Money as a weapon: Financing a winner-take-all competition[*]

Sanjay Banerji[†]
Nottingham University

Dawei Fang[‡]
University of Gothenburg

Abstract

We study capital structure of pioneering startup firms who are often credited with opening of new markets and niches in the digital era and often face the threat of potential entry of successful, cash-rich firms from adjacent markets. Our analysis is made in the context of a winner-take-all competition in the form of an all-pay auction for the monopolistic position in a new market. We show that in such scenarios, a pioneer's optimal capital structure exhibits widespread diversity and is determined by a tradeoff between entry deterrence and post-entry competition intensification. In particular, our results show that a pure-equity (a mixture of equity and risky debt) structure is optimal when (1) barriers to entry are small (large), (2) the future prospect of the new market is fairly certain and/or (3) the externality of winning/losing the new market on the potential entrant's existing business is large (small). The post-entry competition is likely to engender large losses on both the winner and the loser.

Keywords: all-pay auctions, capital structure, financial constraints

[†]Nottingham University Business School. Email: sanjay.banerji@nottingham.ac.uk.

[‡]Centre for Finance and Department of Economics, University of Gothenburg. Email: dawei.fang@economics.gu.se.

1 Introduction

"The world's most valuable resource is no longer oil, but data." This appealing and eventually quite popular statement was made originally by The Economist magazine (2017) in an issue devoted to the emergence of "new" firms in the digital era. Internet firms, like Alibaba, Amazon, Facebook, Google, and others, have established their dominant positions in their respective industries by superior abilities to collect, process, and use a vast amount of data arising from business transactions and social interactions. A salient feature of such industries, as pointed out by Shapiro and Varian (1999, p.182), is that "the lion's share of the rewards will go to the winner, not the number two player who just manages to survive."

When new markets (e.g., Google with search engines, Alibaba and Amazon with all-encompassing online business), new niches, or new ways of conducting an old business (e.g., Netflix with movies, Uber and Lyft with transportation services) are being created by pioneering firms who move first, the potential large rewards offered by these new markets can invite entry of other firms who are often cash-rich and successful in adjacent markets. Entry, in turn, leads to fierce competition between fledgling pioneers and cash-rich entrants from adjacent markets, in multiple dimensions like pricing, quality, advertisements, product developments, etc. In the end, very few wins. For example, Netscape, founded in 1994, was a pioneering startup in the browser market and dominated the market in the mid-90s. However, its success soon drew the attention of Microsoft, a dominant player in the operating system market by that time. After entering the browser market, Microsoft launched a browser war against Netscape and eventually took away Netscape's market share in a couple of years (cf. Hoffmann, 2017). For another example, Renren, founded in December 2005, was a pioneer in the social networking market in China before 2009, but it soon lost its market share after the entry of two Chinese internet giants, Tencent and Sina, into the market around 2010 (Liang, 2018).[1] Of course, there are also examples of pioneering firms retaining their first-mover advantage and dominant market position. Amazon is one such example. Amazon was originally a pioneer in the online bookstore market; traditional book retailers never successfully challenged Amazon's dominant force in this market.

Two distinguished features mark such a battle between a young incumbent and a cash-rich, established firm entering from an adjacent market. First, to win the battle, both firms often have to spend profligately on marketing and advertising and heavily subsidize consumers or even provide consumers completely free services, products, and technologies.[2] Second, as we mentioned earlier, the winner

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[1]Renren was originally named Xiaonei.

[2]For example, according to a Financial Times article (Clover, 2016), "Burning cash has become alarmingly fashionable among Chinese internet companies, many of whom have taken to paying customers massive subsidies to use their services in hopes that their competitors go out of business before they run out of money."

could be either the pioneering firm or the cash-rich entrant but hardly both, even though both firms may throw massive resources into competition.

Given that this sort of winner-take-all competition features "cash burning" while pioneering startups often lack internal cash and, thus, need to raise cash externally to be prepared for the competition, it is natural to ask the following questions: What is the optimal financial arrangement (debt and/or equity financing) for a pioneering firm under the threat of a potential winner-take-all battle? How does a pioneering firm's capital structure affect its potential (cash-rich) rival's decision to enter and the post-entry competition strategies? Does a pioneering firm's first-mover advantage affect its preference between debt and equity financing? How does uncertainty in the future prospect of the market affect a pioneering firm's capital structure? Does a pioneering firm prefer high or low leverage when its development in the new market is likely to pose a threat to the existing business of a potential, cash-rich entrant?

We address these questions based on a three-stage model. In the first stage, a pioneering firm, who is the incumbent in a new market (or niche), raises external capital through debt and/or equity financing, anticipating potential entry of a cash-rich rival (e.g., an established firm in an adjacent market) in the second stage and a winner-take-all competition in the third stage if entry takes place in the second stage. To enter the market, the cash-rich firm has to pay an entry cost, which represents the pioneering firm's first-mover advantage. Post-entry competition takes the form of an *all-pay* auction (Baye et al., 1996; Che and Gale, 1998): the competing firms simultaneously submit a bid, whose cost is sunk regardless of which firm wins (so called "all-pay"), and the firm that submits a higher bid wins a prize (e.g., future monopoly profits in the new market). One can think of a bid a firm chooses as representing the amount of money it spends on advertising and marketing or the quality of free services it offers to consumers. In the context of races, such as R&D races and high-frequency trading arms races (Budish et al., 2015), a bid can be interpreted as investment in (innovation or trading) speed. In fact, all-pay auction models have been applied in the literature to study internet firm competitions (Noe and Parker, 2005) and R&D races (Leininger, 1991). We assume that the cash-rich firm has abundant internal cash that allows it to submit any bid, whereas the maximum bid the pioneering firm can submit depends on how much capital it has raised externally.

In our model, when making its financial decision in the first stage, the pioneering firm faces two types of uncertainties. First, it faces (ex post) uncertainty in its prize value. We assume that a firm's prize value depends on an uncertain economic condition and this uncertainty is resolved only after the winner is identified. Because the pioneering firm's prize is low when the economic condition is poor, this prize value uncertainty makes it possible that a highly levered pioneering firm defaults on its debt even if

it wins the prize. As we demonstrate in the paper, this strategic default can increase the pioneering firm's incentives to submit a high bid in post-entry competition, since under strategic default, the pioneering firm does not fully internalize the cost of a high winning bid. Second, the pioneering firm is uncertain about the potential entrant's relative strength (although this uncertainty is assumed to be resolved post entry), modeled as the ratio between the two firms' prize values.

We find that these two types of uncertainties introduce a tradeoff that shapes the pioneering firm's capital structure. This tradeoff stems from the limited liability effect of debt on the post-entry all-pay auction. As is well known (cf. Siegel, 2009), in such an auction (in our context, the post-entry subgame), both firms randomize their bids on a common interval, between zero and the minimum of the two firms' willingness to pay.[3] We show that increasing the debt level of the pioneering firm tends to increase the pioneering firm's willingness to pay, thereby increasing its aggressiveness (in the sense of choosing a high bid with higher probability) against its rival. This effect tends to deter entry and favors the use of debt. However, when the rival is sufficiently strong and decides to enter anyway, the rival will respond to increased aggressiveness of the pioneering firm by also being more aggressive (i.e., choosing a high bid with higher probability). Intensified post-entry competition reduces the pioneering firm's profit whenever entry preemption fails, which effect disfavors the use of debt. The pioneering firm's optimal debt level is thus determined by the tradeoff between entry deterrence and post-entry competition intensification.

The primary contribution of our paper is that using this tradeoff, we provide a number of novel, testable hypotheses linking a pioneer's capital structure with market environment and competition strategies. Below we provide a brief discussion of our results on the optimal capital structure of a pioneering firm, followed by a collection of implications. First, the optimal capital structure is either a high-leverage structure, which better deters entry but intensifies post-entry competition, or a pure-equity structure. A low-leverage structure with risky debt is never optimal, because, as we demonstrate in the paper, a low level of risky debt is insufficient to increase the pioneering firm's willingness to pay but is sufficient to induce the rival to behave more aggressively post entry. Consequently, a low level of risky debt issued by the pioneer intensifies post-entry competition without providing any entry-deterrence benefit. Second, a pure-equity (leveraged) capital structure is optimal when the potential rival's entry cost is small (large) and/or when the potential rival is likely (unlikely) to value the monopolistic position in the new

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[3]A firm's "willingness to pay" is defined in our paper as the maximum bid at or below which the firm is willing to pay for the prize. Our description of firms' bidding strategies here assumes that, in the post-entry subgame, firms have a sufficient amount of capital to submit a bid equal to their willingness to pay, which will be the case on an equilibrium path of the full game.

market highly. Next, a pure-equity capital structure is optimal when the pioneering firm's prize value is sufficiently insensitive to the future economic condition. The reason for this result is that if there is only a small difference in the prize value between good and bad economic states, the entry deterrence effect (discussed above) of debt is very limited and is dominated by the competition intensification effect. Finally, the pioneering firm always raises a sufficient amount of external capital that allows it to submit a bid equal to its willingness to pay.

These results give rise to rich empirical implications. First, our results imply that young firms pioneering in winner-take-all markets usually have extreme capital structures, with either no risky debt or high leverage. Second, our analysis predicts for a positive association between a pioneering firm's leverage and its first-mover advantage (modeled as the entry cost a rival has to pay to possibly be on par with the pioneering firm). Next, in practice, a cash-rich firm in an adjacent market is likely to highly value the dominant position in the new market if winning such a dominant position creates a synergy with its existing business and/or if the pioneering firm's development in the new market is likely to pose a threat to its existing business. Thus, our result that a pure-equity (high-leverage) capital structure is optimal for the pioneering firm if the potential rival is likely (unlikely) to highly value the monopolistic position in the new market implies a negative association between a pioneering firm's leverage and the externality of the new market on a potential entrant's existing business. Our analysis might shed light on why pioneering firms who anticipate to be challenged by established firms in adjacent markets often use equity financing.[4] Moreover, our model predicts that pioneering firms are likely to use equity financing in winner-take-all markets whose prospect is fairly insensitive to economic conditions. Finally, our results imply that, when a "cash-burning" battle takes place in a winner-take-all market, who wins the battle and how much cash is "burnt" in the battle are both highly uncertain. It is likely that competition engenders losses on all competitors, including the winner, and this is most likely to be the case if pioneering firms are highly levered.

Related literature The literature most closely related to this paper is on the interaction between financial structure and market competition. Brander and Lewis (1986) show that firms taking on more debt behave more aggressively in Cournot quantity competition and enjoy a strategic advantage, in that higher leverage of a firm causes the rival's equilibrium output to fall. McAndrews and Nakamura (1992) further argue that this strategic advantage can be used by an incumbent monopolist to effectively deter entry without deviating from the (all-equity) monopoly output. Showalter (1995) shows that, under

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[4]For example, Netscape raised capital via IPO in 1995 right before Microsoft launched the browser war by releasing version 1.0 of Internet Explorer.

Bertrand price competition, the use of debt leads to a rise (fall) in prices and firm profits when demand is (costs are) uncertain. Showalter (1999) further argues that, if post-entry competition is Bertrand and costs are uncertain, an incumbent monopolist can use debt to commit to an aggressive post-entry pricing strategy and thereby deter entry.

We contribute to this literature by considering market competition that features "cash burning," modeled as an all-pay auction. Unlike in Cournot (quantity) and Bertrand (price) competitions, where firms' strategies are strategic substitutes and complements, respectively, in an all-pay auction, firms' strategies exhibit certain degree of both substitutability (in that submitting a sufficiently high bid can force rivals to quit) and complementarity (in that simply increasing a bid may encourage rivals to do the same). Thus, the existing insight derived from Cournot and Bertrand models does not provide a straightforward answer to our research questions. We find that the interaction between financial structure and market competition for an all-pay auction looks most like the interaction for Bertrand competition with cost uncertainty (Showalter, 1995, 1999), in that in both cases, an incumbent's use of debt can produce an entry deterrent effect and a post-entry competition intensification effect. However, the differences are also evident. While in Bertrand competition with cost uncertainty, an increase in a firm's debt leads to each firm choosing a more aggressive strategy (i.e., a lower price) with certainty, in an all-pay auction, an increase in a firm's debt at most leads to each firm choosing a more aggressive strategy (i.e., a higher bid) in a stochastic sense. Moreover, in an all-pay auction, it is possible that an increase in a firm's debt has no effect on the firm's own strategy but induces its rival to be more aggressive (in a stochastic sense), a case that is absent in both Bertrand and Cournot models.

Part of the literature emphasizes the strategic role of cash in product market competition (cf. Ma et al., 2019) and patent races (cf. Schroth and Szalay, 2009). While these papers do allow firms to finance externally, their focus is on the costs of external financing (rather than the effects of debt/equity financing) and the importance of firms' internal cash. In contrast, in our model, although pre-competition cash also plays a strategic role, how much initial cash a firm holds internally is not important; what is important is the financing method a firm uses to raise external capital.

Our paper also relates to the literature on the effects of financial constraints and capital structures in settings that resemble auctions. Chowdhry and Nanda (1993) show that, in a takeover contest modeled as an English auction, a bidding firm that can expropriate its existing debtholders by issuing new debt with equal or senior priority can commit to bidding more than its valuation of the target, thereby deterring entry of subsequent bidders. In contrast, Clayton and Ravid (2002) find that, when the value of a firm's existing debt cannot be usurped through the issuance of new debt, increasing a firm's leverage weakly

reduces a firm's bid in English auctions and first-price auctions. Although, like Chowdhry and Nanda (1993), we also identify an entry deterrence effect of debt, in our setting, this effect does not require the first mover to be able to issue new debt to expropriate its current debtholders. Moreover, in our setting, because of the "all-pay" nature of competition, the first mover can never commit to "overbidding" (i.e., bidding more than the expected value of its prize); the use of debt at most induces the first mover to overbid with some probability. The reason why an increase in a firm's leverage never increases a firm's bid in Clayton and Ravid (2002) is because there is no ex post uncertainty in each firm's prize value in their model. In fact, absent ex post prize value uncertainty, leverage would also never induce a firm to overbid in our model. Rhodes-Kropf and Viswanathan (2005) study the effects of external financing of bids in first-price auctions. They show that, under pre-auction financing, bidders tend to overbid when financing from a competitive debt market but not when financing from a competitive equity market, and consequently, only pre-auction equity (but not debt) financing can ensure the auction winner to be the highest-valuation bidder. In contrast, while in our all-pay auctions, it is also the case that a bidder can overbid if it uses debt financing but not if it uses pure-equity financing, the winner is never guaranteed to be the highest-valuation bidder, regardless of the financing method used. Gorbenko and Malenko (2017) show that, when bidders in English auctions can pay their bids in both cash and stock, a bidder's financial constraint does not affect the bidder's aggressiveness in the auction.[5] In contrast, in our "cash-burning" type of all-pay auction, a firm's bid has to be in cash and, if after fundraising, a firm is still financially constrained, it will be less aggressive than what it would be otherwise.

Modeling the competition between Web-based firms as an all-pay auction has also been adopted by Noe and Parker (2005), who focus on characterizing the strategies and return structures resulting from an all-pay auction between two financially unconstrained, ex-ante symmetric firms. In contrast, we study competition between a financially constrained and a financially unconstrained firm with different prize valuations and we focus on the strategic role of the financing choice of the financially constrained firm.

The remainder of the paper is organized as follows: Section 2 describes the setup of the basic, multistage model. Section 3 solves the model using backward induction. In particular, Section 3.1 derives an equilibrium (in mixed strategies) of a post-entry subgame and establishes a link between a pioneering firm's leverage, its willingness to pay, and equilibrium properties of firms' bidding strategies. Section 3.2 analyzes a potential entrant's entry decision in a pre-entry stage. Section 3.3 incorporates

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[5]The literature on security-bid auctions (cf. DeMarzo et al., 2005) studies auctions where bids can be made in securities. This literature is interested in the effect of security bids on the seller's revenue and uses information-based models where bidders have private information on their valuations. In contrast, in our setting, bids must be made in cash, bidders have no private information, and there is no seller who designs the auction to maximize revenue.

the analysis of Sections 3.1 and 3.2 into the pioneering firm's (incumbent's) capital structure decision made in the initial stage. Section 4 considers two extensions of the basic model. Section 5 concludes. All of the formal proofs are relegated to the Online Appendix.

2 The model

Consider the following situation faced by firm A, a pioneering firm in a new market (or niche). This market has a winner-take-all feature, i.e., it creates natural monopoly. Firm A has spent up its internal cash developing the market and is now facing the potential threat from firm B, a cash-rich established company in an adjacent market.[6] Firm B is potentially interested in entering the new market to compete for the monopolistic position in the new market. If firm A retains its monopolistic position (either because firm B stays away from competition or because firm A bests firm B in the new market), firm A will receive a prize, represented by future monopoly profits in the new market. It is common knowledge that the value of firm A's prize, denoted by $\tilde{v}_A$, depends on the future state of the economy. If the future state of the economy is good, which happens with probability $\lambda \in (0, 1)$, $\tilde{v}_A = v_H$; if the future state of the economy is bad, which happens with probability $1 - \lambda$, $\tilde{v}_A = v_L$, where $0 < v_L < v_H$. If firm B takes over the monopolistic position in the new market, firm B will receive a prize, whose value is denoted by $\tilde{v}_B$ and is in proportion to $\tilde{v}_A$, i.e., $\tilde{v}_B = \rho \tilde{v}_A$, where ρ represents the *relative strength* of firm B. ρ is initially unknown to firm A but there is no asymmetric information regarding ρ. We will elaborate on the information structure regarding ρ below. The future state of the economy is realized at the end of the game. All players are risk neutral. The risk-free interest rate is zero. There is no discounting of future.

Our model has three dates, date 0, 1, and 2. At date 0, firm A, which has no debt obligation initially but has spent up its internal capital, anticipates potential future competition and raises external capital, through debt and/or equity financing from a competitive capital market, to be prepared for the potential competition.[7] Firm A's capital structure after fundraising is summarized by its capital, K_A, and its *debt obligation* (i.e., the face value of the debt security held by firm A's creditors), D_A. Firm A chooses K_A and D_A to maximize its internal shareholders' expected profit. The choice of the pair, (K_A, D_A), is subject to two constraints. First, we assume that firm A is only allowed to choose a capital structure with capital exceeding debt obligation (i.e., with $K_A > D_A$). This is a simplifying assumption and we will show in Section 4.2 that relaxing this assumption will not alter our conclusions qualitatively. The second constraint is the participation constraint of external investors (i.e., creditors and external shareholders).

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[6]Allowing firm A to own some internal capital does not change the qualitative nature of any of our results.

[7]Throughout, we use "cash" and "capital" interchangeably.

To finance firm A, external investors need at least break even. In fact, as we will show in Section 3.3, for any choice of (K_A, D_A), $K_A > D_A$, that produces a positive surplus to firm A (i.e., a positive expected joint profit to all firm A's stakeholders), there always exists a financial arrangement (a way of raising capital through debt and/or equity financing) under which all external investors just break even, leaving the entire surplus to the internal shareholders of firm A. Thus, any choice of (K_A, D_A) that produces a positive surplus to firm A will be consistent with the external investors' participation constraint. We assume that, at date 0, when firm A chooses its capital structure, (K_A, D_A), it does not know firm B's exact relative strength, ρ; it only knows that firm B's relative strength is uniformly distributed over $[0, z]$, where $z > 1$ represents the maximum type of firm B.

At date 1, firm B's relative strength, drawn uniformly from $[0, z]$, becomes public information. Firm B observes firm A's capital structure and decides whether to enter the new market or not. To simplify our analysis, we assume that firm B has abundant internal capital, with the precise condition given by equation (1) below. This assumption fits the case in which firm B has other business that has generated cash over an extended period of time. To enter, firm B has to spend a fixed cost c. The entry cost, c, can be considered as representing firm A's first-mover advantage in the new market. If firm B does not enter, the game ends and firm A wins the prize automatically, at which point the future state of the economy is realized and determines the value of firm A's prize. If firm B enters, the game moves on to date 2. If firm B is indifferent between entering and not entering, we assume that firm B does not enter. Firm B has a sufficient amount of capital at hand in the sense that its capital, K_B, and debt obligation, D_B, in its existing capital structure satisfy that

$$K_B \geq z v_H + c + D_B. \tag{1}$$

Condition (1) means that, even if firm B pays the entry cost and submits a bid equal to $z v_H$ (an upper bound on the value of firm B's prize), firm B can still fully pay its debt obligation. While we do not rule out the possibility that firm B raises extra money, as will be clear, under condition (1), firm B has no incentive to raise extra money and will always fully pay its debt obligation.

If firm B enters, at date 2, the two firms compete for the monopolistic position in the new market in the form of an all-pay auction. In particular, both firms simultaneously submit a costly bid. A firm's bid can be interpreted as the money the firm spends to win the market, e.g., a firm's marketing and advertising expenses, consumer subsidies, R&D expenditures. The cost of a bid equals the amount bid. Each firm can submit any nonnegative bid no greater than the firm's capital. The firm that submits a higher bid wins the prize and the losing firm wins no prize. We assume an arbitrary tie-breaking rule
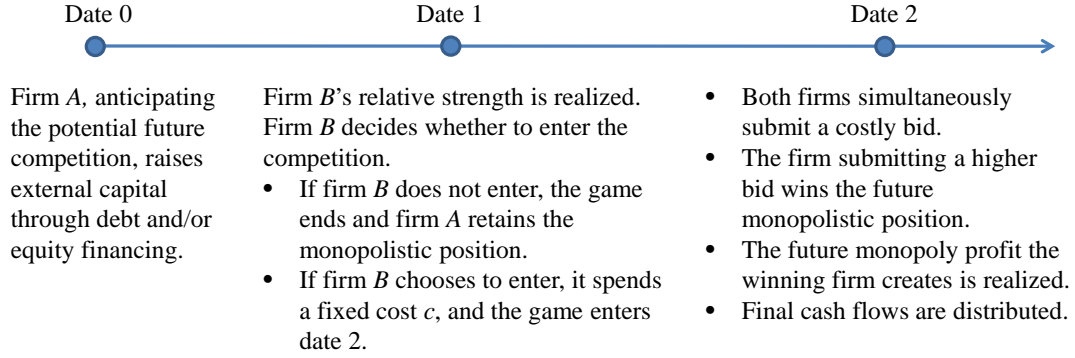
Date 0	Date 1	Date 2

Firm *A,* anticipating the potential future competition, raises external capital through debt and/or equity financing.

Firm *B*'s relative strength is realized. Firm *B* decides whether to enter the competition.
- If firm *B* does not enter, the game ends and firm *A* retains the monopolistic position.
- If firm *B* chooses to enter, it spends a fixed cost *c*, and the game enters date 2.

- Both firms simultaneously submit a costly bid.
- The firm submitting a higher bid wins the future monopolistic position.
- The future monopoly profit the winning firm creates is realized.
- Final cash flows are distributed.

Figure 1: Timeline of the model

except for the following case: if a tie occurs at K_A, firm *B* wins with certainty.[8] After the prize is allocated, the future state of the economy is realized, which, together with the identity of the winner, determines the realized value of the prize. Afterward, each firm's final cash flow is divided between the firm's shareholders and creditors (if any) as follows: if the firm's debt obligation is D and its final cash flow is x, the firm's creditors obtain $\min[D, x]$ and the firm's shareholders receive $\max[x - D, 0]$. Figure 1 summarizes the timeline of our model.

We assume that each firm, at any point in time, acts on behalf of its shareholders at that point in time. Because, whenever a firm issues equity to external shareholders, the interests of the firm's internal and external shareholders will be aligned after the equity issuance, our assumption is equivalent to assuming that each firm always acts on behalf of its internal shareholders. To obtain a sharper prediction of a firm's capital structure, we assume that, if raising extra capital does not benefit a firm, the firm raises no extra capital. The equilibrium concept we adopt is subgame perfect Nash equilibrium.[9]

3 Model analysis

3.1 Date 2: The competition stage

The game can be solved backward. In what follows, we first analyze the subgame at date 2 (the competition stage), which is an all-pay auction with the two firms' prize valuations and capital structures

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[8]In fact, as long as an equilibrium exists, the choice of the tie-breaking rule does not affect equilibrium outcomes because, on an equilibrium path, a tie never occurs. The extra care we give to the tie-breaking rule is to ensure equilibrium existence for subgames on an off-equilibrium path (in particular, the subgames where firm *A* does not raise a sufficient amount of capital).

[9]While firm *A* knows firm *B*'s type at date 1 but not at date 0, there is no asymmetric information in our model. It is the nature who picks firm *B*'s type at the beginning of date 1 that determines which date-1 subgame to play (conditional on firm *A*'s capital structure choice at date 0).

being public information.[10] For ease of exposition, we will focus our discussion on any date-2 subgame in which firm B raises no extra capital at date 1 and firm A's capital after fundraising, K_A, is sufficiently large in that

$$K_A \geq \max[\lambda\, v_H + (1-\lambda)\, v_L, \lambda\, v_H + (1-\lambda)\, D_A], \qquad (2)$$

where D_A is firm A's debt obligation after fundraising. As we will see from our analysis below, condition (2) ensures that firm A's capital constraint never binds in any equilibrium of the date-2 subgame. As will be demonstrated later, any date-2 subgame that is on an equilibrium path of the full game must satisfy these conditions. We relegate a more complete analysis of the date-2 subgame (including subgames on an off-equilibrium path) to the Online Appendix.[11] Because, to enter the competition, firm B has to spend a fixed cost c at date 1, firm B, without raising extra capital, has capital $K_B - c$ at the beginning of our date-2 subgame.

To provide an equilibrium characterization of this subgame, first note that any equilibrium of this subgame must be in mixed strategies. This is because each firm has a desire to "win small," i.e., win by submitting a bid slightly higher than its rival's. Consequently, any deterministic low bid would be profitably topped by the rival and any deterministic high bid that causes the rival to concede victory (i.e., to bid zero) could be profitably lowered by the firm, both of which preclude the emergence of a pure-strategy equilibrium.

To characterize the equilibrium bid distributions of firm A and firm B, denoted by F_A and F_B respectively, it is helpful to first find an upper bound of the support of each firm's bid distribution. We define each firm's *willingness to pay* as the bid (or the highest bid if there are multiple such bids) at which the firm is indifferent between winning at this bid and quitting the competition (or equivalently, losing at a zero bid).[12] By the definition of willingness to pay, for each firm, submitting any bid higher than its willingness to pay will be strictly dominated by submitting a zero bid. Thus, for each firm, its willingness to pay serves as an upper bound of the support of the firm's bid distribution. Because the concept of willingness to pay will play a critical role in our analysis, we embark on our analysis by first computing each firm's willingness to pay. Afterward, we show that, in any equilibrium of the date-2 subgame under consideration, the upper bound of the support of F_A and F_B always equals the minimum of the two firms' willingness to pay.

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[10]Recall that, although firm A does not know firm B's exact relative strength, ρ, at date 0, ρ becomes public information at date 1.

[11]See Section A.1 and the proof of Proposition 5 in the Online Appendix.

[12]Our definition of willingness to pay coincides with the concept of *reach* (cf. Siegel, 2009) known in the all-pay contest literature.

Firm *A*'s willingness to pay: Let r_A denote the willingness to pay of firm *A*, whose capital structure after fundraising is given by the pair (K_A, D_A), with $K_A > D_A$ by assumption. Because each firm acts on behalf of its shareholders, by the definition of r_A, firm *A*'s shareholder payoff from winning at r_A must equal firm *A*'s shareholder payoff from quitting the competition. The latter payoff is simply $K_A - D_A > 0$, whereas, to compute the former, let $w_A^s(b)$ be firm *A*'s shareholder payoff from winning at b.[13] Note that $w_A^s(b)$ is given by

$$w_A^s(b) = \lambda \max[v_H + K_A - b - D_A, 0] + (1 - \lambda) \max[v_L + K_A - b - D_A, 0]. \tag{3}$$

Equation (3) is not difficult to understand. If firm *A* wins at bid b, firm *A*'s final cash flow equals $v_H + K_A - b$ if the future state of the economy is good and equals $v_L + K_A - b$ if the future state of the economy is bad. Because firm *A* is obliged to pay its creditors D_A out of its final cash flow before giving its shareholders any cash flow, the final cash flow retained by firm *A*'s shareholders equals $\max[v_H + K_A - b - D_A, 0]$ if the future state of the economy is good and equals $\max[v_L + K_A - b - D_A, 0]$ if the future economy of the state is bad. Equation (3) then follows from the fact that the future state of the economy is good with probability λ and bad with probability $1 - \lambda$.

The definition of r_A implies that $w_A^s(r_A) = K_A - D_A$. Clearly, firm *A*'s willingness to pay, r_A, must be no greater than its prize value in the good state, v_H, since, if firm *A* submitted a bid strictly more than v_H, firm *A*'s final cash flow would always be strictly below K_A (even if firm *A* wins the prize), in which case firm *A*'s shareholders would always receive a payoff strictly lower than their payoff from quitting the competition, $K_A - D_A > 0$. Given that $r_A \leq v_H$ and $K_A - D_A > 0$, it must be that $\max[v_H + K_A - r_A - D_A, 0] = v_H + K_A - r_A - D_A$. Thus, equation (3) and the fact that $w_A^s(r_A) = K_A - D_A$ imply that

$$\lambda (v_H + K_A - r_A - D_A) + (1 - \lambda) \max[v_L + K_A - r_A - D_A, 0] = K_A - D_A. \tag{4}$$

Now we use equation (4) to solve for r_A. Suppose $v_L + K_A - r_A - D_A \geq 0$. Then equation (4) implies that $r_A = \lambda v_H + (1 - \lambda) v_L$, in which case the hypothesis that $v_L + K_A - r_A - D_A \geq 0$ is equivalent to $K_A - D_A \geq \lambda (v_H - v_L)$. Suppose $v_L + K_A - r_A - D_A < 0$. Then equation (4) implies that $r_A = v_H - \left(\frac{1 - \lambda}{\lambda}\right) (K_A - D_A)$, in which case the hypothesis that $v_L + K_A - r_A - D_A < 0$ is equivalent to $K_A - D_A <$

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[13]The word "payoff" stands for "expected payoff" whenever the payoff is random.

$\lambda (v_H - v_L)$. Therefore, we can express r_A as follows:

$$r_A = \begin{cases} \lambda v_H + (1-\lambda) v_L & \text{if } K_A - D_A \geq \lambda (v_H - v_L) \\ v_H - \left(\frac{1-\lambda}{\lambda} \right) (K_A - D_A) & \text{if } K_A - D_A < \lambda (v_H - v_L) \end{cases} \tag{5}$$

$$= \max \left[\lambda v_H + (1-\lambda) v_L, v_H - \left(\frac{1-\lambda}{\lambda} \right) (K_A - D_A) \right], \tag{6}$$

where the equality in (6) follows from the fact that the right hand side of equation (5) is equivalent to $v_H - \left(\frac{1-\lambda}{\lambda} \right) \min[K_A - D_A, \lambda (v_H - v_L)]$, which, given the fact that $a - b \min[c, d] = \max[a - bc, a - bd]$, further equates to the right hand side of (6).

Equations (5) and (6) give two equivalent expressions for firm A's willingness to pay, r_A. In equation (6), the first value in the max function, $\lambda v_H + (1 - \lambda) v_L$, is the expected value of firm A's prize. If firm A were not protected by limited liability, in which case firm A's debt obligation were always fully paid, firm A's willingness to pay would always equal the expected value of its prize. This can be seen from equation (4). If firm A were not protected by limited liability, the max function in (4) would be replaced by its first value, in which case the value of r_A that solves (4) would equal $\lambda v_H + (1 - \lambda) v_L$. However, because of limited liability, as equation (5) shows, firm A's willingness to pay equals the expected value of firm A's prize if and only if firm A's leverage is small, in that firm A's capital, K_A, exceeds its debt obligation, D_A, by at least $\lambda (v_H - v_L)$. When firm A's leverage is large, i.e., when $K_A - D_A < \lambda (v_H - v_L)$, firm A, conditional on winning at the expected value of its prize, lacks sufficient cash flow to fully pay its debt obligation if the future state of the economy is bad.[14] In this case, limited liability protects firm A's shareholders against a negative payoff that would have been received under unlimited liability. This protection raises firm A's willingness to pay.

An inspection of equations (5) and (6) reveals that increasing firm A's leverage, represented by a reduction in $K_A - D_A$, has a nonnegative effect on firm A's willingness to pay, with the effect being strictly positive if the resulting leverage is sufficiently high (i.e., if the resulting $K_A - D_A$ falls below $\lambda (v_H - v_L)$) and with the effect being zero if otherwise. As we will see from our later analysis, this result has significant implications on firms' bidding strategies and firm A's capital structure choice.

By the definition of r_A, bidding more than r_A is suboptimal for firm A. Thus, even absent capital constraint, firm A's bid would never exceed r_A, implying that having $K_A \geq r_A$ ensures that firm A's capital constraint never binds in equilibrium.[15] By equation (6), $K_A \geq r_A$ is equivalent to condition (2).

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[14]This can be seen by noting that, when $K_A - D_A < \lambda (v_H - v_L)$, the first value in the second max function in equation (3) will be negative for $b = \lambda v_H + (1 - \lambda) v_L$.

[15]By equation (6), what determines r_A is $K_A - D_A$ but not K_A per se. We can think of firm A's choice at date 0 as a choice of

Thus, condition (2), which we have imposed for our current analysis, essentially means that we are focusing on the date-2 subgame where firm A's capital constraint never binds in equilibrium.

Firm B's willingness to pay: Let r_B denote the willingness to pay of firm B. Let $w_B^s(b)$ be firm B's shareholder payoff from winning at b. By an argument analogous to the one for deriving the expression for w_A^s, noting that firm B's prize equals firm A's prize multiplied by firm B's relative strength, ρ, and firm B has capital $K_B - c$ at the beginning of date 2, we obtain the following expression for w_B^s:

$$w_B^s(b) = \lambda \max[\rho v_H + K_B - c - b - D_B, 0] + (1 - \lambda) \max[\rho v_L + K_B - c - b - D_B, 0]. \qquad (7)$$

Because the value of firm B's prize is at most ρv_H and $\rho \leq z$, for firm B, bidding more than $z v_H$ clearly destroys its cash flow even if firm B wins. Thus, firm B will never choose any bid greater than $z v_H$. Condition (1) implies that $K_B - c - b - D_B \geq 0$ for all $b \in [0, z v_H]$. Thus, excluding those high bids that firm B never chooses, we can rewrite equation (7) as

$$w_B^s(b) = \rho \left(\lambda v_H + (1 - \lambda) v_L\right) + K_B - c - b - D_B, \quad b \in [0, z v_H]. \qquad (8)$$

Given that firm B has capital $K_B - c$ at the beginning of date 2, if it quits the competition at date 2, its shareholder payoff will be $K_B - c - D_B$. The definition of r_B then implies that $w_B^s(r_B) = K_B - c - D_B$. Hence, by equation (8),

$$r_B = \rho \left(\lambda v_H + (1 - \lambda) v_L\right), \qquad (9)$$

i.e., firm B's willingness to pay simply equals the expected value of firm B's prize. Intuitively, given that firm B's capital is sufficiently larger than its debt obligation, the competition never renders firm B insolvent. Thus, firm B fully internalizes the cost of its bid and behaves as if it had a pure-equity structure. Consequently, by winning at the expected value of firm B's prize, firm B's shareholders receive zero expected profit.

In what follows, we split our analysis of the equilibrium outcomes of the date-2 subgame by first analyzing the case in which firm B has higher willingness to pay and then the opposite case.

3.1.1 The case in which firm B has higher willingness to pay

Suppose that firm B's willingness to pay is higher, i.e., $r_B > r_A$. When firm B has higher willingness to pay, it can secure its shareholders a positive expected profit by submitting a bid equal to firm A's

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K_A and $K_A - D_A$, where $K_A - D_A$ determines r_A.

willingness to pay, r_A, so as to force firm A to concede. If firm B chooses to do so, firm B's shareholders will obtain a payoff equal to $w_B^s(r_A)$, where, by equations (8) and (9), $w_B^s(r_A) = r_B - r_A + K_B - c - D_B$, and firm A's shareholders will obtain their "reservation payoff" (i.e., their payoff from quitting the competition or equivalently, from losing at a zero bid), $K_A - D_A$. However, if firm A concedes, firm B can ensure winning with an arbitrarily small bid, which will in turn induce firm A to compete. Thus, there is no equilibrium in which firm A completely concedes. As our first proposition shows, while in equilibrium, firm B's shareholders do obtain a payoff equal to $r_B - r_A + K_B - c - D_B$ and firm A's shareholders a payoff equal to $K_A - D_A$, both parties receive their associated payoff through a randomization of their bid over the interval between 0 and the minimum of the two firms' willingness to pay (that is, between 0 and firm A's willingness to pay given that $r_B > r_A$). We relegate all the proofs to the Online Appendix.

Proposition 1. *Suppose that at date 2, firm B's willingness to pay is strictly higher, i.e., $r_B > r_A$, where r_A and r_B are given by equations (5) and (9), respectively. Suppose that, at the beginning of date 2, firm A's capital, K_A, satisfies condition (2) and firm B's capital and debt obligation equal $K_B - c$ and D_B respectively. The subgame at date 2 has a unique equilibrium.*

In this equilibrium, firm A places point mass on 0 and randomizes continuously on $(0, r_A]$. Firm B randomizes continuously on $[0, r_A]$. Firm A's shareholder payoff equals $K_A - D_A$ while firm B's shareholder payoff equals $r_B - r_A + K_B - c - D_B$.

Because, as has been shown before, an increase in firm A's leverage weakly increases firm A's willingness to pay, r_A, Proposition 1 has two immediate implications. First, an increase in firm A's leverage weakly increases the upper bound of the support of each firm's bid distribution, implying that an increase in firm A's leverage weakly increases each firm's probability of submitting a very aggressive bid. Second, as shown by Proposition 1, firm B's shareholder payoff equals $r_B - r_A + K_B - c - D_B$, which is negatively associated with firm A's willingness to pay, r_A. Thus, an increase in firm A's leverage weakly reduces firm B's shareholder payoff.

We define a firm's *surplus created at date 2* as the post-competition cash flow that is to be distributed to the firm's stakeholders *less* the capital the firm holds at the beginning of date 2. This surplus represents the expected joint profits the firm creates for its stakeholders at date 2. Because firm B's debt is riskless, the result that an increase in firm A's leverage weakly reduces firm B's shareholder payoff further implies that an increase in firm A's leverage weakly reduces firm B's surplus created at date 2.

In fact, as we will show below, an increase in firm A's leverage also weakly reduces firm A's surplus created at date 2, and increased firm A's leverage tends to increase each firm's bid in the sense of

first-order stochastic dominance (FOSD).[16] Establishing these results requires a closer investigation of the equilibrium bid distribution for firm A, F_A, and for firm B, F_B. While Proposition 1 provides no expression for F_A or F_B, the proposition suggests two indifference conditions that can be used to derive F_A and F_B. In particular, Proposition 1 shows that, in equilibrium, firm B does not place any point mass, implying zero chance of a tie. Thus, given that firm A places point mass on 0 and randomizes continuously on $(0, r_A]$, any bid $b \in [0, r_A]$ has to be a best response for firm A. Because each firm acts on behalf of its shareholders and firm A's shareholders receive a payoff of $K_A - D_A$, it must be that any bid $b \in [0, r_A]$ gives firm A's shareholders a payoff of $K_A - D_A$. We thus obtain the following firm A's indifference condition:

$$F_B(b)w_A^s(b) + (1 - F_B(b))l_A^s(b) = K_A - D_A, \quad b \in [0, r_A], \tag{10}$$

where $w_A^s(b)$, given by (3), represents firm A's shareholder payoff conditional on winning at b, and

$$l_A^s(b) = \max[K_A - D_A - b, 0] \tag{11}$$

represents firm A's shareholder payoff conditional on losing at b. Because firm B does not place any point mass, firm A's probability of winning at bid b simply equals $F_B(b)$. The interpretation of equation (10) is thus straightforward.

Analogously, we can establish firm B's indifference condition to solve for firm A's equilibrium bid distribution, F_A. By Proposition 1, in equilibrium, firm B randomizes continuously on $[0, r_A]$. Because firm A places point mass on and only on 0, for firm B, choosing any bid $b \in (0, r_A]$ never results in a tie. Thus, any bid $b \in (0, r_A]$ has to be a best response for firm B. Because firm B's shareholders receive a payoff of $r_B - r_A + K_B - c - D_B$, it must be that any bid $b \in (0, r_A]$ gives firm B's shareholders a payoff of $r_B - r_A + K_B - c - D_B$. We thus obtain the following firm B's indifference condition:

$$F_A(b)w_B^s(b) + (1 - F_A(b))(K_B - c - b - D_B) = r_B - r_A + K_B - c - D_B, \quad b \in [0, r_A], \tag{12}$$

where $w_B^s(b)$, given by equation (8), represents firm B's shareholder payoff conditional on winning at b, and $K_B - c - b - D_B$ is firm B's shareholder payoff conditional on losing at $b \in [0, r_A]$. In (12), we have applied the fact that F_A, being a probability distribution function, is right-continuous and thus, given that the equality in (12) holds as b approaches zero from the right, the equality also holds when $b = 0$.

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[16]For any two random variables, X and Y, with the cumulative distribution functions denoted by F_X and F_Y, respectively, X (strictly) first-order stochastically dominates Y if and only if $F_X(b) \leq F_Y(b)$ for all b (with strict inequality for some b).

Plug the values of $w_A^s(b)$, $l_A^s(b)$, $w_B^s(b)$, and r_B, using (3), (11), (8), and (9), respectively, into (10) and (12), and rearrange the resulting expressions. This yields the following result on F_A and F_B:

Lemma 1. *The two firms' bid distributions in the equilibrium of the date-2 subgame presented in Proposition 1 both have support equal to $[0, r_A]$, where r_A is given by equation (5), and over this support,*

$$F_A(b) = 1 - \frac{r_A - b}{\rho \left(\lambda v_H + (1 - \lambda) v_L\right)}, \tag{13}$$

$$F_B(b) = 1 - \frac{1}{1 + \frac{\min[b, K_A - D_A]}{\lambda (v_H - b) + (1 - \lambda) \max[v_L - b, -(K_A - D_A)]}}. \tag{14}$$

By equation (6), an increase in firm A's leverage, represented by a reduction in $K_A - D_A$, weakly increases firm A's willingness to pay, r_A. Thus, an inspection of equations (13) and (14) shows that increasing firm A's leverage not only tends to expand the upper bound of the support of the two firms' bid distributions but, for any bid within the old support, F_A and F_B tend to be smaller, implying that both firms' equilibrium bids tend to be larger in the sense of FOSD.

The next proposition provides a more detailed characterization of the effect of firm A's leverage on each firm's bidding strategy. In this proposition, we also show that increasing firm A's leverage tends to reduce firm A's surplus created at date 2, denoted by $\pi_A^g(K_A, D_A; \rho)$ for every $\rho \in (0, z]$ and computed as

$$\pi_A^g(K_A, D_A; \rho) = \int_0^{r_A} \left[(\lambda v_H + (1 - \lambda) v_L) F_B(b) - b\right] dF_A(b)$$

$$= \frac{1}{\rho \left(\lambda v_H + (1 - \lambda) v_L\right)} \int_0^{r_A} \left[(\lambda v_H + (1 - \lambda) v_L) F_B(b) - b\right] db, \quad (15)$$

where the second line follows from equation (13). The interpretation of the integrand in the first line of equation (15) is straightforward. $\lambda v_H + (1 - \lambda) v_L$ is the expected value of firm A's prize, which is captured by firm A if and only if firm A wins. Thus, by bidding b, firm A captures this prize value with probability $F_B(b)$. By bidding b, firm A pays the cost b no matter whether it wins or not. Hence, the integrand in the first line of (15) is firm A's surplus at date 2 conditional on bidding $b \in [0, r_A]$. Integrating with respect to firm A's bid distribution, F_A, gives firm A's surplus at date 2 produced by its random bid.

Proposition 2. *Treat firm A's willingness to pay, r_A, defined in equation (5), as a function of $K_A - D_A$. Let $0 < y'' < y'$ be two distinct values of $K_A - D_A$. Note that r_A is weakly decreasing in $K_A - D_A$, so $r_A(y') \leq r_A(y'')$. Assume that $r_A(y'') < r_B$, where r_B is given by equation (9). Then under the assumptions made in Proposition 1, the effect on the two firms' equilibrium (random) bids submitted at date 2 and*

firm A's surplus created at date 2 of increasing firm A's leverage, represented by a reduction of $K_A - D_A$ from y' to y'', is given as follows:

   i. if $y', y'' \in (0, \lambda(v_H - v_L)]$, both firms' equilibrium bids are strictly larger in the sense of FOSD when $K_A - D_A = y''$ than when $K_A - D_A = y'$; firm A's surplus created at date 2 is strictly lower when $K_A - D_A = y''$ than when $K_A - D_A = y'$.

  ii. If $y', y'' \in [\lambda(v_H - v_L), \lambda v_H + (1 - \lambda)v_L]$, firm B's equilibrium bid is strictly larger in the sense of FOSD when $K_A - D_A = y''$ than when $K_A - D_A = y'$, whereas firm A's equilibrium bid distribution is the same when $K_A - D_A = y''$ as when $K_A - D_A = y'$; firm A's surplus created at date 2 is strictly lower when $K_A - D_A = y''$ than when $K_A - D_A = y'$.

  iii. If $y', y'' \geq \lambda v_H + (1 - \lambda)v_L$, both firms' equilibrium bid distributions are the same when $K_A - D_A = y''$ as when $K_A - D_A = y'$; firm A's surplus created at date 2 is the same when $K_A - D_A = y''$ as when $K_A - D_A = y'$.

Part i of Proposition 2 shows that, when $K_A - D_A$ falls below a low threshold, $\lambda(v_H - v_L)$, further increasing firm A's leverage by reducing $K_A - D_A$ induces both firms to bid more aggressively and reduces firm A's surplus created at date 2. The logic behind this result is clear. When firm A's leverage is sufficiently high (i.e., when $K_A - D_A$ is sufficiently small), because of limited liability, firm A does not fully internalize the cost of a high winning bid. Consequently, as implied by equation (5), increasing firm A's leverage to a sufficiently high level will increase firm A's willingness to pay. When firm B is strong, firm A's increased reluctance to concede results in both firms competing more aggressively. More aggressive rival behavior reduces firm A's surplus created at date 2. The reduction in firm A's surplus is further exacerbated by the fact that, when firm A's leverage is raised to a sufficiently high level, firm A chooses a bid greater than the expected value of its prize with higher probability and such a bid destroys firm A's surplus even if firm A wins.

Part ii of Proposition 2 shows that, when $K_A - D_A$ lies between the low threshold, $\lambda(v_H - v_L)$, and a high threshold, which equals the expected value of firm A's prize, $\lambda v_H + (1 - \lambda)v_L$, increasing firm A's leverage by reducing $K_A - D_A$ induces firm B to bid more aggressively but has no effect on firm A's bidding strategy (provided that the reduced $K_A - D_A$ still lies between the two thresholds). More aggressive rival behavior reduces firm A's surplus created at date 2.

It might appear puzzling that, while in part i of Proposition 2, increasing firm A's leverage induces firm A to adopt a more aggressive bidding strategy, in part ii, increased firm A's leverage has no effect on firm A's bidding strategy. To understand the underlying logic, note that, because $K_A - D_A$ falls below $\lambda(v_H - v_L)$ in part i while exceeds $\lambda(v_H - v_L)$ in part ii, by equation (5), firm A's willingness to pay, r_A,

is greater than the expected value of firm A's prize, $\lambda v_H + (1-\lambda)v_L$, in part i while equals this expected prize value in part ii. Thus, unlike part i, in part ii, increasing firm A's leverage does not extend the right tail of firm A's bid distribution.

While one might conjecture that, even absent an extension of the right tail, increased firm A's leverage should still affect firm A's behavior, this conjecture has a flaw because it ignores the adjustment of firm B's behavior. In fact, under the condition in part ii, holding firm B's bid distribution fixed, it is indeed the case that increasing firm A's leverage will make low bids less preferred to higher bids for firm A. This can be seen from firm A's indifference condition, i.e., equation (10). Suppose that we increase firm A's leverage by reducing $K_A - D_A$ by an infinitesimal amount $\varepsilon > 0$. Then the right-hand side of equation (10), which represents the reservation payoff of firm A's shareholders at date 2, drops by ε. However, holding firm B's bid distribution, F_B, fixed, because of limited liability, the left-hand side of equation (10) drops by ε only for low bids, whereas for high bids, the left-hand side of equation (10) drops by less than ε. Thus, holding F_B fixed, increased firm A's leverage does make high bids (that are still below firm A's willingness to pay) more attractive to firm A than low bids. However, such a change of firm A's incentive tends to motivate firm B (whose willingness to pay is higher) to behave more aggressively once we allow firm B to adjust its strategy. To reach a new equilibrium associated with a higher leverage of firm A under the condition in part ii, firm B's (random) bid adjusts in a way that recovers firm A's indifference condition, i.e., equation (10). Because, holding F_B fixed, reducing $K_A - D_A$ by ε invalidates equation (10) for high bids but not for low bids, to recover equation (10), F_B adjusts for high bids but not for low bids. Because $F_B(b)$ equals firm A's probability of winning at bid b and because, without the adjustment of F_B, firm A prefers high bids (that are still below firm A's willingness to pay) to low bids, F_B decreases for high bids (and thus, firm B's bid becomes larger in FOSD) so as to equalize firm A's preference between high and low bids. The recovery of firm A's indifference condition in turn justifies the optimality of firm A's maintenance of its bidding strategy.

Part iii of Proposition 2 shows that, when $K_A - D_A$ exceeds the high threshold, $\lambda v_H + (1-\lambda)v_L$, reducing $K_A - D_A$ has no effect on any firm's bidding strategy or firm A's surplus created at date 2, provided that the reduced $K_A - D_A$ still exceeds this high threshold. The intuition is straightforward. Whenever firm A's capital, K_A, is sufficiently large relative to its debt obligation, D_A, firm A, by submitting any bid no greater than its willingness to pay, can always meet its debt obligation, even if it loses the competition. In this case, firm A's debt is riskless and firm A behaves as if it were purely equity financed.

Because external investors need at least break even and, as we will show later, in fact, they just break

even in equilibrium, any loss of firm *A*'s surplus created at date 2 will eventually be borne by firm *A*'s internal shareholders. Thus, our above analysis implies a cost of firm *A*'s leverage on the welfare of firm *A*'s internal shareholders. However, as shown by our following analysis of the case in which firm *A* has higher willingness to pay than firm *B*, a high leverage of firm *A* also produces a benefit to firm *A*'s internal shareholders.

3.1.2 The case in which firm *A* has higher willingness to pay

Suppose that firm *A*'s willingness to pay is weakly higher, i.e., $r_A \geq r_B$. Like the case in which firm *B*'s willingness to pay is higher, when firm *A* has higher willingness to pay, as long as both firms have capital no less than the minimum of the two firms' willingness to pay, the firms' desire to "win small" will result in both firms randomizing their bids up to the minimum of the two firms' willingness to pay (that is, up to r_B when $r_A \geq r_B$). The shareholders of the firm with lower willingness to pay receive their reservation payoff (i.e., their payoff from quitting the competition) while the shareholders of the firm with higher willingness to pay receive a payoff equal to their payoff from winning at their rival's willingness to pay. The next proposition provides a characterization of the equilibrium strategies and shareholder payoffs for the case of $r_A \geq r_B$.

Proposition 3. *Suppose that at date 2, firm A's willingness to pay is weakly higher, i.e., $r_A \geq r_B$, where r_A and r_B are given by equations (5) and (9), respectively. Suppose that, at the beginning of date 2, firm A's capital, K_A, satisfies condition (2) and firm B's capital and debt obligation equal $K_B - c$ and D_B respectively. The subgame at date 2 has a unique equilibrium.*

In this equilibrium, firm A randomizes continuously on $[0, r_B]$. Firm B randomizes continuously on $[0, r_B]$ if $r_A = r_B$, while if $r_A > r_B$, firm B places point mass on 0 and randomizes continuously on $(0, r_B]$. Firm A's shareholder payoff equals $w_A^s(r_B)$, where w_A^s is given in equation (3), while firm B's shareholder payoff equals $K_B - c - D_B$.

Recall that firm *B*'s willingness to pay, r_B, is fixed at the expected value of firm *B*'s prize. Thus, Proposition 3 implies that, when firm *A*'s willingness to pay is no less than the expected value of firm *B*'s prize, firm *B*'s shareholders receive an expected payoff of $K_B - c - D_B$ at the end of date 2. Note that firm *B*'s shareholders receiving a payoff of $K_B - c - D_B$ implies that firm *B*'s entry into the competition makes its shareholders worse off, because had firm *B* not paid the entry cost to enter, its shareholders would have obtained a payoff of $K_B - D_B$. Given that increasing firm *A*'s leverage tends to increase firm *A*'s willingness to pay, an immediate implication we obtain here is the *entry deterrent effect* of

leverage: the higher the leverage of firm A, the more likely the potential cash-rich challenger is to find it unprofitable to enter the competition and, consequently, the more likely the potential cash-rich challenger is to stay away from competition. Proposition 3, combined with Proposition 2, implies that firm A's choice of capital structure at date 0 hinges on a tradeoff: while a high leverage of firm A better deters entry, a high leverage lowers firm A's surplus created at date 2 conditional on rival entry.

Like Proposition 1, Proposition 3 also implies a pair of indifference conditions that can be used to solve for the equilibrium bid distributions. However, when firm A has higher willingness to pay, firm B will be deterred from entry once we endogenize firm B's entry decision at date 1, implying that the subgame at date 2 will be on an off-equilibrium path when firm A has higher willingness to pay. Thus, for the sake of brevity, we refrain from presenting the equilibrium bid distributions when firm A has higher willingness to pay.

3.2 Date 1: Firm B's entry decision

Now we analyze the subgame starting at date 1, at which point, firm B observes its relative strength, ρ, and firm A's choice of capital structure, (K_A, D_A), and decides whether to enter or not. We focus on the date-1 subgame where the capital firm A has raised at date 0, K_A, satisfies condition (2). As has been discussed in the previous section, this condition is equivalent to firm A's capital, K_A, being no less than firm A's willingness to pay, r_A. Imposing this condition is innocuous for our equilibrium outcome predictions, because, as we will show in the next section, where we study firm A's capital structure choice at date 0, it is never optimal for firm A to choose $K_A < r_A$.

Because, at date 2, firm A never bids more than its willingness to pay, r_A, and because, by equation (1), firm B has abundant capital even without fundraising, if firm B raised extra capital, the extra capital would simply be stored and returned to external investors at the end of date 2. Raising such redundant capital would bring no benefit to firm B. Thus, firm B raises no extra capital at date 1. Hence, we only need to focus on firm B's entry decision.

As is clear from the discussion in the previous section, firm B enters only if its willingness to pay, r_B, which equals the expected value of firm B's prize, exceeds firm A's willingness to pay, r_A. By Proposition 1, conditional on $r_B > r_A$, if firm B enters, firm B's shareholder payoff at the end of date 2 equals $r_B - r_A + K_B - c - D_B$. If firm B does not enter, firm B's shareholder payoff equals $K_B - D_B$. Thus, given that firm B acts on behalf of its shareholders, firm B enters if and only if $r_B - r_A - c > 0$. The next result is, hence, straightforward from the expressions for r_A and r_B given by equations (6) and (9) respectively.

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Proposition 4. *Suppose that at date 1, firm A's capital, K_A, satisfies condition* (2). *Then in any equilibrium of the subgame starting at date 1, firm B enters the competition if and only if firm B's relative strength, ρ, satisfies that*

$$\rho > \frac{r_A + c}{\lambda v_H + (1 - \lambda) v_L},\tag{16}$$

where r_A is given by equation (6).

The right-hand side of the entry condition (16) is increasing in firm A's willingness to pay, r_A. Because r_A is weakly increasing in firm A's leverage, Proposition 4 verifies the entry deterrent effect of leverage. Based on this result and the results we derived from our analysis of the date-2 competition, in the next section, we examine firm A's choice of capital structure at date 0.

3.3 Date 0: Firm A's capita structure choice

At date 0, firm A knows that firm B's relative strength, ρ, is uniformly distributed over $[0, z]$ but does not know the exact value of ρ. Firm A also knows that, by raising a sufficient amount of capital that satisfies condition (2), there exists a threshold in ρ, given by the right hand side of equation (16), below which firm B will not enter. Because ρ is bounded above by z and firm A's willingness to pay, r_A, given by equation (6), is bounded below by $\lambda v_H + (1 - \lambda) v_L$, the set of ρ that satisfies the entry condition (16) is nonempty only if

$$z > 1 + \frac{c}{\lambda v_H + (1 - \lambda) v_L}.\tag{17}$$

If condition (17) is violated, then the choice between debt and equity financing *per se* is not important to firm A; as long as firm A raises a sufficient amount of capital at date 0 that satisfies condition (2), firm A completely preempts rival entry.

To make our analysis of firm A's capital structure choice nontrivial, in what follows, we assume that condition (17) holds. The next proposition shows that, in any equilibrium of the full game, condition (2) binds and firm A uses either pure equity or a high-leverage capital structure.

Proposition 5. *Suppose that condition* (17) *holds. In any equilibrium of the full game, condition* (2) *binds. Moreover, either $K_A = \lambda v_H + (1 - \lambda) v_L$ and $D_A = 0$ (a pure-equity structure) or $K_A = \lambda v_H + (1 - \lambda) D_A$ and $D_A > v_L$ (a high-leverage structure).*

Note that condition (2) binds if and only if firm A's capital, K_A, equals firm A's willingness to pay, r_A, given by equation (6). The intuition for why $K_A = r_A$ in equilibrium is not difficult to understand. Firm A has no incentive to bid more than its willingness to pay, so raising more than its willingness to

pay is unnecessary. Raising less than its willingness to pay lowers the highest bid firm A could possibly submit in the competition, which weakens firm A's ability to deter entry.[17] Consequently, firm A raises exactly $K_A = r_A$ in equilibrium.

Proposition 5 implies that firm A never chooses a capital structure with a low level of risky debt. The reason is because, compared to a pure-equity structure, having a low level of risky debt does not change firm A's willingness to pay, r_A, but only induces the rival to behave more aggressively conditional on rival entry.[18] Thus, compared to a pure-equity structure, having low but nonzero leverage intensifies post-entry competition without bringing any entry-deterrent benefit to firm A. Consequently, as shown by Proposition 5, firm A uses either a pure-equity structure or a high-leverage structure.

To investigate firm A's preference between a pure-equity structure and a high-leverage structure, we first examine *firm A's surplus*, i.e., the expected joint profits of firm A's stakeholders, under these capital structures. Then we argue that, as long as a capital structure creates a positive surplus to firm A, there always exists some financial arrangement (fundraising through debt and/or equity financing) that makes this capital structure feasible to firm A and enables firm A's internal shareholders to capture the entire surplus created by firm A. Thus, between a pure-equity structure and a high-leverage structure, the one that produces a higher surplus to firm A is preferred by firm A's internal shareholders.

Let $\pi_A(K_A, D_A)$ be firm A's surplus when firm A's capital and debt obligation at the end of date 0 equal K_A and D_A, respectively. First, consider the pure-equity structure specified in Proposition 5, i.e., $K_A = \lambda v_H + (1 - \lambda) v_L$ and $D_A = 0$. Without leverage, firm A's willingness to pay, r_A, simply equals the expected value of firm A's prize, $\lambda v_H + (1 - \lambda) v_L$. Thus, by Proposition 4, firm A preempts the entry of firm B whenever firm B's relative strength, ρ, is no greater than $1 + c/(\lambda v_H + (1 - \lambda) v_L)$. Conditional on that firm B does not enter, firm A's surplus is simply the expected value of firm A's prize, $\lambda v_H + (1 - \lambda) v_L$. By Propositions 1 and 4, conditional on $\rho > 1 + c/(\lambda v_H + (1 - \lambda) v_L)$, firm B enters and firm A's surplus becomes zero. Because ρ is uniformly distributed over $[0, z]$, where, by hypothesis (17), $z > 1 + c/(\lambda v_H + (1 - \lambda) v_L)$, firm A deters rival entry with probability $(1/z) \times (1 + c/(\lambda v_H + (1 - \lambda) v_L))$. The following result, which characterizes firm A's surplus under the pure-equity structure, is thus straightforward.

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[17]Lowering K_A may also affect the two firms' bidding strategies conditional on the entry of firm B. Thus, a formal proof of $K_A = r_A$ in equilibrium is more involved and requires an investigation of the date-2 subgame on off-equilibrium paths. See the proof Proposition 5 in the Online Appendix. In particular, Lemma A-13 in this proof shows that, when condition (17) is satisfied, it is never optimal for firm A to have $K_A < r_A$.

[18]To see this, note that, under a pure-equity structure, r_A simply equals $\lambda v_H + (1 - \lambda) v_L$. By equation (5), using leverage raises r_A above $\lambda v_H + (1 - \lambda) v_L$ if and only if $K_A - D_A < \lambda (v_H - v_L)$, which condition is equivalent to $D_A > v_L$ when condition (2) binds. When condition (2) binds and $D_A \leq v_L$, $K_A = \lambda v_H + (1 - \lambda) v_L$. By Proposition 2.ii, given $K_A = \lambda v_H + (1 - \lambda) v_L$, firm B's bid is larger in the sense of FOSD if $D_A \in (0, v_L]$ than if $D_A = 0$.

Proposition 6. *Suppose that condition* (17) *holds. Firm A's surplus under the pure-equity structure,* $K_A = \lambda v_H + (1 - \lambda) v_L$ *and* $D_A = 0$, *is given by*

$$\pi_A(K_A = \lambda v_H + (1 - \lambda) v_L, D_A = 0) = \frac{\lambda v_H + (1 - \lambda) v_L + c}{z}. \tag{18}$$

Next, consider the high-leverage structures specified in Proposition 5, i.e., $K_A = \lambda v_H + (1 - \lambda) D_A$ and $D_A > v_L$. Under a high leverage, firm A's willingness to pay, r_A, follows the expression in the second line of equation (5), in which case, given that $K_A = \lambda v_H + (1 - \lambda) D_A$, we have

$$r_A = \lambda v_H + (1 - \lambda) D_A. \tag{19}$$

Thus, by Proposition 4, firm B enters if and only if

$$\rho > \frac{\lambda v_H + (1 - \lambda) D_A + c}{\lambda v_H + (1 - \lambda) v_L}. \tag{20}$$

The right-hand side of (20) is increasing in D_A. Thus, among the high-leverage structures specified in Proposition 5, those with a higher D_A better deter entry. Under the assumption that $K_A > D_A$ and the high-leverage structure specification in Proposition 5 that $K_A = \lambda v_H + (1 - \lambda) D_A$, D_A has an upper bound given by $D_A < v_H$. By equation (20), by choosing D_A arbitrarily close to v_H, firm A can deter firm B from entering whenever firm B's relative strength, ρ, satisfies that $\rho < (v_H + c)/(\lambda v_H + (1 - \lambda) v_L)$. Thus, when the maximum type of firm B, z, falls below $(v_H + c)/(\lambda v_H + (1 - \lambda) v_L)$, firm A can completely avoid competition by having a sufficiently high leverage, in which case firm A's surplus equals the expected value of firm A's prize, $\lambda v_H + (1 - \lambda) v_L$. This observation leads to the following result:

Lemma 2. *If the maximum type of firm B, z, satisfies that* $z < (v_H + c)/(\lambda v_H + (1 - \lambda) v_L)$, *there always exists a high-leverage structure for firm A under which firm B enters with zero probability and firm A's surplus equals* $\lambda v_H + (1 - \lambda) v_L$.

Clearly, when a high-leverage structure enables firm A to completely preempt the entry of firm B, firm A's surplus under this high-leverage structure will be greater than firm A's surplus under the pure-equity structure given by equation (18). Thus, Lemma 2 implies that, when the maximum type of firm B, z, is less than $(v_H + c)/(\lambda v_H + (1 - \lambda) v_L)$, a high-leverage structure produces higher surplus to firm A than a pure-equity structure.

Now consider the case in which the maximum type of firm B, z, is no less than $(v_H + c)/(\lambda v_H + (1 - \lambda) v_L)$. In this case, given $D_A < v_H$, none of the high-leverage structures specified in Proposition 5

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can completely preempt entry. The choice of leverage thus involves a tradeoff: higher leverage better deters entry but, conditional on rival entry, higher leverage exacerbates competition. Firm A's surplus in this case is given in the next proposition.

Proposition 7. *Let $\pi_A^g(K_A, D_A; \rho)$, which denotes firm A's surplus created at date 2 conditional on the relative strength of firm B being ρ, be defined as in equation (15), with r_A and F_B in this equation given by (19) and (14) respectively. If the maximum type of firm B, z, satisfies that $z \geq (v_H + c)/(\lambda v_H + (1 - \lambda) v_L)$, under a high-leverage capital structure, $K_A = \lambda v_H + (1 - \lambda) D_A$ and $D_A > v_L$ (in which case the assumption $K_A > D_A$ is satisfied if and only if $D_A < v_H$), firm A's surplus is given by*

$$
\pi_A(K_A = \lambda v_H + (1 - \lambda) D_A, D_A) = \left[\frac{\lambda v_H + (1 - \lambda) D_A + c}{z(\lambda v_H + (1 - \lambda) v_L)} \right] (\lambda v_H + (1 - \lambda) v_L)
$$

$$
+ \int_{\frac{\lambda v_H + (1 - \lambda) D_A + c}{\lambda v_H + (1 - \lambda) v_L}}^{z} \frac{1}{z} \pi_A^g(K_A = \lambda v_H + (1 - \lambda) D_A, D_A; \rho) \, d\rho \qquad (21)
$$

$$
= \frac{1}{z} \left\{ \lambda v_H + (1 - \lambda) D_A + c + \left(\log z - \log \left[\frac{\lambda v_H + (1 - \lambda) D_A + c}{\lambda v_H + (1 - \lambda) v_L} \right] \right) \zeta(D_A) \right\}, \qquad (22)
$$

where

$$
\zeta(D_A) = \lambda (v_H - D_A) \log \left(\frac{\lambda v_H + (1 - \lambda) v_L}{\lambda (v_H - v_L)} \right) + (v_H - D_A) \log \left(\frac{v_H - v_L}{v_H - D_A} \right)
$$

$$
- \frac{1}{2(\lambda v_H + (1 - \lambda) v_L)} \left[(v_H - (1 - \lambda)(v_H - D_A))^2 - \lambda^2 (v_H - D_A)^2 \right].
$$

To understand equation (21), first note that the whole term in the square bracket in the first line of (21) represents the probability that firm B does not enter. This whole term follows from the entry condition (20) and the hypothesis that ρ is drawn uniformly from $[0, z]$, with $z \geq (v_H + c)/(\lambda v_H + (1 - \lambda) v_L)$. Conditional on that firm B does not enter, firm A's surplus equals $\lambda v_H + (1 - \lambda) v_L$. When $\rho \in (\frac{\lambda v_H + (1 - \lambda) D_A + c}{\lambda v_H + (1 - \lambda) v_L}, z]$, firm B enters. Because ρ is drawn uniformly from $[0, z]$, its density at any point over this interval is $1/z$. For each specific value of $\rho \in (\frac{\lambda v_H + (1 - \lambda) D_A + c}{\lambda v_H + (1 - \lambda) v_L}, z]$, the game moves on to date 2 and firm A's surplus created at date 2 is $\pi_A^g(K_A, D_A; \rho)$. Given the high-leverage structures specified in Proposition 5, in the expression for $\pi_A^g(K_A, D_A; \rho)$ given by equation (15), r_A and F_B are given by (19) and (14) respectively and $K_A = \lambda v_H + (1 - \lambda) D_A$. Equation (21) thus follows. Computing the integral in equation (21) gives equation (22).

There exists a high-leverage capital structure specified in Proposition 5 that produces a higher firm A's surplus than the pure-equity structure specified in Proposition 5 if and only if there exists some debt level, $D_A \in (v_L, v_H)$, which produces a value of $\pi_A(K_A = \lambda v_H + (1 - \lambda) D_A, D_A)$ given by equation (22)

that exceeds firm A's surplus under the pure-equity structure given in Proposition 6. The result from this comparison gives firm A's preference between a high-leverage structure and a pure-equity structure, because as long as a capital structure (K_A, D_A), $K_A > D_A$, produces a positive surplus for firm A, firm A's internal shareholders can always obtain this capital structure and capture the entire surplus through debt and/or equity financing. To see this, note that, creditors of the debt security with face value D_A at most receive a cash flow of D_A at the end of the game. Thus, the value of their debt, denoted by V_D, is at most D_A. By having the creditors provide an amount of external capital equal to V_D, firm A can make these creditors just break even. Because $V_D \leq D_A$ and $K_A > D_A$, after the creditors provide an amount of capital equal to V_D, to raise a total amount of external capital K_A, firm A still needs to raise an amount of capital equal to $K_A - V_D > 0$ from external shareholders. When firm A's surplus under (K_A, D_A) is positive, raising this amount of capital from external shareholders by giving all the firm's shares to them will make these external shareholders more than break even. Thus, when firm A's surplus under (K_A, D_A) is positive, there always exists some fraction of firm A's shares such that, by exchanging this fraction of shares for the capital $K_A - V_D$ from external shareholders, firm A can make external shareholders just break even. Given that all external investors just break even, firm A's internal shareholders capture the entire surplus. The next result is thus straightforward.

Proposition 8. *If the maximum type of firm B, z, satisfies that $z \geq (v_H + c)/(\lambda v_H + (1 - \lambda) v_L)$, then the following results hold:*

i. *if there exists no $d \in (v_L, v_H)$ under which the value of $\pi_A(K_A = \lambda v_H + (1 - \lambda) d, D_A = d)$ in equation (22) exceeds the value of $\pi_A(K_A = \lambda v_H + (1 - \lambda) v_L, D_A = 0)$ in equation (18), then choosing the pure-equity structure, $K_A = \lambda v_H + (1 - \lambda) v_L$ and $D_A = 0$, is optimal for firm A.*

ii. *If there exists some $d \in (v_L, v_H)$ under which the value of $\pi_A(K_A = \lambda v_H + (1 - \lambda) d, D_A = d)$ in equation (22) exceeds the value of $\pi_A(K_A = \lambda v_H + (1 - \lambda) v_L, D_A = 0)$ in equation (18), then choosing the high-leverage structure, $K_A = \lambda v_H + (1 - \lambda) d$ and $D_A = d$, is strictly preferred by firm A to choosing the pure-equity structure, $K_A = \lambda v_H + (1 - \lambda) v_L$ and $D_A = 0$.*

When the pure-equity structure, $K_A = \lambda v_H + (1 - \lambda) v_L$ and $D_A = 0$, is not optimal for firm A, finding out firm A's optimal capital structure requires solving for an optimal value of D_A that maximizes firm A's surplus in equation (22) subject to $D_A \in (v_L, v_H)$. Because we do not allow D_A to take on the value of v_H, a solution may not exist. This non-existence problem can be avoided if we slightly modify the model by restricting firm A's choice of D_A to $D_A \in (v_L, v_H - \varepsilon]$, where $\varepsilon > 0$ is infinitesimal.[19] Unfortunately, there

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[19]When d is sufficiently close to v_L, any high-leverage structure, $K_A = \lambda v_H + (1 - \lambda) d$ and $D_A = d$, is dominated by the pure-equity structure, $K_A = \lambda v_H + (1 - \lambda) v_L$ and $D_A = 0$. Thus, conditional on the existence of some $d \in (v_L, v_H - \varepsilon]$ under which

is no closed-form solution to this maximization problem. However, without solving the maximization problem, we are still able to derive some general conclusions on firm A's optimal capital structure choice based on Proposition 8.

First, as the next proposition shows, firm A is more inclined to use a high-leverage capital structure the higher the entry cost, c, of firm B.

Proposition 9. *The effect of a change in firm B's entry cost, c, on firm A's capital structure choice is as follows:*

i. *if firm A finds it optimal to use the pure-equity structure, $K_A = \lambda v_H + (1 - \lambda) v_L$ and $D_A = 0$, when $c = c'$, then everything else being equal, firm A will also find it optimal to use this pure-equity structure for any $c < c'$.*

ii. *If firm A finds it optimal to use a high-leverage structure when $c = c'$, then everything else being equal, firm A will also find it optimal to use a high-leverage structure for any $c > c'$.*

iii. *When c is sufficiently large, firm A finds it optimal to use a high-leverage structure.*

The intuition for Proposition 9 is straightforward. For firm A, the drawback of adopting a high-leverage structure is that, once its rival enters, a high-leverage structure intensifies the competition and creates a loss for firm A. Everything else being equal, an increase in firm B's entry cost makes it less likely that firm B enters the competition. Thus, the drawback of using a high-leverage structure is mitigated if firm B faces a high entry cost. The result here is consistent with Lemma 2. Lemma 2 implies that, when firm B's entry cost is sufficiently large, firm A can completely preempt firm B's entry by using a high-leverage structure, in which case the drawback of a high-leverage structure is eliminated. If we think of c as representing firm A's first-mover advantage in the new market, Proposition 9 implies that the pioneering firm (firm A) is more likely to adopt a high-leverage structure if it possesses a large first-mover advantage (because of, e.g., its consumers' high switching costs).

It is worth mentioning that, if firm A uses a high-leverage structure, it is likely that, when firm B enters to compete, the post-entry competition will result in both firms losing money. This is because, if firm B enters when firm A uses a high-leverage structure, firm A is likely to "overbid," i.e., bid more than the expected value of its prize. Whenever firm A overbids, firm A will lose money in expectation even if it wins. Moreover, whenever firm A overbids, it is likely that firm A will defeat firm B, in which case firm B will lose money. Thus, although a cash-burning battle is unlikely to take place when the pioneering

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$\pi_A(K_A = \lambda v_H + (1 - \lambda) d, D_A = d)$ in equation (22) is strictly greater than the value of $\pi_A(K_A = \lambda v_H + (1 - \lambda) v_L, D_A = 0)$ in Proposition 6, the fact that, in the maximization problem for solving for the optimal high-leverage structure, the choice set for D_A does not include the lower endpoint, v_L, does not invalidate the existence of an optimal structure.

firm uses a high-leverage structure, whenever it takes place, it is likely that the battle engenders losses on both the winner and the loser.

Next, we present a couple of conditions under which firm *A* finds it optimal to use a pure-equity structure.

Proposition 10. *Firm A finds it optimal to use the pure-equity structure, $K_A = \lambda v_H + (1 - \lambda) v_L$ and $D_A = 0$, if any of the following conditions holds:*

  i. *$v_H - v_L$ is sufficiently small;*

  ii. *λ is sufficiently close to 1.*

Proposition 10 implies that a pure-equity structure is optimal for firm *A* if (ex post) prize valuation uncertainty is sufficiently low, represented by a sufficiently small difference in the value of firm *A*'s prize between the two future states of the economy, $v_H - v_L$, and/or by a good future state of the economy being sufficiently certain, i.e., λ being sufficiently close to 1. The reason why low prize-valuation uncertainty tends to induce firm *A* to adopt a pure-equity structure is fairly straightforward. Low prize-valuation uncertainty limits firm *A*'s ability to increase its willingness to pay by exploiting limited liability under a high-leverage structure. Because the extent to which a high-leverage structure deters entry is determined by the extent to which a high-leverage structure increases firm *A*'s willingness to pay, low prize-valuation uncertainty weakens the entry-deterrent benefit provided by high-leverage structures. If we interpret a small $v_H - v_L$ as the monopoly profits in the new market being insensitive to economic conditions, our result here implies that the pioneering firm in a new market (firm *A*) will use pure-equity financing if the future monopoly profits in the new market is sufficiently insensitive to economic conditions.

4 Extensions

4.1 The effect of an upward shift of firm *B*'s prize value

In our model, we assumed that the value of firm *B*'s prize equals ρv_H if the future state of the economy is good and equals ρv_L if the future state of the economy is bad, where ρ, known to be drawn uniformly from $[0, z]$ by firm *A* at date 0, represents firm *B*'s relative strength in the new market. In this section, we are interested in the effect on firm *A*'s capital structure choice of an increase in firm *B*'s prize value, modeled as follows: Firm *B*'s prize value equals $\rho (v_H + \tau)$ in the good future state and $\rho (v_L + \tau)$ in the bad future state, where $\tau \geq 0$ represents an upward shift of firm *B*'s prize value and is common knowledge at date 0. By introducing τ, we aim to investigate situations in which firm *B* may value and be

perceived by firm A ex ante as valuing the monopolistic position in the new market highly, represented by a large τ. Such situations happen if the new market (or niche) firm A is pioneering in is likely to produce an externality on firm B's existing business. For example, τ is large if firm B's winning of the new market (or niche) creates a synergy with firm B's existing business and/or firm B's losing of the new market exposes firm B to the risk of having its existing market enveloped by the new market.

We continue to assume that firm B has a sufficiently large amount of capital such that it is always solvent at the end of date 2. In this case, firm B behaves as if it had a pure-equity structure. Thus, firm B's shareholders' valuation of the prize equals

$$\lambda(\rho v_H + \tau) + (1-\lambda)(\rho v_L + \tau) = \left(\rho + \frac{\tau}{\lambda v_H + (1-\lambda)v_L}\right)(\lambda v_H + (1-\lambda)v_L).$$

An inspection of the above equation shows that the effect of introducing an upward shift of firm B's prize value is essentially the same as the effect of increasing firm B's relative strength by $\tau/(\lambda v_H + (1-\lambda)v_L)$. Therefore, to study the effect of an increase in firm B's prize value, we only need to slightly modify our model by assuming that firm B's relative strength, ρ, is uniformly distributed on $[s, z+s]$ (rather than on $[0, z]$), where

$$s = \frac{\tau}{\lambda v_H + (1-\lambda)v_L}$$

measures the magnitude of the upward shift of firm B's prize value. We assume that $s < 1$, so in the modified setting, it is possible that the value of firm A's prize exceeds the value of firm B's prize.

This modification does not alter any of our analysis of dates 1 and 2 made in Sections 3.2 and 3.1 respectively, because the analysis there was made conditional on each realization of ρ. Moreover, the modification does not change the conclusion in Proposition 5, which shows that, at date 0, low-leverage structures are never optimal for firm A. Compared to a pure-equity structure, these low-leverage structures provide no entry-deterrent benefit but only induce the entrant to compete more aggressively. In fact, this modification only affects our analysis of date 0 through its effect on firm A's surplus under the pure-equity structure, $K_A = \lambda v_H + (1-\lambda)v_L$ and $D_A = 0$, and firm A's surplus under a high-leverage structure, $K_A = \lambda v_H + (1-\lambda)D_A$ and $D_A \in (v_L, v_H)$.

Note that, by Proposition 4 and the fact that, under the pure-equity structure, firm A's willingness to pay, r_A, equals $\lambda v_H + (1-\lambda)v_L$, firm B enters if and only if $\rho > (\lambda v_H + (1-\lambda)v_L + c)/(\lambda v_H + (1-\lambda)v_L)$. Thus, given that now ρ is uniformly distributed on $[s, z+s]$, $s < 1$, under the pure-equity structure, with a probability equal to $(\frac{\lambda v_H + (1-\lambda)v_L + c}{\lambda v_H + (1-\lambda)v_L} - s)/z$, firm B stays away from competition. Be-cause, conditional on that firm B stays away from competition, firm A's surplus is $\lambda v_H + (1-\lambda)v_L$,

and conditional on that firm B enters, firm A's surplus is zero under the pure-equity structure, firm A's surplus under the pure-equity structure in the modified model now becomes

$$\pi_A(K_A = \lambda\, v_H + (1-\lambda)\, v_L, D_A = 0) = \frac{(1-s)(\lambda\, v_H + (1-\lambda)\, v_L) + c}{z}. \qquad (23)$$

By a similar argument used for deriving equation (21) (provided in the paragraph after Proposition 7), in the modified model, firm A's surplus under a high-leverage structure, $K_A = \lambda\, v_H + (1-\lambda)\, D_A$ and $D_A \in (v_L, v_H)$, is given by

$$\pi_A(K_A = \lambda\, v_H + (1-\lambda)\, D_A, D_A) = \frac{\lambda\, v_H + (1-\lambda)\, D_A + c - s(\lambda\, v_H + (1-\lambda)\, v_L)}{z}$$
$$+ \int_{\frac{\lambda v_H + (1-\lambda) D_A + c}{\lambda v_H + (1-\lambda) v_L}}^{z+s} \frac{1}{z} \pi_A^g(K_A = \lambda\, v_H + (1-\lambda)\, D_A, D_A; \rho)\, d\rho. \quad (24)$$

Because firm A's surplus conditional on firm B finding it profitable to enter is negative under a high-leverage structure (i.e., $\pi_A^g(K_A = \lambda\, v_H + (1-\lambda)\, D_A, D_A; \rho) < 0$ conditional on $\rho > \frac{\lambda v_H + (1-\lambda) D_A + c}{\lambda v_H + (1-\lambda) v_L}$), an inspection of equations (23) and (24) reveals that the marginal effect of an increase in s on firm A's surplus is less negative if firm A uses the pure-equity structure than if firm A uses a high-leverage structure. Thus, an increase in s tends to induce firm A to adopt a pure-equity structure.

Proposition 11. *The effect on firm A's capital structure choice of a change in s, where $s < 1$ represents the magnitude of an upward shift of firm B's prize value, is as follows:*

  i. *if firm A finds it optimal to use the pure-equity structure, $K_A = \lambda\, v_H + (1-\lambda)\, v_L$ and $D_A = 0$, when $s = s'$, then everything else being equal, firm A will also find it optimal to use this pure-equity structure for any $s \in (s', 1)$.*

  ii. *If firm A finds it optimal to use a high-leverage structure when $s = s'$, then everything else being equal, firm A will also find it optimal to use a high-leverage structure for any $s < s'$.*

Proposition 11 is intuitive. If firm B is likely to value the prize highly, firm B is likely to enter even if firm A adopts a high-leverage structure. In this case, given that a high-leverage structure of firm A intensifies post-entry competition, adopting such a structure will be very costly to firm A. Consequently, firm A chooses a pure-equity structure to soften post-entry competition when firm B is likely to highly value the prize. Proposition 11 implies that a pioneering firm in a new market (or niche) is likely to adopt a pure-equity capital structure if the new market (or niche) is likely to produce a large externality on a potential entrant's existing business.

4.2 Allowing for $K_A \leq D_A$

In our basic model, we restricted firm A's capital structure choice at date 0 to any capital structure with capital, K_A, exceeding debt obligation, D_A. Now we relax this assumption by allowing for $K_A \leq D_A$. Note that, when $K_A \leq D_A$, at date 2, firm A's shareholder payoff from quitting the competition equals 0. Because of limited liability, zero is also the lowest possible payoff for firm A's shareholders, regardless of what bids the two firms submit. Thus, when $K_A \leq D_A$, firm A's willingness to pay does not exist, i.e., there is no threshold bid above which firm A strictly prefers quitting the competition. As we show below, the non-existence of firm A's willingness to pay can lead to multiple equilibria for our date-2 subgame, and firm A's capital structure choice at date 0 can be affected by which equilibrium we select for the date-2 subgame. However, we will also argue that under commonly used equilibrium refinements, allowing for $K_A \leq D_A$ does not change the qualitative nature of our previous analysis.

For example, suppose $K_A = D_A \geq v_H$, in which case firm A has a sufficient amount of capital to bid v_H. Because v_H is the value of firm A's prize when the future state of the economy is good, by bidding v_H or more than v_H, firm A destroys its final cash flow. However, because of limited liability, regardless of how much money firm A burns, firm A's shareholder payoff is at least zero, that is, at least equal to their payoff from surrendering under $K_A = D_A$. Thus, if a strong, cash-rich firm B, with willingness to pay $r_B \in [v_H, K_A]$, enters, there exists a pure-strategy equilibrium (or a pure-strategy epsilon equilibrium) in which firm B chooses r_B deterministically and wins the prize with certainty whereas firm A chooses r_B deterministically (or a bid slightly below r_B if a tie at r_B is broken in a way that gives firm A a positive probability of winning) and loses the competition with certainty. Because firm B wins the prize at its willingness to pay, r_B, firm B's shareholders earn no profit from entering. Thus, if we select this pure-strategy equilibrium for our date-2 subgame, firm B will not enter when its willingness to pay falls within $[v_H, K_A]$. It can be shown that, if firm B's willingness to pay falls below v_H, entering is also unprofitable to firm B's shareholders. Hence, if the prescribed pure-strategy equilibrium is selected for $r_B \in [v_H, K_A]$, by having $K_A = D_A \geq v_H$, firm A can preempt entry of any firm B with $r_B \leq K_A$. In this case, given that, by equation (9) and the assumption that $\rho \leq z$, $r_B \leq z(\lambda v_H + (1-\lambda)v_L)$, firm A can completely preempt any entry by having $K_A = D_A > z(\lambda v_H + (1-\lambda)v_L)$.

However, the date-2 subgame in which firm A with $K_A = D_A \geq v_H$ competes against a strong, cash-rich firm B with $r_B \geq v_H$ also has a mixed-strategy equilibrium. In this mixed-strategy equilibrium, firm A places point mass on 0 and randomizes continuously on $(0, v_H]$, while firm B randomizes continuously on $[0, v_H]$. Firm B's shareholders receive an expected profit of $r_B - v_H \geq 0$. In fact, this mixed-strategy

equilibrium corresponds to the limit of the date-2 equilibrium presented in Proposition 1 as $K_A - D_A \rightarrow$ 0. Thus, picking this mixed-strategy equilibrium is a continuous selection. Continuity is a natural requirement for the selection from the equilibrium correspondence and has been adopted in the all-pay auction literature.[20] Under this equilibrium selection, when $K_A = D_A \geq v_H$, firm A never bids more than v_H. Thus, equilibrium outcomes will be the same as those under firm A's choice of $K_A = D_A = v_H$. Continuity then implies that firm A's surplus under the choice of $K_A = D_A = v_H$ simply equals the limit of firm A's surplus presented in equation (22) as $D_A \rightarrow v_H$.

Equilibrium multiplicity also arises when $v_H \leq K_A < D_A$. In particular, the aforementioned pure-strategy equilibrium also exists for $v_H \leq K_A < D_A$. If this equilibrium is selected, then just like the case of $v_H \leq K_A = D_A$, firm A can completely preempt entry by raising a sufficiently large amount of capital. There however also exist mixed-strategy equilibria for the date-2 subgame which lead to a tradeoff between entry deterrence and post-entry competition intensification that firm A need consider when choosing its capital structure at date 0.

It is worth mentioning that, to support equilibria of the full game in which complete entry deterrence happens for $K_A \leq D_A$, we need both firms to compete extremely aggressively in the date-2 subgame when a strong, cash-rich firm B enters. Such equilibria for the date-2 subgame, however, are Pareto dominated by mixed-strategy equilibria in which both firms randomize their bids over a lower bid interval. Thus, while allowing for $K_A \leq D_A$ can produce equilibria in which firm A completely preempts any entry, such equilibria do not satisfy commonly used equilibrium refinement, such as Pareto dominance or continuous equilibrium selection. Moreover, because firm A's final cash flow after bidding b is at most $\max[v_H + K_A - D_A - b, 0]$, any bid $b \geq v_H + K_A - D_A$ will give firm A's shareholders a zero payoff regardless of firm B's strategy. Choosing such a bid is a weakly dominated strategy for firm A. Thus, if $K_A \leq D_A$, then in any equilibrium of the date-2 subgame where a firm does not play its weakly dominated strategy with a positive probability, the upper bound of the support of firm A's bid distribution must not exceed v_H. Hence, once we rule out equilibria in which firm A plays a weakly dominated strategy with a positive probability, complete entry deterrence does not happen in any other equilibrium when $z > (v_H + c)/(\lambda v_H + (1-\lambda)v_L)$. Whenever complete entry deterrence fails, firm A's capital structure choice involves a consideration of the tradeoff between entry deterrence and post-entry competition intensification. Thus, allowing for $K_A \leq D_A$ does not change the qualitative nature of our analysis.

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[20]See, e.g., Fang et al. (2019).

5   Conclusion

In this paper, we studied the optimal capital structure of a pioneering firm in a new market (or niche) who faces the threat of a cash-rich potential entrant from an adjacent market. The post-entry competition considered in our paper takes the form of a winner-take-all, all-pay auction. This type of competition resembles competitions in the internet world, where Web-based firms often compete by spending profligately on marketing and advertising in an attempt to acquire consumers and consumer data and to gain a (near) monopoly position through data-related advantage. R&D races (Leininger, 1991) and high-frequency trading arms races (Budish et al., 2015) also have a winner-take-all, all-pay feature, in that only the race winner is likely to reap a large benefit from its innovation or increased speed of response to market events, while all players have to bear the cost of investment in speed. We found that the optimal capital structure of the pioneering firm hinges on a tradeoff: high leverage better deters entry but intensifies competition if entry deterrence fails. We showed that, depending on a couple of factors, the pioneering firm uses either a pure-equity structure or a high-leverage structure. The pioneering firm is more likely to adopt a pure-equity structure if barriers to entry are low, the externality of the new market on the cash-rich potential entrant's existing business is high, and/or the uncertainty of the new market's prospect is low. We also found that, due to strategic uncertainty in firms' post-entry behavior, who wins the market is highly uncertain and, when the pioneering firm is highly levered, it is likely that post-entry competition results in both the winner and the loser bearing large losses.

Two extensions of our model could be fruitful. First, it can be interesting to analyze a pioneering firm's financial arrangement from a more general security design perspective or by expanding the set of securities the pioneering firm can choose from. For example, one can show that, if the pioneering firm issues a risky call option (a mirror image of a risky debt) to external investors, the post-entry competition can be softer compared to the case under debt and/or equity financing.[21] However, entry is more likely to happen if the pioneer issues risky call options rather than using debt and/or equity financing. Second, one could assume that the potential entrant also lacks cash and needs external financing to fight in the competition. This extension could lead to predictions of the entrant's capital structure choice. We plan to investigate these possible extensions in our future research.

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[21]We thank an anonymous referee for this insight.

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Online Appendix to "Money as a weapon: Financing a winner-take-all competition"

A.1 Preliminary results for analyzing the date-2 subgame

Propositions 1 and 3 provide a characterization of the equilibrium of the date-2 subgame on an equilibrium path of the full game. Verifying this equilibrium is not difficult and can be done by constructing a pair of strategies that are mutually best responses and satisfy the properties characterized in the associated proposition. What is less straightforward is a proof of equilibrium uniqueness and a characterization of the equilibrium of the date-2 subgame on an off-equilibrium path. In this section, we establish several results that are useful for analyzing the date-2 subgame.

Throughout this section, we assume that, at the beginning of date 2, firm A's capital, K_A, and debt obligation, D_A, satisfy that $K_A > D_A \geq 0$, and firm B's capital, $K_B - c$, and debt obligation, D_B, satisfy condition (1). Firm A's willingness to pay, r_A, and firm B's willingness to pay, r_B, are given by equations (6) and (9) respectively. Let $\underline{b}_A$ and $\bar{b}_A$ be, respectively, the lower and the upper bound of the support of firm A's bid distribution, F_A. Define $\underline{b}_B$ and $\bar{b}_B$ for firm B's bid distribution, F_B, analogously.

Lemma A-1. *Firm A's shareholder payoff from winning at b, $w_A^s(b)$, and firm B's shareholder payoff from winning at b, $w_B^s(b)$, given by equations (3) and (8), respectively, satisfy that $b \hookrightarrow w_A^s(b)$ is strictly decreasing for $b \in [0, v_H]$ and $b \hookrightarrow w_B^s(b)$ is strictly decreasing for $b \in [0, z v_H]$.*

Proof. The assumption that $K_A > D_A$ implies that $\max[v_H + K_A - b - D_A, 0] = v_H + K_A - b - D_A$ for all $b \in [0, v_H]$. Thus, by equation (3), when $b \in [0, v_H]$, $w_A^s(b) = \lambda (v_H + K_A - b - D_A) + (1 - \lambda) \max[v_L + K_A - b - D_A, 0]$, which is strictly decreasing in b. By equation (8), it is obvious that $b \hookrightarrow w_B^s(b)$ is strictly decreasing for $b \in [0, z v_H]$. $\square$

Lemma A-2. *In any equilibrium of the date-2 subgame, $\bar{b}_A \leq \min[K_A, r_A] \leq v_H$ and $\bar{b}_B \leq r_B$.*

Proof. For each firm, choosing a bid strictly greater than its willingness to pay is strictly dominated by choosing a zero bid. Thus, $\bar{b}_A \leq r_A$ and $\bar{b}_B \leq r_B$. For firm A, it cannot bid more than its capital, K_A. Hence, $\bar{b}_A \leq K_A$. Equation (6) and the assumptions that $v_H > v_L$ and $K_A > D_A$ imply that $r_A \leq v_H$. Thus, $\min[K_A, r_A] \leq v_H$. $\square$

Lemma A-3. *In any equilibrium of the date-2 subgame, $\bar{b}_A = \bar{b}_B$.*

Proof. Suppose, to the contrary, that $\bar{b}_A \neq \bar{b}_B$ in an equilibrium of this subgame. Then, in this equilibrium, either $\bar{b}_A < \bar{b}_B$ or $\bar{b}_A > \bar{b}_B$. If $\bar{b}_A < \bar{b}_B$, then for firm B, choosing any $b \in (\bar{b}_A, \bar{b}_B]$ would ensure winning. In this case, firm B's shareholder payoff from choosing any $b \in (\bar{b}_A, \bar{b}_B]$ would equal $w_B^s(b)$. Because, by Lemma A-2, $\bar{b}_B \leq r_B$ and because $r_B \leq z v_H$, $\bar{b}_B \leq z v_H$. Thus, by Lemma A-1, $b \hookrightarrow w_B^s(b)$ is strictly decreasing for $b \leq \bar{b}_B$. Hence, for any

pair of bids over $(\bar{b}_A, \bar{b}_B]$, firm B would strictly prefer the lower one, implying that $\bar{b}_B$ cannot be the upper bound of the support of F_B, a contradiction. This contradiction implies that, in any equilibrium of the date-2 subgame, it cannot be that $\bar{b}_A < \bar{b}_B$. Applying a similar argument to the case of $\bar{b}_A > \bar{b}_B$, noting that, by Lemmas A-1 and A-2, $b \mapsto w_A^s(b)$ is strictly decreasing for $b \in [0, \bar{b}_A]$, shows that, in any equilibrium of the date-2 subgame, it cannot be that $\bar{b}_A > \bar{b}_B$. Therefore, in any equilibrium of the date-2 subgame, $\bar{b}_A = \bar{b}_B$. $\qquad\square$

Given Lemma A-3, in what follows, let $\bar{b} = \bar{b}_A = \bar{b}_B$ denote the common upper bound of the support of each firm's bid distribution for any equilibrium of the date-2 subgame.

Lemma A-4. *In any equilibrium of the date-2 subgame, at most one firm places point mass on zero and neither firm places point mass on any $b \in (0, \bar{b})$, where $\bar{b} = \bar{b}_A = \bar{b}_B$ denotes the common upper bound of the support of each firm's bid distribution.*

Proof. We first show that, in any equilibrium of the date-2 subgame, for any given $b \in [0, \bar{b})$, at most one firm places point mass on b. Note that, at any $b \in [0, \bar{b})$, neither firm's capital constraint is binding, since otherwise, $\bar{b}$ could not be the common upper bound of the support of each firm's bid distribution. The Tie Lemma in Siegel (2009) can be applied to any bid at which neither firm's capital constraint is binding. By the Tie Lemma, if both firms placed point mass on $b \in [0, \bar{b})$, then they either both win with certainty or both lose with certainty when choosing b, contradicting the fact that one and only one of the two firms wins.

Next we show that, in any equilibrium of the date-2 subgame, neither firm places point mass on any $b \in (0, \bar{b})$. To prove this result, given what we just established, it suffices to show that there exists no equilibrium of the date-2 subgame in which one firm, say i, places point mass on $b' \in (0, \bar{b})$ and the other firm, $j \neq i$, does not place point mass on b'. Suppose, to the contrary, that such an equilibrium exists. In this case, by Lemmas A-2 and A-3, the bid $b' \in (0, \bar{b})$ on which firm i places point mass must satisfy that $b' < \min[r_A, r_B]$. Because for any $b < \min[r_A, r_B]$, each firm's shareholders are strictly better off winning at b than losing at b and because each firm's shareholder payoff conditional on winning at b and that conditional on losing at b are both continuous in b, given that firm i places point mass on $b' \in (0, \bar{b})$, where $b' < \min[r_A, r_B]$, firm j strictly prefers choosing a bid slightly above b' than choosing any bid $b \in [b' - \varepsilon, b')$ for $\varepsilon > 0$ sufficiently small. Thus, given that firm j does not place point mass on b', firm j places no weight over $[b' - \varepsilon, b']$. But then firm i could not be optimizing since, by Lemma A-1, firm i could give its shareholders a strictly higher payoff by transferring mass from b' to $b' - \varepsilon$, a contradiction. $\qquad\square$

Lemma A-5. *In any equilibrium of the date-2 subgame, $\underline{b}_A = \underline{b}_B = 0$.*

Proof. First, note that there is no equilibrium in which a firm submits a deterministic bid. Thus, in any equilibrium of the date-2 subgame, $\underline{b}_A < \bar{b}_A$ and $\underline{b}_B < \bar{b}_B$. Hence, by Lemma A-4, $\max[\underline{b}_A, \underline{b}_B] < \bar{b} = \bar{b}_A = \bar{b}_B$.

Next, we show that, in any equilibrium of the date-2 subgame, $\underline{b}_A = \underline{b}_B$. Suppose, to the contrary, that $\underline{b}_A \neq \underline{b}_B$ in an equilibrium of this subgame. Then, in this equilibrium, either $\underline{b}_A < \underline{b}_B$ or $\underline{b}_A > \underline{b}_B$. If $\underline{b}_A < \underline{b}_B$, given that $\underline{b}_A \geq 0$, it must be that $\underline{b}_B > 0$. Then the fact that $\underline{b}_B < \bar{b}$, together with Lemma A-4, implies that firm B places no mass on $\underline{b}_B$. By the definition of $\underline{b}_B$, firm B places no weight over $[0, \underline{b}_B)$. Thus, firm B places no weight over

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$[0, \underline{b}_B]$. This implies that, for firm A, choosing any bid $b \in [0, \underline{b}_B]$ would ensure losing. When $K_A > D_A$, losing at a zero bid gives firm A's shareholders a strictly higher payoff than losing at any strictly positive bid. Thus, given that $\underline{b}_A < \underline{b}_B$, it must be that $\underline{b}_A = 0$ and firm A places point mass on 0 but places no weight over $(0, \underline{b}_B]$. In this case, for firm B, its probability of winning by choosing a bid arbitrarily close to $\underline{b}_B > 0$ is arbitrarily close to the probability of winning by choosing a bid slightly above zero. Thus, given that, by Lemma A-1, firm B's valuation of winning is strictly decreasing in its winning bid, by having $\underline{b}_B > 0$, firm B could not be optimizing, a contradiction. Therefore, in any equilibrium of this subgame, it cannot be that $\underline{b}_A < \underline{b}_B$. The case in which $\underline{b}_A > \underline{b}_B$ can be ruled out analogously. Hence, in any equilibrium of the subgame, $\underline{b}_A = \underline{b}_B$.

To complete the proof of the lemma, it suffices to show that, in any equilibrium of the date-2 subgame, it cannot be that $\underline{b}_A = \underline{b}_B > 0$. Let $\underline{b}$ denote the common lower bound of the support of each firm's bid distribution. Suppose, to the contrary, that $\underline{b} > 0$. Then Lemma A-4 implies that neither firm places point mass on any $b \in [\underline{b}, \bar{b})$. Thus, a firm's probability of winning by choosing a bid arbitrarily close to $\underline{b} > 0$ would be arbitrarily close to zero. Such a bid would be dominated by a zero bid. Hence, by having a strictly positive lower bound of the support of its bid distribution, a firm could not be optimizing, a contradiction. □

Lemma A-6. *In any equilibrium of the date-2 subgame, at least one firm's shareholders receive a payoff equal to their payoff from losing at 0.*

Proof. By Lemma A-4, at most one firm places point mass on 0. Thus, we prove the lemma by considering two cases: (1) one and only one firm places point mass on 0 and (2) neither firm places point mass on 0.

First, consider the case in which one and only one firm places point mass on 0. Call this firm i and call the firm that places no mass on 0 j. Because firm i places point mass on 0, choosing 0 must be a best response for firm i. Because firm $j \neq i$ places no mass on 0, firm i, by choosing 0, ensures losing. Given that choosing 0 is a best response for firm i, firm i's shareholders must receive a payoff equal to the payoff from losing at 0 in this equilibrium.

Next, consider the case in which neither firm places point mass on 0. Because, by Lemma A-5, 0 is the common lower bound of the support of each firm's bid distribution, given that neither firm places point mass on 0, for each firm, choosing 0 must be a best response and give the firm's shareholders a payoff from losing at 0. □

Lemma A-7. *In any equilibrium of the date-2 subgame, $\bar{b} = \min[K_A, r_A, r_B]$, where $\bar{b}$ denotes the common upper bound of the support of each firm's bid distribution.*

Proof. First consider the case in which $K_A \leq \min[r_A, r_B]$. In this case, $\min[K_A, r_A, r_B] = K_A$. Note that it must be that in any equilibrium of the date-2 subgame, $\bar{b} \leq K_A$. This is because firm A cannot bid more than K_A and, given that the tie-breaking rule specifies firm B as the winner if a tie occurs at K_A, firm B, by bidding K_A, ensures winning. Now we argue that, in any equilibrium of the date-2 subgame, it cannot be that $\bar{b} < K_A$. Suppose otherwise. Then given that $K_A \leq \min[r_A, r_B]$, for each firm $i = A, B$, bidding slightly more than $\bar{b}$ could ensure that firm i wins and firm i's shareholders receive a payoff strictly greater than $w_i^s(\min[r_A, r_B])$. Because for each

firm $i = A, B$, $w_i^s(\min[r_A, r_B]) \geq w_i^s(r_i)$ and, by the definition of r_i, $w_i^s(r_i)$ equals firm i's shareholder payoff from losing at 0, by bidding slightly more than $\bar{b}$, firm i could ensure a shareholder payoff strictly greater than the one from losing at 0, contradicting Lemma A-6. Thus, if $K_A \leq \min[r_A, r_B]$, then in any equilibrium of the subgame, it cannot be that $\bar{b} < K_A$. Therefore, it must be that $\bar{b} = K_A$.

Next, consider the case in which $K_A > \min[r_A, r_B]$. In this case, $\min[K_A, r_A, r_B] = \min[r_A, r_B]$. By an argument similar to the one just used, we can show that there is no equilibrium in which $\bar{b} < \min[r_A, r_B]$. Lemma A-2 implies that $\bar{b} \leq \min[r_A, r_B]$. Thus, if $K_A > \min[r_A, r_B]$, in any equilibrium of the subgame at date 2, it must be that $\bar{b} = \min[r_A, r_B]$. $\qquad\square$

Lemma A-8. *In any equilibrium of the date-2 subgame, the support of each firm's bid distribution equals* $[0, \min[K_A, r_A, r_B]]$.

Proof. By Lemmas A-5 and A-7, to establish the result, it suffices to show that each firm's bid distribution has a connected support. Suppose, to the contrary, that one firm, say i, has a gap, (b', b''), in the support of its bid distribution, where $0 \leq b' < b'' \leq \min[K_A, r_A, r_B]$ and b' and b'' are both in the support of firm i's bid distribution. Because (b', b'') is not part of the support of firm i's bid distribution, firm i places no weight over (b', b''). This implies that there exists $\varepsilon > 0$ such that, for firm $j \neq i$, bidding slightly above b' dominates any bid $b \in [b'' - \varepsilon, b'')$. Thus, firm j places no weight over $[b'' - \varepsilon, b'')$. Because b'' is in the support of firm i's bid distribution, it must be that bidding b'' or arbitrarily close to b'' is a best response for firm i. Because firm j places no weight over $[b'' - \varepsilon, b'')$, bidding b'' or arbitrarily close to b'' is a best response for firm i only if firm j places point mass on b''. Because firm i places no weight over $[b'' - \varepsilon, b'')$ either, to justify firm j's point mass on b'', it must be that firm i also places point mass on b''. Because $b'' \leq \min[K_A, r_A, r_B]$, by Lemmas A-4 and A-7, both firms placing point mass on b'' could happen only if $b'' = \min[K_A, r_A, r_B]$. If both firms placing point mass on $b'' = \min[K_A, r_A, r_B]$, firm B must win with certainty at b'', because otherwise, given that firm B's capital constraint is slack at b'', firm B could be better off transferring mass from b'' to $b'' + \varepsilon$ for $\varepsilon > 0$ sufficiently small. However, if firm B wins with certainty at b'', firm A, which also places point mass on b'', could not be optimizing, because given that firm B places no weight over $[b'' - \varepsilon, b'')$, firm A's probability of winning would remain the same if it transferred the mass from b'' to $b'' - \varepsilon$. Such a transfer would make firm A's shareholders strictly better off, a contradiction. $\qquad\square$

A.2 Proofs of results

Proof of Proposition 1. We first show equilibrium shareholder payoffs. Condition (2) implies that $K_A \geq r_A$. Equation (1) implies that $K_B \geq r_B$. Because a firm, even if it has unlimited capital, has no incentive to bid more than its willingness to pay, the hypotheses that $K_A \geq r_A$ and $K_B \geq r_B$ ensure that the two firms behave as if they had no capital constraint. The assumption that $K_A > D_A$ and equation (1) imply that, when $r_B > r_A$, the date-2 competition satisfies the Generic Conditions in Siegel (2009). In this case, Theorem 1 in Siegel (2009), which requires an all-pay auction to satisfy the Generic Conditions, can be applied. This theorem implies that, when $r_B > r_A$, in any

equilibrium, firm A's shareholders receive a payoff equal to the one from losing at 0 (i.e., equal to $K_A - D_A$) while firm B's shareholders receive a payoff equal to the one from winning at its rival's willingness to pay (i.e., equal to $w_B^s(r_A) = r_B - r_A + K_B - c - D_B$, where the equality follows from equations (8) and (9)).

Next we show equilibrium strategies. Given that $K_A \geq r_A$ and the hypothesis that $r_B > r_A$, $\min[K_A, r_A, r_B] = r_A$. Thus, by Lemmas A-4 and A-8, in any equilibrium of this date-2 subgame, the support of each firm's bid distribution is $[0, r_A]$, neither firm places point mass on the interior of this support, and at most one firm places point mass on 0. Now we argue that, in any equilibrium of this date-2 subgame, firm A must place point mass on 0 and neither firm places point mass on r_A. To see that firm A must place point mass on 0, simply note that had firm A placed no mass on 0, firm B's shareholder payoff from bidding zero or bidding arbitrarily close to 0 would be equal to or arbitrarily close to their payoff from losing at 0, $K_B - c - D_B$. Given that firm B's shareholders receive a payoff equal to $r_B - r_A + K_B - c - D_B$, which is strictly greater than $K_B - c - D_B$ under the hypothesis that $r_B > r_A$, bidding zero or arbitrarily close to zero could not be a best response for firm B, contradicting that $\underline{b}_B = 0$. To see that neither firm places point mass on r_A, note that, if one firm chose r_A with probability $p > 0$, the other firm's probability of winning by bidding less than r_A would be less than $1 - p$. In this case, by bidding arbitrarily close to r_A, the other firm's shareholders could not receive an expected payoff that is arbitrarily close to their payoff from winning at r_A, implying the existence of $\varepsilon > 0$ such that any bid $b \in [r_A - \varepsilon, r_A)$ is not a best response for the other firm, contradicting that the support of the other firm's bid distribution is $[0, r_A]$.

The above argument shows that, in any equilibrium of this subgame, firm A places point mass on 0 and randomizes continuously on $(0, r_A]$. Firm B randomizes continuously on $[0, r_A]$. The uniqueness of the associated equilibrium strategies follows immediately from Lemma 1, which presents the associated equilibrium bid distributions. $\qquad\square$

Proof of Lemma 1. Follows from the argument before this lemma in the main text. $\qquad\square$

Proof of Proposition 2. Because a change in $K_A - D_A$ can change both the shape and the support of each firm's bid distribution, to simplify our proof, it is useful to first establish expressions for the bid distributions over a fixed interval that always contains the support of each firm's bid distribution. Because, by equation (6) and the hypothesis that $K_A > D_A$, $r_A \leq v_H$, the support $[0, r_A]$ is always contained in the fixed interval $[0, v_H]$. Thus, we first establish the expressions for F_A and F_B over $[0, v_H]$. By Lemma 1,

$$F_A(b; K_A - D_A) = \min\left[1 - \frac{r_A - b}{\rho\,(\lambda\,v_H + (1-\lambda)\,v_L)}, 1\right], \quad b \in [0, v_H], \tag{A-1}$$

where r_A is given by equation (6), and

$$F_B(b; K_A - D_A) = \min\left[1 - \frac{1}{1 + \frac{\min[b,\,K_A-D_A]}{\lambda\,(v_H-b)+(1-\lambda)\max[v_L-b,\,-(K_A-D_A)]}}, 1\right], \quad b \in [0, v_H]. \tag{A-2}$$

In what follows, let y' and y'', where $0 < y'' < y'$, be two distinct values of $K_A - D_A$.

We first establish the results on how a change of $K_A - D_A$ changes F_A. By equations (A-1) and (6), a change in $K_A - D_A$ changes F_A if and only if the change in $K_A - D_A$ changes r_A. By equation (5), if $y', y'' \geq \lambda (v_H - v_L)$, $r_A(y'') = r_A(y')$. Thus, by equation (A-1), if $y', y'' \geq \lambda (v_H - v_L)$, $F_A(b; y'') = F_A(b; y')$ for all $b \in [0, v_H]$, i.e., firm A's bid distribution under $K_A - D_A = y''$ is the same as the one under $K_A - D_A = y'$. If $y', y'' \leq \lambda (v_H - v_L)$, by equation (5) and the hypothesis that $0 < y'' < y'$, $r_A(y'') > r_A(y')$. Because, by equation (A-1), holding $b \in [0, v_H]$ fixed, F_A is nonincreasing in r_A for all b and strictly decreasing in r_A for some b, the fact that $r_A(y'') > r_A(y')$ implies that $F_A(b; y'') \leq F_A(b; y')$ for all $b \in [0, v_H]$, with strictly inequality for some $b \in [0, v_H]$. Hence, if $y', y'' \leq \lambda (v_H - v_L)$, firm A's bid distribution under $K_A - D_A = y''$ is strictly larger than the one under $K_A - D_A = y'$ in the sense of FOSD.

Next, we establish the results on how a change of $K_A - D_A$ changes F_B. First, an inspection of equation (A-2) reveals that, holding $b \in [0, v_H]$ fixed, increasing $K_A - D_A$ weakly increases F_B. Thus, firm B's bid distribution under $K_A - D_A = y''$ is always weakly larger than the one under $K_A - D_A = y'$ in the sense of FOSD. Because, when $y', y'' \leq \lambda (v_H - v_L)$, the upper bound of the support of F_B under $K_A - D_A = y''$ is strictly higher than that under $K_A - D_A = y'$, implying that the shape of F_B must change, given that reducing $K_A - D_A$ always weakly increases firm B's bid in the sense of FOSD, firm B's bid distribution under $K_A - D_A = y''$ must be strictly larger than the one under $K_A - D_A = y'$ in the sense of FOSD if $y', y'' \leq \lambda (v_H - v_L)$. Now consider the case in which $y', y'' \geq \lambda (v_H - v_L)$. In this case, by equation (5), $r_A(r'') = r_A(r') = \lambda v_H + (1 - \lambda) v_L$. Thus, the support of F_B under $K_A - D_A = y''$ and that under $K_A - D_A = y'$ are both $[0, \lambda v_H + (1 - \lambda) v_L]$. By equation (14), for any $K_A - D_A \in [\lambda (v_H - v_L), \lambda v_H + (1 - \lambda) v_L]$, over the support $[0, \lambda v_H + (1 - \lambda) v_L]$,

$$F_B(b; K_A - D_A) = \begin{cases} \dfrac{b}{\lambda v_H + (1 - \lambda) v_L} & b \in [0, K_A - D_A] \\ \dfrac{K_A - D_A}{\lambda v_H + (1 - \lambda) v_L + K_A - D_A - b} & b \in [K_A - D_A, \lambda v_H + (1 - \lambda) v_L] \end{cases},$$

in which case a change of $K_A - D_A$ changes the shape of F_B. Thus, given that reducing $K_A - D_A$ always weakly increases firm B's bid in the sense of FOSD, firm B's bid distribution under $K_A - D_A = y''$ must be strictly larger than the one under $K_A - D_A = y'$ in the sense of FOSD if $y', y'' \in [\lambda (v_H - v_L), \lambda v_H + (1 - \lambda) v_L]$. By equation (14), for any $K_A - D_A \geq \lambda v_H + (1 - \lambda) v_L$, over the support $[0, \lambda v_H + (1 - \lambda) v_L]$,

$$F_g(b) = \frac{b}{\lambda v_H + (1 - \lambda) v_L}, \quad b \in [0, \lambda v_H + (1 - \lambda) v_L],$$

which expression is independent of $K_A - D_A$. Thus, for any $y', y'' \geq \lambda v_H + (1 - \lambda) v_L$, firm B's bid distribution under $K_A - D_A = y''$ is the same as that under $K_A - D_A = y'$. This establishes the results on how a change of $K_A - D_A$ changes F_B.

Finally, we establish the results on how a change of $K_A - D_A$ changes firm A's surplus created at date 2, denoted by $\pi_A^g(K_A, D_A; \rho)$. First, consider the case in which $K_A - D_A \leq \lambda (v_H - v_L)$. In this case, by equation (5),

$r_A = v_H - \left(\frac{1-\lambda}{\lambda}\right)(K_A - D_A)$. Then by Lemma 1 and the first line of equation (15), we have

$$\pi_A^g(K_A, D_A; \rho) = \frac{1}{\rho}\left[\int_{K_A-D_A}^{v_L+K_A-D_A} \frac{K_A - D_A}{\lambda\, v_H + (1-\lambda)\, v_L + K_A - D_A - b}\, db + \int_{v_L+K_A-D_A}^{v_H-\left(\frac{1-\lambda}{\lambda}\right)(K_A-D_A)} \frac{K_A - D_A}{\lambda\,(v_H + K_A - D_A - b)}\, db\right.$$
$$\left. - \int_{K_A-D_A}^{v_H-\left(\frac{1-\lambda}{\lambda}\right)(K_A-D_A)} \frac{b}{\lambda\, v_H + (1-\lambda)\, v_L}\, db\right]$$
$$= \frac{1}{\rho}\left[(K_A - D_A)\log\left(\frac{\lambda\, v_H + (1-\lambda)\, v_L}{\lambda\,(v_H - v_L)}\right) + \left(\frac{K_A - D_A}{\lambda}\right)\log\left(\frac{\lambda\,(v_H - v_L)}{K_A - D_A}\right)\right.$$
$$\left. - \frac{1}{2(\lambda\, v_H + (1-\lambda)\, v_L)}\left[\left(v_H - \left(\frac{1-\lambda}{\lambda}\right)(K_A - D_A)\right)^2 - (K_A - D_A)^2\right]\right]. \quad \text{(A-3)}$$

Differentiating π_A^g with respect to $K_A - D_A$, using equation (A-3), and multiplying the result by ρ yield

$$\rho\,\frac{\partial\,\pi_A^g}{\partial\,(K_A - D_A)} = \log\left(\frac{\lambda\, v_H + (1-\lambda)\, v_L}{\lambda\,(v_H - v_L)}\right) + \frac{1}{\lambda}\log\left(\frac{\lambda\,(v_H - v_L)}{K_A - D_A}\right) - \frac{1}{\lambda}$$
$$+ \frac{1}{\lambda\, v_H + (1-\lambda)\, v_L}\left[\left(v_H - \left(\frac{1-\lambda}{\lambda}\right)(K_A - D_A)\right)\left(\frac{1-\lambda}{\lambda}\right) + K_A - D_A\right]. \quad \text{(A-4)}$$

Note that $\log x \geq 1 - (1/x)$ for all $x \geq 1$, with strict inequality for all $x > 1$. Thus, because $\lambda\, v_H + (1-\lambda)\, v_L > \lambda\,(v_H - v_L)$, and because, by the hypothesis in part i that $K_A - D_A \leq \lambda\,(v_H - v_L)$, we have

$$\log\left(\frac{\lambda\, v_H + (1-\lambda)\, v_L}{\lambda\,(v_H - v_L)}\right) > \frac{v_L}{\lambda\, v_H + (1-\lambda)\, v_L} \quad \text{(A-5)}$$
$$\log\left(\frac{\lambda\,(v_H - v_L)}{K_A - D_A}\right) \geq 1 - \frac{K_A - D_A}{\lambda\,(v_H - v_L)}. \quad \text{(A-6)}$$

Equations (A-4), (A-5), and (A-6) imply that, for all $K_A - D_A \leq \lambda\,(v_H - v_L)$,

$$\rho\,\frac{\partial\,\pi_A^g}{\partial\,(K_A - D_A)} > \frac{v_L}{\lambda\, v_H + (1-\lambda)\, v_L} + \frac{1}{\lambda}\left(1 - \frac{K_A - D_A}{\lambda\,(v_H - v_L)}\right) - \frac{1}{\lambda}$$
$$+ \frac{1}{\lambda\, v_H + (1-\lambda)\, v_L}\left[\left(v_H - \left(\frac{1-\lambda}{\lambda}\right)(K_A - D_A)\right)\left(\frac{1-\lambda}{\lambda}\right) + K_A - D_A\right]$$
$$= \frac{(1-\lambda)\, v_H + \lambda\, v_L}{\lambda\,(\lambda\, v_H + (1-\lambda)\, v_L)} - \frac{(1-\lambda)\, v_H + \lambda\, v_L}{\lambda^2\,(v_H - v_L)(\lambda\, v_H + (1-\lambda)\, v_L)}(K_A - D_A). \quad \text{(A-7)}$$

The expression in the last line of (A-7) is clearly decreasing in $K_A - D_A$. Thus, for all $K_A - D_A \leq \lambda\,(v_H - v_L)$, this expression reaches its minimum when $K_A - D_A = \lambda\,(v_H - v_L)$ and this minimum value equals 0. Hence, equation (A-7) implies that $\frac{\partial\,\pi_A^g}{\partial\,(K_A-D_A)} > 0$ for all $K_A - D_A \leq \lambda\,(v_H - v_L)$. Thus, firm A's surplus created at date 2 under $K_A - D_A = y''$ is strictly lower than under $K_A - D_A = y'$ if $y', y'' \leq \lambda\,(v_H - v_L)$ and $y'' < y'$.

Next consider the case in which $K_A - D_A \in [\lambda\,(v_H - v_L), \lambda\, v_H + (1-\lambda)\, v_L]$. In this case, by equation (5), $r_A = \lambda\, v_H + (1-\lambda)\, v_L$. Then by Lemma 1 and the first line of equation (15), we have

$$\pi_A^g(K_A, D_A; \rho) = \int_0^{\lambda\, v_H + (1-\lambda)\, v_L} [(\lambda\, v_H + (1-\lambda)\, v_L)\, F_B(b) - b]\, dF_A(b), \quad \text{(A-8)}$$

where F_A and F_B are given in Lemma 1. As has been established above, a reduction in $K_A - D_A$ from $K_A - D_A = y'$ to $K_A - D_A = y''$ such that $y', y'' \in [\lambda(v_H - v_L), \lambda v_H + (1 - \lambda)v_L]$ has no effect on F_A but strictly increases firm B's bid in the sense of FOSD (i.e., weakly reduces F_B for all b and strictly reduces F_B for some b). Thus, by equation (A-8), this reduction in $K_A - D_A$ strictly reduces π_A^g.

The result that for any $y', y'' \geq \lambda v_H + (1 - \lambda)v_L$, firm A's surplus created at date 2 under $K_A - D_A = y''$ is the same as that under $K_A - D_A = y'$ follows immediately from the result we have established that the prescribed change of $K_A - D_A$ does not change any firm's equilibrium strategy. This completes the proof of the results on how a change in $K_A - D_A$ changes firm A's surplus created at date 2 and completes the proof of the proposition. $\square$

Proof of Proposition 3. The proof for the case of $r_A > r_B$ is similar to the proof of Proposition 1, so we shall be brief. Equations (1) and (2), combined with the fact that neither firm has an incentive to bid more than its willingness to pay, imply that both firms behave as if they had unlimited capital. The assumption that $K_A > D_A$ and equation (1) imply that, when $r_A > r_B$, the date-2 competition satisfies the Generic Conditions in Siegel (2009), under which Theorem 1 in Siegel (2009) can be applied. By Theorem 1 in Siegel (2009), in any equilibrium, firm B's shareholders receive a payoff equal to the one from losing at 0 (i.e., equal to $K_B - c - D_B$) while firm A's shareholders receive a payoff equal to the one from winning at its rival's willingness to pay (i.e., equal to $w_A^s(r_B)$). The result that, when $r_A > r_B$, firm A randomizes continuously on $[0, r_B]$ while firm B places point mass on 0 and randomizes continuously on $(0, r_B]$ follows from an argument analogous to the one used in the proof of Proposition 1.

Now consider the case in which $r_A = r_B$. By Lemma A-8 and the fact that both firms behave as if they had unlimited capital, in any equilibrium, the support of each firm's bid distribution is $[0, r]$, where $r = r_A = r_B$ denotes the common willingness to pay. By the definition of willingness to pay, each firm's shareholder payoff from winning at $r = r_A = r_B$ equals the firm's shareholder payoff from losing at 0. Thus, in any equilibrium, it cannot be that both firms place point mass on r, because otherwise, at least one of the two firms could not ensure winning at r and would strictly prefer moving mass from r to 0. In fact, it also cannot be that one and only one firm places point mass on r, because otherwise, the firm that does not place point mass on r could not obtain a probability of winning arbitrarily close to 1 by bidding less than but arbitrarily close to r, in which case for this firm, bidding 0 would strictly dominate bidding arbitrarily close to r. Given that this firm, by hypothesis, does not place point mass on r, there would exist $\varepsilon > 0$ such that this firm places no weight over $[r - \varepsilon, r]$, contradicting that r is the upper bound of the support of this firm's bid distribution. Therefore, when $r_A = r_B$, neither firm places point mass on $r = r_A = r_B$. Then by Lemma A-4, neither firm places point mass on any $b \in (0, r]$. This result, combined with the fact that the support of each firm's bid distribution is $[0, r]$, implies that, when $r_A = r_B$, in any equilibrium, bidding $r = r_A = r_B$ is a best response for each firm and each firm's shareholder payoff equals the payoff from losing at 0. This further implies that neither firm places point mass on 0, because had one firm placed point mass on 0, the other firm could give its shareholders a payoff strictly greater than their payoff from losing at 0 by bidding slightly more than 0. Hence, when $r_A = r_B$, both firms randomize continuously on $[0, r]$, where $r = r_A = r_B$.

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Equilibrium uniqueness follows from the fact that the bid randomization behavior of each firm implies an indifference condition that uniquely pins down the bid distribution of the rival firm. The detailed argument is analogous to the one provided before Lemma 1 in the main text. □

Proof of Proposition 4. Follows from the argument right before the proposition in the main text. □

Proof of Proposition 5. At date 0, firm A chooses K_A and D_A subject to $K_A > D_A$. We first argue that, as long as the choice of K_A and D_A, where $K_A > D_A$, creates a positive surplus to firm A, there always exists a financial arrangement, through debt and/or equity financing, that leads to the capital structure, K_A and D_A, makes external investors just break even, and gives the entire surplus to firm A's internal shareholders. Let $K_{A,e}$ and $K_{A,d}$ be the capital raised by firm A from external shareholders and creditors, respectively. Given that, before fundraising, firm A is penniless, $K_A = K_{A,e} + K_{A,d}$. Let $\pi_A(K_A, D_A)$ be the surplus firm A creates under the capital structure, K_A and D_A. Under the capital structure, K_A and D_A, the expected final cash flow that is to be distributed to firm A's stakeholders is $K_A + \pi_A(K_A, D_A)$, which equals the sum of the value of the debt, denoted by V_D, the value of the external equity, denoted by V_{EE}, and the value of the internal equity, denoted by V_{IE}. Because creditors holding a debt security with face value D_A can never receive more than D_A, the value of the debt security with face value D_A is always no greater than its face value, D_A, i.e., $V_D \le D_A$. Thus, to obtain the capital structure, K_A and D_A, firm A can first compute V_D and raise exactly $K_{A,d} = V_D$ from creditors, in which case creditors just break even. Because $K_A > D_A$, given that $D_A \ge V_D$, after raising $K_{A,d} = V_D$ from creditors, firm A still needs to raise $K_{A,e} = K_A - V_D > 0$ from external shareholders to make the total capital equal to K_A. When $\pi_A(K_A, D_A) > 0$, by giving all the residual cash flows (after paying creditors D_A) to external shareholders, firm A can make external shareholders more than break even. Thus, when $\pi_A(K_A, D_A) > 0$, there always exists some fraction of firm A's shares such that, by giving external shareholders this fraction of shares, firm A can make external shareholders just break even, i.e., under this fraction of shares, $V_{EE} = K_{A,e}$. When firm A's external shareholders and creditors all just break even, firm A's internal shareholders capture all the surplus $\pi_A(K_A, D_A)$.

Thus, in the following proof, we can ignore external investors' participation constraints but focus on firm A's surplus associated with the choice of K_A and D_A. To continue, we first establish a couple of lemmas that will be used in our proof.

Lemma A-9. *In any equilibrium of the subgame starting at date 1, firm B enters the competition if and only if*

$$r_B > \min[K_A, r_A] + c,$$

where r_A and r_B are given by equations (6) *and* (9) *respectively.*

Proof. The "if" part is clear. By Lemma A-2, firm A never bids more than $\min[K_A, r_A]$. Thus, by equations (8) and (9), by entering and bidding more than and arbitrarily close to $\min[K_A, r_A]$, firm B can ensure winning and giving its shareholders a payoff arbitrarily close to $r_B + K_B - c - \min[K_A, r_A] - D_B$. Without entering, firm B's shareholder payoff is $K_B - D_B$. Thus, when $r_B > \min[K_A, r_A] + c$, firm B strictly prefers entering.

Now we establish the "only if" part. First, suppose $\min[K_A, r_A] \leq r_B \leq \min[K_A, r_A] + c$. In this case, if firm B enters, by Lemma A-8, in any equilibrium of the subgame at date 2, the two firms' bid distributions both have support equal to $[0, \min[K_A, r_A]]$. This implies that, in any equilibrium of the subgame at date 2, firm B's shareholder payoff post competition equals at most the payoff from winning at $\min[K_A, r_A]$. By equations (8) and (9), this payoff equals at most $r_B + K_B - c - \min[K_A, r_A] - D_B$, which is no greater than $K_B - D_B$ (the payoff from not entering) when $r_B \leq \min[K_A, r_A] + c$. Thus, when $\min[K_A, r_A] \leq r_B \leq \min[K_A, r_A] + c$, firm B does not enter.

Next, suppose $r_B < \min[K_A, r_A]$. In this case, if firm B enters, by Lemma A-8, in any equilibrium of the subgame at date 2, the two firms' bid distributions both have support equal to $[0, r_B]$. Because $r_B < \min[K_A, r_A]$, firm A can ensure winning and giving its shareholders a payoff strictly greater than the payoff from losing at zero by bidding more than and arbitrarily close to r_B. Then by Lemma A-6, in any such equilibrium, firm B's shareholders receive a payoff equal to the one from losing at 0, implying that firm B's shareholders cannot earn a positive expected profit by entering the competition. Thus, when $r_B < \min[K_A, r_A]$, firm B does not enter. This completes the proof of the "only if" part. $\square$

Lemma A-10. *Suppose that condition* (17) *holds. Let* (K_A, D_A) *and* (K'_A, D'_A) *be two different capital structures for firm A, where* $K_A > D_A$ *and* $K'_A > D'_A$. *Let* r_A *and* r'_A *be firm A's willingness to pay associated with* (K_A, D_A) *and* (K'_A, D'_A) *respectively, defined according to equation* (6). *If* $K'_A = r_A = K_A < r'_A$, *then firm A's surplus is strictly higher under* (K_A, D_A) *than under* (K'_A, D'_A).

Proof. By Lemma A-9, because $K'_A = r_A = K_A < r'_A$, under both (K_A, D_A) and (K'_A, D'_A), firm B enters if and only if $r_B > r_A + c$. Thus, to show that firm A's surplus is strictly higher under (K_A, D_A) than under (K'_A, D'_A), it suffices to show that, for any given type of firm B, conditional on that firm B's type is sufficiently high such that $r_B > r_A + c$, firm A's surplus is strictly higher under (K_A, D_A) than under (K'_A, D'_A). By Lemma A-8 and the hypothesis that $K'_A = r_A = K_A < r'_A$, under both (K_A, D_A) and (K'_A, D'_A), both firms have the support of their bid distributions equal $[0, r_A]$. Because in both cases, by bidding more than and arbitrarily close to r_A, firm B ensures winning and giving its shareholders a payoff arbitrarily close to $w_B^s(r_A)$, where w_B^s is given by equation (8), and because the support of firm B's bid distribution is $[0, r_A]$, firm B's shareholder payoff in both cases equals $w_B^s(r_A)$. Thus, in both cases, the indifference condition for firm B is the same. Because firm B's indifference condition pins down firm A's bid distribution, in both cases, firm A's bid distribution is the same.

Now consider firm A's indifference condition. Because, under both (K_A, D_A) and (K'_A, D'_A), firm B's shareholder payoff equals $w_B^s(r_A)$ and because $r_B > r_A + c$, in both cases, firm B's shareholder payoff is strictly greater than that from losing at 0. Thus, by Lemma A-6, in both cases, firm A's shareholder payoff equals the one from losing at 0. That is, under (K_A, D_A), firm A's shareholder payoff equals $K_A - D_A$, while under (K'_A, D'_A), firm A's shareholder payoff equals $K'_A - D'_A$. Because, by Lemma A-4, at most one firm places point mass on 0, if firm B placed point mass on 0, firm A would place no mass on 0, in which case firm B would lose for sure by bidding 0. Thus, given that firm B can give its shareholders a payoff strictly greater than the payoff from losing at 0 by bidding more than and arbitrarily close to r_A, by placing point mass on 0, firm B could not be optimizing. Hence,

firm B places no mass on 0. Lemma A-4 also implies that firm B places no mass on any $b \in (0, r_A)$. Thus, under both (K_A, D_A) and (K'_A, D'_A), firm B places no mass on any $b \in [0, r_A)$. Hence, by an argument analogous to the one for establishing equation (A-2), noting that, by hypothesis, $r_A < r'_A$, firm B's bid distribution under (K_A, D_A), which we denote by F_B, and the one under (K'_A, D'_A), which we denote by $\hat{F}_B$, satisfy that

$$F_B(b) = 1 - \frac{1}{1 + \frac{\min[b, K_A - D_A]}{\lambda (v_H - b) + (1 - \lambda) \max[v_L - b, -(K_A - D_A)]}}, \quad b \in [0, r_A) \tag{A-9}$$

$$\hat{F}_B(b) = 1 - \frac{1}{1 + \frac{\min[b, K'_A - D'_A]}{\lambda (v_H - b) + (1 - \lambda) \max[v_L - b, -(K'_A - D'_A)]}}, \quad b \in [0, r_A). \tag{A-10}$$

Because $r_A < r'_A$, by equation (5), it must be that $K'_A - D'_A < \lambda (v_H - v_L)$ and $K'_A - D'_A < K_A - D_A$. Because, by equation (6), $r_A > \lambda (v_H - v_L)$, an inspection of equations (A-9) and (A-10) then shows that, for any fixed $b \in [0, r_A)$, $F_B(b) \geq \hat{F}_B(b)$, while for $b \in [0, r_A)$ sufficiently close to r_A, $F_B(b) > \hat{F}_B(b)$. Thus, F_B is strictly dominated by $\hat{F}_B$ in FOSD, implying that firm B bids more aggressively under (K'_A, D'_A) than under (K_A, D_A).

Therefore, given that firm B bids more aggressively under (K'_A, D'_A) than under (K_A, D_A) while firm A's bid distribution remains the same, firm A's surplus must be strictly lower under (K'_A, D'_A) than under (K_A, D_A). $\square$

Lemma A-11. *Suppose that condition* (17) *holds. For any* $k < \lambda v_H + (1 - \lambda) v_L$, *firm A's surplus is weakly higher under* $(K_A = k, D_A = 0)$ *than under* $(K_A = k, D_A > 0)$.

Proof. By equation (6), firm A's willingness to pay is always no less than $\lambda v_H + (1 - \lambda) v_L$. Thus, given that $k < \lambda v_H + (1 - \lambda) v_L$, by Lemma A-9, under both $(K_A = k, D_A = 0)$ and $(K_A = k, D_A > 0)$, firm B enters if and only if $r_B > k + c$. By Lemma A-8, conditional on $r_B > k + c$, at date 2, both firms' bid distributions have support equal to $[0, k]$ in both cases. Then by an argument similar to the one used in the proof of Lemma A-10, we can show that firm A's bid distribution is the same in both cases while firm B's bid distribution under $(K_A = k, D_A > 0)$ weakly dominates that under $(K_A = k, D_A = 0)$ in FOSD. This result implies that firm A's surplus is weakly higher under $(K_A = k, D_A = 0)$ than under $(K_A = k, D_A > 0)$. $\square$

Lemma A-12. *Suppose that condition* (17) *holds. Firm A's surplus is strictly higher under the pure-equity structure,* $(K_A = \lambda v_H + (1 - \lambda) v_L, D_A = 0)$, *than under any structure* $(K_A = k, D_A)$, *where* $k < \lambda v_H + (1 - \lambda) v_L$.

Proof. By Lemma A-11, to prove the result, it suffices to show that firm A's surplus is strictly higher under the pure-equity structure $(K_A = \lambda v_H + (1 - \lambda) v_L, D_A = 0)$ than under any pure-equity structure $(K_A = k, D_A = 0)$, where $k < \lambda v_H + (1 - \lambda) v_L$. Note that, by Lemma A-9, the pure-equity structure $(K_A = \lambda v_H + (1 - \lambda) v_L, D_A = 0)$ better deters entry than any pure-equity structure $(K_A = k, D_A = 0)$, where $k < \lambda v_H + (1 - \lambda) v_L$. By Lemma A-6, conditional on that the entry condition in Lemma A-9 is satisfied (and, hence, firm B finds it profitable to enter), firm A's surplus is zero whenever a pure-equity structure is used. The result thus follows from the fact that the pure-equity structure $(K_A = \lambda v_H + (1 - \lambda) v_L, D_A = 0)$ better deters entry than any pure-equity structure $(K_A = k, D_A = 0)$, where $k < \lambda v_H + (1 - \lambda) v_L$. $\square$

Lemma A-13. *Suppose that condition* (17) *holds. For firm A, having any capital structure* (K_A, D_A) *such that* $K_A < r_A$, *where* r_A *is given by equation* (6)*, is never optimal.*

Proof. Lemma A-12 implies that, for firm A, having any capital structure (K_A, D_A) such that $K_A < \lambda\, v_H + (1 - \lambda)\, v_L$ is never optimal. Thus, given that, by equation (6), $r_A \geq \lambda\, v_H + (1 - \lambda)\, v_L$, to prove the result, it suffices to show that, for firm A, having any capital structure (K_A, D_A) such that $\lambda\, v_H + (1 - \lambda)\, v_L \leq K_A < r_A$ is never optimal. The latter follows immediately from Lemma A-10. $\square$

Now we use the above lemmas to prove the proposition. The choice of K_A and D_A uniquely determines the choice of K_A and $K_A - D_A$, and vice versa. Thus, we can think of firm A as choosing K_A and $K_A - D_A$. By equation (6), the choice of $K_A - D_A$ uniquely determines r_A. Because firm A has no incentive to bid more than r_A and, by hypothesis, firm A does not raise redundant capital, in any equilibrium, $K_A \leq r_A$. Then by Lemma A-13, in any equilibrium, $K_A = r_A$, which condition is equivalent to condition (2). To complete the proof, it suffices to rule out the optimality of low-leverage structures, structures with $0 < D_A \leq v_L$. By condition (2), these low-leverage structures all have $K_A = \lambda\, v_H + (1 - \lambda)\, v_L$, with the associated firm A's willingness to pay being $\lambda\, v_H + (1 - \lambda)\, v_L$. By Lemma A-9, firm B's entry decision is the same under these low-leverage structures and the pure-equity structure $(K_A = \lambda\, v_H + (1 - \lambda)\, v_L, D_A = 0)$. By Proposition 2.ii, conditional on that the entry deterrent condition in Lemma A-9 is satisfied, these low-leverage structures produce strictly lower firm A's surplus than the pure-equity structure. Thus, given that, as we have established at the beginning of this proof, as long as firm A's surplus is positive, firm A's internal shareholders can always capture the entire surplus through some financial arrangement, these low-leverage structures are strictly dominated by the pure-equity structure. $\square$

Proof of Proposition 6. Follows immediately from the argument right before the proposition in the main text. $\square$

Proof of Lemma 2. Follows immediately from the argument right before the lemma in the main text. $\square$

Proof of Proposition 7. Under the specified high-leverage structures, $K_A = \lambda\, v_H + (1 - \lambda)\, D_A$, implying that $K_A - D_A = \lambda\, (v_H - v_L)$, and $\pi_A^g(K_A, D_A; \rho)$ is given by equation (A-3). The proposition then follows from the argument provided in the paragraph after the proposition, equations (21) and (A-3), a straightforward integration, and a substitution for $K_A - D_A$ using $K_A - D_A = \lambda\, (v_H - v_L)$. $\square$

Proof of Proposition 8. Follows immediately from the argument right before the proposition in the main text. $\square$

Proof of Proposition 9. To establish the proposition, we first show that, conditional on the entry condition in Proposition 4 being satisfied, firm A's surplus created at date 2, π_A^g, under any high-leverage structure, $(K_A = \lambda\, v_H + (1 - \lambda)\, D_A, D_A)$, where $D_A \in (v_L, v_H)$, must be strictly negative. This result follows from Proposition 2 and the fact that, had firm A used the pure-equity structure, $(K_A = \lambda\, v_H + (1 - \lambda)\, v_L, D_A = 0)$, its surplus created at date 2 would be 0.

Then note that, by Proposition 6, the marginal effect of increasing c on firm A's surplus under the pure-equity structure is $1/z$. By equation (21), the marginal effect of increasing c on firm A's surplus under any fixed high-leverage structure is strictly greater than $1/z$. This is because the integrand in the second line of equation (21) is negative, implying that the marginal effect on the integral of increasing its lower limit through an increase in c is strictly positive, while the derivative with respect to c of the right hand side in the first line of equation (21) equals $1/z$.

Therefore, the marginal effect of increasing c on firm A's surplus under the pure-equity structure is strictly lower than that under any fixed high-leverage structure. The first two parts of the proposition thus follow from this result. To establish the last part of the proposition, simply note that, for any fixed high-leverage structure, when c satisfies that

$$\frac{\lambda v_H + (1-\lambda) D_A + c}{\lambda v_H + (1-\lambda) v_L} = z,$$

the integral in equation (21) vanishes. In this case, firm A's surplus under the high-leverage structure equals $(\lambda v_H + (1-\lambda) D_A + c)/z$, which, given that the high-leverage structure satisfies $D_A > v_L$, is strictly greater than $(\lambda v_H + (1-\lambda) v_L + c)/z$, firm A's surplus under the pure-equity structure. □

Proof of Proposition 10. First, we show that the pure-equity structure, $(K_A = \lambda v_H + (1-\lambda) v_L, D_A = 0)$, is optimal when $v_H - v_L$ is sufficiently small. Let $v_L = v_H - \varepsilon$, $\varepsilon > 0$. Let $D_A = v_H - \delta \varepsilon$, $\delta \in (0,1)$. Our construction covers any $D_A \in (v_L, v_H)$. As $\varepsilon \to 0$, $v_L \to v_H$ and $D_A \to v_H$. Note that firm A's surplus under the pure-equity structure presented in Proposition 6 tends to $(v_H + c)/z$ as $\varepsilon \to 0$. In contrast, because the integral in equation (21) is negative under a high-leverage structure, firm A's surplus under a high-leverage structure given by equation (21) tends to a value strictly lower than $(v_H + c)/z$ as $\varepsilon \to 0$. Thus, by Propositions 5 and 8, as $\varepsilon \to 0$ (or equivalently, as $v_H - v_L \to 0$), the pure-equity structure, $(K_A = \lambda v_H + (1-\lambda) v_L, D_A = 0)$, is optimal.

Next, we show that the pure-equity structure, $(K_A = \lambda v_H + (1-\lambda) v_L, D_A = 0)$, is optimal when λ is sufficiently close to 1. Note that firm A's surplus under the pure-equity structure presented in Proposition 6 tends to $(v_H + c)/z$ as $\lambda \to 1$. In contrast, because the integral in equation (21) is negative under a high-leverage structure, firm A's surplus under a high-leverage structure given by equation (21) tends to a value strictly lower than $(v_H + c)/z$ as $\lambda \to 1$. Thus, by Proposition 5 and 8, when λ is sufficiently close to 1, the pure-equity structure, $(K_A = \lambda v_H + (1-\lambda) v_L, D_A = 0)$, is optimal. □

Proof of Proposition 11. Follows immediately from the argument right before the proposition in the main text.
□