There and Back Again: Mapping and Factorizing Cosmological Observables

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Cosmological correlators encode invaluable information about the wave function of the primordial Universe. In this Letter we present a duality between correlators and wave function coefficients that is valid to all orders in the loop expansion and manifests itself as a $\mathbb{Z}_4$ symmetry. To demonstrate the power of the duality, we derive a correlator-to-correlator factorization formula for the parity-odd part of cosmological correlators that relates n-point observables to lower-point ones via a series of diagrammatic cuts. These relations serve as the first example of physically testable cutting rules as they involve observables defined for arbitrary physical kinematics. We further show how the duality allows us to translate the cosmological optical theorem for wave function coefficients into statements about cosmological correlators.

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Introduction—One of the ultimate goals of physics is to understand the laws of nature at the beginning of time. A cosmologist's approach to this problem is to measure spatial correlations in the cosmic microwave background radiation and the large scale structure of the Universe. These correlations are seeded by primordial "cosmological correlators" of quantum fields evolving during inflation, with their momentum dependence encoding the secrets of the underlying physics at play during the Universe's first moments. For example, the soft limit of correlators probes the cosmic expansion history [1,2] and the inflationary particle spectrum [3–30], the equilateral limit probes higher-dimensional self-interactions of the inflaton [31–35], while the collinear limit probes the initial state [32,36–40] and environmental effects [41]. Understanding the structure of cosmological correlators is therefore of upmost importance in our quest to understand the early Universe and therefore physics at extreme energy scales.

In recent years, however, much attention has been paid to more primitive objects, namely "wave function coefficients" that encode the inflationary dynamics in the perturbative expansion of the wave function of the Universe (given that we work on a fixed background geometry, "field-theoretic wave function" might be a more appropriate name) [42,43]. Although these objects are not directly observable, cosmological correlators can be extracted from them by applying the Born rule, and their somewhat simpler kinematic dependence means that constraints from cherished physical principles such as symmetries, locality, and unitarity turn out to be more transparent [44–64]. They also play an important role in defining cosmological amplitudes [58,65–68], can be used to understand the origin of IR divergences in de Sitter space [69–71], contain (boost-breaking) flat-space amplitudes in a certain singular kinematic limit [72–74], and have neat connections to geometry [49,75–77].

In this Letter, we derive a duality between cosmological correlators B_n and the "physical" part of wave function coefficients $\rho_n \equiv \psi_n + \psi_n^\sharp$, where $\sharp$ stands for complex conjugation and momentum reversal [78], that is valid to all orders in perturbation theory (i.e., to all orders in the loop expansion). We show that in the dictionary that translates the $\{\rho_n\}$ to the $\{B_n\}$, there exists a $\mathbb{Z}_4$ symmetry that "syntactically" swaps $\rho_n \leftrightarrow B_n$ and maps any valid equation to another valid equation within the dictionary. The

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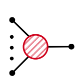
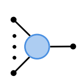
duality therefore takes us from wave function coefficients to cosmological correlators, *and back again*.

We use this duality, in combination with the results of [79] where we showed that parity-odd correlators are factorized, to prove that under a set of mild assumptions, parity-odd correlators of inflatons and gravitons factorize into a *structured sum of products of lower-point correlators*. This factorization holds for physical kinematics (no need for analytic continuation) and for generic momentum dependence (no need to take specific kinematic limits) and is in principle a relation that can be tested with observations. We show that it also has a neat interpretation in terms of diagrammatic cuts. The first nontrivial example of this correlator-to-correlator factorization (CCF) relates the trispectrum of primordial perturbations to the bispectra involving two curvature perturbations and one additional state with integer spin and a complementary series mass, and the power spectrum of this state. The power of this relation lies in the fact that it maps an observable to a combination of other observables.

We further show the usefulness of our duality in the context of unitarity and the cosmological optical theorem (COT) [51]. The COT is most naturally derived for wave function coefficients since unitary time evolution imposes a set of conditions on the wave function of the Universe [47,51–53,61]. It manifests itself as a relation between analytically continued wave function coefficients. It is desirable, however, to derive conditions on cosmological correlators since these are ultimately the fundamental observables and, while some progress has been made in this direction [58] (see also [80–83] for cutting rules that focus on the cosmological collider signal), the full set of conditions has not been derived (even at tree level). In this Letter we show how our duality can play an important role in this regard by converting the COT for wave function coefficients into statements for cosmological correlators.

Notations and conventions   For conciseness, we use the DeWitt notation [84], where both field indices and spatial coordinates are abbreviated as a single Latin index as $\varphi^A(\mathbf{x}) \equiv \varphi_i$. Contractions are interpreted as $\varphi_i\chi^i \equiv \sum_A \int \mathrm{d}^3x\, \varphi_A(\mathbf{x})\chi^A(\mathbf{x})$. We adopt the following diagrammatic notations:

$B_n \equiv$  , $\rho_n \equiv$  .   (1)

These blobs represent the abstract notion of correlators and wave function coefficients, respectively, without specific perturbation theory structures inside. They should be distinguished from the typical Schwinger-Keldysh and Witten diagrams that compute B_n and ρ_n. For conciseness, all momentum and tensor indices have been suppressed.

Correlator-wave function duality—We start with a lightning review of the wave function approach to primordial perturbations. Consider a set of weakly interacting quantum fields collectively denoted by $\Phi^i(\eta)$ evolving in a classical inflationary spacetime $g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu},\ a(\eta) = -1/(H\eta)$. In the Schrödinger picture, we define the wave function of the Universe by projecting the time-evolved Bunch-Davies (BD) vacuum onto a field-value eigenstate,

$$\Psi[\varphi] = \langle\varphi|U(\eta_0,-\infty)|\mathrm{BD}\rangle = \int_{\mathrm{BD}}^{\Phi(\eta_0)=\varphi} \mathcal{D}\Phi e^{iS[\Phi]}. \quad (2)$$

In practice, the Bunch-Davies vacuum choice means that we integrate over fields that vanish in the far past (with a suitable contour deformation). In perturbation theory, the wave function is conventionally parametrized by

$$\Psi[\varphi] = \exp\left[+\sum_{n=1}^{\infty}\frac{1}{n!}(\psi_n)^{i_1\cdots i_n}\varphi_{i_1}\cdots\varphi_{i_n}\right], \quad (3)$$

with the coefficients ψ_n computed via Witten diagrammatics. Given our notation, there is an implicit sum over different fields in (3), thereby ensuring that each wave function coefficient has the appropriate normalization, e.g., for ψ_3 with only two identical fields, we have an overall factor of $(3/3!) = (1/2!)$. We unconventionally start the summation with $n = 1$. This tadpole term starts out at loop level and is necessary for the cancellation of the monopole moment in observables.

The full n-point correlators of quantum fields are computed by the Born rule as [85]

$$\langle\varphi_{i_1}\cdots\varphi_{i_n}\rangle = \frac{\int\mathcal{D}\varphi|\Psi[\varphi]|^2\varphi_{i_1}\cdots\varphi_{i_n}}{\int\mathcal{D}\varphi|\Psi[\varphi]|^2}. \quad (4)$$

However, the quantities of more observational relevance are their connected part, which we will denote as B_n, in which only one momentum-conserving δ function is present in momentum space. These are computed from the generating functional

$$Z[J] = \int\mathcal{D}\varphi|\Psi[\varphi]|^2 e^{iJ^i\varphi_i} \quad (5)$$

by taking derivatives with respect to the auxiliary current, i.e.,

$$(B_n)_{i_1\cdots i_n} = \frac{\partial}{i\partial J^{i_1}}\cdots\frac{\partial}{i\partial J^{i_n}}\ln Z[J]\bigg|_{J=0}. \quad (6)$$

This yields the correct normalization since the denominator in (4) is simply $Z[0]$, and we emphasize that the derivatives act on $\ln Z[J]$ rather than $Z[J]$, thereby ensuring we only extract connected correlators. Alternatively, we can integrate (6) to obtain

$$\exp\left[+\sum_{n=1}^{\infty}\frac{i^n}{n!}(B_n)_{i_1\cdots i_n}J^{i_1}\cdots J^{i_n}\right] = \int\mathcal{D}\varphi\,\exp\left[+\sum_{n=1}^{\infty}\frac{1}{n!}(\rho_n)^{i_1\cdots i_n}\varphi_{i_1}\cdots\varphi_{i_n}\right]e^{iJ^i\varphi_i}, \quad (7)$$

where $\rho_n \equiv \psi_n + \psi_n^\sharp$ is the physical part of a wave function coefficient. Therefore, the connected correlators can be viewed as a Fourier transformation of the physical wave function coefficients. Now notice that the two sides of (7) take completely analogous forms. We can therefore perform an inverse Fourier transformation to rewrite (7) as

$$\exp\left[+\sum_{n=1}^{\infty}\frac{1}{n!}(\rho_n)^{i_1\cdots i_n}\phi_{i_1}\cdots\phi_{i_n}\right] = \int \mathcal{D}J \exp\left[+\sum_{n=1}^{\infty}\frac{i^n}{n!}(B_n)_{i_1\cdots i_n}J^{i_1}\cdots J^{i_n}\right]e^{-iJ^i\phi_i}. \quad (8)$$

After a change of the dummy variable $J \to -J$, we obtain

$$\exp\left[+\sum_{n=1}^{\infty}\frac{1}{n!}(\rho_n)^{i_1\cdots i_n}\phi_{i_1}\cdots\phi_{i_n}\right] = \int \mathcal{D}J \exp\left[+\sum_{n=1}^{\infty}\frac{(-i)^n}{n!}(B_n)_{i_1\cdots i_n}J^{i_1}\cdots J^{i_n}\right]e^{iJ^i\phi_i}. \quad (9)$$

Comparing (7) and (9), we see that a syntactic replacement [86],

$$g\colon \begin{pmatrix}\rho_n \\ B_n\end{pmatrix} \mapsto (-i)^n \begin{pmatrix}B_n \\ \rho_n\end{pmatrix}, \quad (10)$$

maps them to each other (after some labeling). Since both (7) and (9) are equivalent statements of the correlator-wave function dictionary, we deduce that g is an exact symmetry of the dictionary. Since $g^4 = 1$, the duality mapping generates a $\mathbb{Z}_4$ group that maps the dictionary to itself. The formal proof above shows that such a $\mathbb{Z}_4$ symmetry is valid to "arbitrary" finite orders in perturbation theory, since the Gaussian integrals in both (7) and (9) are well-defined as long as $\rho_2 < 0 < B_2$. Note that in this derivation we have dropped an integration constant since ultimately the relationship between the ρ_n and B_n comes from taking derivatives with respect to an auxiliary current and therefore an overall constant in (9) is inconsequential.

To demonstrate the duality, let us inspect some simple examples. Up to $n = 5$ at tree level, we have

$$B_2 = -\frac{1}{\rho_2}, \quad (11a)$$

$$B_3 = -\frac{1}{\rho_2^3}\rho_3, \quad (11b)$$

$$B_4 = \frac{1}{\rho_2^4}\left[\rho_4 - \left(\rho_3\frac{1}{\rho_2}\rho_3 + 2\,\text{perms}\right)\right], \quad (11c)$$

$$B_5 = -\frac{1}{\rho_2^5}\left[\rho_5 - \left(\rho_4\frac{1}{\rho_2}\rho_3 + 9\,\text{perms}\right) + \left(\rho_3\frac{1}{\rho_2}\rho_3\frac{1}{\rho_2}\rho_3 + 14\,\text{perms}\right)\right], \quad (11d)$$

where the internal DeWitt indices are understood as contracted. Note that, up to minus signs, the coefficient of each term is unity thanks to the normalization of the wave function. Under the duality mapping g, Eqs. (11) become

$$-\rho_2 = -\frac{1}{-B_2}, \quad (12a)$$

$$i\rho_3 = -\frac{1}{-B_2^3}iB_3, \quad (12b)$$

$$\rho_4 = \frac{1}{B_2^4}\left[B_4 - \left(iB_3\frac{1}{-B_2}iB_3 + 2\,\text{perms}\right)\right], \quad (12c)$$

$$-i\rho_5 = -\frac{1}{-B_2^5}\left[-iB_5 - \left(B_4\frac{1}{-B_2}iB_3 + 9\,\text{perms}\right) + \left(iB_3\frac{1}{-B_2}iB_3\frac{1}{-B_2}iB_3 + 14\,\text{perms}\right)\right], \quad (12d)$$

which are equivalent to solving the original Eqs. (11) for the ρ's. Note also that since the $n = 1$ entry starts out at loop level, we have neglected it here. In practice, it is convenient to remove the tree-level tadpoles, so that the Gaussian term dominates the typical field fluctuations. However, we note that the duality (10) works for any values of ρ_1 and B_1, since convergence is always guaranteed by the Gaussian term at nontypically large field fluctuations. In the Supplemental Material [87], we explicitly verify the duality including the $n = 1$ tadpole terms up to four-point one-loop order, and show that consistently keeping tadpole terms is essential for the duality to work. We have further successfully confirmed the validity of the duality with a channel-insensitive check at nine-point four-loop order using a computer algorithm [88].

In general, the tree-level dictionary translating physical wave function coefficients to correlators (in the absence of linear mixings that we assume throughout) reads

$$B_n = \frac{1}{(-\rho_2)^n}\sum_{k=0}^{n-3}(-1)^k\left(k\,\text{cuts}\right)_\rho, \quad (13)$$

with

$$\left(k\,\text{cuts}\right)_\rho \equiv \sum_{n-k\geq n_1\cdots n_{k+1}\geq 3}\left[\rho_{n_1}\frac{1}{\rho_2}\rho_{n_2}\cdots\rho_{n_k}\frac{1}{\rho_2}\rho_{n_{k+1}} + (\pi_{n_1\cdots n_{k+1}} - 1)\,\text{perms}\right]. \quad (14)$$

The correlator-wave function duality then implies the reciprocal formula

$$\rho_n = \frac{1}{B_2^n}\sum_{k=0}^{n-3}(-1)^k\big(k\,\text{cuts}\big)_B, \qquad (15)$$

where $\big(k\,\text{cuts}\big)_B$ is obtained from (14) via a syntactic substitution $\rho \to B$. Notice that here we have applied the tree-level topology to write $(-i)^{n_1+\cdots+n_{k+1}-n} = (-1)^k$.

In summary, the power of the duality allows us to directly invert the dictionary without the need of ever performing the algebraic inversion in practice, and all the combinatorics are automatically left intact.

Factorization of parity-odd correlators—Let us now see how the duality we derived above can be put to good use. In [79], we derived a factorization theorem for cosmological correlators of the inflaton and graviton that states that n-point functions of these states are factorized into lower-point objects if these observables are parity-odd (PO) [89]. This theorem allows for correlators arising from the exchange of additional states of *any* mass and integer spin (in addition to contact diagrams), and relies on the following small set of mild assumptions: (i) unitarity and locality, (ii) the tree-level approximation, (iii) Bunch-Davies vacuum conditions, (iv) IR convergence of the nested time integrals that compute cosmological correlators, and (v) scale invariance of the interactions.

An immediate consequence of the theorem is the absence of total-energy singularities in the PO sector of primordial perturbations that leads to a very nice distinction between the PO and parity-even sectors [90]. The theorem does not, however, state what objects such correlators factorize into. This is where the duality we have derived here comes to fruition. Indeed, the basis of the factorization theorem of [79] is a proof that for the PO sector we have $\rho_n^{\text{PO}} = 0$ (as long as the external states are the inflaton and/or the helicity-summed graviton, and any internal states that are produced during inflation and decay into these massless states are in the complementary series of de Sitter representations or the $SO(3)$ representations of [26]). This follows from the fact that for the PO sector we have $\rho_n^{\text{PO}} = \psi_n(\mathbf{k}) - \psi_n^*(\mathbf{k})$, i.e., it is the imaginary part of wave function coefficients that contribute to cosmological observables, yet under the above assumptions, wave function coefficients are purely real. This can be proven on very general grounds by performing Wick rotations of the time variables and using the fact that the time-ordered part of the bulk-bulk propagator is purely real after this rotation [92], as are the vertices and the bulk-boundary propagator [79]. The result holds for exchanging fields of arbitrary integer spin [95].

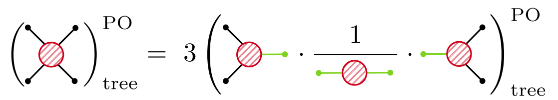
Turning our attention to (15), in the PO sector the factorization theorem of [79] therefore implies

$$\frac{1}{B_2^n}\sum_{k=0}^{n-3}(-1)^k\Big[\big(k\,\text{cuts}\big)_B\Big]^{\text{PO}} = \rho_n^{\text{PO}} = 0, \qquad (16)$$

which can be rearranged to yield a formula for B_n^{PO} in terms of lower-point correlators,

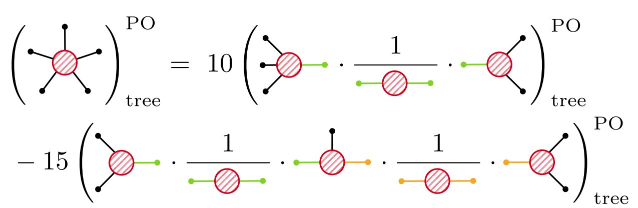
$$B_n^{\text{PO}} = \sum_{k=1}^{n-3}(-1)^{k-1}\Big[\big(k\,\text{cuts}\big)_B\Big]^{\text{PO}}, \quad \forall\; n \geq 4. \qquad (17)$$

In these expressions $[\ldots]^{\text{PO}}$ indicates that we are projecting correlators onto their PO part, which for correlators of the inflaton means that we take the imaginary part [97]. Note that this formula holds regardless of how the parity violation arises, which could be due to parity-violating vertices or due to the exchange of a spinning field with a parity-violating two-point function. We therefore see that the correlator-wave function duality has enabled us to derive, using the factorization theorem of [79], a correlator-to-correlator factorization (CCF) formula for the PO sector of primordial perturbations that has a neat interpretation in terms of correlator cuts. As an example, for $n = 4$ there is only one possibility where an exchange diagram is cut into two cubic diagrams and (17) can be diagrammatically represented by



(18)

with the factor of 3 symbolizing the three different channels. The absence of a contact diagram contribution was already noted in [98–100]. For $n = 5$ there is more structure with single and double cuts possible. We have



(19)

where again the numerical factors are counting distinct channels, and the different colors allow for the exchange of different fields.

In the above derivation we took additional states to be in the complementary series as this allowed us to set $\rho_n^{\text{PO}} = 0$ [79]. If there are also principle series fields, then this is not possible since then $\rho_n^{\text{PO}} \neq 0$. Correlators are indeed still factorized, but they do not factorize into other correlators. Our CCF formula is still useful for principle series fields, however, since if we have an explicit expression for B_n^{PO} due to the exchange of complementary series fields (which can be computed using the CCF formula), we can analytically continue the mass parameter to derive corresponding expressions for the exchange of principle series fields [106].

The most relevant case for phenomenology is $n = 4$ (corresponding to the trispectrum) and with curvature perturbations as the external states. This observable has received much attention recently due to the purported detection of parity violation in the galaxy four-point function [107,108] (see also [109,110], however). In this case our CCF formula reads

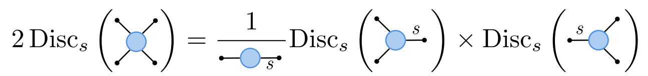
$$B_4^{\mathrm{PO}}(\{\mathbf{k}\}) = \left[B_3(\mathbf{k}_1, \mathbf{k}_2, -\mathbf{s}) \cdot \frac{1}{B_2(\mathbf{s})} \cdot B_3(\mathbf{s}, \mathbf{k}_3, \mathbf{k}_4)\right]^{\mathrm{PO}} + (\mathbf{t} + \mathbf{u}) \text{ channels}, \tag{20}$$

which provides a neat connection between distinct observables, namely between the PO part of the trispectrum of curvature perturbations, the bispectrum consisting of two curvature perturbations and one additional state with integer spin and mass in the complementary series, and the power spectrum of this new state. This relation is in principle testable, and any violation of this relation would imply one of the above listed assumptions is violated. One case of particular interest is where the exchanged field is the graviton. Parity violation in such a setup can come from a number of sources, e.g., as a dynamical Chern-Simons correction to the graviton propagator [111] or due to a mixing between the graviton and an $SU(2)$ gauge field [112,113]. It would be interesting to study these cases in more detail given our CCF formula.

Finally, let us point out that although the CCF relation (17) is almost completely transparent in the wave function language thanks the correlator-wave function duality, it seems rather mysterious from the traditional in-in and Schwinger-Keldysh diagrammatics of [114]. One can indeed derive CCF relations using Schwinger-Keldysh diagrammatics for specific lower-point examples, as we demonstrate for $n = 4$ in the Supplemental Material, yet the general proof seems to be hidden from sight. It remains an intriguing question as to why boundary correlators should factorize into other correlators even away from specific kinematic limits (e.g., the operator product expansion limit).

Unitarity and cosmological correlators—We now turn our attention to unitarity. Understanding the consequences of unitarity on observables is a vital component of a bootstrap toolbox. For scattering amplitudes unitarity requires tree-level processes to factorize near poles, and consistent factorization across multiple channels can heavily constrain the space of admissible amplitudes and therefore admissible theories [74,115]. More generally, unitarity imposes a set of Cutkosky cutting rules for scattering amplitudes [116]. For cosmology the constraints imposed by unitarity are best understood at the level of wave function coefficients where a set of conditions imposed by unitary time evolution impose relations between analytically continued wave function coefficients [47,51–53,61]. Given that correlators rather than wave function coefficients are the true observables that are probed by cosmological surveys, it is desirable to find conditions that unitarity imposes directly on correlators. Progress in this direction has been made in, e.g., [58]; however, the general relations are not yet known. In this section we show how the duality we have derived in this Letter can be used to convert the COT for wave function coefficients into relations between analytically continued cosmological correlators, and as an example we derive the general relation for the tree-level five-point function of massless scalars fields, with IR-finite interactions, in de Sitter space. To complement the other discussions in this Letter, we will assume parity-even interactions.

The above assumptions are useful since they imply that the wave function coefficients are purely real [79,99]. We therefore have $\rho_n = 2\psi_n$, which we can use to write the COT in terms of ρ_n followed by using our duality to convert the rules into statements about B_n. Let us illustrate this procedure for $n = 4$ where unitarity imposes the following relations between ρ_4 and ρ_3 [51]:

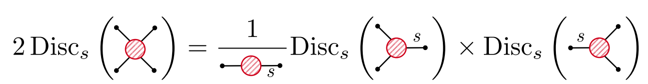


(21)

where $s = |\mathbf{k}_1 + \mathbf{k}_2|$, and

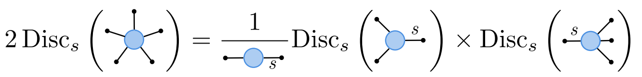
$$\mathrm{Disc}_y f(\{x\}, y) = f(\{x\}, y) - f(\{x\}, -y), \tag{22}$$

with $\{x\}$ a set of variables that do not flip sign under the discontinuity. Note that we do not include a complex conjugation on the right-hand side of (22), in contrast to, e.g., [53], since the wave function coefficients are real, and we have suppressed the dependence on spatial momenta and only included the dependence on the energies [117]. By acting on the fully symmetric ρ_4 with Disc_s we project onto the s-channel exchange diagram only. The relation (21) can be derived from the assumption of real couplings and factorization properties of the bulk-bulk propagator [51]. We can now use the duality, namely the relations between the ρ_n and B_n in (12) on both sides of (21) to yield



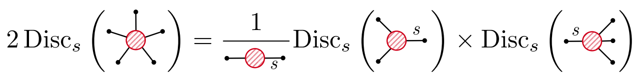
(23)

where we have used $B_2(-s) = -B_2(s)$, which holds for massless fields. This recovers the expression derived in [51]. We emphasize that although this relation shares some similarities with the CCF formula, it requires us to work with nonphysical momenta, whereas the CCF formula is a true statement also for physical momenta. We can follow the same procedure for $n = 5$, where unitarity imposes the following relation between ρ_5, ρ_4, and ρ_3:



(24)

If we now use the relations (12) on both sides of (24) we arrive at



(25)

which relies on a very nontrivial cancellation between the contributions with three copies of B_3. We emphasize that we have not been restricted to particular diagrammatics or channels in the above relations: they hold for the full $\{\rho_n\}$ and $\{B_n\}$ at tree level, with the duality automatically keeping track of the combinatorics. We find it very interesting that the form of the COT is universal, i.e., the ρ relations are identical to the B ones. This seems to suggest that the COT commutes with a naive application of the duality. We believe that this deserves further attention and we plan to return to it in the future.

Summary—In this Letter we have derived a duality in the dictionary between cosmological correlators $\{B_n\}$ and the physical part of wave function coefficients $\{\rho_n\}$ that is valid to all orders in perturbation theory. This duality allows us to derive a reciprocal formula that reconstructs $\{\rho_n\}$ from $\{B_n\}$ in a syntactic fashion. When combined with the results of [79], which states that $\rho_n^{\rm PO} = 0$ for massless scalar and graviton external states, and complementary series internal states, we obtain an infinite set of correlator-to-correlator factorization (CCF) formulas. These relations state that n-point PO correlators at tree level are factorized into structured combinations of lower-point correlators. These CCF relations involve observables defined for physical kinematics and can therefore in principle be tested observationally. Any violation of these relations would directly point to the failure of the tree-level assumption, unitarity, locality, scale invariance, or the Bunch-Davies vacuum. We showed how our CCF formulas can be understood in terms of diagrammatic cuts, and since taking these cuts does not require any analytical continuation, they serve as the first example to understand the general structure of cosmological observables in a physically accessible manner.

In addition to deriving physically testable relations, we also showed how the duality can be used to map the COT for wave function coefficients, which follows from unitary time evolution, into statements about cosmological correlators. Intriguingly, the form of the COT remains the same before and after application of the duality. We believe that this observation deserves further attention. It would also be interesting to use the duality to derive the COT for all n, thereby generalizing what we have focused on in this Letter for $n = 4$, 5.

Our Letter certainly opens up many more avenues for future exploration. For instance, the $\mathbb{Z}_4$ symmetry goes beyond the context of cosmology all the way to connected Green functions and effective field theory Wilson coefficients in general quantum field theories. It would be interesting to see if one can make general statements there, too. In addition, it would be interesting to extend our CCF formula (or something akin to CCF) to loop level. Finally, it would be neat to find a full proof of the CCF formula directly using the Schwinger-Keldysh formalism, where only observables are involved from the get-go.

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