Supplementary Material

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1 Qubit Response to Photon Scattering

In this section we provide the theory and additional results in support of Fig. 3 in the paper. When the collectively encoded qubit is resonantly driven on the $|g\rangle \leftrightarrow |e\rangle$ transition by a laser pulse (referred to as a scattering pulse in the main text) with Rabi frequency Ω_s, the density matrix ρ of each atom undergoes dissipative dynamics set by the master equation

$$\partial_t \rho = -\mathrm{i}[\mathcal{H}, \rho] + \gamma \mathcal{L} \rho \mathcal{L}^\dagger - \frac{\gamma}{2} \left\{ \mathcal{L}^\dagger \mathcal{L}, \rho \right\}, \tag{1}$$

$$\mathcal{H} = \frac{\Omega_s}{2} \left(|g\rangle \langle e| + |e\rangle \langle g| \right), \tag{2}$$

$$\mathcal{L} = |g\rangle \langle e| . \tag{3}$$

The laser pulse is turned on for a given time t. It is then switched off ($\Omega = 0$) at which point all $|e\rangle$ population decays to $|g\rangle$.

In the following derivation we will calculate the temporal evolution of a Rydberg polariton shared amongst three atoms, and we will then generalise our results to larger ensembles. To prepare the Rydberg polariton we first initialise the atoms in the spin wave $|e\rangle = \frac{1}{\sqrt{3}}(|gge\rangle + |geg\rangle + |egg\rangle$ which is then adiabatically transferred to the polariton state $|r\rangle = \frac{1}{\sqrt{3}}(|ggr\rangle + |grg\rangle + |rgg\rangle)$.

The density matrix of the initial state $|i\rangle$ is

$$|i\rangle \langle i| = \frac{1}{3} \left(|ggr\rangle \langle ggr| + |ggr\rangle \langle grg| + |ggr\rangle \langle rgg| + ... \right), \tag{4}$$

where $|ggr\rangle \langle ggr| = |g\rangle \langle g| \otimes |g\rangle \langle g| \otimes |r\rangle \langle r|$. The figure of merit quantifying the robustness of the Rydberg polariton is $\mathcal{F} = \langle i|\rho_\mathrm{f}|i\rangle$, which defines the readout fidelity. Here ρ_f is the density matrix after evolution during the laser pulse. In the case of no dissipative dynamics the fidelity is 1 for all times (this assumes that no other dissipative effects are present).

To calculate $\mathcal{F}$, we notice that the element $|r\rangle \langle r|$ of the density matrix does not evolve, since dissipation acts in a different subspace. On the other hand, $|e\rangle \langle e|$ does evolve in time, but decays back to $|g\rangle \langle g|$ after the laser is switched off. This process is much faster than the experimental timescale. Thus the only element of the above density matrix with non trivial dynamics is $|g\rangle \langle r|$.

$$\partial_t |g\rangle\langle r| = -\mathrm{i}\Omega\,|e\rangle\langle r|\,, \tag{5}$$

$$\partial_t |e\rangle\langle r| = -\mathrm{i}\Omega\,|g\rangle\langle r| - \frac{\gamma}{2}\,|e\rangle\langle r|\,. \tag{6}$$

Solving these coupled differential equations with the initial conditions $|g\rangle\langle r|_{\mathrm{t=0}} = |g\rangle\langle r|$ and $|e\rangle\langle r|_{\mathrm{t=0}} = 0$ gives

$$|g\rangle\langle r|_{\mathrm{t}} = \mathrm{e}^{-\frac{\gamma}{4}t}\left(\cosh(\omega t) + \frac{\gamma}{4\omega}\sinh(\omega t)\right)|g\rangle\langle r| - \mathrm{i}e^{-\gamma t}\frac{\Omega}{\omega}\sinh(\omega t)\,|e\rangle\langle r|\,, \tag{7}$$

where $\omega = \frac{1}{4}\sqrt{\gamma^2 - 16\Omega^2}$. When switching of the laser all $|e\rangle$ population decays to $|g\rangle$ leaving only the $|g\rangle\langle r|$ coherence, such that the application of the scattering pulse effects the following transformation:

$$|g\rangle\langle r| \rightarrow e^{-\frac{\gamma}{4}t}\left(\cosh(\omega t) + \frac{\gamma}{4\omega}\sinh(\omega t)\right)|g\rangle\langle r| = \alpha(t)\,|g\rangle\langle r| \tag{8}$$

So the three-atom density matrix undergoes the transformation

$$|i\rangle\langle i| \rightarrow \frac{1}{3}\left(|ggr\rangle\langle ggr| + |g\rangle\langle g| \otimes \alpha(t)\,|g\rangle\langle r| \otimes \alpha^*(t)\,|r\rangle\langle g| + ...\right) \tag{9}$$

$$= \frac{1}{3}\left(|ggr\rangle\langle ggr| + |\alpha^2(t)|\,|ggr\rangle\langle grg| + ..\right). \tag{10}$$

The structure of this final density matrix ρ_{f} becomes more apparent in matrix form:

$$\rho_{\mathrm{f}} = \frac{1}{3}\begin{pmatrix} 1 & |\alpha(t)|^2 & |\alpha(t)|^2 \\ |\alpha(t)|^2 & 1 & |\alpha(t)|^2 \\ |\alpha(t)|^2 & |\alpha(t)|^2 & 1 \end{pmatrix}. \tag{11}$$

This result can then be generalised to N particles, where

$$\rho_{\mathrm{f}} = \frac{1}{N}|\alpha(t)|^2\begin{pmatrix} 1 & . & . & 1 \\ . & 1 & . & . \\ . & . & 1 & . \\ 1 & . & . & 1 \end{pmatrix} + \frac{1}{N}\left(1 - |\alpha(t)|^2\right)\begin{pmatrix} 1 & . & . & 0 \\ . & 1 & . & . \\ . & . & 1 & . \\ 0 & . & . & 1 \end{pmatrix}. \tag{12}$$

Evaluating the fidelity then yields

$$\mathcal{F} = \langle i|\rho_{\mathrm{f}}|i\rangle = \frac{1}{N} + \frac{N-1}{N}|\alpha(t)|^2. \tag{13}$$

Which for $\Omega \gg \gamma$ and $N \gg 1$ reduces to

$$\mathcal{F} = \exp\left[-4\frac{\Omega_s^2}{\gamma_s}t\right]. \tag{14}$$

Experimental results in Fig. S1 support this theory. A qualitative match to the above prediction of an decay of the scattering field as a function of Ω_s is observed.

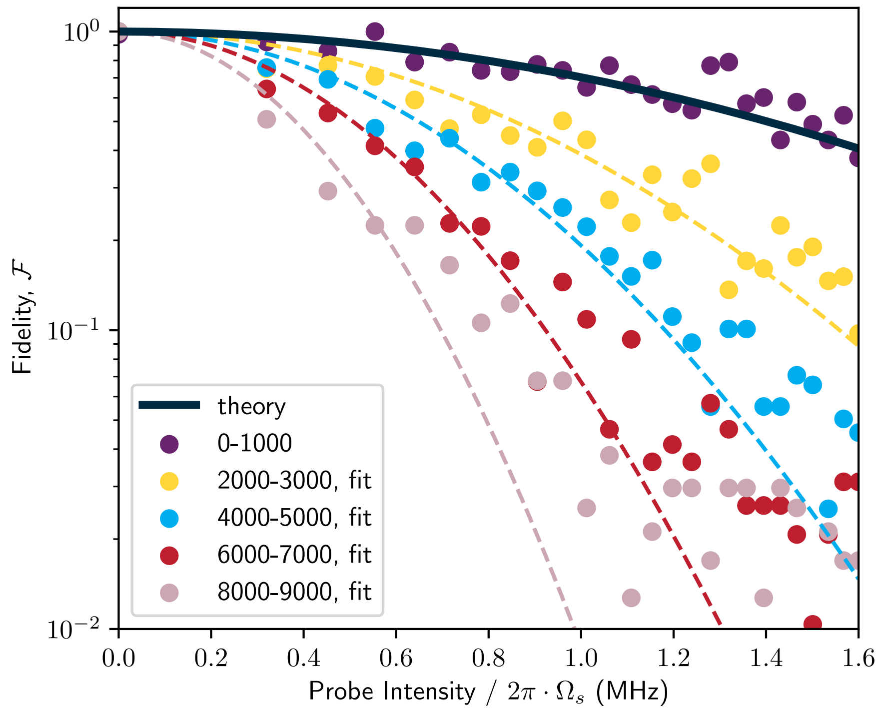

Figure S1: As the intensity of the scattering field is increased, the magnitude of the retrieval decreases due to a combination of loss of OD and fidelity reduction. Each ensemble is recycled for ten thousand experiments and the scattering field causes a progressive loss of optical depth which affects the magnitude of the retrieval. By grouping the data by shot number (1 being the first experiment performed upon an ensemble and 10,000 being the last, see legend inset) we are able to show that at low shot numbers, where OD is high, our observed retrieval converges upon the theory outlined in equation (14).

2 Qubit Response to Electrical Noise

In this section we provide theory and additional results in support of Fig. 4 in the paper. The coupling of the atomic Rydberg states to the externally applied electric field is modelled as a quadratic Stark perturbation described by the Hamiltonian

$$\mathcal{H} = \frac{1}{2}\begin{pmatrix} \alpha_{\mathrm{r}'} & 0 \\ 0 & \alpha_{\mathrm{r}} \end{pmatrix} E^2(t). \tag{15}$$

Here $\alpha_{\mathrm{r,r}'}$ are the polarisabilities of the Rydberg $|r\rangle$ and $|r'\rangle$ states of the collective qubit. The electric field strength $E(t)$ is assumed to take the form of random variable over the interval $[-E_0, E_0]$.

Application of this perturbation introduces a relative, time dependent energy shift between the Rydberg states $|r\rangle$ and $|r'\rangle$. It also leads to decoherence of quantum superpositions of states in $|r\rangle$ and $|r'\rangle$. To discriminate between the two effects, we can decompose the squared electric field term in equation (15) as

$$E^2(t) = \langle E^2(t)\rangle + \xi(t). \tag{16}$$

Here $\langle E^2(t)\rangle$ is the time-averaged value of $E^2(t)$

$$\langle E^2(t)\rangle = \frac{1}{2E_0}\int_{-E_0}^{E_0} E^2(t)\, dE(t) = \frac{E_0^2}{3}. \tag{17}$$

$\xi(t) = E^2(t) - \langle E^2(t)\rangle$ represents time dependent fluctuations of E about this mean value.

The Hamiltonian of equation (15) can now be rewritten as a sum of a constant part (causing the quadratic Stark shift) and a time-dependent perturbation which is a fluctuating term with mean value zero:

$$\mathcal{H} = \frac{1}{2}\begin{pmatrix} \alpha_{\mathrm{r}'} & 0 \\ 0 & \alpha_{\mathrm{r}} \end{pmatrix}\frac{E_0^2}{3} + \frac{1}{2}\begin{pmatrix} \alpha_{\mathrm{r}'} & 0 \\ 0 & \alpha_{\mathrm{r}} \end{pmatrix}\xi(t). \tag{18}$$

The temporal correlations of $\xi(t)$ can be rewritten in terms of $E(t)$:

$$\begin{aligned}
\langle \xi(t)\xi(t')\rangle &= \langle (E^2(t) - \langle E^2(t)\rangle)(E^2(t') - \langle E^2(t')\rangle)\rangle \\
&= \langle E^2(t)E^2(t')\rangle - 2\langle E^2(t)\rangle\langle E^2(t')\rangle + \langle E^2(t)\rangle\langle E^2(t')\rangle \\
&= \langle E^2(t)E^2(t')\rangle - \langle E^2(t)\rangle^2 .
\end{aligned}$$

Then, using

$$\langle E^2(t)E^2(t')\rangle = \frac{1}{2E_0}\int_{-E_0}^{E_0} E^2(t)E^2(t')dE(t) = \frac{E_0^4}{5} \tag{19}$$

we find that for equal times ($t = t'$):

$$\langle \xi(t)\xi(t)\rangle = \frac{E_0^4}{5} - \frac{E_0^4}{9} = \frac{4}{45}E_0^4. \tag{20}$$

For unequal time, we assume that the correlation function decays exponentially with a correlation time τ_{corr}:

$$\langle \xi(t)\xi(t')\rangle \approx \langle \xi(t)\xi(t)\rangle \exp\left[-\frac{|t-t'|}{\tau_{\mathrm{corr}}}\right] = \frac{4}{45}E_0^4 \exp\left[-\frac{|t-t'|}{\tau_{\mathrm{corr}}}\right]. \tag{21}$$

We assume that the noise correlation time is much shorter than the typical timescales governing the evolution of the collective qubit. In this case we can approximate the rapidly decaying correlations with a delta-function:

$$\langle \xi(t)\xi(t')\rangle \approx 2\tau_{\mathrm{corr}}\frac{4}{45}E_0^4\,\delta(t-t'), \tag{22}$$

Where the factor of $2\tau_{\mathrm{corr}}$ ensures that the integral over the noise correlations is unchanged. The evolution of the density matrix of the system

$$\rho = \begin{pmatrix} \rho_{\mathrm{r'r'}} & \rho_{\mathrm{rr'}} \\ \rho_{\mathrm{r'r}} & \rho_{\mathrm{rr}} \end{pmatrix} \tag{23}$$

is then obtained via a Lindblad master equation of the form

$$\frac{\partial}{\partial t}\rho(t) = -\frac{\mathrm{i}}{\hbar}\left[\mathcal{H},\rho(t)\right] + \mathcal{D}(\rho(t)). \tag{24}$$

After Chenu et al [1], the dissipator $\mathcal{D}(\rho(t))$ is given by

$$\mathcal{D}(\rho(t)) = -2\tau_{\mathrm{corr}}\frac{4}{45}E_0^4\frac{1}{\hbar^2}\left[\frac{1}{2}\begin{pmatrix} \alpha_{\mathrm{r'}} & 0 \\ 0 & \alpha_{\mathrm{r}} \end{pmatrix}, \left[\frac{1}{2}\begin{pmatrix} \alpha_{\mathrm{r'}} & 0 \\ 0 & \alpha_{\mathrm{r}} \end{pmatrix}, \rho(t)\right]\right] \tag{25}$$

$$= -\frac{1}{2}\tau_{\mathrm{corr}}\frac{4}{45}E_0^4\frac{1}{\hbar^2}\begin{pmatrix} 0 & (\alpha_{\mathrm{r}}-\alpha_{\mathrm{r}})^2\rho_{\mathrm{rr'}} \\ (\alpha_{\mathrm{r'}}-\alpha_{\mathrm{r}})^2\rho_{\mathrm{rr'}} & 0 \end{pmatrix}. \tag{26}$$

Coherences between the qubit states are expressed by the operator

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{27}$$

which evolves according to

$$\frac{\partial}{\partial t}\sigma_x(t) = -\frac{\gamma_{\mathrm{deph}}}{2}\sigma_x(t), \tag{28}$$

where

$$\gamma_{\mathrm{deph}} = \tau_{\mathrm{corr}}\frac{4}{45}\cdot\frac{(\alpha_{\mathrm{r'}}-\alpha_{\mathrm{r}})^2\,E_0^4}{\hbar^2}. \tag{29}$$

is the dephasing rate.

In order to investigate the dependence of shift and dephasing on the amplitude of electrical noise applied to the collective qubit, the qubit was modelled as a two level system as with equation (15), where an energy offset is included to give a symmetric symmetric level shift $\Delta_{\text{int}} = (\alpha_{\text{r}'} - \alpha_{\text{r}})E_0^2/3$ and dephasing rate γ_{dep}.

$$\dot{\rho} = -\frac{\mathrm{i}}{\hbar}[\mathcal{H}, \rho] + \gamma_{\text{dep}} \left[L\rho L^\dagger - \frac{1}{2}\left\{L^\dagger L, \rho\right\}\right] \tag{30}$$

where the Hamiltonian of the qubit was parameterised with both the interferometer shift, and the detuning of the microwave field coupling S and P.

$$\mathcal{H} = \frac{1}{2}\begin{pmatrix} \Delta_{\text{int}} + \delta_{\text{int}} & 0 \\ 0 & -\Delta_{\text{int}} - \delta_{\text{int}} \end{pmatrix} \tag{31}$$

Fringe visibilities were used to determine the degree of dephasing. Visibilities are calculated with simple sinusoidal fits of the form $A_{\text{fit}} \sin(w_{\text{fit}} * \Delta_{\text{int}} + \phi_{\text{fit}}) + O_{\text{fit}}$, where A_{fit} is the fringe amplitude, w_{fit} the frequency, ϕ_{fit} the phase offset and O_{fit} is the amplitude offset.

$$\text{Vis} = A_{\text{fit}}/O_{\text{fit}}. \tag{32}$$

This simple calculation compensates systematic errors arising due to shot-to-shot fluctuations in storage/retrieval efficiency observed during operation of the experiment.

In order to compare quartic and quadratic models, a series of fits were performed on datasets comprising of N data points spanning $E^{(0)} = 0$ V/cm to $E^{(n)}$, where $E^{(n)}$ is the electric field at the n'th data point. The goodness of these fits is summarised in figure (S2)b. From these fits and residuals, it can be seen that the reduction in visibility is initially quartic, diverging from this relationship only at higher electric fields $E_0 > 2$ V/cm. Comparison of this data with the stark maps for Rubidium (see figure (S3)) show that the breakdown of the quartic model occurs due to the complex stark splitting with an onset of $E_0 = 4$V/cm. This complex stark splitting also causes a reduction in retrieval efficiency after exposure to strong electrical noise.

References

[1] Chenu, A., Beau, M., Cao, J. & del Campo, A. Quantum simulation of generic many-body open system dynamics using classical noise. *Phys. Rev. Lett.* **118**, 140403 (2017).

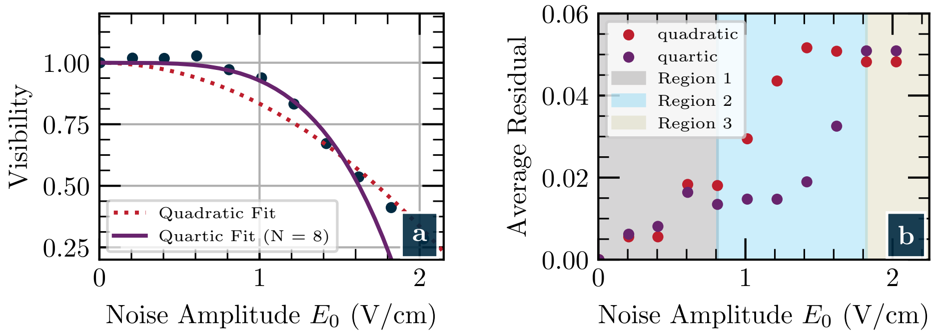

Figure S2: Comparing Quadratic and Quartic models to fringe visibility. **a** Quadratic (red) and quartic (purple) fits to the normalised fringe visibility of the interferometer as a function of electric field. The quadratic fit outperforms the quartic model when fitting the whole dataset. However the quartic model performs significantly better when fitting only the first N data points, where $N < 8$ and the visibility remains above 50 percent. **b** Average residuals of quadratic (red) and quartic (purple) fits to the first N visibility data points. Three regions are highlighted in grey (R1, $E_0 < 0.8$ V/cm), cyan (R2, 0.8 V/cm $< E_0 < 1.8$ V/cm), and gold (R3, 1.8 V/cm$< E_0$). In Region 2, a quartic dependence is a much better fit to the data, evidenced by smaller average residuals. In region 3, the visibility drops to 20 percent of the visibility at $E_0 = 0$. A departure from strong quartic scaling is observed as visibility approaches the limiting value of zero.

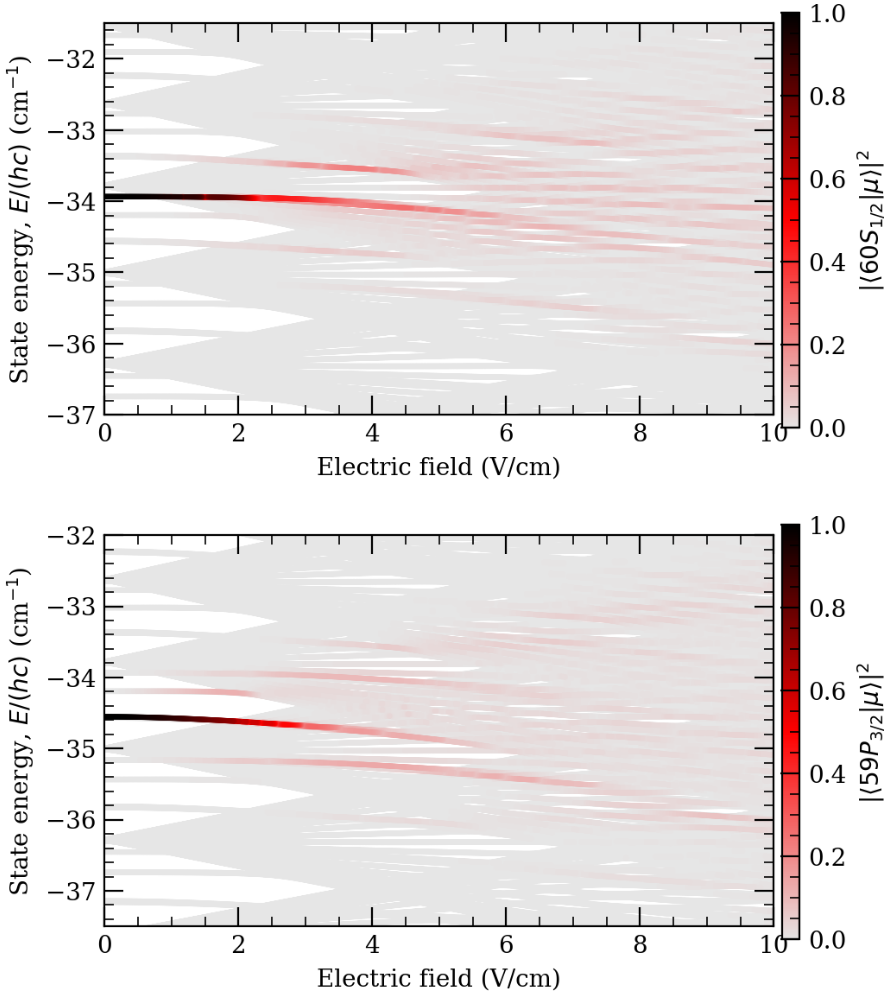

Figure S3: Stark shifts of the $60S_{\frac{1}{2}}$,$59P_{\frac{3}{2}}$ energy level of Rubidium. The stark effect is only quadratic for small E_0. Divergence from a simple E^2 relationship begins at around around $E_0 = 2$ V/cm, and is clearly present at $E_0 = 4$ V/cm for both states. This effect can be seen in the divergence of the observed fringe visibility from the quartic model of figure (S2).