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ScienceDirect

Procedia Manufacturing 51 (2020) 1038–1045

Procedia MANUFACTURING

www.elsevier.com/locate/procedia

30th International Conference on Flexible Automation and Intelligent Manufacturing (FAIM2021)
15-18 June 2021, Athens, Greece.

# Tolerance allocation: A reliability based optimisation approach

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**Abstract**

Tolerance analysis and allocation are two activities of great importance in product development. The mathematical formulation of the latter concerns the establishment and solution of a constraint optimisation problem. In this work, making one step forward, a probabilistic framework is developed and the tolerance synthesis problem is reformulated to a reliability based optimisation one introducing probabilistic constraints. Advanced reliability methods are merged with professional computer aided tolerance tools to estimate the distribution of the assembly key characteristic. Cost-tolerance relationships based on the variability of the manufacturing resources rather than on empirical formulas were adopted in a process based cost modelling methodology. The suggested framework is compared to the classical tolerance allocation approaches of the worst case scenario and the root sum square. It was found that despite the increased computational cost, further relaxation in the design tolerance can be achieved using reliability based optimisation techniques driving down the product cost.

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*Keywords:* Tolerance synthesis, Tolerance analysis, Reliability based Optimisation, Cost modelling, Mechanical assemblies

## 1. Introduction

The main core of any dimensional management strategy in product development is the ability to perform tolerance analysis and synthesis at the early design stage. That is, to predict the variance or the entire distribution of specified assembly key characteristics (AKC) as well as to optimise and allocate design tolerances for the assembly features in the various parts by minimising manufacturing cost. For the latter, several studies have been performed, as listed in [1], in which the optimisation problem was formulated using mainly one objective function, the manufacturing cost (or the total cost combining manufacturing cost and Taguchi quality loss function) and constraint functions based on the worst case (WC) error or the root sum square (RSS) of the AKC. State of the art commercial Computer Aided Tolerance tools (CAT), e.g. 3DCS variation analyst [2], use similar approach to allocate tolerances.

Despite the significant contribution of the developed methods to allocate tolerances to the various features in the parts of an assembly, there are still important limitations introduced with these approaches. More specifically, the estimation of the variance of the AKC following RSS, assumes a linearization of the assembly function $f$ whilst the probability distribution of the AKC is not taken into account in the tolerance allocation calculations. For this reason, classical tolerance allocation approaches become less accurate for certain problems.

In an attempt to consider more statistical information about the AKC as well as the actual form of the assembly model function to the tolerance synthesis problem, probabilistic tolerance allocation schemes have been proposed. In this approach, the constraint functions of the optimisation problem, usually involve the estimation of the probabilities, i.e. the probability that products cannot meet predefined specification limits. Lending the terminology from the structural reliability analysis field, the probabilistic tolerance synthesis problem can

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10.1016/j.promfg.2020.10.146

be treated as a Reliability-Based Optimisation (RBO) problem [3]. The task becomes quite complex because an uncertainty quantification analysis is nested inside an optimisation problem. That is, probability estimation should be performed for every iteration of the optimisation algorithm. The computational cost can increase dramatically.

Several methodologies exist to solve the RBO problem, e.g. in [3]. The key for a successful RBO methodology is to balance between computational time and accuracy in both the optimization and the uncertainty quantification problem. Studies implementing reliability analysis techniques on the tolerance allocation problem can be found in literature e.g. in [4]-[9]. Several works, e.g. in [4]-[6], concern the implementation of crude Monte Carlo analysis in combination with genetic or gradient based optimization algorithms. Although Crude Monte Carlo is quite efficient to handle high dimensional and non-linear problems however it suffers from the need of large samples to estimate relatively small probability values. In order to alleviate this problem, different strategies have been implemented. Variable size of samples were generated in crude Monte Carlo depending on the accuracy needed during the progress of the optimisation algorithm in [4], variance reduction techniques were implemented, e.g. Importance Sampling and correlation method in [5], as well as popular gradient based reliability methods, First Order Reliability Method (FORM) and Second Order Reliability Method SORM [10], were used to accelerate probability estimations, e.g. in [7]. It is highlighted, however, that tolerance analysis problems can be quite complex problems and thus, the associated tolerance models or in RBO terminology the specified state functions can be highly non-linear, implicit functions with respect to (wrt) the contributors, i.e. dimensions that form the tolerance chain, and expensive to evaluate them. This non-linearity might introduce the possibility of disjoint failure domains [11] in the random variable space and thus, many of the well-established reliability methods become insufficient to deal with the uncertainty quantification problem as demonstrated, e.g. in [12]. This includes typical formulation of the FORM and any other reliability method that is based on FORM as part of its strategy to estimate probabilities. Thus, most of the probabilistic tolerance allocation schemes proposed in the literature have limited applicability in tolerance modelling. An alternative, for time consuming and implicit state functions, is to use metamodeling techniques. Thus, Response Surface Method (RSM), has been introduced into the tolerance allocation problem in [8]. Generally, in this technique, simple models are built to substitute the limit state function and thus accelerate the limit state function evaluations. Nevertheless, it is doubtful whether second order polynomial model, usually employed in RSM method, is the most appropriate selection for highly non-linear state functions and deal with disjoint failure domains in the uncertainty quantification problem. Additionally, the high dimensionality of a tolerance analysis problem can greatly increase the number of state function evaluation when performing the design of experiment and thus building RSM model can become computationally quite expensive activity. For similar reasons, design of experiment techniques in combination with Pearson systems proposed in [9], can have reduced applicability.

Concerning the cost-tolerance relationships, the majority of the works found in the literature, e.g. in [4]-[7], is based on empirical cost-tolerance functions that capture the change of the cost of the product wrt the tolerances while calibration of these models is performed by fitting these models to appropriate experimental data. Despite the simplicity of the empirical models, there is a strong criticism about the effectiveness of these mathematical models, the quality and the type of the data necessary to build these models, the accessibility to these data as well as the applicability of the models [13].

From the previous discussion, the research activity on developing a suitable framework that combines (1) appropriate RBO methodologies capable to deal with complex assemblies and non-linearities with (2) cost-tolerance relationships based on reliable, easily accessed manufacturing data and (3) suitable optimization schemes is still growing. The aim of this work is to present the development of such a framework that allows to explore in details methods, schemes and functions for complex mechanical assemblies. As an outcome of the current investigation, the Quasi Monte Carlo method based on Sobol sequence (QMC-S) [10] combined with Sequential Quadratic Programming (SQP) [14] addressed successfully a simple benchmark case study that characterised by disjoint failure domains in the uncertainty quantification problem. Therefore, in section 2, the formulation of the tolerance allocation problem as a two-level reliability based optimisation one is presented. The developed framework and the case study are introduced in section 3 and 4 respectively. Results are presented and compared to the classical tolerance allocation approach in section 5. Useful conclusions are drawn in section 6.

**Nomenclature**

| | |
|---|---|
| AKC | Assembly Key Characteristic |
| CAT | Computer Aided Tolerance |
| FORM | First Order Reliability Method |
| GA | Genetic Algorithm |
| PBCM | Process Based Cost Model |
| RBO | Reliability Based Optimisation |
| RNG | Random Number Generation |
| RSM | Response Surface Methodology |
| RSS | Root Sum Square |
| QMC-S | Quasi Monte Carlo method based on Sobol sequence |
| SORM | Second Order reliability Method |
| SQP | Sequential Quadratic Programming |
| WC | Worst Case |
| wrt | with respect to |

## 2. Reliability based optimization formulation

One possible formulation of the tolerance allocation problem following a classical approach, i.e. WC or RSS, can be expressed by

$$\begin{gathered} minimise\ C(t_i) \\ subject\ to\ \ \Delta A(t_i) \leq USL - LSL, \quad (WC) \\ or\ subject\ to\ \ 6 \cdot \sigma_{AKC}(t_i) \leq USL - LSL, \quad (RSS) \end{gathered} \tag{1}$$

where $C(t_i)$ is the manufacturing cost and is a function of the design parameters of the optimization problem i.e. the specified design tolerances $t_i$ where $i = 1,2, \dots n$. $t_i$ corresponds to the range that one dimension $d_i$ can fluctuate about its respective nominal value $\overline{d_i}$ . $USL$ and $LSL$ are the upper and lower specification limits usually defined by customer requirements and $\Delta A$ corresponds to the WC variation

$$\Delta A = \frac{\partial f}{\partial d_1} t_1 + \frac{\partial f}{\partial d_2} t_2 + \cdots + \frac{\partial f}{\partial d_n} t_n \quad (2)$$

and $\sigma_{AKC}(t_1, t_2 \dots, t_n)$ corresponds to the RSS variation given by

$$\sigma_{AKC} = \sqrt{\left(\frac{\partial f}{\partial d_1}\right)^2 \sigma_{d_1}^2 + \left(\frac{\partial f}{\partial d_2}\right)^2 \sigma_{d_2}^2 + \cdots \left(\frac{\partial f}{\partial d_n}\right)^2 \sigma_{d_n}^2} \quad (3)$$

where the AKC can be expressed as a function of the contributors $d_i$ , i.e. other dimensions on the parts of the assembly formulating the tolerance chain, as $AKC = f(d_1, d_2, \dots, d_n)$ . The function $f$ establishes the assembly model. $\partial f / \partial d_i$ is the sensitivity of the AKC to the contributors $d_i$ and $\sigma_{d_i}^2$ is the variance of contributor $d_i$. The tolerance $t_i$ is related to the standard deviation of the contributor $\sigma_{d_i}$ by $t_i = N\sigma_{d_i}$ where $N$ usually equals to 6. Furthermore, considering geometrical tolerances, $t_i$ can be associated with more than one contributors, for example the positional tolerance of a hole which is generally a function of two parameters in two different directions (or a function of the magnitude and the angle of the positional vector of the varied centre wrt to the nominal one).

The assumptions related to the Eq. (2) and (3) can lead to less accurate solutions due to non-linearity in the assembly model and non-consideration of the distribution type of the AKC in the tolerance allocation process. The problem of interest can be reformulated as a RBO problem, its form is given by

$$\begin{gathered} minimise\ C(t_i) \\ subject\ to\ \ P_D\left[g_j\left(AKC\left(d_i(t_i)\right), SLs\right) \leq 0\right] \leq P_t,\ j = 1, .., m \end{gathered} \quad (4)$$

where $P_D$ is the probability of defected products i.e. the probability of the event that the specified AKC do not conform to the specification limits, *SLs*. This is equivalent to the probability of failure introduced in structural reliability analysis field. $g_j\left(AKC\left(d_i(\mathrm{t_i})\right), SLs\right)$ stands for the state function i.e. the relationship between the AKC and the specification limits and determines whether the product is a good or a defected one. $m$ corresponds to the different number of limit state functions. The mathematical formulation of the state functions wrt the tolerances can be an explicit mathematical expression or can be given implicitly e.g. by the use of a CAT tool. $P_t$ is the target probability of defected products and is usually calculated by its complementary event, i.e. the yield of the process which generally equals to 99.7% following six-sigma quality approach. It is highlighted that design variables of the optimisation problem are the tolerances $t_i$ which are deterministic variables whilst the random variables of the uncertainty quantification problem are the contributors $d_i$. Tolerances and contributors are related to each other by tolerances being the range of each contributor. Therefore, by adopting different tolerance values, the distribution of the AKC is changing and thus, the probability of defected products is changing as well. The optimum tolerances that satisfy the constraints for specific probability levels of defected products, $P_D$ , while the manufacturing cost is mimimised can be derived by solving Eq. (4).

## 3. Suggested tolerance allocation framework

There are four major elements to be determined in order to solve the tolerance allocation problem described by Eq. (4). That is, there is a need to develop tolerance models that establish a mathematical relationship between the tolerances in the various features of a product and the specified AKC. Having this relationship, uncertainty quantification methods should be implemented to estimate statistics of the AKC and thus, being able to estimate the constraints in Eq. (4) given the specification limits. A cost model capable of capturing the effect of tolerances is necessary to be defined to estimate the total manufacturing cost of the product in relationship to the desired tolerance in the various features whilst all the previous activities should be included in an optimisation loop to optimise the assigned design tolerances.

The proposed scheme developed herein is depicted in Fig. 1 along with the tools used to implement it. The tools used are a combination of in-house developed codes along with off the self software.

### *3.1. Tolerance modelling*

The tolerance models are developed, herein, using the professional CAT tool, 3DCS variation analyst [2]. The decision of using an off the self software is based on the capabilities of the specific software to analyse quite complex mechanical assemblies including mechanisms and compliant parts as well as on the fact that CAT tools are adopted more and more in the every-day practice of industry.

The specific software gives the possibility to run tolerance analyses in a batch mode. Thus, 3DCS analysis can be executed using e.g. a MATLAB script [15]. Furthermore, 3DCS gives the possibility, to import externally generated samples for every contributor in a vectorised form. Therefore, it is feasible to generate samples for the tolerance analysis problem under investigation using other tools than 3DCS itself, e.g. the statistical toolbox of MATLAB, and feed the generated samples into 3DCS for further processing. It is worth noting that the various uncertainty quantification methods and especially the simulation techniques estimate probabilities by evaluating the state function several times. The key element that differentiate the various uncertainty quantification methods is the way that the samples are generated and thus, this gives the capability to implement any type of reliability method in collaboration with 3DCS by generating the appropriate samples externally.

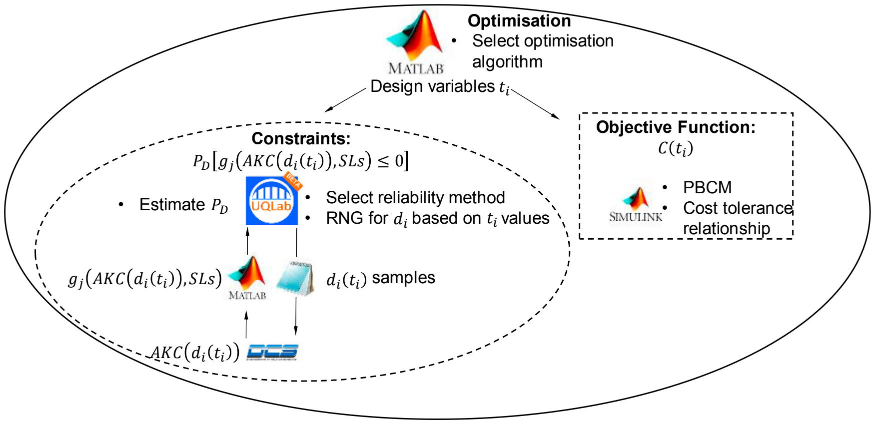


Fig. 1. Proposed tolerance allocation framework

Thus, 3DCS is only used to evaluate the tolerance model, i.e. the term $AKC(d_i(t_i))$ in Eq. (4) as shown in Fig. 1. Due to the vectorised form of the samples that can be fed in 3DCS, advanced simulation techniques can be implemented without much additional computational effort.

Assuming one AKC under investigation as well as the existence of both upper (USL) and lower specification limits (LSL), the state functions, $g_j(AKC(d_i(t_i)), SLs)$, of Eq. (4), are defined by

$$g_1 = AKC(d(t_i)) - LSL$$
$$g_2 = USL - AKC(d(t_i)) \quad (5)$$

where AKC is the Assembly Key Characteristic calculated by 3DCS analysis. When $g < 0$, the system is in fail state, i.e. a defected product has been produced. When $g \geq 0$, the system is in safe state and a good product has been produced. An example of the distribution of an AKC is depicted in Fig. 2 along with the two probabilities values that need to be estimated and summed together in order to specify $P_D$ in Eq. (4).

## 3.2. Uncertainty quantification

Having specified the state function of the problem, probability of defected products for every state function in Eq. (5) can be estimated using, theoretically, any uncertainty quantification method. For the proposed framework, UQLab [16] was used, an advanced general-purpose uncertainty quantification tool developed by ETH Zurich. UQLab is based on MATLAB functions and includes several state of the art reliability methods including gradient-based, simulation and meta-model based ones. However, due to complications related to the tolerance modeling, e.g. multiple failure domains, as discussed earlier, only simulation techniques were considered in this study. Other advanced reliability methods should be investigated in the future.

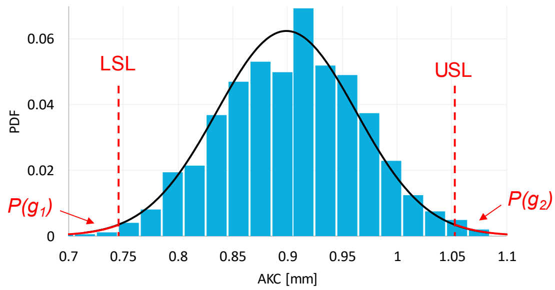


Fig. 2. Probability distribution of AKC of a product

The link between UQLab and 3DCS is based on the fact that the state functions in UQLab can be specified in a MATLAB script. Therefore, it is possible to call and execute, in a batch mode, a 3DCS analysis via UQLab using MATLAB script files. Additionally, UQLab offers a variety of random number generators including generators for random variables with prescribed marginal distributions and correlation matrix and thus, samples are generated and fed directly into 3DCS in the form of text files. The constraint functions of Eq. (4) thus can be estimated as depicted in Fig. 1.

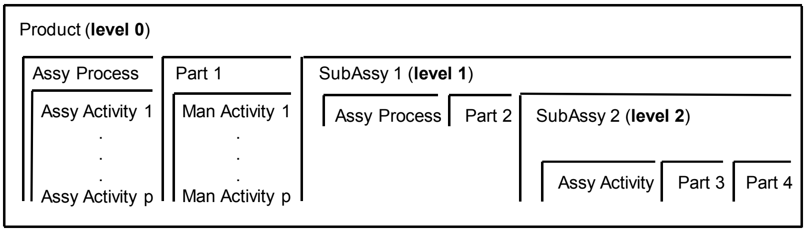


Fig. 3. Overview of CAMcost structure

### 3.3. Cost modelling

For the cost modelling, an in-house cost tool (CAMcost) was developed using SIMULINK and is based on the process based cost modelling technique (PBCM) [17]. Cost modelling in SIMULINK provides a user friendly interface whilst the cost modelling activity becomes quite simple by dragging and dropping dedicated blocks already developed in a customized cost library.

In CAMcost, the total manufacturing cost per part as well as the total assembly cost per product can be estimated by the sum of specific recurring and non-recurring costs. An overview of the structure of CAMcost is presented in Fig. 3. That is, in Fig. 3, there is the product block that contains all the other blocks (subassembly, part, process and activity blocks). At this level industrial parameters are defined such as scheduled operation time, number of shifts etc. whilst the total product cost is estimated. Inside the product block exists the subassembly and part blocks as well as the assembly process block necessary to join the parts and subassemblies involved at that level. Subassembly blocks can contain part and other subassembly blocks as well as a new assembly process blocks. Part blocks contain manufacturing activity blocks whilst assembly process block contains assembly activity blocks. Necessary manufacturing, cost and design input parameters are provided for every block at every level in the cost model.

Cost estimations can be extracted at any block or level. Recurring costs involve the material, the labour and the energy resources to produce parts and products. Their calculation is based on the unit method. Non-recurring costs such as cost to buy and maintain machines/equipment, tools/fixtures, or cost to accommodate the production, i.e. floor-space cost are estimated in CAMcost. The time value of money is considered, and the annual worth value is computed by summing the capital recovery and the annual operating and maintenance costs for every asset.

It should be highlighted that the resources needed to achieve the annual production volume is of high importance. Thus, the number of lines (or workstations) necessary to achieve the effective annual production volume is estimated based on simple average capacity equations. Machines and equipment can be dedicated or not to the activity. Tools are always considered to be dedicated to the activities that are involved. The time needed to perform one manufacturing or assembly activity is provided either directly by the user or is estimated using simple equations based on industrial reports and textbooks.

Finally, the applicability of the developed cost model greatly depends on the validity of the cost–tolerance relationship captured by the model. Cost-tolerance relationship, herein, is established based on the approach suggested in [13]. The adopted approach applies to a single tolerance at a time by considering the existing variability of manufacturing resources. This variability can be easily quantified by designing and performing dedicated experimental tests, e.g. for drilling process, drilling holes on a plate and determining the variability of the drilling machine on the size or the position of the holes. The model, then, compares the variability of the resource against the assigned design tolerance and the number of parts out of tolerances are determined. Thus, the extra number of parts that must be fabricated to compensate for this loss is determined. For this reason, dedicated activity blocks, e.g. pilot-hole drilling block, were developed in CAMcost that estimate the yield of the specific activity based on the variability of the resources involved, e.g. drilling machine, and the associated tolerance on the feature of interest to be fabricated. It is highlighted that the yield of each activity determines the effective production volume which affect the total cost of the product.

### 3.4. Optimisation algorithm

The final element that completes the probabilistic tolerance allocation framework concerns the optimization algorithms. MATLAB optimization toolbox [15] was used and thus, both global and gradient based algorithms are available to search for the optimum solution. There are specific pros and cons related to each optimization algorithm. It is important to mention that gradient based algorithms may converge and be trapped into a local minimum for non-convex problems whilst not so common for explorative based algorithms such as the genetic algorithm (GA) method with the appropriate parameter tuning. Thus, the selection of the initial design point when using, e.g. the SQP method is quite critical for the convergence to a local or a global minimum. SQP algorithm was selected in this work to set up the RBO methodology due to its fast convergences to a local (or global) minimum. In order to have a chance that a SQP algorithm will find values close to the global minimum, a heuristic approach was followed and a few iterations of the whole optimisation process were performed by starting the optimisation algorithm from several initial points in the design space.

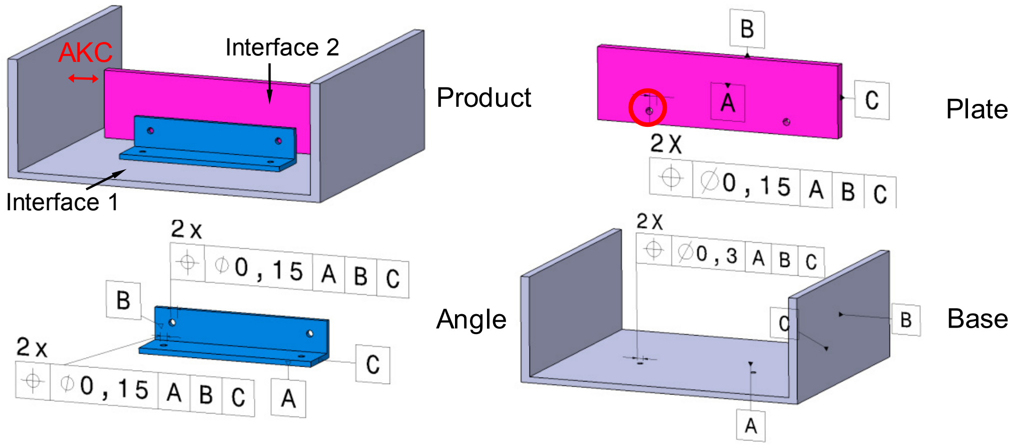


Fig. 4. Product overview along with design tolerances

The search for the global minimum is an interesting ongoing research topic whilst both deterministic and stochastic strategies have been devised and suggested in the literature [18]. However, the objective of this work is to develop the framework that this type of studies can be implemented on real complex assemblies and thus, it was not further investigated.

It is pointed out that the probability $P_D$ in the constraint function in Eq. (4) can vary across several order of magnitude. This might cause numerical issues for some optimisation algorithms. Therefore, probability values can be expressed alternatively with respect to the reliability index β [10] using the relationship $\beta_D = \Phi^{-1}(1 - P_D)$, where Φ is the standard Gaussian cumulative distribution function and thus, avoiding this dramatic change in the values of the constraint function.

Summarizing, the various activities of the developed framework, the objective function of Eq. (4) is calculated using the cost model described in section 3.3 whilst the two constraint functions of the problem are estimated based on the combination of the tolerance model and the uncertainty quantification methods presented in section 3.1 and 3.2 respectively. The optimization problem defined by Eq. (4) is solved using algorithms listed in section 3.4.

## 4. Case study

In aerospace industry, numerous strict requirements dictate the built of a wing box of a fixed wing aircraft. One of these requirements concerns the gaps between two adjacent parts for example the distance between the flanges of the ribs to the shear webs of the spar beams. Due to manufacturing variability, the rib should be designed slightly smaller in order to fit between the two spar beams. Therefore, small gaps exist between the rib and the spars interfaces. Strict requirements are applied to the values of these gaps in order to preserve structural integrity of the wing box structure whilst shimming process is performed to fill in those gaps. Without loss of generality, a simplified representation of the wing box with only three parts is presented in Fig. 4 and studied to prove the applicability of the developed framework. The product consists of three parts namely the base, the plate and the angle and those three parts correspond to a lower panel and two spar beams subassembly, a rib and a shear tie respectively. The shear tie connects the rib to the lower panel whilst rib, i.e. the plate in Fig. 4, has been designed much smaller to visualize the distance under study. This distance defines the AKC for the case study of this work.

It is assumed that all the parts are made from carbon/epoxy composite material (prepregs) using hand lay up manufacturing method. Thus, in this method, prepregs in the form of rolls are cut to fit in the mould usually by a CNC cutting machine and then they are kitted and stored in a freezer. The mould is carefully cleaned while a coating of release agent is applied to the mould to facilitate the removal of the finished part. Prepregs plies are thaw for few hours and then they are manually laid up on the mould one by one until the desired thickness is reached. The de-bulking process is taking place applying vacuum to remove volatiles and trapped air whilst curing process is performed in an autoclave. From Fig. 4, there are two interfaces with two mating holes per interface. The joining method of the three parts is assumed to be realized by mechanical fasteners. Thus, after demoulding of the parts, final size holes are drilled and inspected. Initial design tolerances are presented in Fig. 4. It is assumed that the base, generally a large structure and difficult to handle, will be drilled manually using a hand drill and some drilling templates whilst for the other two parts, holes will be opened using a CNC drill machine. For this reason two different positional tolerance values are depicted in Fig. 4. The assembly process flow consists of simple manual operations putting together one part at a time and forming the final product. A fixture is used to provide support to the base. It is highlighted that the developed cost model in SIMULINK captures the intended manufacturing and assembly plan described herein.

A 3DCS model is built using mainly six plane and rigid pattern moves simulating the assembly sequence and the selected indexing plan. 3DCS simulates positional tolerances using two random variables, i.e. the magnitude and the angle of the positional vector of the varied centre wrt to the nominal one as depicted in Fig. 5. Due to the trigonometric functions inserted in the simulation when expressing the varied position of the holes, the case study in Fig. 4 results in a disjoint failure domain in the space of the random variables [12].

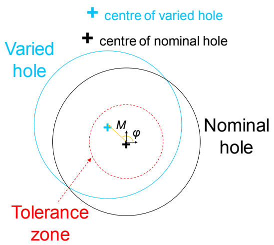


Fig. 5 Nominal and varied form of a hole

To demonstrate this, the state function, $g_1$, of Eq. (5), is analysed and depicted in Fig. 6 for the case study considering variation only for one hole and more specifically positional variation for the hole in the plate highlighted with the red circle in Fig. 4. This simplification reduces the reliability problem in a two random variable reliability problem and thus, makes possible to visualise the limit state function. The two random variables of the problem are the magnitude ($M$) and the angle ($\varphi$) of the positional vector that give the location of the varied centre of the hole in the plate wrt to its nominal position. A Rayleigh distribution is usually assumed for the magnitude, $M$, and a Uniform one for the angle, $\varphi$. The parameter of the Rayleigh distribution is defined such that three standard deviations result in half of the tolerance range. The parameters for the Uniform distribution are set equal to 0 and 360 degrees respectively. The 3D graph and the contour plot for the limit state function $g_1$ in the physical space are directly estimated using 3DCS and they are presented in Fig. 6. For clearer visualization, only two contour lines were plotted in Fig. 6 whilst the axis limits were modified appropriately. Clearly, the failure domain is not a uniform one and this fact makes most of the well-established reliability methods to provide less accurate results or even fail, e.g. in case of typical formulation of FORM.

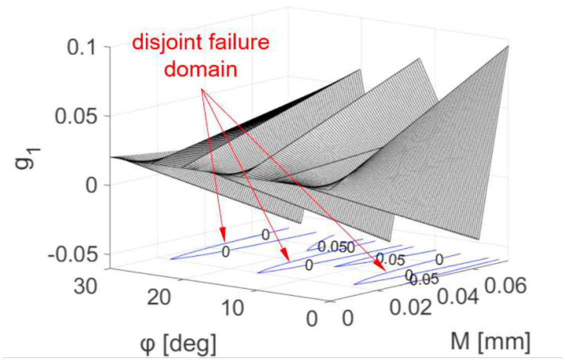


Fig. 6 3D graph and contour plot of the state function $g_1$ for the simplified case study

Due to the existence of disjoint failure domains and in order to accelerate the probability estimations, QMC-S method was selected to derive statistics for the AKC. According to [12], approximately half of the sample size is sufficient to achieve the same accuracy and repeatability in the probability estimation with respect to crude Monte Carlo method. Sobol sequences belong to the family of low-discrepancy sequences. Discrepancy is the measure that characterises the lumpiness of a sequence of points in a multidimensional space. Samples made from a finite subset of such sequences are called quasi-random samples and they are as uniform as possible in the random variable space. Thus, the random variable space is explored more efficiently, a good characteristic to deal with multiple failure domains.

## 5. Results and discussion

Having set up the framework, the RBO problem defined by Eq. (4) can be solved for the case study of Fig. 4. The target probability value, $P_t$, in Eq. (4) is assumed to be equal to 1.5E-03 for both constraints with limit state functions given by Eq. (5). Regarding the uncertainty quantification method, samples of 3,500 values were used every time QMC-S was called. According to [12], the length of the sample was judged sufficient giving a fair estimate on the probability estimates of Eq. (4). Tolerance bounds were set up between 0.05 mm and 1mm. The AKC of the assembly in Fig. 4 is equal to 21.571mm at nominal form whilst the LSL and USL were assumed equal to 21.361mm and 21.781mm respectively. The assigned tolerances in Fig. 4 result in a distribution of AKC nested inside the specification limits as depicted in Fig. 7 (Ref curve). The fabrication product cost using the tolerances of Fig. 4 defines the reference cost $C_{ref}$. In order to establish the cost tolerance relationship, typical values of the variability related to a CNC drilling machine as well as to drilling templates were used in this case study.

Results following the suggested RBO approach are presented in Table 1 under the heading RBO. Only few iterations were needed for SQP to converge to a local solution implementing the whole process, however, few times in order to obtain confidence about the type of the optimum found. The cost of the product following RBO approach was estimated $0.69 \cdot C_{ref}$ achieving great reduction in the cost whilst satisfying the probabilistic constraints as presented in Fig 7 (RBO curve). Regarding the positional tolerances in the various parts, the tolerance related to the holes in the base component has been slightly tighten while for all the other holes more relaxed tolerances are suggested.

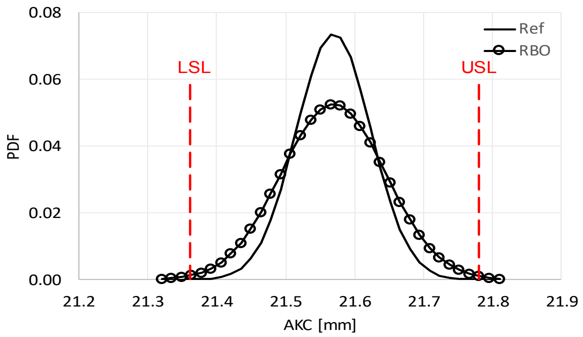


Fig. 7. Distribution of the AKC for different design tolerance values

The tolerance allocation process was performed two more times using traditional approaches expressed by Eq. (1)-(3). The optimization problem in Eq. (1) was solved in MATLAB calling the developed cost model whilst the WC and RSS variability were estimated directly by 3DCS computations and thus, the same tolerance models were used for all the tolerance allocation approaches. Results are depicted in Table 1. The cost of the product was estimated $3.97 \cdot C_{ref}$ when WC scenario was applied and $0.77 \cdot C_{ref}$ when the RSS was used.

Table 1. Tolerance values for the simple example using various tolerance allocation approaches

| Positional tolerance | Ref (Fig. 4) | WC | RSS | RBO |
|---|---|---|---|---|
| Base | 0.30 | 0.09 | 0.23 | 0.27 |
| Angle-Interface 1 | 0.15 | 0.11 | 0.25 | 0.28 |
| Angle-Interface 2 | 0.15 | 0.08 | 0.20 | 0.28 |
| Plate | 0.15 | 0.05 | 0.05 | 0.26 |

Tolerance allocation using RBO framework seems to reduce even more the cost of the product when compared to worst case and the root sum square approach. The reason for this reduction is attributed to the fact that the actual non-linear tolerance model is captured by the analysis in the reliability analysis and optimization process.

## 6. Conclusions

A probabilistic framework has been developed to allocate tolerances to the various features of a product. The tolerance allocation scheme is formulated as a RBO problem and a computational tool has been created based on in-house codes as well as on off the self software giving great applicability to any type of mechanical assembly.

The RBO methodology presented herein was based on a Quasi Monte Carlo method. Equally, other advanced reliability methods that deal with disjoint failure domains could have been used to estimate probabilities, e.g. Subset Simulation method readily available in UQLab. The same holds true for the optimization algorithms. Nevertheless, exhaustive studies should be performed to identify the best combination of methods and strategies to deal with probabilistic tolerance allocation problems.

Possible limitation of the developed framework could be the transfer of the random samples from the statistical tool to the CAT tool. For every tolerance specification assigned to a contributor, at least one text file needs to be generated and thus, for high dimensional problems several files might need to be created, an activity that decelerate the whole process.

The reported findings concerned a specific simplified case study. More complex assemblies should be analysed to prove the efficiency of the developed numerical tool as well as the observed tolerance relaxation pattern when using RBO method

## 7. Acknowledgements

This project has received funding from the Clean Sky 2 Joint Undertaking under the European Union's Horizon 2020 and innovation programme under grant agreement No 738221 (VADIS). The VADIS call proposal developed under LEONARDO.

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