The Scandal of Deduction and Aristotle's Method for Discovering Syllogisms

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1. Introduction

(1) If a deductive argument is valid, then the conclusion is not novel. (2) If the conclusion of an argument is not novel, the argument is not useful. So, (3) if a deductive argument is valid, it is not useful. The conclusion is unacceptable. Since the argument is valid, we must reject at least one premise. So, should we reject (1) or (2)? This puzzle is usually known as the 'scandal of deduction'.²

Analytic philosophers have tried to reject (1) but have assumed premise (2). I argue here that Aristotle would deny (2). Aristotle thinks that at least some deductive arguments are useful, even though they present no new conclusions. Thus, Aristotle's view contrasts with analytic philosophers of logic, who assume that all useful deductive arguments present novel conclusions. I don't claim that Aristotle 'solves' the problem: it was never posed in Aristotle's time. Rather, I suggest that Aristotle does not face the problem because he assumes deductions can be useful, without presenting novel conclusions. Aristotle's view of deduction tames the scandal.

¹ Since antiquity, some authors have challenged the epistemic value of deductive reasoning (e.g. Sextus Empiricus, *Outlines of Pyrrhonism* II.195-197). Similar issues re-surfaced in the 19th century in Mill (*System of Logic*, II 3.2). For discussion of Mill see Walton (1977), Woods and Walton (1982), pp. 92-97 and Woods (1999), pp. 318-321. There are some similarities between the paradox I outline here and earlier challenges, but they are beyond the scope of this paper.

² Hintikka (1970), p. 135, coins the expression.

Section 2 of this paper outlines scandal of deduction's tacit commitment to (2).

Section 3 argues that Aristotle assumes (2) to be false. I show that Aristotle thinks at least some syllogisms do not present a novel conclusion, but are nonetheless useful. My argument has two premises. First, Aristotle's method for discovering syllogisms (*Prior Analytics* I. 27-31) finds at least one syllogism for a given conclusion. Because the method works 'backwards' from conclusion to premises, the conclusion is not novel. Second, Aristotle considers the syllogisms generated by the method useful in a range of contexts. Section 4 draws some conclusions.

2. The Scandal of Deduction

I will discuss two ways analytic philosophers have approached the scandal. The first is Hempel's 'psychological novelty' approach. The second is Hintikka's 'objective novelty' approach.³ Both reject (1) by trying to show how a valid argument can conclude something new. Both deny (1), but tacitly assume (2).

But first, why might we think that premise (1) holds? Hempel puts it thus:

To illustrate the point by a very elementary example, from the premise 'This figure is a right angled triangle', we can deduce the conclusion, 'this figure is a triangle'; but this conclusion clearly reiterates part of the information already contained within the premise.⁴

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³ More recent philosophers have developed approaches that don't fit neatly into either camp (e.g. Sequoiah-Grayson (2008); Primiero (2008), chap. 2; Sillari (2008); D'Agostino and Floridi (2009); Duží (2010), and Jago (2013).

⁴ Hempel (1945), cited in D'Agostino and Floridi (2009), p. 7. Cf. Dummett (1993), p. 195.

Hempel's idea is that the information contained within the conclusion is already present in the premises; but for an argument to have a novel conclusion, the conclusion must contain more information than the premises. 'Containment' is a metaphor. Hempel's point is obviously not that the argument is uninformative because the expression 'this figure is a triangle' is part of the expression 'this figure is a right angled triangle'. But without this literal sense of 'containment', what could Hempel mean?

One common way to cash out the idea that the premises contain all the 'information' of the conclusion invokes the theory of semantic information.⁶ The theory posits an inverse relationship between probability and informativeness. An information bearer is informative just in so far as that it is unlikely.⁷ Tautologies have a probability of 1, so tautologies carry no information. But, in classical logic, a sentence q is deducible from a set of premises $p_1, ..., p_n$ iff the conditional $p_1 \& , ..., \& p_n \rightarrow q$ is a tautology. Since tautologies carry no information, no deductive inference can increase information.⁸ So if an argument is classically valid, it cannot present a novel conclusion.

This is one way of making sense of Hempel's claim that a deductive inference does not contain any new information. If that is right, it is clear why Hempel accepts the first premise of the scandal. But could Hempel and other Logical Positivists deny premise (2)? Hempel continues:

⁵ Cohen and Nagel (1934), p. 175.

⁶ Popper (1959), p. 96. Shannon and Weaver (1949), Bar-Hillel and Carnap (1953).

⁷ Floridi (2006).

⁸ D'Agostino and Floridi (2009), p. 3; D'Agostino (2013), p. 40. Note that there are other ways, arguably independent of the theory of semantic information, to motivate (1). Wittgenstein's *Tractatus* (e.g. Sections 6.125-6.127) prominently features the idea that logical proofs are tautologies and the results cannot be surprising, without discussing semantic information. Others motivate (1) in a more informal way (e.g. Cohen and Nagel (1934), p. 173; Hempel (1945); Hintikka (1973), p. 135; Dummett (1993), p. 195 and Dutilh Novaes (2010), p. 1. Aristotle himself seems, at least in places, sympathetic to (1). *Prior Analytics* II.21 67a5-67b11 and *Posterior Analytics* II. 71a17-71b8 both suggest that we already know, in some sense of 'know', the conclusion of an argument when we know the premises. Aristotle, of course, denies that tautologies are inferences.

[Deduction] discloses what assertions are concealed in a given set of premises, and it makes us realize to what we committed ourselves in accepting those premises; but none of the results obtained by this technique ever goes one iota beyond the information already contained in the initial assumptions ... although [the conclusion's] content may well be psychologically new in the sense that we are not aware of its being implicitly contained in the postulates.⁹

Hempel claims that the conclusion may be psychologically new, in the sense that some agent may not have realized that a particular conclusion follows from the premises. A conclusion can be novel to me, even though it is contained within the premises. The axioms of Euclidean geometry contain Pythagoras' theorem. But simply knowing those axioms does not mean that I know Pythagoras' theorem. A logically omniscient agent would believe all the consequences of her beliefs and know all the consequences of her knowledge. But, Hempel points out, since we are not logically omniscient, a deductive argument might still be useful. It can tell us some consequences of a belief or piece of knowledge that we have. And therein lies the usefulness of deduction: deduction is useful for drawing novel beliefs out of beliefs we already hold. To extend the metaphor of 'containment', we might say that deduction 'unpacks' information contained within the premises.

Hintikka rejects this approach as it makes deduction 'merely psychological conditioning, some sort of intellectual psycho-analysis, calculated to bring us to see better and without inhibitions what objectively speaking is already before our eyes'. Hempel's

⁹ Hempel (1945), cited in D'Agostino and Floridi (2009), p. 7. Cf. Dummett (1993), p. 195.

¹⁰ Hintikka (1970), 'A Priori', p. 135.

approach disturbs Hintikka, because Hempel robs any deductive discipline of objective novelty. Mathematics, philosophy and logic do not increase knowledge in an objective sense. Novelty depends only on the contingent features of an individual's psychological history. I will mention Hintikka's own solution below, but rather than further follow his criticism of Hempel here, I wish to note something about Hempel's strategy. Hempel accepts the link between novelty and usefulness of deduction because usefulness consists in a certain kind of novelty, namely, psychological novelty. Hempel, like many other analytic philosophers, has assumed that the usefulness of deduction consists in its ability to produce novelty of some sort or another.

This assumed connection between novelty and usefulness comes out most clearly in Dummett, when he comments on Hempel's solution: 'deductive inference has here been justified at the expense of its power to extend our knowledge and hence of any genuine utility'. Dummett explicitly connects the ability of deduction to produce novel conclusions with the usefulness of deduction. This connection is implicit in Hempel. If Hempel were not making this assumption, he would not try to salvage some sense in which a conclusion of a deductive argument is novel. In fact, it seems that the second premise of the scandal is what gives the scandal bite: without the link between novelty and usefulness, it is not worrisome that deduction does not produce new information.

Hintikka's 'objective novelty' approach to the scandal reveals a similar pattern.

Hintikka tries to show that deduction can be valid and produce novel conclusions. 12 Hintikka holds the novelty to be objective, but shares with Hempel the view that novelty is necessary for usefulness. Hintikka distinguishes between 'surface' and 'depth' information. Both sorts

¹¹ Dummett (1993), p. 195.

¹² Hintikka (1970). Hintikka (1973) develops the philosophical consequences of this perspective.

of information are objective. Depth information is potential information and so cannot increase with deductive inference. Surface information, on the other hand, can grow with deductive inference since it is, so to speak, actualized information. Hintikka's distinction between two sorts of information is only present in non-decidable logics, so only such logics can 'increase' information, by actualizing the potential information. In a decidable logic, all the potential information is actual.¹³

Hintikka's approach has been criticized on various grounds. ¹⁴ For now, I stress that Hintikka cannot say how propositional logic, monadic predicate calculus and, indeed, syllogistic logic increase information, since they are all decidable. But what interests me here is Hintikka's strategy. In response to the scandal he denies (1) by trying to give a sense in which a deductive argument, in this case, an argument in predicate calculus with embedded quantification, can be both valid and have a conclusion with more objective information than the premises. ¹⁵ Like Hempel, then, he assumes a connection between validity and usefulness.

Both the psychological and objective approaches try to show that a conclusion can be novel, even though a deduction is valid. That is, they deny (1). But premise (2) is what makes the scandal troubling. Without (2), an assumed connection between novelty and usefulness, the worst that we can conclude is that valid arguments do not yield new knowledge. But lack of novelty is only worrying in a context where we think deduction yields new knowledge, like mathematics or science. In short, there is a hidden pragmatic assumption, connecting novelty with usefulness, at play in the scandal, which I made explicit in (2). In the next section, I will discuss Aristotle's attitude towards the usefulness of deduction. I will show

¹³ See D'Agostino and Floridi (2009), p. 9 for this reading of Hintikka's approach.

¹⁴ Sequoiah-Grayson (2008).

¹⁵ D'Agostino and Floridi (2009) develop an approach distinct from Hintikka's. See also D'Agostino (2014).

that for Aristotle, novelty is not necessary for usefulness. In this way, he has a different view of the usefulness of deduction to analytic philosophers of logic, and, because he assumes that (2) is false, he would not face the scandal of deduction.

3. Aristotle on the Novelty and Usefulness of Deduction

For Aristotle in some cases, a useful deduction does not need a novel conclusion. I do not need to prove the stronger claim that, for Aristotle, the usefulness of a deduction is never the novelty of its conclusion. Indeed, Aristotle occasionally locates usefulness in psychological novelty. But in many cases, a deduction is useful, for Aristotle, despite lacking a novel conclusion.

My argument for this has two premises. First, Aristotle thinks some deductions lack a novel conclusion. Second, those deductions are useful in certain contexts. I argue for the first premise on the basis of an important stretch of *Prior Analytics* I, where Aristotle presents his method for discovering syllogisms.¹⁷ The method moves 'backwards' to discover premises for certain conclusions. Hence, the conclusions are not novel. I argue for the second premise by showing that Aristotle thinks the method discovers syllogisms that are useful in a range of contexts. But in none of these contexts does their usefulness depend on producing novel conclusions.

Aristotelian science proceeds from previous knowledge. First, demonstration requires some prior knowledge of the meanings of terms (*Posterior Analytics* I 1 71a1-15). Second,

¹⁷ I use the term 'discovery' and cognates, rather than 'heuristic', to distinguish the method I identify from the 'heuristic' use of teleology in Aristotle's science discussed by Wieland (1975b)'. Cf. Leunissen (2010), pp. 125-27.

¹⁶ For example, *Posterior Analytics* I.1 71^a25-71^b11; *Prior Analytics* II.21; *Topics* VIII.1 155^b1-15.

demonstrations begin with propositions clearer to us and proceed to those clearer by nature (*Physics* I 1 184a15-25). It seems that Aristotle already has a perspective on demonstration that cannot be scandalized. If so, is my point worth arguing for in detail? I suggest that it is, since neither of these well-known features of Aristotle's science defuses the scandal.

To develop an Aristotelian approach to the scandal, we need to do more than show that each demonstration begins with some prior knowledge. A scandalizer could agree with Aristotle that to perform a deduction, we need to know what, for example, a triangle isor know that the principle of non-contradiction and law of the excluded middle hold. No one holds that deductions can proceed from no information. The rather different claim, that a deduction can be useful, even though the conclusion is not novel, is disputed. Moreover, a scandalizer would not be satisfied by Aristotle's point that science begins with propositions that are better known to us and upgrades this epistemic achievement. The scandal raises a general worry about whether a valid deductive argument can be useful. Clearly, we can use a demonstration to increase our epistemic standing (a point I will return to below). But to provide a convincing Aristotelian perspective, Aristotle needs general story about how deductions, not just demonstrations, can be useful. My discussion shows that Aristotle has such an account at his disposal.

3.1 Some deductions lack novel conclusions

The bulk of *Prior Analytics* I, 23-32 is what I call, following Aristotle, a discovery method for syllogisms. The discovery method finds, for a given conclusion, a middle term to deduce that conclusion. Once we have the middle term, along with the other two terms which feature in the conclusion, we can determine the syllogism we need to prove that conclusion. Aristotle's discovery method depends on some formal properties of his syllogistic, which I

will mention here. Syllogisms have two premises and a conclusion. One subject term and one predicate term form each premise. The predicate term (A) and subject term (B) relate in one of four ways:

- 1. A belongs to every B (written 'AaB');
- 2. A belongs to some B (written 'AiB');
- 3. A belongs to no B (written 'AeB');
- 4. A does not belong to some B (written 'AoB').

Valid syllogistic forms, the moods, are known by their medieval mnemonic names, Barbara, Celarent, Darii, Ferio and so on. Barbara, for instance, has the form AaM; MaB; so, AaB. Key to giving a valid syllogistic argument is the middle term, labeled above as 'M'. This term occurs in each premise, but not in the conclusion. The discovery method finds, for a given conclusion, a middle term to prove that conclusion. Once we have the middle term, along with the other two terms which feature in the conclusion, we can determine the syllogism we need to prove that conclusion.

As Aristotle describes it, the discovery method has two steps. The first step lists terms and the relations between the terms. The second step applies a mechanical procedure to those lists. The procedure finds a middle term that we need to deduce a given conclusion. At *Prior Analytics* I.27 43^b1-11, Aristotle describes the first part of the method. He tells us to set down the target term, call it "T". ¹⁸ We then list terms that relate to T in three categories: what

the table.

¹⁸ Cf. Smith (1989), p. 150 and Striker (2009), p. 192. Striker interprets 'T' as being only the terms in the conclusion that we are interested in establishing. But that cannot be correct, since we are still gathering information into the tables at this stage: we do not yet know what we will be arguing for, hence do not know what terms will be in the conclusion. As I read Aristotle, 'T' is just any term that we are interested in adding to

follows T, what is followed by T and what cannot belong to T. This 'follows' relationship picks out predication. Aristotle tells us to collect basic information about which terms relate to T, but also information about the kind of relation. Is the predication definitional, peculiar or accidental? Is it true or reputable? I will return to the way in which the tables are organized in section 3.2, when I discuss how the resulting syllogisms are useful. Below I give a table containing a sample of the lists.

Term	is subject	is predicate	Terms that
	of	of	cannot
			belong
biologist	rational, human, scientist	geneticist, zoologist	stone, dog
scholar	rational, human	geneticist, physicist, scientist	stone, dog

Table 1: Example lists for target terms 'biologist' and 'scholar'.

Aristotle gives two more requirements on the basic lists. First, the universal predication requirement. The lists should contain only predicates of T that belong to all T and subjects of T such that T belongs to all of the subjects (*Prior Analytics* I.27 43^b11-13). Aristotle explains that the syllogistic system justifies the requirement. Each valid mood involves at least one universal predication: no valid mood has two particular premises. So, to generate valid syllogisms, we need to guarantee that at least one premise involves a universal predication. We ensure this by recording only universal predications in our table.

Aristotle also states the specificity requirement: Do not list terms that apply to every item (terms such as 'being', or 'one') (*Prior Analytics* I.27 43^b35-39). At *Prior Analytics* 44 ^b 20-4, Aristotle points out that the syllogistic justifies this requirement, as it is not possible to prove a privative conclusion from a middle term that follows everything. We can see this from the following argument. 'A middle term that follows everything' can be glossed this

way: where 'M' is a middle term and 'T' is any term, M belongs to all T. Only the second figure is relevant here, since Aristotle presupposes that the conclusion we are aiming at does not contain any terms of universal application. With this assumption, terms of universal application will appear in the table only in the column of terms that follow both the subject and predicate term. So we only need consider the second figure. In the second figure, we have premises M belongs to all A and M belongs to all B, but in this case, no conclusion can follow as there is no second figure syllogism with two universal affirmative premises. If the middle term belongs to any term whatsoever, there is no syllogism which will derive a conclusion. In other words, to prove any conclusion, the middle term must not follow universally both other terms. So the tables should not list any such terms.

Aristotle includes the method for discovering syllogisms for practical purposes. An individual in a discursive context could use the method. Aristotle is clear that the lists record neither all the predications nor only true ones involving the target term T. ¹⁹ First, an exhaustive collection of predications would be impossible for any actual agent. Moreover, Aristotle implicitly denies that the lists of terms will be exhaustive: 'the more such terms one has available, the faster one will hit upon a conclusion and the more true, the more one will demonstrate' (*Prior Analytics* I.27 43^b9-11). If Aristotle conceived of the lists as exhaustive, his advice to get as many terms on the list as possible would be irrelevant. So the lists of predication terms can be non-exhaustive. Second, the recorded predications need not be true, since Aristotle tells us that we should record which predications hold 'only as a matter of opinion and which according to truth' (*Prior Analytics* I.27 43^b8-10). This remark would be

¹⁹ *Pace* Smith (1989), p. 150.

moot if Aristotle held that the lists record only true predications. So Aristotle does not think the tables collect all the true predications nor only the true ones.

Generating such tables is the first part of the discovery method. The second part of the method is the mechanical procedure for finding the middle term (and hence premises) for a given conclusion. Aristotle gives us four rules, one corresponding to each form the conclusion can take (a, i, e or o). *Prior Analytics* I.28 43^b39-44^a1 describes the rule for finding the middle term for an a-conclusion. Call this the 'a-rule':

(A) If one wants to establish a conclusion of the form 'A belongs to all B', then (i) take the list of terms that A is the predicate of, and (ii) take the list of terms B is the subject of. Any term on both lists will be a middle term for the conclusion 'A belongs to all B'.

This procedure finds a middle term for the 'a' conclusion. Barbara is the only mood that can establish a universal affirmative conclusion. The procedure tells us to take both lists of terms: those that A is predicate of and those that B is subject of. Any item on both lists will be a middle term for the conclusion. The procedure may, of course, find more than one possible middle term for the conclusion.

For example, suppose we are trying to find an argument for the conclusion that scholar belongs to every biologist. We can extend the table given above to note the role each target term plays in the conclusion (i.e. whether it is a subject or a predicate). This gives the following table:

Place in the	Term	is subject of	is predicate	Terms that
conclusion:			of	cannot belong

Subject	biologist	rational, human,	geneticist,	stone, dog
		scientist	zoologist	
Predicate	scholar	rational, human	geneticist, physicist,	stone, dog
			scientist	

Table 2: Example lists for target terms 'biologist' and 'scholar' organized to find a middle term for 'scholar belongs to every biologist'.

Since the conclusion is a universal affirmative, we apply the a-rule. First, we look at the list of terms which the predicate of the conclusion, 'scholar', is predicate of. We find there the terms 'geneticist', 'physicist' and 'scientist'. Next we look at all the terms that the subject of the conclusion, 'biologist', is subject of. We find 'rational', 'human' and 'scientist'. We now see that 'scientist' is a term on both lists. According to the mechanical procedure, 'scientist' will be a middle term for a syllogism with 'scholar belongs to every biologist' as the conclusion. Indeed it is: Scholar belongs to every scientist; scientist belongs to every biologist; so, scholar belongs to every biologist. Note that the procedure looks for one possible middle term for the conclusion. The procedure may not, and need not, find every possible middle term for the conclusion.

There are a few points to note about the procedure. First, it relies on the lists of terms generated in the first step of the discovery method. The procedure cannot do anything without the lists telling us how terms truly or apparently relate, because the procedure operates on the lists. Second, the procedure is mechanical in the sense that it gives detailed instructions for finding a syllogism for a particular conclusion, which can be applied without insight. The conclusion fully determines which rule to apply; if the tables contain sufficient information, the rule determines at least one middle term that will give that conclusion. Facts about the syllogistic system determine how to relate the middle term to the others to produce a valid syllogism. In contrast, generating the lists of terms is not obviously a mechanical procedure: it seems to require insight into, at least, the meanings of the terms.

I have given just one simple example, using the 'a-rule'. Here are the other three rules that Aristotle gives:

- (I) If one wants to establish a conclusion of the form 'A belongs to some B', then (i) look at the terms that A is the *predicate of*, and (ii) look at the terms B is the *predicate of*. Any term on both lists will be a middle term for the conclusion 'A belongs to some B' (*Prior Analytics* I.28 44a1-3).²⁰
- (E) If one wants to establish a conclusion of the form 'A belongs to no B', then (i) look at the terms that *cannot belong to* A, and (ii) look at the terms B is the *subject of*.

 Any term on both lists will be a middle term for the conclusion 'A belongs to no B'

 (*Prior Analytics* I.28 44a4-7).²¹
- (O) If one wants to establish a conclusion of the form 'A does not belong to some B', then (i) look at the terms that *cannot belong to* A, and (ii) look at the terms that B is *the predicate of*. Any term on both lists will be a middle term for the conclusion 'A does not belong to some B' (*Prior Analytics* I.28 44^a8-12).²²

Each rule is a conditional, which tells us what to do given that we want to find a certain conclusion. When we apply any of the rules, comparing the two lists may not yield any terms.

²⁰ Given that the lists only include terms that belong to all other terms, this rule will only generate syllogisms in the third figure Darapti. There are other syllogisms with 'i' conclusions, but they cannot be found with the discovery method. This is further evidence that the method is for discovering some syllogism for a given conclusion, not every syllogism.

²¹ This rule will give us a syllogism in Cesare. But, of course, there are other syllogisms with an 'e' conclusion. Aristotle gives us an alternative rule, where we can look at terms that cannot belong to the subject term, but which belong to the predicate term. If we do this, we get a second figure syllogism, Camestres. But again, this leaves out other ways of proving an 'e' conclusion, such as Celarent.

²² This rule can only produce syllogisms in Felapton. But, again, there are other ways to prove the 'o' conclusion, e.g. Ferio, Festino, Baroco, Bocardo and Ferison.

If so, we cannot find a syllogism for the conclusion using the information contained in the tables we have. But if comparing the lists does yield at least one term, there will be a middle term for that conclusion. If the two lists share more than one term, then a further step is required, namely to select one of the terms to be a middle term for this conclusion. I will discuss below the reasons one might have to select one middle term rather than another.²³

It is now easy to see that, according to Aristotle, some arguments do not present novel conclusions. The discovery method finds deductive arguments of a certain sort, namely syllogisms. The conclusions are not novel. Each of the four rules begins with the conclusion and tells how to find an argument to establish it. The rules give a formal procedure to find a syllogism to a given conclusion. The form of the argument is determined by the rules, but the content of that argument, of course, will depend on using the tables of terms gathered. But the conclusion that an agent wants to establish has already been identified. The conclusion cannot be novel in the psychological sense, since the discoverer already knows what the conclusion is before she discovers the syllogism. You might say that there is some sense in which the conclusion is novel, namely, it has gone from being merely a proposition to being a conclusion of a syllogism. But this is mistaken: the arguer already takes the conclusion to be a conclusion, and searches for the syllogism. The conclusion is not novel in some objective sense either. The conclusion contains no more (objective) information than the premises built around the middle term, whatever this notion of objective information might be. Aristotle

²³ I disagree with Smith's reading of the method. Smith takes the method as comparable to a 'decision procedure' in modern logic. Smith (1989), p. 154 attributes to Aristotle the claim that the 'procedure will find a deduction if and only if a deduction is possible'. If this bi-conditional holds, the method will find all and only possible deductions, which would be sufficient to determine what can be proved from a set of truths. If Aristotle thinks that the bi-conditional holds, then Aristotle thinks the method will find all conclusions that can be proved, not just from information in the tables, but in general. But Aristotle never asserts the biconditional, and it is in fact false. One direction of the bi-conditional holds trivially: if the method finds a deduction then the deduction is possible. But the other direction is frighteningly strong: if a deduction is possible, then the method will find it. The method, of course, cannot find all possible deductions, but only a very small class of deductions, if any: those deductions that will yield a given conclusion, based on information contained within a certain set of tables.

admits that at least some syllogistic arguments, in certain situations, do not present novel conclusions. In the next section, I argue that Aristotle, nonetheless, considers such arguments useful. This will show that, for Aristotle, some valid arguments don't present novel conclusions, but are nonetheless useful, i.e. that Aristotle holds (2) to be false.

3.2 How the syllogisms discovered are useful

Whether something is useful depends on what we intend to use it for. The way Aristotle sets out the discovery method points to two contexts of use for the discovered syllogisms. The tables record information about predications. But Aristotle also suggests that we group that information according to whether those predications are true or reputable (*endoxon*) (*Prior Analytics* I.27 43^b2-4). The dichotomy between true and reputable predications recalls the beginning of the *Prior Analytics*, where Aristotle divides syllogistic premises, and hence syllogisms, into demonstrative and dialectical sorts at *Prior Analytics* I.1 24^a30-^b11 (cf. *Topics* I.1 100^a27-30). Aristotle says that a premise is demonstrative if 'it is true and accepted on the basis of the starting points' (*Prior Analytics* I.1 24^a30), while a dialectical premise is 'apparent and reputable (*endoxon*)' (*Prior Analytics* I.1 24b10). This dichotomy suggests, again, that the method is useful in the context of dialectic, but also in the context of demonstration.

When he reflects on the application of the discovery method, Aristotle confirms that there are two contexts of use, demonstration and dialectic, for the syllogisms discovered by the method:

When it is a question of truth [one must select] from the lists of those things that belong truly, but when in a dialectical syllogism [one must select] premises from the premises according to opinion. (*Prior Analytics* I.30 46^a8-10)

Aristotle does not say that the discovery method itself is useful in the context of demonstration and the context of dialectic. Rather, the resulting syllogism is useful: we select from the list of true predications when we want to find demonstrations, and from the list of opine predications when we want a dialectical syllogism. This is important, since I have claimed that the usefulness of deductions, not the usefulness of some method for finding syllogisms, is what is at stake in the scandal of deduction. Aristotle here confirms that he is interested in using the resulting deductions.

Aristotle suggests demonstration and dialectic as two broad contexts of use for syllogisms discovered by the method. Dialectical contexts are necessarily multi-agent, while demonstrative contexts need not be. This is clear from *Prior Analytics* I.1 24a22-25, where Aristotle says that in a demonstration premises are assumed, while in dialectic they must be asked for from an interlocutor. Since in dialectic, premises must be asked for, at least two agents are involved in the argument, but the same does not hold of demonstration. This pragmatic difference, however, does not affect the underlying logic of the argument (*Prior Analytics* I.1 24a25-28), so the syllogisms found by the discovery method are available for use in both contexts. So I must now show how the syllogisms, which are produced by the discovery method and which necessarily do not present novel conclusions, are useful in

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²⁴ Cf. also *Topics* I.1 100^a27-30 and *Sophistical Refutations* 2 165^a38-^b8.

Aristotle's mono- and multi-agent contexts. I begin with mono-agent contexts of demonstration.

The first use of a syllogism without a novel conclusion is simply to be certain that the conclusion is the case. A syllogism is an indefeasible, necessarily truth-preserving argument: if the premises are true, then the conclusion must be true. Syllogisms necessarily preserve truth. Since they preserve truth, I can be certain that the conclusion is true, if the premises are true. Aristotle's remarks in *Topics* I.1 100^b18, which stress that demonstrations have true premises, suggest this use of demonstration. I might *think* that, for example, all biologists are scientists, but I might wish to *convince* myself that all biologists are scientists. If I have sufficiently comprehensive tables of true predications concerning the practitioners of science, then I will be able to discover whether all biologists are scientists. Then, given that the premises are true, I can be certain that the conclusion is true, since the syllogisms discovered by the method have the property of necessary truth-preservation.

The second use for syllogisms in demonstrative contexts, is to help us understand the conclusion. A syllogism could integrate the conclusion into a systematic body of knowledge, such as one of the special sciences.²⁵ We may, of course, wish to integrate a conclusion which we already believe or know to be true. *Posterior Analytics* 71^b17-22 claims that demonstrations are deductions of a certain kind. Demonstrations are deductions which give rise to understanding.²⁶ Aristotle lists six individually necessary conditions for such

²⁵ Cf. *Physics* I.1 184^a16-20, *Posterior Analytics* I.1 71^a1-30, and *Metaphysics* A.9 992^b29-993^a1. In these passages, Aristotle suggests that we start with propositions that are better known to us, and progress to those that are better known 'in themselves'. Wieland (1975), pp. 129-30, interprets this as Aristotle pointing out that we begin with one sort of knowledge and 'upgrade' it to another. This is precisely the process we see in the discovery method, as I describe it.

²⁶ In translating *episteme* and cognates as 'understanding', I follow Barnes (1975), who himself follows Burnyeat (1981), pp. 97-108. Burnyeat's insight is that there is something holistic about *episteme* in Aristotle's Greek that is not captured by our noun 'knowledge'. That holism is, incidentally, captured by our adjective 'knowledgeable'.

demonstrative syllogisms. Demonstrative syllogisms are (a) true, (b) primitive, (c) unmediated, (d) better known than, (e) prior to and (f) explanatory of the conclusion.²⁷ I focus here on the condition that the starting points are 'better known' than the conclusion. I suggest that demonstrations are useful, according to Aristotle, because they make the conclusions better known.

How should we understand Aristotle's idea that the premises are 'better known' than the conclusion?²⁸ At *Topics* VI.6 141^b30-34, Aristotle argues that both genus and differentia are better known than the species. In his example, 'man' names the species, 'animal' the genus and 'terrestrial' the differentia.²⁹ Aristotle explains that an agent can know the genus, being an animal, without necessarily knowing the species, being a man. Likewise with the differentia, terrestrial. But it is not possible to know the species without knowing the differentia. We could formulate Aristotle's idea of being better known this way:

(BK) X is better known to a than Y iff it is possible that (a knows X and a does not know Y).³⁰

²⁷ If (a) to (f) are conditions on the principles appropriate to some domain of investigation, it does not follow that every premise in every proof in that domain must meet all six conditions. The six conditions Aristotle gives on 'demonstration' do not apply to the premises of each and every deduction; rather they 'properly characterize the principles of axioms of a demonstrative science' (Barnes 1975, p. 98).

²⁸ Aristotle also mentions that a demonstration can fail because the premises are not 'prior' to the conclusion (*Prior Analytics* II.21 64^b33-34), but in *Posterior Analytics* I.2 71^b31-72^a5, Aristotle seems to identify the being better known and priority requirements. In any case, discussing being better known is sufficient for the point I wish to make. This brief explanation is also given in my (2014).

²⁹ 'pezon' which probably means 'terrestrial', rather than 'footed'. Cf. *Historia Animalium* 490a31, where snakes are described as without feet (*apoun*) and terrestrial (*pezon*). Thanks to Istvan Bonar for this point.

³⁰ The brackets in this formulation indicate that the possibility operator has a wide scope. Aristotle does not, in general, distinguish between individuals, terms and propositions when discussing knowledge. Hence 'X' and 'Y' can range over any of these classes.

If (BK) states Aristotle's principle, the 'better known than' relation is irreflexive. When we replace 'X' and 'Y' in the schema with the same expression, the right-hand side of the resulting biconditional is false. For (BK) to hold, whatever replaces 'X' cannot also replace 'Y'. In the case where we better know the premises, it follows from BK that we do not know the conclusion. In that case, at least, there is a hierarchical structure between the things a demonstration proceeds from (the premises) and what it proceeds to (the conclusions): the premises differ from the conclusion and better known than it.

But we upgrade our cognition of the conclusion to understanding by showing how it relates to this hierarchy of better-known propositions. Aristotle's idea is that we know p better, once we have shown how it relates to other propositions, q, and r, which are different from p. Suppose q and r are better known than p: it is possible to know q and r but not p. Assuming that deduction is closed under known implication, if we know q and r, p can become known once we know how to relate it deductively to q and r. We get to know p by integrating p into the systematic structure of knowledge we already possess. But to do this we already have p available; so, p is not novel. What we need is a deduction of p. That deduction will be useful, as it will help us understand p by learning how p fits with propositions we know. So a deduction can be useful for increasing understanding, but without producing a novel conclusion.

These features of Aristotelian science are widely acknowledged. But it is less often remarked that Aristotle tells us that his discovery method can help integrate a given conclusion into a wider body of knowledge (*Prior Analytics* 46^a18-27). Suppose we have extensive tables of predicates in the domain of number theory, and I want to understand why it is the case that:

(C) Ten is the product of primes numbers..

I have a true belief that (C) but I want to upgrade my cognitive achievement to understanding. Using my tables and discovery method I can discover a syllogism to (C), involving the following two premises:

- (P1) Any number is the product of prime numbers.
- (P2) Ten is a number.

It is clear, given (BK), that (P1) and (P2) are better known than (C). I can know each of (P1) and (P2) without knowing (C). But once I have a syllogism from (P1) and (P2) to (C), (C) becomes as well-known as (P1) and (P2), because I cannot know (P1) and (P2) together without knowing (C), again assuming that knowledge is closed under known implication. Here the syllogism results from the discovery method and so cannot have a novel conclusion. But the deduction in question is still useful in Aristotle's epistemic framework, since it allows me to relate a conclusion to other propositions I know and hence to better know the conclusion.

³¹ Although the method allows us to understand the conclusion through a demonstration, other conditions Aristotle puts on demonstration cannot reliably be met by the deductions discovered in the discovery method. Condition (f), for example, cannot reliably be met by the method. The method discovers some middle term through which a given conclusion can be demonstrated. The method may not find terms that explain a conclusion, merely those that can be used to deduce it. For example, scholar belongs to all geneticist can be deduced using two different middle terms: scientist or lab-worker. But only the former middle term explains why scholar belongs to all geneticist. Geneticists are scholars because they are scientists, not because they are lab workers. This should not concern us. Aristotle is well aware that there is a difference between demonstrations *that* something is the case and demonstrations which explain *why* something is the case. For the distinction between 'factual' and 'explanatory' syllogisms see *Posterior Analytics* I.13 78^b35-79^a6; *Posterior Analytics* II.2 89^b44-90^a34; *Progression of Animals* 1 704^b9-10; *Parts of Animals* II.1 646^a7-12 and *Prior Analytics* II.2 53^b9.

The syllogisms discovered by the method, then, are useful in demonstrative contexts, even though they do not present novel conclusions. The same is true of some multi-agent contexts. At *Sophistical Refutations* 2 165^a37-^b9, Aristotle distinguishes four sorts of multi-agent encounter. The first, didactic, is the context of teaching. The second, dialectic in a narrow sense, is a game to drive one's interlocutor into a contradiction, starting with reputable opinions (*endoxa*).³² The third is peirastic, roughly, Socratic-style questioning. The fourth is eristic, contexts of exclusively adversarial argument. In this paper, I discuss the first two sorts of encounter and show how the arguments in such encounters need not present novel conclusions, although the arguments are useful.

I begin with narrow dialectical contexts. The *Topics* says that two agents participate in a dialectical syllogism and proceed by question-and-answer.³³ The two participants are known as the questioner and the answerer. The syllogisms proceed from reputable opinions. The questioner aims to drive the answerer into a contradiction.³⁴ The game begins when the questioner presents a problem (*problema*) to the answerer. Where 'X' and 'Y' can range over individuals or terms, all problems are questions of the form 'is X Y or not?', offering the answerer the choice of one of a pair of contradictories. For example, 'Are knowledge and perception the same or not?' (*Topics* I.5 102^a8). The answerer selects one of the contradictories. This is the 'starting point' (*to en archêi*). I will label it *p*. The questioner tries to compel the answerer, by means of 'yes or no' questions, to concede the contradictory of

³² By 'narrow' dialectical contexts, I mean the dialectical game that Aristotle describes in the *Topics*. I take it that 'broad' dialectical contexts are all those that involve reasoning from *endoxa*. There is a well-known debate about whether dialectic, in what I call the broad sense, somehow grounds enquiry in Aristotle. Defenders of this view include Irwin (1988) and Nussbaum (1986). Criticism of this view is found in Hamlyn (1990), Bolton (1999) and Smith (1993). What I say here should be compatible with either view about the relation of broad dialectic to inquiry in Aristotle.

³³ See Smith (1993), p. 337.

³⁴ For details of the structure of the *Topics* game, see Moraux (1968), 'La Joute'; Slomkowski (1997), *Aristotle's Topics*. Castelnérac and Marion (2009), 'Arguing for Inconsistency', suggest a rather different 'rational reconstruction' of the *Topics* game, in light of Plato's Socratic dialogues.

the starting point, namely, not-*p*.³⁵ Each statement that the answerer accepts along the way is called a 'proposition' or 'premise' (*protasis*). Eventually, the answerer is forced to accept the conclusion (*sumperasma*), won by a series of such questions.³⁶ We can see in the example below that (1) represents the *problema*, presented by the questioner, and starting point, chosen by the answerer, (2)-(6) the *protaseis* and (7) the *sumperasma*:

- 1. Q: Are knowledge and perception the same, or not? A: They are the same.
- 2. Q: Is it possible to both know and not know the same thing at the same time? A:

 No.
- 3. Q: If I remember something, do I know it? A: Yes.
- 4. Q: If I remember something, do I perceive it? A: No.
- 5. Q: So, if I remember something, I both know it and do not perceive it? A: Yes
- 6. Q: But knowing and perceiving are the same? A: Yes
- 7. Q: So if I remember something, I both know and do not know it. Which you agreed is impossible. A: Oh ...
- 8. Q: So knowledge and perception are not the same.³⁷

It is clear that the conclusion is not psychologically or objectively novel. Both parties, the questioner and the answerer, already know where the argument is heading. Because of the rules of the game, the questioner aims to force the answerer to accept the contradictory of the

concealment discussed in the *Topics* are recommended only in eristic contexts.

³⁵ Cf. Bolton (1999), p. 103. Bolton wrongly suggests that the conclusion is always concealed in dialectic. The rules imply that both questioner and answerer know what the overall conclusion should be, and the skill is in forcing or avoiding this conclusion. Some intermediate conclusions, which are needed as part of a larger argument for the overall conclusion, may be concealed. Owen (1968), p. 107 claims that the tactics for

³⁶ Ryle points out that neither competitor is committed to the truth or falsehood of *p in propria persona*. It is accepted or denied merely for the sake of argument (see Ryle (1968), pp. 74-75; Moraux (1968), p. 302; and Slomkowski (1999), pp. 107ff.).

proposition the answerer chose as the 'starting point' (p). But, also because of the rules of the game, both parties know what p is and what the aim of the game is. There is no question of this game producing a new conclusion; the only question is whether the questioner can successfully force the answerer to accept the sumperasma. The game is only worth playing if both parties already know what the sumperasma is, at least after the first move at (1). A Topics game will, therefore, never head towards a novel conclusion. 38

But is a syllogism generated by the discovery method, which we know does not present a novel conclusion, useful in the context of dialectic? Notice first that Aristotle thinks that the underlying logical structure, the syllogism, is the same in a dialectical argument as in a mono-agent argument(*Prior Analytics* I.1 24°25-28). Further evidence that the structure is the same is the way Aristotle distinguishes the contexts of didactic, dialectic, peirastic and eristic at *Sophistical Refutations* 2 164°37-165^b9. Each context involves syllogisms. The difference between these syllogisms is not logical; it is epistemic. Dialectical syllogisms have merely reputable opinions as premises and need not have true ones; didactic syllogisms differ from the other kinds of syllogisms because their premises are 'principles appropriate to each branch of learning'. Only the epistemic status of the premises differs. The logical structure underlying each sort of argument is the same, so the method can deliver syllogisms that are useful in each of the contexts mentioned.

In dialectic, the questioner can use the method to discover the premises she needs to force the answerer to accept a particular conclusion. Moreover, the method will tell the questioner the doxastic status of the premises needed (i.e. whether they are true, reputable or

³⁸ For the view that these games are the conceptual or historical antecedents of deductions for Aristotle, see Kapp (1942), pp. 14-16 and Kapp (1975), p. 40.

hold for the most part).³⁹ From this, the questioner can tell which ones the answerer is more likely to grant.⁴⁰ To see this, look again at the example game above. (1) is the starting point, and 'all known things are perceived' follows directly from it. Where 'K' stands for 'known thing' and 'P' stands for 'perceived thing', we write this as 'PaK'.⁴¹ The questioner knows that, due to the rules of the game, she must refute PaK. She knows that to do this, she can prove the contradictory, namely PoK (i.e. perceived thing does not belong to some known thing).

Suppose the questioner has tables of predications in the domain of epistemology. She needs to prove an o-conclusion, so she applies the o-rule to the tables. She discovers that 'memory' ('M') will be the middle term she needs for a syllogism in Felapton (PeM, KaM, so PoK). But this syllogisms does not give a novel conclusion. The conclusion was already known to the questioner. In fact, the rules of the dialectical game determine what the conclusion has to be. But the questioner has discovered which premises she needs to force the answerer to admit PoK, and hence refute the answerer's starting-point of PaK. In particular, the questioner needs the answerer to admit PeM and KaM. The syllogism discovered does not present a novel conclusion, but the syllogism is useful because it can force the answerer to accept a certain conclusion. Thus, a syllogism without a novel conclusion can be useful in a narrow dialectical case.

Not all question-and-answer interactions aim at driving the answerer into contradiction, but even so syllogisms without novel conclusions are useful. Didactic discussions aim at one

³⁹ It is not always the case that any reputable opinion will be admitted by the answerer: it depends on the dialectical context (see *Topics* VIII.5 159^a37-^b25). But, whatever the attitude of the answerer towards reputable opinions, it is important information for the questioner to have available.

⁴⁰ See *Topics* VIII.1 155^b1-15 for these sorts of consideration.

⁴¹ Recall that the predicate term is first in this notation.

party, the answerer, increasing their knowledge. Didactic arguments have premises that are better known to the learner/answerer than the conclusion.⁴² Didactic discussion is not spelled out in just one location in Aristotle's corpus, but it seems that (i) didactic arguments proceed to truths, because no one would teach a falsehood;⁴³ (ii) the starting points for didactic are proper to the branch of science in question (*Sophistical Refutations* 2 165^b1-3). Moreover, (iii) the learner/answerer should always agree to reputable opinions, which are moves on the way to the conclusion (*Topics* VIII.5 159^a30). But clearly, the central feature of didactic is connecting the conclusion, which is less well known to the learner/answerer, to the premises, which are better known to her.

Syllogisms discovered by the method will be useful for teaching, even though the conclusion is already known. Suppose I am trying to teach someone that (C) all triangles have angles that sum to two right-angles. (C) could be thought of as the conclusion of a certain syllogism. But that conclusion is not novel to me (and, although it may be novel to my student, it need not be: she may have taken the answer on trust, for example, without having understood it). As long as the pupil accepts the axioms of geometry, she should become convinced of (C) on the basis of the syllogisms which connect the axioms to the conclusion. The syllogism which has (C) as its conclusion is useful because it enables the pupil to see not only that (C) holds, but also why (C) holds, which is one necessary part of learning (C). So the discovery method could discover a didactically useful syllogism which does not present a novel conclusion.

⁴² Topics VIII.3 159a10-14; cf. Smith (1997), pp. 127-28.

⁴³ Topics VIII.4 159^a29-30; cf. Topics VIII.7 161^a25.

4. Conclusion

I began with a problem that has occupied philosophers of logic. (1) If a deductive argument is valid, then the conclusion is not novel. (2) If the conclusion of an argument is not novel, the argument is not useful. So, (3) if a deductive argument is valid, then it is not useful. Certain analytic philosophers denied the first premises of the scandal. This strategy betrays an assumption about what constitutes a useful deduction, an assumption captured by premise (2) of the argument.

Aristotle does not assume that a deduction is useful just in so far as its conclusion is novel. In many contexts Aristotle considers a syllogism to be useful, despite not presenting a novel conclusion. Aristotle's method discovers middle terms for syllogisms based on the conclusion of that syllogism. If this is the case, the syllogism discovered cannot have a novel conclusion. But, as I went on to argue, in the demonstrative and dialectical contexts considered by Aristotle, these deductions are still useful. Aristotle does not assume that having a novel conclusion is necessary for a syllogism to be useful. This shows that Aristotle would reject (2). (2) is a hidden assumption about the use of deduction.

What, if anything, has this paper shown about the relationship between analytic philosophy and ancient philosophy? Even if I am right that (a) Aristotle holds that some deductions without novel conclusions are useful and so (b) would not face the problematic conclusion, has this detour into ancient philosophy revealed anything valuable to an analytic philosopher, as a philosopher, rather than, say, an historian?

We have learned one philosophically valuable thing about the scandal of deduction. I argued in section 1 that the scandal of deduction, as framed by analytic philosophers, contains a hidden assumption that novelty is necessary for usefulness. Aristotle rejects this assumption. This does not yet show that Aristotle's approach is philosophically valuable, since one could

reveal and reject that assumption without having read a word of Aristotle. But Aristotle's thought adds a formal framework showing that it is legitimate to reject the connection between novelty and usefulness. This is precisely what Aristotle's discovery method allows. Not only would Aristotle reject premise (2) of the scandal, but he also has the formal resources to do so legitimately. Without some underpinning philosophical and formal work, rejecting the connection between novelty and usefulness would seem *ad hoc*, a move simply designed to avoid the problematic conclusion of the scandal. But within the philosophical and formal framework Aristotle has, rejecting the connection between novelty and usefulness is not *ad hoc*: rather it is simply an obvious consequence. 44

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⁴⁴ I would like to thank Istvan Bodnar, Catarina Dutilh Novaes, Leon Geerdink, Mariska Leunissen, Catherine Rowett and Yue Lu for comments on earlier drafts. As well as audiences in Cambridge and Edinburgh, I would like to thank the organizers and participants of the conference 'Analytic Philosophy and Ancient Philosophy', held in Oxford in October 2013, for their comments. This paper was produced while I held a post-doctoral position on the NWO funded project 'The Roots of Deduction'.

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