. The SRSs with higher train frequency are also the ones with higher levels of redundancy as they all contain two, three and four parallel tracks. When the availability target is made more restrictive (lower values of Q^*), better strategies are needed first for those SRSs with no redundancy and higher centrality so to satisfy the availability constraints. When a feasible solution is still to be found, the impact on the objective function which depends on the train frequency, is of secondary importance. Once a feasible solution satisfying both budget and availability constraints is found, then the remaining budget is used to improve the strategy selected for SRSs with higher train frequency.

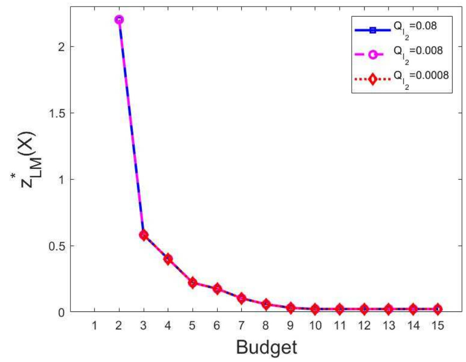
5.3. Sensitivity analysis

In this section the sensitivity of the optimal solution with respect to the unavailability threshold Q_l^* for each line is analysed. Parameter Q_l^* is decreased from 0.08 to 0.0008 one line at a time, while the threshold for the remaining lines is kept at 0.08. Results are shown in Figure 7. The purpose of this analysis is to identify the most stringent constraint(s), namely the lines which are most sensitive to an increase in the availability requirement. These will be the lines for which an increase in the availability target will determine a change in the optimial solution. Figures 7a, 7b, 7c, 7d and 7e show the optimal solution z^*_{LM} plotted against the 15 values of budget from $b = 350$ to $b = 700$, and obtained by varying the unavailability threshold of line l_1, l_2, l_3, l_4 and l_5 respectively. From figures 7a, 7b and 7d it can be seen that no change in solution is registered when decreasing the unavailability threshold for lines l_1, l_2 and l_4. Figure 7c shows how the optimal solution changes only when the threshold on line l_3 is decreased from 0.008 to 0.0008 and for budget lower than $b = 425$. Finally, figure 7e shows a change in the optimal solution when decreasing the unavailability threhsold for line l_5 from 0.08 to 0.008, and again from 0.008 to 0.0008. Results of this sensitivity analysis show that when varying the budget and the threshold values Q_l^* the constraint on the availability of line l_5 is always stringent. This is because SRS 06 is single track, thus constituting a bottleneck for the network. Indeed, if any section of track within SRS 06 fails, than the entire line will be unavailable. The maximum level of availability that can be ensured for each line is of the same order as the minimum availability of the SRS with lowest redundancy. The insight obtained through this sensitivity analysis can be useful to the infrastructure manager when decisions on a redistribution of maintenance resources accross different SRSs has to be made.

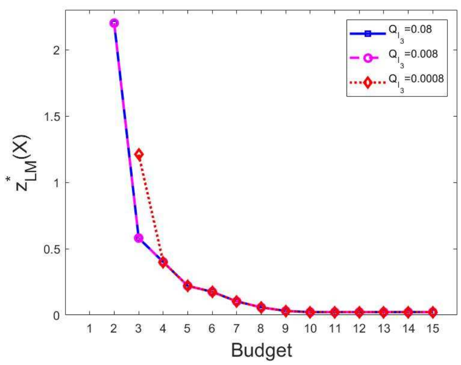
The scenario and sensitivity analysis have shown how the level of redundancy and the centrality of the sections determine whether the affordable solutions are actually feasible for the availability targets. The more restrictive these availability targets are, the bigger the influence of redundancy and centrality of each section. Then, as long as both budget and availability constraints are satisfied, better strategies



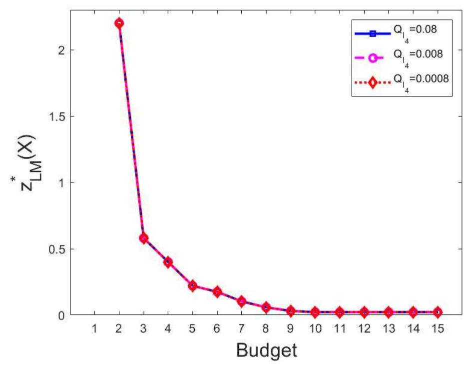
(a) Optimal solutions z^*_{LM} when only Q_{l_1} is varied between 0.08 and 0.008



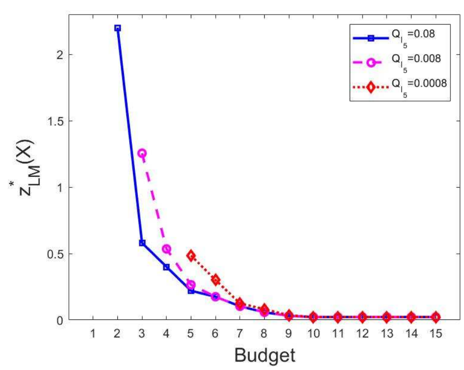
(b) Optimal solutions z^*_{LM} when only Q_{l_2} is varied between 0.08 and 0.008



(c) Optimal solutions z^*_{LM} when only Q_{l_3} is varied between 0.08 and 0.008



(d) Optimal solutions z^*_{LM} when only Q_{l_4} is varied between 0.08 and 0.008



(e) Optimal solutions z^*_{LM} when only Q_{l_5} is varied between 0.08 and 0.008

Figure 7: Optimal solutions z^*_{LM} when unavailability threshold is varied one line at a time, and for different budget values.

are applied first to sections with higher train frequency. The maximum level of availability that can be ensured in each line is of the same order of the maximum availability of the section with lower redundancy. Single track sections constitute a bottleneck for the network because any failure here would result in the entire line shutting down.

6. Conclusions

This paper presents a nonlinear integer-programming model and a tailored solution approach to support the selection of maintenance strategies for a railway network. The logic behind the assignment of strategies to track sections makes use of the network segmentation and sections aggregation approach used in practice for strategic planning purposes. Strategies are selected which collectively minimise the impact of section conditions on service, given network availability and budget constraints. Different metrics related to the network topology, sections' availability, service frequency, performance requirements and maintenance costs, are combined into a quantitative approach with a holistic view and a clear and simple structure rather than a black box. The approach is simple yet effective to model large networks and makes use of standard solvers, thus facilitating its implementation in practice. Based on the *Rare Event Approximation* principle, both an *ad hoc* heuristic solution and relaxation methods are developed. The latter enables the quality of the heuristic solution to be estimated by calculation of the upper bound to the percentage error. With this information the decision maker can decide whether the current approximate solution is acceptable or a better solution needs to be searched for. Furthermore it enables to quantify the performance of the proposed heuristic. To show the potential for implementation to real world scenarios, the model and its relaxed counterpart have been implemented on an illustrative example considering a portion of the UK railway network, and solved for a number of scenarios where the available budget and the thresholds on lines availability are varied. This scenario analysis gives insight on the robustness of the solution with respect to the model parameters, as well as on the relative and combined contribution of different factors on the selection of strategies, thus enabling more informed decisions. These factors include track redundancy, centrality of sections, service frequency, budget and lines availability targets.

The infrastructure manager can leverage on this model to explore how a portfolio of strategies can affect lines availability and service, and how performance requirements can best be achieved by an optimal distribution of the maintenance budget accross different railway sections. Due to its simple structure, the model can be easily adapted to address similar problems arising in the maintenance of other types of networks, such as road and bridges networks, where long-term maintenance strategies must be choosen for individual or groups of road segments or bridges respectively, when deciding on the strategic allocation of maintenance efforts.

Acknowledgments

John Andrews is the Network Rail Professor of Infrastructure Asset Management and Director of Lloyds Register Foundation (Lloyds Register Foundation supports the advancement of engineering-related education, and funds research and development that enhances safety of life at sea, on land and in the air) Resilience Engineering Research Group at the University of Nottingham. The authors gratefully acknowledge the support of these organizations.

References

[1] S. Petchrompo, A. Parlikad, A review of asset management literature on multi-asset systems, Reliability Engineering and System Safety 181 (2019) 181–201.

[2] B. de Jonge, P. Scarf, A review on maintenance optimisation, European Journal of Operational Research 3 (285) (2020) 805–824.

[3] M. O. Keizer, S. Flapper, R. Teunter, Condition-based maintenance policies for systems with multiple dependent components: a review, European Journal of Operations Research 261 (2017) 405–420.

[4] R. Nicolai, R. Dekker, Optimal maintenance of multi-component systems: a review. Complex System Maintenance Handbook. Springer Series in Reliability Engineering, Springer, London, 2008.

[5] H. Yeo, Y. Yoon, S. Madanat, Algorithms for bottom up maintenance optimization for heterogeneous infrastructure systems, Structure and Infrastructure Engineering 1 (2012) 1–12.

[6] A. Furuya, S. Madanat, Accounting for network for effects on railway asset management, Journal of Transportation Engineering 139 (2013) 92–100.

[7] C. A. Robelin, S. M. Madanat, Reliability-based system-level optimization of bridge maintenance and replacement decisions, Tansportation Science 42 (4) (2008) 1–6.

[8] C. Robelin, S. Madanat, History-dependent bridge deck maintenance and replacement optimization with markov decision process, Journal of Infrastructure Systems 3 (13) (2007) 195–201.

[9] K. Verbert, B. D. Schutter, R. Babuška, Timely condition-based maintenance planning for multi-component systems, Reliability Engineering and System Safety 159 (2017) 310–321.

[10] Y. Shi, Y. Xiang, H. Xiao, L. Xing, Joint optimization of budget allocation and maintenance planning of multi-facility transportation infrastructure systems, European Journal of Operational Research 288 (2021) 382–393.

[11] N. Lethanh, B. Adey, D. Fernando, Optimal intervention strategies for multiple objects affetcted by manifest and latent deterioration, Structure and Infrastructure Engineering 11 (3) (2015) 389–401.

[12] M. Burkhalter, C. Martani, B. Adey, Determination of risk-reducing intervention programs for railwy lines and the significance of simplification, Journal of Infrastructure Systems 24 (2018) 1:04017038.

[13] M. Burkhalter, B. Adey, A network flow model approach to determining optimal intervention programs for railway infrastructure networks, Infratsructures 3 (3) (2018) 31.

[14] M. Burkhalter, B. Adey, Modelling the complex relationship between interventions, intervention costs and the service provided when evaluating intervention programs on railway infrastructure networks, Infratsructures 5 (12) (2020) 113.

[15] G. Budai, D. Huisman, R. Dekker, Scheduling preventive railway maintenance activities, Journal of the Operational Research Society 57 (9) (2006) 1035–1044.

[16] A. V. Horenbeek, L. Pintelon, A dynamic predictive maintenance policy for multi-component systems, Reliability Engineering and System Safety 120 (2013) 39–50.

[17] P. D. Van, A. Barros, C. Bérenguer, K. Bouvard, F. Brissaoud, Dynamic grouping maintenance with time limited opportunities, Reliability Engineering and System Safety 120 (2013) 51–59.

[18] K. Nguyen, P. D. Van, A. Grall, Multi-level predictive maintenance of multi-component systems, Reliability Engineering and System Safety 144 (2015) 83–94.

[19] P. D. Van, H. C. Vu, A. Barros, C. Bérenguer, Maintenance grouping for multi-component systems with availability constraints and limited maintenance teams, Reliability Engineering and System Safety 142 (2015) 56–67.

[20] L. Caetano, P. Teixeira, Availability approach to optimizing railway track renewal operations, Journal of Transportation Engineering 139 (2013) 941948.

[21] S. Chen, T. Ho, B. Mao, Maintenance schedule optimisation for a railway power supply system, International Journal of Production Research 51 (2013) 48964910.

[22] S. Chen, T. Ho, B. Mao, Y. Bai, A bi-objective maintenance scheduling for power feeding substations in electrified railways, Transportation Research Part C: Emerging Technologies 44 (2014) 350362.

[23] A. R. Andrade, P. F. Teixeira, Biobjective optimisation model for maintenance and renewal decisions related to rail track geometry, Transportation Research Record: Journal of Transportation Research Board (2261) (2011) 163–170.

[24] L. Podofillini, E. Zio, J. Vatn, Risk-informed optimisation of railway tracks inspection and maintenance procedures., Reliability Enginering and System Safety 91 (2006) 20–35.

[25] M. Marseguerra, E. Zio, L. Podofillini, Condition-based maintenance optimization by means of genetic algorithms and monte carlo simulation, Reliability Engineering and System Safety 77 (2002) 151–166.

[26] J. Zhao, A. H. C. Chao, A. Stirling, K. Madelin, Optimising policies of railway ballast tamping and renewal, Transportation Research Record 1943 (2006) 50–56.

[27] Q. Zhu, H. Peng, B. Timmermans, G. van Houtum, A condition-based maintenance model for a single component in a system with scheduled and unscheduled downs, International Journal of Production Economics 193 (2017) 365–380.

[28] J. Liesia, A. Salo, J. M. Keisler, A. Morton, Portfolio decision analysis: Recent developments and future prospects, European Journal of Operational Research 293 (2021) 811–825.

[29] T. Sacco, M. Compare, E. Zio, G. Sansavini, Portfolio decision analysis for risk-based maintenance of gas networks, Journal of Loss Prevention in the Process Industries 60 (2019) 269–281.

[30] M. Compare, L. Bellani, E. Zio, Optimal allocation of prognostics and health management capabilities to improve the reliability of a power transmission network, Reliability Engineering and System Safety 184 (2019) 164–180.

[31] C. Fecarotti, J. Andrews, A mathematical programming approach to railway network asset management, in: Proceedings of ESREL Conference: Safety and Reliability - Safe Societies in a Changing World - Haugen et al. (Eds), 2018, Trondheim, Norway.

[32] R. David, H. Alla, Discrete, continuous and hybrid Petri nets, Springer Science & Business Media, 2010.

[33] J. Andrews, D. Prescott, F. De Rozieres, A stochastic model for railway track asset management, Reliability Enginering and System Safety 130 (2014) 76–84.

[34] B. Le, J. Andrews, C. Fecarotti, A petri net model for railway bridge maintenance, Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability 3 (231) (2017) 306–323.

[35] C. Fecarotti, J. Andrews, Producing an effective maintenance strategy to control railway risk, in: Proceedings of Probabilistic Safety Assessment and Management, UCLA Los Angeles, 16-21 September 2018.

[36] C. Fecarotti, J. Andrews, A petri net approach to assess the effects of railway maintenance on track availability, Infrastructure Asset Management 3 (7) (2020) 201–220 ISSN 2053–0242 — E–ISSN 2053–0250.

[37] L. Bai, R. Liu, Q. Sun, F. Wang, P. Xu, Markov-based model for the prediction of railway track irregularities., Proceeding of the Institution of Mechanical Engineers, Part F: J Rail Rapid Transit 229 (2) (2013) 150–159.

[38] D. Prescott, J. Andrews, Investigating railway track asset management using a markov analysis, Proceeding of the Institution of Mechanical Engineers, Part F: J Rail Rapid Transit 229 (4) (2015) 402–416.

[39] A. K. M. Skinner, J. Williams, Challenges of developing whole life cycle cost models for network rail top 30 assets, in: In: IET and IAM asset management conference, 30 November - 1 December 2011, London, UK.

[40] Track Asset Policy, Tech. rep., Network Rail (2012).

[41] Electrical power asset policy. internal network rail documentation, Tech. rep., Network Rail (2012).

[42] Route Specifications: London North Eastern and East Midlands, Tech. rep., Network Rail (2018).

[43] J. D. Andrews, T. R. Moss, Reliability and Risk Assessment, Professional Engineering Publishing, 2002.

[44] F. S. Hillier, G. J. Lieberman, Introduction to Operations Research, 9th Edition, McGrow-Hill Higher Education, 2009.