COMPARISON OF EQUIVALENT PLATE MODELS USING WAVENUMBER APPROACH

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ABSTRACT

Mutli-layered structures (such as sandwich panels) are commonly used in engineering applications for the improved sound comfort and noise reduction. Finite Element modelling of these layered structures would often results in increased total number of degrees of freedom which would lead to high computation time. An alternative to the full modelling of multi-layered system is the use of equivalent plate models to simulate the multi-layered system as a single layer. These models would aims at reducing the number of degrees of freedom in finite element models. Applicability of these models would be limited up to certain frequency range due to underlying assumptions of wave propagation in each layer. In this work, these equivalent plate models are compared using wavenumber analysis. Sandwich panels of different types (soft and stiffer cores) are considered for this comparison study and behaviour of each model are analysed.

1. INTRODUCTION

Multi-layer or sandwich composite panels which exhibit high stiffness with light weight are commonly used in the civil and transportation industries. Since the multi-layered system comprises of diversified materials, FE modelling requires suitable mesh types based on the material used in each layer, which would increase total mesh count and thereby increasing the computer power required to do the calculation. Equivalent plate models are used as an alternative which condense the behaviour of the multi-layer system in to a single layer material. This would reduce the computational power significantly when it is used in FE modelling.

In this paper, two frequently used equivalent plate models : Guyader model [1, 2] and RKU model [3–6], are discussed briefly with their underlying assumptions. Using wavenumber analysis, these models are compared to discuss their validity over different sandwich plate types. Finally, the influences of symmetric and antisymmetric motions of the sandwich plates are studied as the present equivalent plate models exhibit only antisymmetric motion of the system.

2. WAVENUMBER ANALYSIS OF PLATE MODELS

2.1 Single isotropic layer

Wave propagation with respect to thin and thick plate theories for a single isotropic layer is briefly explained in this section. Following the usual notations, the propagating wave number solution for the thin and thick plate theories are given as:

$$k_{\text{thin}} = k_b = \sqrt{\omega\sqrt{\frac{m_s}{D}}}, \tag{1}$$

$$k_{\text{thick}} = \pm\sqrt{\frac{1}{2}\left(k_s^2 + k_r^2 + \sqrt{4k_b^4 + (k_s^2 - k_r^2)^2}\right)}, \tag{2}$$

where, $k_s = \omega\sqrt{\frac{m_s}{G^* h}}$ and $k_r = \omega\sqrt{\frac{I_z}{D}}$ represent the corrected shear and membrane wavenumbers respectively. Here, $\omega = 2\pi f$ is the circular frequency, D is the bending stiffness, m_s is the mass per unit area, G^* is the corrected shear modulus and I_z is mass moment of inertia of the layer.

From Eqn. (2), it can be seen that k_{thick} approaches k_{thin} and k_s when $\omega \to 0$ and $\omega \to \infty$ respectively. The same can be observed, for example, in Fig. 1 for a plasterboard plate of 25 mm. From these two asymptotes of k_{thick}, it is inferred that the thin plate assumption is valid up to certain frequency after which the thick plate theory must be considered to include the shear motion of the plate along with bending motion. If the plate is considerably thick and/or soft in terms of shearing motion, the thick plate theory need to be applied. In this context, an analytical frequency limit for the thin plate theory is given in the Ref. [7].

2.2 Equivalent plate theories

By condensing the free vibration behaviour of the multi-layer system, equivalent plate models provide the sin-

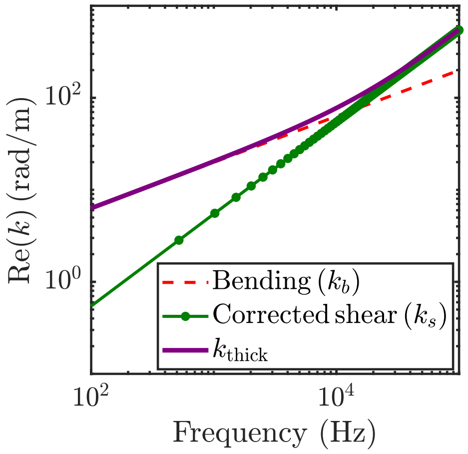

Figure 1. Propagating wavenumbers of a Reissner-Mindlin plate and its asymptotic behaviours. The plate is made of plasterboard with h = 25 mm, E = 3 GPa, ρ = 700 kg/m^3 and ν = 0.22.

gle layer material properties to simulate the same vibro-acoustic behaviour of the multi-layers. Guyader [1, 2] and RKU [3–6] are the two equivalent plate models which are widely used in engineering applications. Though both theories provide equivalent isotropic thin plate properties of a single layer, Guyader model assumes both bending and shear wave propagation in each layer whereas RKU model assumes propagation of bending and shear waves only in the outer and core layers respectively. On the applicability front, Guyader model can be used for any number of multi-layer system while RKU model is used mainly on three-layer sandwich plates.

Equivalent dynamic bending rigidity ($D_{eq}(\omega)$) is computed from these models and substituted in Eqn. (1) to find the propagating wavenumber as D_{eq} corresponds to equivalent single layer isotropic thin plate. In the following subsections, these equivalent plate models are compared in the wavenumber domain for different configurations of three-layer sandwich plates. Sandwich plates from Tab. 1 are considered for comparing these two models.

	Skin	Soft core	Stiff core
h (mm)	5	10	50
ρ (kg/mm^3)	2780	200	2300
E (GPa)	71	0.1	30
ν	0.3	0.3	0.3

Table 1. Material properties of isotropic layers used in this article.

In Tab. 1, h, ρ, E and ν are thickness, density, Young's modulus and Poission's ratio of the isotropic layer respectively.

2.2.1 *Sandwich plate with stiff core*

In this section, a three-layer sandwich plate with stiff core (concrete) bonded by two aluminium skins are considered for the comparison study and the equivalent bending wavenumber computed by different models are presented in Fig. 2. Along with Guyader and RKU models, added stiffness model is also presented here for comparison. In this model, D_{eq} is computed by adding the individual bending stiffness of each layer that is obtained with respect to the neutral axis of the multi-layer plate. This is similar to the approach followed for beams with multi-layers, as described in the Ref. [8].

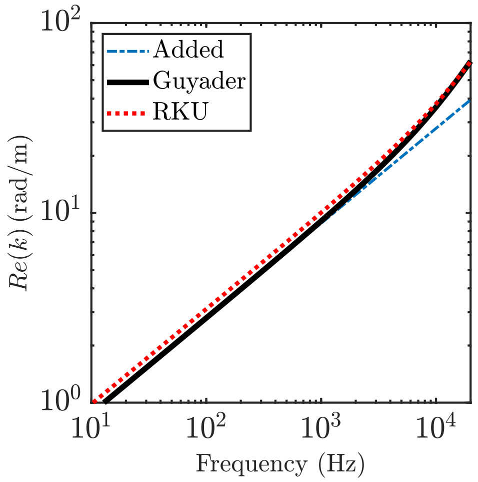

Figure 2. Wavenumbers of equivalent plate theories: sandwich plate with stiff core.

From Fig. 2, it is observed that the low frequency response of Guyader model matches well with the added stiffness model whereas RKU model fails to predict the correct low frequency response. This is mainly due to the assumption made in RKU model that the core is assumed to have shearing motion and not bending motion. Since the core is made of stiffer material in this example, the bending contribution from the core layer is ignored by the RKU model which results in deviated response in low frequencies with other equivalent plate models.

2.2.2 *Sandwich plate with soft core without thickness-stretching effect*

For the sandwich panel with soft core, the wavenumbers obtained from the equivalent plate theories are presented in Fig. 3. Although the core material is soft, the thickness-stretching effect (or compressional effect) is ignored for this example. In other words, only anti-symmetric motions (bending, shear and membrane) are considered for this example and contribution from symmetric motion is ignored.

It is observed from Fig. 3 that both Guyader and RKU models are in good agreement with each other throughout the entire frequency range. Another observation (from Figs. 1 and 3) is that the asymptotic behaviour of the propagating wavenumber of a three-layer sandwich plate is different from that of the isotropic single layer plate. Further, three types of region, associated with asymptotic behaviours, are also observed as described by Boutin and

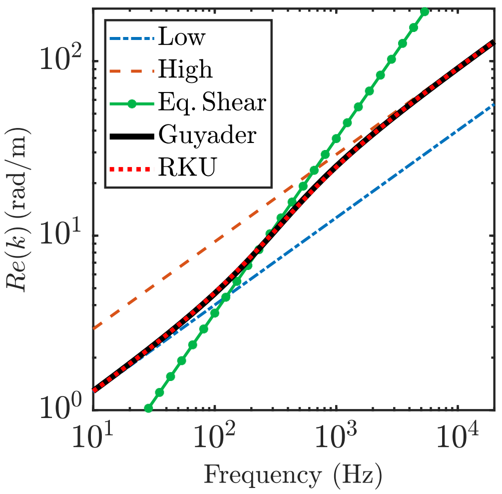

Figure 3. Wavenumbers of equivalent plate theories: sandwich plate with soft core.

Viverge [9]. At low frequencies, the sandwich wave propagation is mainly controlled by the global bending [9] whereas higher frequency region is controlled by the inner bending behaviour of the skins. The transition phase from lower asymptote to upper asymptote is influenced by the shearing behaviour of the core. The bending rigidity expressions to calculate these wavenumber asymptotes are given in Eqn. (3) for symmetric sandwich plate with soft core [9].

$$D_{\text{low}} = D_1 \left(8 + \frac{12h_2}{h_1} + \frac{6h_2^2}{h_1^2} \right);\ D_{\text{high}} = 2D_1, \quad (3)$$

In the above equation, '1' and '2' represents skin and core respectively. The equivalent shear wavenumber is expressed as,

$$k_{\text{eq}_{\text{shear}}} = \omega \sqrt{\frac{M}{G_2 h_t}}, \quad (4)$$

where M and h_t are the total mass per unit area and total thickness of the sandwich plate respectively.

Based on these asymptotic behaviours at three regions (low, high and transition), a simple equivalent plate model can be formulated to compute the dynamic bending stiffness (which would be valid for the entire frequency range) of a three-layer sandwich plate. This model will be proposed in a journal article in the near future [10] and it will be easier for implementation compared to other equivalent plate models.

2.2.3 Sandwich plate with soft core with thickness-stretching effect

In this section, wavenumbers corresponding to anti-symmetric and symmetric motions of three-layer sandwich plates are compared to analyse the influencing effect of symmetric motions in the vibro-acoustic computations. For this purpose, first anti-symmetric (A_0) and symmetric (S_0) wavenumber solutions for the three-layer sandwich panel are computed from the Lamb wave theory [11].

For the three-layer sandwich plate with stiff core, it is observed from Fig. 4 that symmetric wavenumber (correspond to compressional or dilatational motion) starts to propagate only after 20 kHz whereas the anti-symmetric wavenumber (correspond to bending and shear motions) is propagative throughout the frequency range. The reader may note that first anti-symmetric mode solution from Lamb wave theory is well captured by the Guyader model.

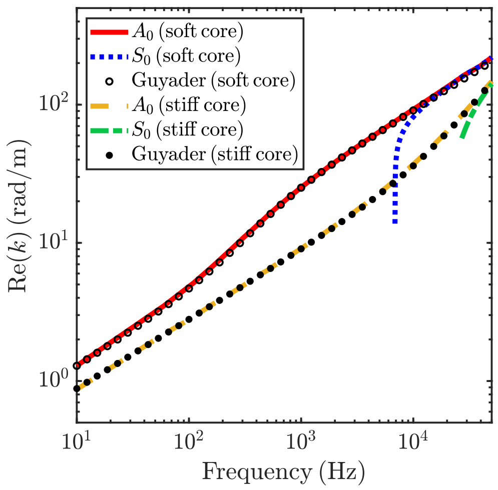

Figure 4. Anti-symmetric (A_0) and symmetric (S_0) wavenumbers of the first mode of a sandwich plate with soft and stiff cores, obtained from Lamb wave theory.

Therefore, for the audible frequency range, the existing equivalent plate models are sufficient to compute the vibro-acoustic indicators if the sandwich core is made of stiffer materials.

In case of a three-layer sandwich plate with soft core, it is observed from Fig. 4 that first symmetric wavenumber starts to propagate from around 4 kHz and progressively reaches the first anti-symmetric wavenumber (or equivalent bending wavenumber from Guyader model) in the higher frequencies.

As the existing equivalent plate models assume constant transverse velocity throughout the plate thickness, these models would not be valid after the frequency at which the symmetric wavenumber starts to propagate. If the core is very soft, the symmetric wavenumber would start to propagate from the low frequencies and therefore, for this kind of system, the existing equivalent plate models would not be suitable candidates to condense or compute the vibro-acoustic behaviours of the multi-layer plates.

In this context, a new condensed model could be developed by including both the symmetric and anti-symmetric admittances of the multi-layer plates by following the work by Dym and Lang [12, 13] to compute the admittances. This model will also be proposed in a journal article [14] in the near future.

3. CONCLUDING REMARKS

Three-layer sandwich panels of different types (stiff and soft cores) are taken for the comparison study of existing equivalent plate models in the wavenumber domain. It is observed that RKU model fails in low frequency region in comparison with Guyader model. Negligence of bending wave propagation inside the core in the RKU model leads to this mismatch at lower frequencies. In case of soft core, both Guyader and RKU model provide well-matched results. Further, from the observations made on this comparison study, two new models would be proposed in the journal articles on the following topics: first, a simple equivalent plate model for three-layer sandwich system that would be formulated based on the physical behaviours observed in the low, high and transition frequency regimes; second, a new condensed model which would take contributions from both symmetric and anti-symmetric motions of the symmetric multi-layer plate to compute the equivalent dynamic response.

4. ACKNOWLEDGEMENT

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