Micro to Macro: Optimal Trade Policy with Firm Heterogeneity*

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Abstract

The empirical observation that "large firms tend to export, whereas small firms do not" has transformed the way economists think about the determinants of international trade. Yet, it has had surprisingly little impact on how economists think about trade policy. Under very general conditions, we show that from the point of view of a country that unilaterally imposes trade taxes to maximize domestic welfare, the self-selection of heterogeneous firms into exports calls for import subsidies on the least profitable foreign firms. In contrast, our analysis does not provide any rationale for export subsidies or taxes on the least profitable domestic firms.

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1 Introduction

There are large firms and small firms. The former tend to export whereas the latter do not. What are the policy implications of that empirical observation?

Models of firm heterogeneity have transformed the way economists think about the determinants of international trade. Yet, the same models have had relatively limited impact on how they think about trade policy. The goal of this paper is to fill this gap on the normative side of the literature and uncover the general principles that should guide the design of optimal trade policy when heterogeneous firms select into exporting.

Our baseline environment is a canonical model of intra-industry trade with monopolistic competition and firm-level heterogeneity. The main building blocks are taken from Melitz (2003). We assume that labor is the only factor production, that all cost functions are linear, and that preferences have Constant Elasticity of Substitution (CES). Compared to Melitz (2003), we allow firms to be heterogeneous in terms of both their variable costs and their fixed costs and impose no restrictions on the joint distribution of these costs across firms and markets. In this sense, the pattern of selection into exports, which is the main focus of our analysis, is unrestricted.

Our baseline results offer a full characterization of the ad-valorem taxes that maximize domestic welfare, which we label unilaterally optimal taxes, when a country's government is free to impose different taxes on different firms and the rest of the world is passive. Our main finding is that the self-selection of heterogeneous firms into exports calls for import subsidies on the least profitable foreign firms. The standard case for an optimal import tariff is based on the idea that neither consumers nor firms internalize the impact of their import decisions on import prices. In a neoclassical environment where import prices increase with quantities, a government should thus impose a positive tax on imports proportional to this negative terms-of-trade effect. Selection does not affect this broad logic, but flips the sign of the relationship between prices and quantities. When firms only export if they can sell enough, an increase in imported quantities lowers the prices of goods sold by firms that would have not selected into exports otherwise. For marginal firms, prices de facto jump from their reservation values to a lower finite price. This creates a motive for import subsidies on the least profitable foreign firms.

In contrast, our baseline analysis does not provide any rationale for export subsidies or taxes on the least profitable domestic firms. At the macro level, a government may want to manipulate the total amount of exports because of general equilibrium considerations. But conditional on the aggregate level of exports, the allocation of resources across firms in the environments that we consider is efficient. Unlike domestic consumers that

ignore that buying more from foreign firms may lower the domestic prices that they face, domestic firms fully internalize the benefit of exporting or not, up to general equilibrium considerations. This leads to optimal export taxes or subsidies that are uniform across all domestic firms.

The last part of our paper establishes the robustness of our conclusions to various generalizations. In terms of the economic environment, we introduce multiple factors of production and variable marginal costs; we allow goods to belong to different groups, without imposing any restriction on how consumers derive utility from those groups; we introduce arbitrarily many countries; and we allow free entry, without imposing any restriction on how firms may also vary in terms of their entry costs. With general technology and preferences, we show that our results continue to apply industry-by-industry, where an industry is either defined with respect to supply considerations, as a subset of firms with the same factor intensity, or with respect to demand considerations, as one of the aforementioned groups of goods. With multiple countries, they apply country-by-country. With free entry, they hold without further qualifications.

In terms of policy instruments, we explore two alternatives to our benchmark environment. The first one is richer and allows firm-specific two-part tariffs; the second one is more restricted and no longer allows taxes to vary across firms from a given origin. In the former case, we still find import subsidies on the less profitable foreign firms and uniform export taxes or subsidies on the domestic firms. Compared to our baseline analysis, the only difference is that import subsidies now take the form of lower fixed fees, which are non-distortionary, rather than lower linear taxes, which are. In the latter case, we demonstrate that uniform import tariffs are necessarily lower when selection is active than when it is not.

We conclude by extending our results to the case of a trade war where both countries are strategic and set taxes in a simultaneous move game. At a Nash equilibrium, we show that our conclusions are unchanged: (i) domestic taxes are uniform across all domestic producers; (ii) export taxes are uniform across all exporters; and (iii) import taxes are uniform across Foreign's most profitable exporters and strictly increasing with profitability across its least profitable ones.

The rest of the paper is organized as follows. Section 2 offers a brief review of the related literature. Section 3 describes our basic environment. Section 4 sets up and solves the micro and macro planning problems of a welfare-maximizing country manipulating its terms-of-trade. Section 5 shows how to decentralize the solution to the planning problems through micro and macro trade taxes when governments are free to discriminate across firms. Section 6 generalizes our results. Section 7 offers some concluding remarks.

2 Related Literature

Few economic mechanisms have received as much empirical support as the selection of heterogeneous firms into exporting; see e.g. Bernard and Jensen (1999), Bernard, Eaton, Jensen and Kortum (2003), Bernard, Jensen, Redding and Schott (2007), and Eaton, Kortum and Kramarz (2011). Policy makers have paid attention. As documented in the World Trade Report 2016, there were only two regional trade agreements (RTA) with provisions related to small- and medium-sized enterprises (SME) prior to 1990. As of March 2016, 133 RTAs, representing 49% of all the notified RTAs, included at least one provision mentioning SMEs explicitly.

Ironically, there has been little academic research to date about the policy implications of the endogenous selection of heterogeneous firms into exporting.[1] Demidova and Rodríguez-Clare (2009) is an early exception. They study the optimal uniform tax in a small open economy version of the Melitz model with Pareto distributions of firm-level productivity, CES preferences and uniform fixed exporting costs. This is a special case of the environment that we consider in Section 6.2. Since Demidova and Rodríguez-Clare (2009) assume that taxes are uniform across firms, their analysis is necessarily silent about whether governments should, from a unilateral standpoint, tax large firms or small firms differently and, in turn, whether trade agreements should ever include provisions related to small firms.

The same restriction to uniform taxes applies to more recent papers analyzing optimal trade policy in environments with heterogeneous firms, including Felbermayr, Jung and Larch (2013), who extend the results of Demidova and Rodríguez-Clare (2009) to economies with two large countries, Ossa (2011), who studies the gains from GATT/WTO negotiations, Haaland and Venables (2016), Bagwell and Lee (2018b), and Campolmi, Fadinger and Forlati (2018), who analyze economies with multiple sectors, and Demidova (2017) and Bagwell and Lee (2018a), who study economies with linear demand.[2] Our analysis, in contrast, focuses on the optimal structure of firm-level trade taxes. Under very general conditions, we show that selection creates a rationale for unilateral beggar-thy-neighbor import subsidies on the least profitable foreign firms. Since this pattern is also a feature of any Nash equilibrium, our findings suggests that rather than expanding the exports of SMEs, an optimal trade agreement may want to restrict them.

[1] The last handbook of international economics, Gopinath, Helpman and Rogoff, eds (2014), is a case in point. Maggi's (2014) chapter on trade policy does not feature any paper about firm heterogeneity. Melitz and Redding's (2014) chapter on heterogeneous firms only features one paper about trade policy.

[2] Ossa (2011) introduces firm heterogeneity in his online appendix available at https://www.econ.uzh.ch/en/people/faculty/ossa/research.html.

Our paper is related to a large body of work exploring the optimality of allocations under monopolistic competition, from the original work of Spence (1976) and Dixit and Stiglitz (1977) to the more recent work of Nocco, Ottaviano and Salto (2014), Dhingra and Morrow (2019) and Nocco, Ottaviano and Salto (2019). Away from the case of CES preferences, it is well-known that there are domestic distortions and that the laissez-faire equilibrium is inefficient. We impose CES preferences to abstract from these considerations. Hence, the allocation in the laissez-faire equilibrium is globally efficient, but countries may individually impose trade taxes to improve their own welfare. We then fully characterize the structure of this policy.[3]

Our analysis shares with Romer (1994) an emphasis on the welfare implications of new foreign goods. He studies a small open economy where foreign firms face fixed costs of exporting differentiated inputs. Starting from free trade, he demonstrates that imposing a tariff creates first-order welfare losses equal to the "Dupuit triangles" associated with the foreign goods no longer imported. He does not, however, analyze optimal trade policy. Our result that optimal import tariffs are decreasing with firms' profitability instead reflects a trade-off between the potential losses from Dupuit triangles and the potential losses from Harberger triangles generated by lower import tariffs.

In terms of methodology, we build on the work of Costinot, Lorenzoni and Werning (2014) and Costinot, Donaldson, Vogel and Werning (2015) who characterize the structure of optimal trade taxes in a dynamic endowment economy and a static Ricardian economy, respectively. Like these papers, we use a primal approach and general Lagrange multiplier methods to characterize optimal wedges rather than explicit policy instruments. The novel aspect of our analysis is to break down the problem of finding optimal wedges into a series of micro subproblems, where we study how to choose quantities across varieties conditional on aggregate quantities, and a macro problem, where we solve for the optimal aggregate quantities. The solutions to the micro problems determines the structure of optimal micro taxes described above, whereas the solutions to the macro problem delivers the overall level of those taxes.

Our approach has two attractive features. First, it is well suited to deal with zeros, a central aspect of any selection model. Despite the fact that the government's optimization problem is infinite dimensional and that firm's technologies are non-convex, our micro

[3]Our paper is also related, though less closely, to political-economy models of trade policy with heterogeneous firms, such as Bombardini (2008) who demonstrates how cross-sectoral differences in firm heterogeneity can explain cross-sectoral differences in the level of trade protection. By assumption, taxes in Bombardini (2008) cannot vary across firms. This is the variation that we are interested in. Likewise, our analysis is related to a large body of work, synthesized in Helpman and Krugman (1989), that studies optimal trade policy under imperfect competition, but abstracts from firm-level considerations.

problems reduce to simple one-dimensional Lagrangian problems. Accordingly, whether or not a government prefers to import or not reduces to comparing the value of that Lagrangian at an interior optimum and at a corner. Second, our approach is well suited to accommodate rich general equilibrium interactions. Since our micro problems are inner problems that take aggregate considerations as given, those can be added without making the core of our analysis more complex nor affecting our main finding, as Section 6 formally demonstrates.

3 Baseline Environment

3.1 Technology, Preferences, and Market Structure

In our baseline analysis, we focus on a world economy with two countries, indexed by $i = H, F$; one factor of production, labor; and a continuum of differentiated goods or varieties. Labor is immobile across countries. w_i and L_i denote the wage and the inelastic supply of labor in country i, respectively.

Technology. In each country there is a continuum of heterogeneous firms, each endowed with a blueprint $\varphi \in \Phi$. N_i denotes the total measure of firms in country i and G_i denotes the distribution of blueprints φ across firms in that country. Each blueprint describes how to produce and deliver a firm's differentiated variety to any country. $l_{ij}(q, \varphi)$ denotes the total amount of labor needed by a firm from country i with blueprint φ in order to produce and deliver $q \geq 0$ units in country j. We assume

$$l_{ij}(q, \varphi) = \begin{cases} a_{ij}(\varphi)q + f_{ij}(\varphi) & \text{if } q > 0, \\ 0 & \text{if } q = 0. \end{cases}$$

Technology in Krugman (1980) corresponds to the special case in which G_i has all its mass at a single blueprint with zero fixed costs of selling in the two markets, $f_{ij} = 0$ for all i, j. Technology in Melitz (2003) corresponds to the special case in which firms have heterogeneous productivity, but face homogenous iceberg trade costs, $a_{ij}(\varphi) \equiv \tau_{ij}/\varphi$, and homogenous fixed costs, $f_{ij}(\varphi) \equiv f_{ij}$ for all $\varphi \in \mathbb{R}^+$. Here, we do not restrict φ to be one-dimensional. Instead we let Φ be any measurable subset of R^{+4}, with $\varphi = (\varphi_1, \varphi_2, \varphi_3, \varphi_4) \in \Phi$ such that $a_{ii}(\varphi) = \varphi_1$, $a_{ij}(\varphi) = \varphi_2$, $f_{ii}(\varphi) = \varphi_3$, and $f_{ij}(\varphi) = \varphi_4$ for $i \neq j, i = H, F$. It follows that the joint distribution of variable and fixed costs across firms and destinations is completely unrestricted in our analysis. For instance, conditional on a

given level of variable costs $a_{ij}(\varphi) = a$, fixed costs $f_{ij}(\varphi)$ may be drawn from a log-normal distribution, as in Eaton, Kortum and Kramarz (2011).

Preferences. In each country there is a representative agent with Constant Elasticity of Substitution (CES) preferences, as in Krugman (1980) and Melitz (2003),

$$U_j = [\sum_{i=H,F} \int_{\Phi} N_i(q_{ij}(\varphi))^{1/\mu} dG_i(\varphi)]^{\mu},$$

where $q_{ij}(\varphi)$ is country j's consumption of a variety with blueprint φ produced in country i and $\mu \equiv \sigma/(\sigma-1)$ with $\sigma > 1$ the elasticity of substitution between varieties. To prepare our "micro to macro" analysis, it is convenient to rearrange the previous expression as a two-level utility function,

$$\begin{aligned} U_j =& U_j(Q_{Hj}, Q_{Fj}) = [Q_{Hj}^{1/\mu} + Q_{Fj}^{1/\mu}]^{\mu}, \\ Q_{ij} =& [\int_{\Phi} N_i(q_{ij}(\varphi))^{1/\mu} dG_i(\varphi)]^{\mu}, \end{aligned}$$

where Q_{ij} is the subutility from consuming varieties from country i in country j.

Market Structure. All goods markets are monopolistically competitive. All labor markets are perfectly competitive. Foreign labor is our numeraire, $w_F = 1$. In our baseline analysis, we assume that entry is restricted so that N_i is exogenously given in all countries. In Section 6.1 we relax this assumption as well as the assumptions that labor is the only factor of production, that there are only two countries, that cost functions are linear, and that the elasticity of substitution across all goods is constant.

3.2 Decentralized Equilibrium with Taxes

The focus of our baseline analysis is on a scenario where governments have access to a full set of ad-valorem consumption and production taxes. That is, we let taxes vary across markets and across firms.[4]

[4]We view the availability of a rich set of ad-valorem taxes as a useful benchmark. In theory, there is a priori no reason within the model that we consider why different goods should face the same taxes. In an Arrow-Debreu economy, imposing the same taxes on arbitrary subsets of goods would be ad-hoc. Changing the market structure from perfect to monopolistic competition does not make it less so. In practice, different firms do face different trade taxes, even within the same narrowly defined industry. Take, for instance, "Cotton, not carded or combed" (HS8 520100). As Kim (2017) notes, the most favoured nation (MFN) tariff rate applied by the United States for firms producing "Cotton, not carded or combed, having staple length of 28.575 mm or more but under 34.925 mm (HS8 52010038)" is 14%, as of 2013, whereas "Cotton, not carded or combed, having a staple length under 19.05mm (3/4 inch), harsh or rough (HS8 52010005)" is duty free.

Formally, we let $t_{ij}(\varphi)$ denote the tax charged by country j on the consumption in country j of a variety with blueprint φ produced in country i. Let $s_{ij}(\varphi)$ denote the subsidy paid by country i on the production by a domestic firm of a variety with blueprint φ sold in country j. For $i \neq j$, $t_{ij}(\varphi) > 0$ corresponds to an import tariff while $t_{ij}(\varphi) < 0$ corresponds to an import subsidy. Similarly, $s_{ij}(\varphi) > 0$ corresponds to an export subsidy while $s_{ij}(\varphi) < 0$ corresponds to an export tax. Tax revenues are rebated to domestic consumers through a lump-sum transfer, T_i. We consider alternative tax instruments in Section 6.2.

In a decentralized equilibrium with taxes, consumers choose consumption to maximize their utility subject to their budget constraint; firms choose their output to maximize profits, taking their residual demand curves as given; markets clear; and the government's budget is balanced in each country. Let $\bar{p}_{ij}(\varphi) \equiv \mu w_i a_{ij}(\varphi)/(1+s_{ij}(\varphi))$ and $\bar{q}_{ij}(\varphi) \equiv [(1+t_{ij}(\varphi))\bar{p}_{ij}(\varphi)/P_{ij}]^{-\sigma} Q_{ij}$. Using the previous notation, we can characterize a decentralized equilibrium with taxes as schedules of output, $\mathbf{q_{ij}} \equiv \{q_{ij}(\varphi)\}$, schedules of prices, $\mathbf{p_{ij}} \equiv \{p_{ij}(\varphi)\}$, aggregate output levels, Q_{ij}, aggregate price indices, P_{ij}, wages, w_i, and aggregate profits, Π_i, such that

$$q_{ij}(\varphi) = \begin{cases} \bar{q}_{ij}(\varphi) & \text{if } \mu a_{ij}(\varphi)\bar{q}_{ij}(\varphi) \geq l_{ij}(\bar{q}_{ij}(\varphi), \varphi), \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

$$p_{ij}(\varphi) = \begin{cases} \bar{p}_{ij}(\varphi) & \text{if } \mu a_{ij}(\varphi) q_{ij}(\varphi) \geq l_{ij}(q_{ij}(\varphi), \varphi), \\ \infty & \text{otherwise,} \end{cases} \tag{2}$$

$$Q_{Hj}, Q_{Fj} \in \arg \max_{\tilde{Q}_{Hj}, \tilde{Q}_{Fj}} \{U_j(\tilde{Q}_{Hj}, \tilde{Q}_{Fj}) | \textstyle\sum_{i=H,F} P_{ij}\tilde{Q}_{ij} = w_j L_j + \Pi_j + T_j\}, \tag{3}$$

$$P_{ij}^{1-\sigma} = \int_\Phi N_i [(1+t_{ij}(\varphi))p_{ij}(\varphi)]^{1-\sigma} dG_i(\varphi), \tag{4}$$

$$\Pi_i = N_i \textstyle\sum_{j=H,F} \int_\Phi [\mu w_i a_{ij}(\varphi) q_{ij}(\varphi) - w_i l_{ij}(q_{ij}(\varphi), \varphi)] dG_i(\varphi), \tag{5}$$

$$L_i = N_i \textstyle\sum_{j=H,F} \int_\Phi l_{ij}(q_{ij}(\varphi), \varphi) dG_i(\varphi), \tag{6}$$

$$T_i = \textstyle\sum_{j=H,F} [\int_\Phi N_j t_{ji}(\varphi) p_{ji}(\varphi) q_{ji}(\varphi) dG_j(\varphi) - \int_\Phi N_i s_{ij}(\varphi) p_{ij}(\varphi) q_{ij}(\varphi) dG_i(\varphi)]. \tag{7}$$

Condition (2) will play a central role in our analysis. It states that firms charge a constant markup over a constant marginal cost whenever they sell in a destination, but that they only do so if their profits are non-negative. This makes selection into exports the sole channel through which governments may manipulate import prices across foreign firms.[5]

[5]Conditions (1) and (2) assume that firms that are indifferent between producing and not producing,

Throughout our analysis, we restrict attention to the interesting nontrivial cases where the utility maximization problem (3) admits an interior solution. This rules out equilibria without trade ($Q_{FH} = Q_{HF} = 0$) or without domestic production ($Q_{HH} = Q_{FF} = 0$).[6]

3.3 Unilaterally Optimal Taxation

We assume that the government of country H, which we refer to as Home's government, is strategic, whereas the government of country F, which we refer to as Foreign's government, is passive. Section 6.3 will analyze the Nash equilibrium of the simultaneous game in which both countries behave strategically. For now, Home's government sets ad-valorem taxes, $\mathbf{t_{HH}} \equiv \{t_{HH}(\varphi)\}$, $\mathbf{t_{FH}} \equiv \{t_{FH}(\varphi)\}$, $\mathbf{s_{HH}} \equiv \{s_{HH}(\varphi)\}$, and $\mathbf{s_{HF}} \equiv \{s_{HF}(\varphi)\}$, and a lump-sum transfer T_H in order to maximize home welfare, whereas Foreign's government sets all taxes to zero. Throughout our analysis, we simply refer to the problem of Home's government as Home's problem.

Definition 1. *Home's problem is*

$$\max_{T_H, \{\mathbf{t_{jH}}, \mathbf{s_{Hj}}\}_{j=H,F}, w_H, \{\mathbf{q_{ij}}, Q_{ij}, \mathbf{p_{ij}}, P_{ij}\}_{i,j=H,F}} U_H(Q_{HH}, Q_{FH})$$

subject to conditions (1)-(7).

The goal of the next two sections is to characterize unilaterally optimal taxes, i.e., taxes that prevail at a solution to Home's problem. To do so we follow the public finance literature and use the primal approach. Namely, we will first approach the optimal policy problem of Home's government in terms of a relaxed planning problem in which domestic consumption and output can be chosen directly (Section 4). We will then establish that the optimal allocation can be implemented through linear taxes and characterize the structure of these taxes (Section 5).

produce. This is without loss of generality since, to simplify, we assume indifference is measure zero. We do so by adopting the sufficient condition that conditional on a positive value for the fixed cost $f_{ij} > 0$ the distribution over the variable cost a_{ij} is continuous. This condition rules out mass points but only for strictly positive fixed costs. The distribution conditional on $f_{ij} = 0$ for a_{ij} may have mass points. Thus, the model in Krugman (1980), which has no heterogeneity but no fixed costs of exporting, satisfies our requirement.

[6]Given the assumption that preferences are CES, this is a mild restriction. It states that for any given origin country i and destination country j, there is a strictly positive measure of firms with variable and fixed costs that are low enough to make non-negative profits.

4 Micro and Macro Planning Problems

In this section, we focus on a relaxed version of Home's problem that abstracts from all constraints in which Home's tax instruments, $T_H, \{\mathbf{t_{jH}}, \mathbf{s_{Hj}}\}_{j=H,F}$, and Home's prices, $w_H, \{\mathbf{p_{Hj}}\}_{j=H,F}$, appear. This relaxed problem can be interpreted as the problem of a fictitious planner who directly controls the quantities demanded by home consumers, $\mathbf{q_{HH}} \equiv \{q_{HH}(\varphi)\}$ and $\mathbf{q_{FH}} \equiv \{q_{FH}(\varphi)\}$, as well as the quantities exported by home firms, $\mathbf{q_{HF}} \equiv \{q_{HF}(\varphi)\}$. Specifically, we drop conditions (2), (5), and (7) for $i = H$; we drop condition (3) for $j = H$; and we relax conditions (1) and (4) for $i = H$ or $j = H$ by imposing instead

$$\int_\Phi N_i(q_{ij}(\varphi))^{1/\mu} dG_i(\varphi) \geq Q_{ij}^{1/\mu}, \text{ for } i = H \text{ or } j = H. \tag{8}$$

We refer to this new problem as Home's relaxed planning problem; see Appendix A.

In order to solve this relaxed problem, we take advantage of the nested structure of preferences in this economy and follow a three-step approach. First, we take Home's local output, Q_{HH}, and exports, Q_{HF}, as given and solve for the domestic micro quantities, $\{\mathbf{q_{Hj}}\}_{j=H,F}$, that deliver these macro quantities at the lowest possible cost. Second, we solve for Foreign's micro quantities, $\{\mathbf{q_{Fj}}\}_{j=H,F}$, as well its domestic sales, Q_{FF}, that maximize Home's imports Q_{FH} conditional on its exports Q_{HF}. Third, we solve for the optimal macro quantities, Q_{HH}, Q_{FH}, and Q_{HF}. The solution to these micro and macro problems will determine the optimal micro and macro taxes, respectively, in Section 5.[7]

4.1 First Micro Problem: Home's Production Possibility Frontier

Consider the problem of minimizing the labor cost of producing Q_{HH} units of aggregate consumption for Home and Q_{HF} units of aggregate consumption for Foreign subject to condition (8) for $i = H$ and $j = H, F$. This can be expressed as

$$L_H(Q_{HH}, Q_{HF}) \equiv \min_{\mathbf{q_{HH}}, \mathbf{q_{HF}}} N_H \sum_{j=H,F} \int_\Phi l_{Hj}(q_{Hj}(\varphi), \varphi) dG_H(\varphi) \tag{9a}$$

$$\int_\Phi N_H(q_{Hj}(\varphi))^{1/\mu} dG_H(\varphi) \geq Q_{Hj}^{1/\mu}, \text{ for } j = H, F. \tag{9b}$$

[7] Together with the foreign equilibrium conditions, the previous variables determine all foreign prices at the solution of Home's relaxed planning problem. For expositional purposes, we omit the description of these variables from the main text and present them in Appendix A.2.

Together with Home's resource constraint, i.e., condition (6) for $i = H$, the previous value function will characterize Home's production possibility frontier.

This minimization problem is infinite dimensional and non-smooth. More precisely, since there are fixed costs, the objective function is neither continuous nor convex around $q_{Hj}(\varphi) = 0$ for any φ such that $f_{Hj}(\varphi) > 0$. To deal with the previous issues and derive necessary properties that any solution to (9) must satisfy, we adopt the following strategy.

First, we consider a planning problem that extends (9) by allowing for randomization: conditional on φ, we let the planner select a distribution of output levels.[8] Since this problem is convex, we can invoke Lagrangian necessity theorems. We then show that randomization is not employed at any solution to the extended problem, so that the planner effectively solves (9). It follows that any solution to (9) must minimize the associated Lagrangian,

$$\mathcal{L}_H \equiv N_H \sum_{j=H,F} \int_\Phi \left(l_{Hj}(q_{Hj}(\varphi), \varphi) - \lambda_{Hj}(q_{Hj}(\varphi))^{1/\mu} \right) dG_H(\varphi),$$

for some Lagrange multipliers, $\lambda_{Hj} > 0$, for $j = H, F$. The complete argument can be found in Appendix A.1.[9]

Second, we use the additive separability of the Lagrangian $\mathcal{L}_H$ in $\{q_{Hj}(\varphi)\}$ to minimize it variety-by-variety and market-by-market, as in Everett (1963). Although the discontinuity at zero remains, it is just a series of one-dimensional minimization problems that can be solved by hand. Namely, for a given variety φ and a market j, consider the one-dimensional subproblem

$$\min_q l_{Hj}(q, \varphi) - \lambda_{Hj} q^{1/\mu}.$$

The solution to this problem follows a simple cut-off rule, which must then apply to any solution, $q_{Hj}(\varphi | Q_{HH}, Q_{HF})$, to the original constrained problem (9),

$$q_{Hj}(\varphi | Q_{HH}, Q_{HF}) = \begin{cases} (\mu a_{Hj}(\varphi)/\lambda_{Hj})^{-\sigma}, & \text{if } \varphi \in \Phi_{Hj}, \\ 0, & \text{otherwise,} \end{cases} \tag{10}$$

[8] There are two interpretations of this randomization. In the first, a firm with a blueprint φ is randomly assigned a q according to this conditional distribution; in the second, there is a continuum of firms for a given φ and each firm is assigned a different q so that the population is distributed according to the conditional distribution.

[9] An alternative strategy would be to show that there exists a solution to the Lagrangian problem that satisfies constraint (9b) and then invoke Lagrangian sufficiency theorems. For this first micro problem, this is not more complex than our approach using randomization and Lagrangian necessity theorems. For the next micro problem, however, the latter approach is the only one that we have been able to implement.

with $\Phi_{Hj} \equiv \{\varphi : (\mu-1)a_{Hj}(\varphi)(\mu a_{Hj}(\varphi)/\lambda_{Hj})^{-\sigma} \geq f_{Hj}(\varphi)\}$ the set of domestically produced varieties sold in country j.[10]

By comparing equations (1), (2), and (4), on the one hand, and equations (9b) and (10), on the other hand, one can check that conditional on Q_{HH} and Q_{HF}, the output levels in the decentralized equilibrium with zero taxes and the solution to the relaxed planning problem coincide. This reflects the efficiency of firm's level decision under monopolistic competition with Constant Elasticity of Substitution (CES) utility conditional on industry size; see Dixit and Stiglitz (1977) and Dhingra and Morrow (2019) for closed economy versions of this result. As shown in Section 5, this feature implies that the home government may want to impose a uniform import tariff or an export tax—in order to manipulate the fraction of labor allocated to domestic production rather than exports—but that it never wants to impose taxes that vary across domestic firms, regardless of whether they sell on the domestic or foreign market.

4.2 Second Micro Problem: Foreign's Offer Curve

Next consider the problem of maximizing Home's imports, $Q_{FH}(Q_{HF})$, conditional on its aggregate exports, Q_{HF}, subject to Foreign's equilibrium conditions, namely conditions (1) and (4) for $i = F$ and $j = F$, (2), (5), (6) for $i = F$, and (3) for $j = F$. As shown in Appendix A.2, this second micro problem can be reduced to

$$Q_{FH}^{1/\mu}(Q_{HF}) \equiv \max_{\mathbf{q_{FH}}, Q_{FF}} \int_{\Phi} N_F q_{FH}^{1/\mu}(\varphi) dG_F(\varphi)), \tag{11a}$$

$$N_F \int [\mu a_{FH}(\varphi) q_{FH}(\varphi)] dG_F(\varphi) = P_{FF}(Q_{FF}) MRS_F(Q_{HF}, Q_{FF}) Q_{HF}, \tag{11b}$$

$$L_F = L_{FF}(Q_{FF}) + N_F \int_{\Phi} l_{FH}(q_{FH}(\varphi), \varphi) dG_F(\varphi), \tag{11c}$$

$$\mu a_{FH}(\varphi) q_{FH}(\varphi) \geq l_{FH}(q_{FH}(\varphi), \varphi), \tag{11d}$$

where $MRS_F(Q_{HF}, Q_{FF}) \equiv (Q_{HF}/Q_{FF})^{-1/\sigma}$ denotes the marginal rate of substitution in Foreign, $L_{FF}(Q_{FF})$ denotes the total employment associated with the local sales of foreign firms, and $P_{FF}(Q_{FF})$ denotes the price index for domestic goods within Foreign. Constraint (11b) is a trade balance condition that equalizes the value of Home's imports, on the left-hand side, with the value of its exports, on the right-hand side.[11] Constraint (11c)

[10] Given the previous definition, equation (10) implies that when indifferent between producing or not, the planner chooses producing. Since indifference is measure zero, this is without loss of generality.

[11] Utility maximization in Foreign implies $P_{HF} = P_{FF} MRS_F$.

is Foreign's labor market clearing condition. Together with the new utility constraint (8) for $i = F$ and $j = H$, the value function in (11) will characterize Foreign's offer curve.

It is convenient to focus first on the subproblem that takes both Q_{HF} and Q_{FF} as given and maximizes over $\mathbf{q_{FH}}$. To deal with the non-smoothness and non-convexities of this minimization problem and derive necessary properties that any of its solution must satisfy, we can follow a similar strategy as in Section 4.1. Technical details can again be found in Appendix A.2.

Consider the one-dimensional subproblem of finding the amount of foreign imports of variety φ that solves

$$\max_q q^{1/\mu} - \lambda_T \mu a_{FH}(\varphi) q - \lambda_L l_{FH}(q, \varphi) \tag{12a}$$

$$\mu a_{FH}(\varphi) q \geq l_{FH}(q, \varphi), \tag{12b}$$

where λ_T and λ_L are the Lagrange multipliers associated with (11b) and (11c). The solution to the unconstrained problem, ignoring inequality (12b), is given by

$$q^u_{FH}(\varphi) = \begin{cases} (\mu \chi_{FH} a_{FH}(\varphi))^{-\sigma}, & \text{if } \theta_{FH}(\varphi) \geq \lambda_L^{1/\sigma} \chi_{FH}^{1/\mu}, \\ 0, & \text{otherwise,} \end{cases}$$

with $\theta_{FH}(\varphi) \equiv [(\mu - 1)(a_{FH}(\varphi))^{1-\sigma} / f_{FH}(\varphi)]^{1/\sigma} / \mu$ and $\chi_{FH} \equiv \lambda_L + \mu \lambda_T > 0$.[12] In what follows, we refer to $\theta_{FH}(\varphi)$ as the "profitability" of foreign varieties in Home's market, which decreases with both their fixed costs, $f_{FH}(\varphi)$, and their variable costs, $a_{FH}(\varphi)$. If $q^u_{FH}(\varphi)$ satisfies constraint (12b), then it is also a solution to (12). If it does not, then the solution to (12) is given either by zero or by $q^c_{FH}(\varphi) > q^u_{FH}(\varphi)$ such that (12b) exactly binds, with $q^c_{FH}(\varphi)$ given by

$$q^c_{FH}(\varphi) = f_{FH}(\varphi) / ((\mu - 1) a_{FH}(\varphi)).$$

The former case occurs if $(q^c_{FH}(\varphi))^{1/\mu} - \chi_{FH} a_{FH}(\varphi) q^c_{FH}(\varphi) - \lambda_L f_{FH}(\varphi) < 0$, while the latter case occurs otherwise.

Based on the previous observations, we can express the solution to our second micro

[12] $\chi_{FH} > 0$ is necessary for the solution of the Lagrangian problem to satisfy constraints (11b) and (11c). Since λ_L and λ_T are associated with equality constraints, however, we cannot rule out at this point that one of these two multipliers is negative. We come back to this issue in detail below.

problem in a compact way as

$$q_{FH}(\varphi|Q_{HF},Q_{FF}) = \begin{cases} (\mu\chi_{FH}a_{FH}(\varphi))^{-\sigma} & \text{, if } \varphi \in \Phi^u_{FH}, \\ f_{FH}(\varphi)/((\mu-1)a_{FH}(\varphi)) & \text{, if } \varphi \in \Phi^c_{FH}, \\ 0 & \text{, otherwise,} \end{cases} \tag{13}$$

with the two sets of imported varieties defined by

$$\Phi^u_{FH} \equiv \{\varphi : \theta_{FH}(\varphi) \in [(\max\{\lambda_L/(\lambda_L+\mu\lambda_T),1\})^{1/\sigma}(\lambda_L+\mu\lambda_T),\infty)\},$$
$$\Phi^c_{FH} \equiv \{\varphi : \theta_{FH}(\varphi) \in [\lambda_L+\lambda_T,\lambda_L+\mu\lambda_T)\}.$$

Given a solution to the inner problem, $\{q_{FH}(\varphi|Q_{HF},Q_{FF})\}$, the optimal level of local output in Foreign, $Q_{FF}(Q_{HF})$, can be solved for in a standard manner. The optimal micro quantities are then given by $q_{FH}(\varphi|Q_{HF}) \equiv q_{FH}(\varphi|Q_{FF}(Q_{HF}))$. Substituting into (11a), we obtain Foreign offer's curve, $Q_{FH}(Q_{HF})$.

The set Φ^c_{FH} will play a key role in our subsequent analysis. For varieties $\varphi \in \Phi^c_{FH}$, Home finds it optimal to raise its imports in order to make sure that the least profitable firms in Foreign are willing to produce and export strictly positive amounts, a situation that we will refer to as positive discrimination. As can be seen from the above expression, whether or not Φ^c_{FH} is empty boils down to whether the Lagrange multiplier on the trade balance condition, λ_T, is strictly positive.[13]At the optimal level of exports, which we characterize next, we will demonstrate that this is necessarily the case.

4.3 Macro Problem: Manipulating Aggregate Terms-of-Trade

Finally, consider the choice of macro quantities, (Q_{HH},Q_{FH},Q_{HF}), that maximize U_H subject to the last two constraints of Home's relaxed planning problem: Home's resource constraint, i.e., condition (6) for $i = H$, and the new utility constraint (8) for $i = F$ and $j = H$. Given the analysis of Sections 4.1 and 4.2, these two constraints (6) and (8) can be expressed as $L_H(Q_{HH},Q_{HF}) = L_H$ and $Q_{FH}(Q_{FF}) \geq Q_{FH}$. Thus, optimal aggregate

[13]If it is, then $\lambda_L+\mu\lambda_T$ must also be strictly greater than $\lambda_L+\lambda_T$. The profitability cut-off between Φ^u_{FH} and Φ^c_{FH} is then given by $\theta_{FH}(\varphi) = \lambda_L+\mu\lambda_T$, at which point $q^u_{FH}(\varphi) = q^c_{FH}(\varphi)$. If instead $\lambda_T \leq 0$, then $\lambda_L+\mu\lambda_T \leq \lambda_L+\lambda_T$, Φ^c_{FH} is empty, and the lowest profitability level in Φ^u_{FH} is given by $\theta_{FH}(\varphi) = \lambda_L^{1/\sigma}(\lambda_L+\mu\lambda_T)^{1/\mu}$, at which point the Lagrangian in (12) is equal to zero.

quantities must solve the following macro problem,

$$\max_{Q_{HH},Q_{FH},Q_{HF}} U_H(Q_{HH},Q_{FH}) \tag{14a}$$

$$Q_{FH} \leq Q_{FH}(Q_{HF}), \tag{14b}$$

$$L_H(Q_{HH},Q_{HF}) = L_H. \tag{14c}$$

At this point, it should be clear that we are back to a standard terms-of-trade manipulation problem with the combinations of Foreign's offer curve (constraint 14b) and Home's production possibility frontier (constraint 14c) determining Home's Consumption Possibility Frontier, as in Baldwin (1948). Like in a perfectly competitive model of international trade, foreign technology, endowments, and preferences only matter through their combined effect on Foreign's offer curve, the elasticity of which will determine the aggregate level of trade protection.

To characterize the solution of Home's macro problem (14), let us define Home's aggregate terms-of-trade as,

$$P(Q_{FH},Q_{HF}) \equiv P_{HF}(Q_{HF})/\tilde{P}_{FH}(Q_{HF},Q_{FH}),$$

where $P_{HF}(Q_{HF})$ and $\tilde{P}_{FH}(Q_{HF},Q_{FH})$ are the price of Home's exports and the average cost of Home's imports, respectively,

$$P_{HF}(Q_{HF}) = P_{FF}(Q_{FF}(Q_{HF}))MRS_F(Q_{HF},Q_{FF}(Q_{HF})),$$
$$\tilde{P}_{FH}(Q_{HF},Q_{FH}) = N_F \int_\Phi \mu a_{FH}(\varphi) q_{FH}(\varphi|Q_{HF}) dG_F(\varphi)/Q_{FH}.$$

The tilde symbol emphasizes the fact that the import price index, P_{FH}, faced by Home's consumer in the decentralized equilibrium will differ from $\tilde{P}_{FH}(Q_{HF},Q_{FH})$: the former is inclusive of trade taxes, whereas the latter is not. As shown in Appendix A.3, at an interior solution to (14), which we focus on throughout our analysis, the necessary first-order conditions imply

$$P^*/(MRS_H^*/MRT_H^*) = 1/\eta^*, \tag{15}$$

where $MRS_H^* \equiv (Q_{HH}^*/Q_{FH}^*)^{-1/\sigma}$ and $MRT_H^* \equiv (\partial L_H/\partial Q_{HH})/(\partial L_H/\partial Q_{HF})$ are the marginal rate of substitution and marginal rate of transformation in Home, respectively, and $\eta^* \equiv d\ln Q_{FH}/d\ln Q_{HF}$ is the elasticity of Foreign's offer curve, all evaluated at the solution to the macro problem.

The left-hand side of equation (15) describes the ratio between the relative price of

Home's aggregate exports before taxes, $P^* = P(Q^*_{FH}, Q^*_{HF})$, and the same relative price after taxes, which is equal to MRS^*_H/MRT^*_H in the decentralized equilibrium. At an optimum, the wedge between these two prices, $P^*/(MRS^*_H/MRT^*_H) - 1$, should be equal to $1/\eta^* - 1$. This is the standard optimal tariff formula in a two-good neoclassical economy, as described in Dixit (1985).

4.4 The Case for Positive Discrimination

We are ready to come back to the issue of whether the solution to Home's relaxed planning problem exhibits positive discrimination.

Mathematically, establishing positive discrimination is formally equivalent to establishing that, at the optimum, $\lambda_T > 0$. We do so by contradiction. Suppose that $\lambda_T \leq 0$. Then starting from an optimum, consider a small decrease in aggregate exports, Q_{HF}, accompanied by adjustments in both domestic output and imports, Q_{HH} and Q_{FH}, such that constraints (14b) and (14c) continue to hold. The increase in Q_{HH} creates a first-order utility gain and since $\lambda_T \leq 0$, the change in Q_{FH} creates, at worst, a second-order utility loss,

$$\frac{d[Q^{1/\mu}_{FH}(Q_{HF})]}{dQ_{HF}} = \lambda_T P_{FF}(Q_{FF}) \frac{\partial(MRS_F(Q_{HF}, Q_{FF})Q_{HF})}{\partial Q_{HF}} = \frac{\lambda_T}{\mu} P_{FF}(Q_{FF}) \left(\frac{Q_{HF}}{Q_{FF}}\right)^{-1/\sigma} \leq 0.$$

This deviation therefore strictly increases Home's utility, thereby contradicting the optimality of the original allocation.

To gain intuition about why positive discrimination occurs at an optimum, it is easiest to start with a small open economy, by which we mean an economy that is too small to affect labor demand in the rest of the world. For such an economy, our second micro problem reduces to

$$Q^{1/\mu}_{FH}(Q_{HF}) \equiv \max_{\mathbf{q_{FH}}} \int_\Phi N_F q^{1/\mu}_{FH}(\varphi) dG_F(\varphi))$$
$$N_F \int p_{FH}(q_{FH}(\varphi);\varphi) q_{FH}(\varphi) dG_F(\varphi) = P_{FF}(Q_{FF}) MRS_F(Q_{HF}, Q_{FF}) Q_{HF},$$

with the import prices of foreign varieties such that

$$p_{FH}(q;\varphi) = \begin{cases} \mu a_{FH}(\varphi), & \text{if } \mu a_{FH}(\varphi) q \geq l_{FH}(q,\varphi), \\ \infty, & \text{otherwise.} \end{cases} \tag{16}$$

Compared to a representative consumer who aims to maximize the utility from imports

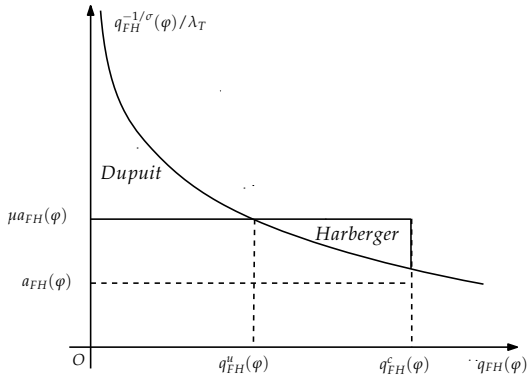


Figure 1: The Case for Positive Discrimination

taking prices as given, Home's government internalizes the fact that import prices are a decreasing function of import quantities. For the prices of imports $p_{FH}(q;\varphi)$ to be equal to $\mu a_{FH}(\varphi)$, imports must be large enough for foreigners' profits to be non-negative. As a result, Home's government may choose to buy more than the unconstrained level of output, $q_{FH}^u(\varphi)$, in order induce firms to self-select into exports.

These considerations are illustrated in Figure 1, where we have plotted the marginal utility of importing a foreign variety, $q_{FH}^{-1/\sigma}(\varphi)/\lambda_T$, as well as its marginal costs from the point of view of Home's government, $\mu a_{FH}(\varphi)$, and the foreign firm, $a_{FH}(\varphi)$. To induce foreign firms to export, Home's government may choose to raise their profits by the rectangle with area $[(\mu-1)a_{FH}(\varphi)] \times [q_{FH}^c(\varphi) - q_{FH}^u(\varphi)]$. The associated benefit is the Dupuit triangle given by the consumer surplus generated by an extra variety, similar to Romer (1994), whereas the cost is the Harberger triangle associated with the purchase of $q_{FH}^c(\varphi) - q_{FH}^u(\varphi)$ units whose marginal benefits are below their marginal costs. For a marginal variety, for which profits at the unconstrained level of output are almost zero, the benefit always exceeds the cost, hence it is optimal to subsidize imports up to $q_{FH}^c(\varphi)$. The required subsidy increases as profitability falls, hence inducing positive discrimination, which ends when fixed costs are so large that the increase in output required for firms to break even leads to a Harberger triangle with area equal to the Dupuit triangle.

Away from the small open economy case, Home's government further internalizes the fact that any increase in imports must be accompanied by an increase in foreign labor demand, which will, in turn, affect the level of local output abroad, Q_{FF}, and Home's terms of trade, $P_{FF}(Q_{FF})MRS_F(Q_{HF}, Q_{FF})$. This creates a second reason for the government to take into account the fixed costs of exporting, above and beyond the non-negativity of foreigners' export profits. Now when deciding whether or not it is worth expanding output beyond its unconstrained level, Home must compare the difference between the Dupuit and Harberger triangle to the shadow cost of raising foreign labor demand by $f_{FH}(\varphi)$. As

formally shown above, however, this general equilibrium consideration can never offset the previous partial equilibrium rationale for positive discrimination.

Because Home's government always takes into account the shadow cost of its import decisions on its trade balance condition, $\lambda_T > 0$, it does not behave like a global social planner, but instead underweights the fixed cost of importing, which only enters constraint (11c), whereas the associated marginal cost enters constraints (11b) and (11c). Thus, for a marginal variety, if foreign firms are indifferent between exporting or not at the unconstrained level of output, Home's government must strictly prefer to import.[14] At a global level, this creates Pareto inefficiencies, with Foreign producing too much of the high fixed-cost varieties associated with low profits (in Foreign) and low social surplus (for the world as a whole), but high consumer surplus (at Home).

5 Optimal Taxes

We have derived three necessary conditions—equations (10), (13), and (15)—that micro quantities, $\{q^*_{HH}(\varphi) \equiv q_{HH}(\varphi|Q^*_{HH}, Q^*_{HF})\}$, $\{q^*_{HF}(\varphi) \equiv q_{HF}(\varphi|Q^*_{HH}, Q^*_{HF})\}$, $\{q^*_{FH}(\varphi) \equiv q_{FH}(\varphi|Q^*_{HF})\}$, and macro quantities, Q^*_{HH}, Q^*_{HF}, and Q^*_{FH}, solving Home's relaxed planning problem must satisfy. We now use these conditions to derive necessary properties that ad-valorem taxes implementing such a solution must satisfy (Sections 5.1-5.3). We will then use these properties to establish the existence of such taxes (Section 5.4). Since they replicate the solution to Home's relaxed planning problem, they a fortiori solve Home's original problem in Definition 1.

5.1 Micro-level Taxes on Domestic Varieties

Consider first a schedule of domestic taxes, $\{s^*_{HH}(\varphi)\}$ and $\{t^*_{HH}(\varphi)\}$, that implements the optimal micro quantities, $\{q^*_{HH}(\varphi)\}$. Fix a benchmark variety φ_{HH} that is sold domestically, $q^*_{HH}(\varphi_{HH}) > 0$. Denote by $s^*_{HH} \equiv s^*_{HH}(\varphi_{HH})$ and $t^*_{HH} \equiv t^*_{HH}(\varphi_{HH})$ the domestic taxes imposed on that variety. Now take any other variety $\varphi \in \Phi_{HH}$ that is sold domestically. By equations (1) and (2), we must have

$$\frac{q^*_{HH}(\varphi_{HH})}{q^*_{HH}(\varphi)} = \left(\frac{(1+t^*_{HH})a_{HH}(\varphi_{HH})}{(1+s^*_{HH})} \frac{(1+s^*_{HH}(\varphi))}{(1+t^*_{HH}(\varphi))a_{HH}(\varphi)} \right)^{-\sigma}.$$

[14] In contrast, from the point of view of a global planner, the social surplus generated by a variety is always proportional to the profits of the firm. So whenever firms are indifferent between exporting or not, a global planner would be indifferent as well.

Combining this expression with equation (10), we obtain our first result.

Lemma 1. *In order to implement an allocation solving the relaxed planning problem, domestic taxes should be such that*

$$\frac{1+s^*_{HH}(\varphi)}{1+t^*_{HH}(\varphi)} = \frac{1+s^*_{HH}}{1+t^*_{HH}} \text{ if } \varphi \in \Phi_{HH}. \tag{17}$$

While we have focused on domestic taxes, there is nothing in the previous proposition that hinges on domestic varieties being sold in the domestic market rather than abroad. Thus, we can use the exact same argument to characterize the structure of export taxes, $\{s^*_{HF}(\varphi)\}$, that implements $\{q^*_{HF}(\varphi)\}$. In line with the previous analysis, let φ_{HF} denote a benchmark variety that is exported, with $s^*_{HF} \equiv s^*_{HF}(\varphi_{HF})$. The following result must hold.

Lemma 2. *In order to implement an allocation solving the relaxed planning problem, export taxes should be such that*

$$s^*_{HF}(\varphi) = s^*_{HF} \text{ if } \varphi \in \Phi_{HF}. \tag{18}$$

5.2 Micro-level Taxes on Foreign Varieties

Now consider a schedule of import taxes, $\{t^*_{FH}(\varphi)\}$, that implements the desired allocation, $\{q^*_{FH}(\varphi)\}$. Fix a benchmark variety $\varphi_{FH} \in \Phi^u_{FH}$ that is imported. In line with our previous analysis, let $t^*_{FH} \equiv t^*_{FH}(\varphi_{FH})$ denote the import tax imposed on that benchmark variety. For any other variety $\varphi \in \Phi_{FH} \equiv \Phi^u_{FH} \cup \Phi^c_{FH}$ that is imported, equations (1) and (2) now imply

$$\frac{q^*_{FH}(\varphi_{FH})}{q^*_{FH}(\varphi)} = \left(\frac{(1+t^*_{FH})a_{FH}(\varphi_{FH})}{(1+t^*_{FH}(\varphi))a_{FH}(\varphi)} \right)^{-\sigma}. \tag{19}$$

There are two possible cases to consider. If $\varphi \in \Phi^u_{FH}$, then equations (13) and (19) imply

$$t^*_{FH}(\varphi) = t^*_{FH}.$$

If $\varphi \in \Phi^c_{FH}$, then equations (13) and (19) imply

$$t^*_{FH}(\varphi) = \frac{(1+t^*_{FH})\theta_{FH}(\varphi)}{\lambda_L + \mu\lambda_T} - 1.$$

This leads to our third result.

Lemma 3. *In order to implement an allocation solving the relaxed planning problem, import taxes*

should be such that

$$t^*_{FH}(\varphi) = (1 + t^*_{FH}) \min\left\{1, \frac{\theta_{FH}(\varphi)}{\lambda_L + \mu\lambda_T}\right\} - 1 \text{ if } \varphi \in \Phi_{FH}, \tag{20}$$

with the profitability index $\theta_{FH}(\varphi) \equiv [(\mu - 1)(a_{FH}(\varphi))^{1-\sigma}/f_{FH}(\varphi)]^{1/\sigma}/\mu$.

5.3 Overall Level of Taxes

Our next goal is to determine the overall level of taxes that is necessary for a decentralized equilibrium to implement the desired allocation. In Sections 5.1 and 5.2, we have already expressed all other taxes as a function of t^*_{HH}, t^*_{FH}, s^*_{HH}, and s^*_{HF}. So, this boils down to characterizing these four taxes. To do so, we compare the ratio between the marginal rates of substitution at home and abroad, evaluated at the solution to Home's relaxed planning problem, and their ratio in the decentralized equilibrium with taxes. As formally established in Appendix B.1, this leads to the following necessary condition.

Lemma 4. *In order to implement an allocation solving the relaxed planning problem, the overall level of optimal taxes,* t^*_{HH}, t^*_{FH}, s^*_{HH}, *and* s^*_{HF}, *should be such that*

$$\frac{(1 + t^*_{FH})/(1 + t^*_{HH})}{(1 + s^*_{HF})/(1 + s^*_{HH})} = \frac{\int_{\Phi_{FH}} \left(\left(\min\left\{1, \frac{\theta_{FH}(\varphi)}{\lambda_L + \mu\lambda_T}\right\}\right)^{\mu} a_{FH}(\varphi)\right)^{1-\sigma} dG_F(\varphi)}{\eta^* \int_{\Phi_{FH}} \left(\left(\min\left\{1, \frac{\theta_{FH}(\varphi)}{\lambda_L + \mu\lambda_T}\right\}\right) a_{FH}(\varphi)\right)^{1-\sigma} dG_F(\varphi)}. \tag{21}$$

According to Lemma 4, if the foreign distribution of blueprints G_F is such that Φ^c_{FH} is measure zero then $\min\left\{1, \frac{\theta_{FH}(\varphi)}{\lambda_L + \mu\lambda_T}\right\} = 1$ for almost all $\varphi \in \Phi_{FH}$, optimal import taxes are uniform, and equation (21) reduces to

$$\frac{(1 + t^*_{FH})/(1 + t^*_{HH})}{(1 + s^*_{HF})/(1 + s^*_{HH})} = 1/\eta^*.$$

This is what would happen in the absence of fixed exporting costs, as in Krugman (1980).[15] If instead Φ^c_{FH} is not measure zero then $\mu > 1$ implies

$$\frac{(1 + t^*_{FH})/(1 + t^*_{HH})}{(1 + s^*_{HF})/(1 + s^*_{HH})} > 1/\eta^*.$$

This derives from our choice of benchmark variety for imports. t^*_{FH} is the tax on varieties $\varphi \in \Phi^u_{FH}$, and we know from Lemma 3 that import taxes should be lower on varieties

[15] In Section 4.4, we have established that Φ^c_{FH} is not empty. For degenerate distributions G_F, however, our analytical results do not rule out the possibility that the measure of blueprints in Φ^c_{FH} is zero.

$\varphi \in \Phi^c_{FH}$. So in order to implement a wedge equal to η^*, the domestic government must now impose import taxes on varieties $\varphi \in \Phi^u_{FH}$ that, relative to other taxes, are strictly greater than $1/\eta^*$.[16]

5.4 Implementation

Lemmas 1-4 provide necessary conditions that linear taxes have to satisfy so that the decentralized equilibrium replicates a solution to the relaxed planning problem. In the next lemma, which is proven in Appendix B.2, we show that that if the previous taxes are augmented with high enough taxes on the goods that are not consumed, $\varphi \notin \Phi_{HH}$, $\varphi \notin \Phi_{HF}$, and $\varphi \notin \Phi_{FH}$, then they are also sufficient to implement any allocation that solves the relaxed planning problem.

Lemma 5. *There exists a decentralized equilibrium with taxes that implements any allocation that solves the relaxed planning problem.*

Since Home's relaxed planning problem is, as its name indicates, a relaxed version of Home's problem in Definition 1, the taxes associated with a decentralized equilibrium that implements a solution to the relaxed planning problem must a fortiori solve Home's problem. Lemmas 2-5 therefore imply that any taxes that solve Home's problem must satisfy conditions (17), (18), (20), and (21). To summarize, we can characterize unilaterally optimal taxes as follows.

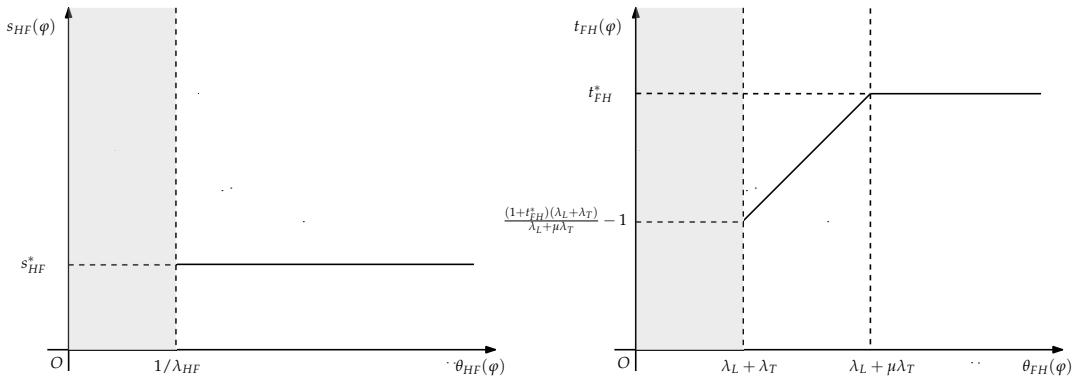
Proposition 1. *At the micro-level, unilaterally optimal taxes are such that: (i) domestic taxes are uniform across all domestic producers (condition 17); (ii) export taxes are uniform across all exporters (condition 18); (iii) import taxes are uniform across Foreign's most profitable exporters and strictly increasing with profitability across its least profitable ones (condition 20). At the macro-level, unilaterally optimal taxes reflect aggregate terms-of-trade considerations (condition 21).*

Figure 2 illustrates our main findings graphically. The left panel (Figure 2a) describes the structure of optimal export taxes, with the grey area representing the region of no exports, while the right panel (Figure 2b) describes the structure of optimal import taxes, with the grey area representing the region of no imports.

[16] Using the characterization of micro-level quantities, one can also solve for η^* as a function of the Lagrange multipliers λ_L and λ_T. This leads to

$$\frac{(1+t^*_{FH})/(1+t^*_{HH})}{(1+s^*_{HF})/(1+s^*_{HH})} = \frac{\mu\lambda_T + \lambda_L}{\mu\lambda_T}\frac{\sigma}{\sigma-1}.$$

For a small open economy, $\lambda_L \to 0$, this simplifies into Gros's (1987) formula. We come back to this connection in Section 6.2 when studying the case of uniform taxes.



(a) Uniform Export Taxes **(b)** Discriminatory Import Taxes

Figure 2: Optimal Trade Taxes

In Figure 2, we have chosen to let both the overall level of export and import taxes s^*_{HF} and t^*_{FH} be non-zero. Note, however, that condition (21) only pins down the relative levels of optimal taxes. Thus, for example, if domestic and export taxes were constrained to be zero, $t^*_{HH} = s^*_{HH} = s^*_{HF} = 0$, the characterization of optimal taxes in Proposition 1 would still be valid with the maximum import tariff t^*_{FH} equal to

$$t^*_{FH} = \frac{\int_{\Phi_{FH}} \left(\left(\min \left\{ 1, \frac{\theta_{FH}(\varphi)}{\lambda_L + \mu\lambda_T} \right\} \right)^{\mu} a_{FH}(\varphi) \right)^{1-\sigma} dG_F(\varphi)}{\eta^* \int_{\Phi_{FH}} \left(\left(\min \left\{ 1, \frac{\theta_{FH}(\varphi)}{\lambda_L + \mu\lambda_T} \right\} \right) a_{FH}(\varphi) \right)^{1-\sigma} dG_F(\varphi)} - 1.$$

Alternatively, one could also impose a uniform export tax, while setting the overall level of other taxes such that $t^*_{HH} = s^*_{HH} = t^*_{FH} = 0$. This is an expression of Lerner symmetry, which must still hold under monopolistic competition, as discussed in Costinot and Werning (2019). Under such an implementation, all varieties $\varphi \in \Phi^c_{FH}$ would have to receive an import subsidy equal to $\theta_{FH}(\varphi)/(\lambda_L + \mu\lambda_T) - 1 < 0$, which is decreasing in profitability. As alluded to in Section 4.1, the fact that domestic taxes can be dispensed with derives from the efficiency of the decentralized equilibrium with monopolistic competition and CES utility. Here, as in Bhagwati (1971), trade taxes are the preferred instruments to exploit monopoly and monopsony power in world markets.

5.5 Firm Heterogeneity, Selection into Exports, and Trade Policy

Since large firms export, whereas small firms do not, it might be tempting to conclude that what policy makers should do is help small firms export as well. As a case in point, the World Trade Report mentioned in Section 2, is entitled "Leveling the Trading Field

for SMEs". The canonical model of trade with self-selection of firms into exports that we have analyzed offers a very different perspective.

From the point of view of a government that unilaterally imposes trade taxes to maximize domestic welfare, Proposition 1 indicates that there is no reason to subsidize the exports of the least profitable domestic firms. Rather, such a government should subsidize the imports of the least profitable foreign firms. Though these prescriptions may run counter to popular narratives about the perceived needs for subsidizing domestic SMEs, the general principle behind our conclusions is an old one. If world prices vary with quantities, but private agents are price-takers, a country should exercise its market power to improve its terms-of-trade, as originally noted by Torrens (1844) and Mill (1844) and later formalized by Johnson (1953).[17] This is true here at the micro-level, where the price of individual varieties $p_{FH}(q;\varphi)$ in equation (16) is a function of import quantities, and this is also true at the macro level, where the elasticity of Foreign's offer curve may differ from one. The selection of heterogeneous firms into exports, however, flips the monotonicity of the relationship between prices and quantities. At the micro-level, prices fall as imports raise to the point where foreign exporters become profitable, which calls for import subsidies rather than taxes.

In contrast to the equivalence result of Arkolakis, Costinot and Rodríguez-Clare (2012), micro-level considerations here lead to very different policy recommendations between Ricardian models, such as Eaton and Kortum (2002), and monopolistically competitive models, such as Melitz (2003). In a Ricardian economy, Costinot, Donaldson, Vogel and Werning (2015) find that optimal export taxes should be heterogeneous across goods, whereas optimal import tariffs should be uniform. This is the exact opposite of what Proposition 1 describes. In a Ricardian economy, goods exported by domestic firms could also be produced by foreign firms. This threat of entry limits the ability of the home government to manipulate prices and leads to lower export taxes on "marginal" goods. Since this threat is absent here, optimal export taxes are uniform instead. On the import side, lower tariffs on "marginal" goods under monopolistic competition derive from the existence of fixed exporting costs, which are necessarily absent under perfect competition.

[17] Bagwell and Staiger (2002) offer an overview of the optimal tariff argument and its implications for the design of trade agreements under perfect competition. Ossa (2011) and Bagwell and Staiger (2012b,a, 2015) discuss whether imperfect competition creates a new rationale for the design of trade agreements.

6 Robustness

The previous analysis makes a number of standard, but strong assumptions. An attractive feature of our method is that it can easily accommodate more general environments. At a technical level, since we focus on inner problems, making the outer problem more complex—because a government may now take into account how its trade taxes affect relative factor prices around the world, demand across industries, or entry—does little to complicate our analysis. In this section, we use this flexibility to establish the robustness of our main findings. To save on space, we focus on sketching alternative environments and summarizing their main implications. A detailed analysis can be found in our Online Appendix

6.1 Economic Environment

Technology. Consider first a strict generalization of our baseline environment with multiple factors of production indexed by n. $L_i \equiv \{L_{i,n}\} \geq 0$ now denotes the exogenous vector of factor endowments in country $i = H, F$, whereas $w_i \equiv \{w_{i,n}\}$ denotes the vector of factor prices in country i. For each origin country i and destination country j, a firm with blueprint φ that uses $l \equiv \{l_n\}$ units of the different factors can produce

$$q_{ij}(l,\varphi) = \left(\frac{\max\{0, g_{ij}(l,\varphi) - f_{ij}(\varphi)\}}{a_{ij}(\varphi)} \right)^{1/(1+\gamma_{ij})},$$

where $g_{ij}(\cdot,\varphi)$ is homogeneous of degree one, strictly quasiconcave, and $\gamma_{ij} > -1/\sigma$. Firms choose their mix of factors to minimize their costs. This leads to the following demand for factor n by a firm with blueprint φ from country i selling q units in country j:

$$l_{ij,n}(q, w_i, \varphi) = \begin{cases} z_{ij,n}(w_i,\varphi)[a_{ij}(\varphi)q^{1+\gamma_{ij}} + f_{ij}(\varphi)] & \text{if } q > 0, \\ 0 & \text{otherwise,} \end{cases}$$

where $z_{ij,n}(w_i,\varphi)$ denotes the solution to $\min_l\{w_i \cdot l | g_{ij}(l,\varphi) \geq 1\}$.[18] The environment of Section 3 corresponds to the special case where firms may vary in terms of their variable and fixed costs, $a_{ij}(\varphi)$ and $f_{ij}(\varphi)$, but there is a single factor, and marginal costs of production are constant, $\gamma_{ij} = 0$. Here, in contrast, firms may vary in their factor intensity, as reflected in different $z_{ij}(w_i,\varphi)$, and, after overhead fixed costs have been paid, there are

[18] Without loss of generality, we normalize $g_{ij}(l,\varphi)$ so that at the equilibrium vector of factor prices, $||z_{ij}(w_i,\varphi)|| = 1$ for all φ.

still increasing or decreasing returns to scale if $\gamma_{ij} \neq 0$.

In this environment, Online Appendix A shows that our characterization of micro-level taxes continues to apply across firms with the same factor intensity. That is, for any given set of firms with the same vector of factor demand $z_{ij}(w_i, \varphi)$, unilaterally optimal taxes should be such that: (i) domestic taxes are uniform across all domestic producers; (ii) export taxes are uniform across all exporters; and (iii) import taxes are uniform across Foreign's most profitable exporters and strictly increasing with profitability across its least profitable ones. If one defines an industry as a group of firms with the same factor intensity, it follows that our previous results hold within each industry, though the level of taxes may now vary across industries as well.

Two restrictions are critical for us to derive this generalized version of Proposition 1. First, cost functions remain separable across markets, which we use to create separate micro problems, like in our baseline analysis. Second, variable costs remain iso-elastic functions of quantities, with the elasticity being common across varieties from the same origin. If not, optimal import tariffs would also reflect differences in the ability to manipulate import prices at different import volumes and, in turn, lead to discriminatory tariffs even when selection is not binding.

Preferences. Consider a strict generalization of our baseline environment where goods may fall into different groups indexed by k. Within a group, we maintain the assumption that preferences are CES, but we do not impose any restriction on the pattern of substitution across these groups. Formally, the utility function of the representative agent in country j is now given by

$$U_j = U_j(\{Q_{Hj}^k, Q_{Fj}^k\}),$$
$$Q_{ij}^k = [\int_\Phi N_i^k (q_{ij}^k(\varphi))^{1/\mu^k} dG_i^k(\varphi)]^{\mu^k},$$

with Q_{ij}^k the subutility from consuming varieties from country i and group k in country j, $\mu^k \equiv \sigma^k/(\sigma^k - 1)$, and $\sigma^k > 1$. The environment of Section 3 corresponds to the special case in which there is only one group and U_j is CES.

Results in this environment echo those obtained under general technologies. As demonstrated in Online Appendix B, our characterization of micro-level taxes continues to apply across firms, but now for those operating within the same group k. In general, one can therefore view the characterization presented in Proposition 1 as a robust "intra-industry" prediction about how the level of trade taxes should vary (or not) with the profitability of firms operating in that industry. This is true regardless of whether one defines an indus-

try as a subset of firms with the same factor intensity, as we did above, or as a subset of firms producing the same group of goods, as in the present extension.[19] Compared to our baseline analysis, as well as the previous and subsequent extensions, the only difference is that we can no longer rule out situations where the set of foreign firms for which there is positive discrimination is empty. Formally, the case for positive discrimination relies on the Lagrange multiplier on Home's trade balance condition being strictly positive, i.e., Home not having incentives to run a trade surplus and transfer money to Foreign. With general preferences, we cannot rule out such "transfer paradoxes", as in Leontief (1936) and Samuelson (1952).

Number of Countries. Consider a strict generalization of our baseline environment with arbitrarily many countries. Home remains the only strategic country, whereas all countries $i \neq H$ are passive. This extension can be thought of as a combination of our two previous generalizations, with each country being endowed with a different factor and producing goods in a different group. In this environment, Online Appendix C shows that Proposition 1 generalizes as follows: (i) domestic taxes are uniform across all domestic producers; (ii) for any given destination country, export taxes are uniform across all domestic exporters; and (iii) for any given origin country, import taxes are uniform across the most profitable exporters and strictly increasing with profitability across the least profitable ones. The "within-country" qualification is the counterpart of the "intra-industry" qualifications in our two previous extensions.

Free Entry Consider a strict generalization of our baseline environment with free entry. In each country, there is a potentially large number of firms that may decide whether to enter or not. As in Melitz (2003), we assume that in order to enter, firms must pay some overhead fixed cost. Once the overhead fixed cost has been paid, firms randomly draw a blueprint $\varphi \in \Phi$, with G_i the distribution of blueprints φ across firms in country i. Compared to Melitz (2003), however, we let firms be ex-ante heterogeneous in terms of their fixed entry costs, with $N_i(f^e)$ the measure of firms with entry costs below some value f^e. Free entry then requires the expected profits of the marginal firm to be equal to its fixed entry cost,

$$f_i^e(N_i) = \sum_{j=H,F} \int_\Phi [\mu a_{ij}(\varphi) q_{ij}(\varphi) - l_{ij}(q_{ij}(\varphi), \varphi)] dG_i(\varphi),$$

[19] The two restrictions that are critical for this generalization also mirror those imposed above. First, preferences are weakly separable across groups, so that we can study each of our micro problems separately. Second, utility remains iso-elastic within each nest, so that markups remain constant and selection remains the only source of terms-of-trade manipulation across foreign firms.

with $f_i^e(\cdot) \equiv N_i^{-1}(\cdot)$. This is the counterpart of equation (5) in our baseline analysis. Melitz (2003) corresponds to the special case where the distribution of fixed entry costs is degenerate, $f_i^e(N_i) = \bar{f}_i^e$, so that entry is perfectly elastic, while our baseline analysis corresponds to the special case where entry is perfectly inelastic. This generalization, with a smooth distribution of fixed entry costs, nests these two extreme situations as limit cases.

Online Appendix D shows that Proposition 1 continues to hold under free entry without further qualifications. For a small open economy, the argument is exactly the same as that in Section 4.4. For a large country, free entry creates new general equilibrium considerations. In addition to manipulating its relative wage, Home can now manipulate the measure of foreign entrants, N_F. This new channel, however, reinforces Home's incentives to tilt Foreign's exports towards varieties with lower profitability. By doing so, Home can reduce profits (gross of entry costs) and hence the equilibrium measure of entrants in Foreign. Since the price of Foreign's varieties abroad, $P_{FF}(Q_{FF}, N_F)$, is decreasing in N_F, lowering the measure of foreign entrants raises the price of Home's exports, $P_{FF}(Q_{FF}, N_F)MRS_F(Q_{HF}, Q_{FF})$, and relaxes Home's trade balance condition, making positive discrimination again optimal.

6.2 Tax Instruments

Two-Part Tariffs. Our baseline analysis allows a rich set of linear taxes that may vary across firms, but rules our the possibility of two-part tariffs. In addition to the ad-valorem taxes available in Section 3, $\{s_{Hj}(\varphi), t_{jH}(\varphi)\}_{j=H,F}$, we now assume that Home's government also has access to firm-specific fixed fees, $\{s_{Hj}^f(\varphi), t_{jH}^f(\varphi)\}_{j=H,F}$. In order to sell any amount in Home's market, a firm with blueprint φ from country j needs to pay $t_{jH}^f(\varphi)$. Conversely, any firm from Home that sells any amount in market j receives $s_{Hj}^f(\varphi)$.

For domestic firms, the introduction of this new instrument does not affect any of our results. Regardless of whether non-linear taxes are available, the cost of producing Q_{HH} and Q_{HF} cannot be lower than $L_H(Q_{HH}, Q_{HF})$. This implies that the solution to Home's planning problem is unaffected and, in turn, that optimal domestic and export taxes remain uniform. Compared to our earlier analysis, the only difference is Foreign's

offer curve, which is now given by

$$
\begin{aligned}
Q_{FH}^{1/\mu}(Q_{HF}) \equiv \max_{\mathbf{t_{FH}^f}, \mathbf{q_{FH}}, Q_{FF}} & \int_{\Phi} N_F q_{FH}^{1/\mu}(\varphi) dG_F(\varphi)), \\
& N_F \int [\mu a_{FH}(\varphi) q_{FH}(\varphi) - t_{FH}^f(\varphi)] dG_F(\varphi) = P_{FF}(Q_{FF}) MRS_F(Q_{HF}, Q_{FF}) Q_{HF}, \\
& L_F = L_{FF}(Q_{FF}) + N_F \int_{\Phi} l_{FH}(q_{FH}(\varphi), \varphi) dG_F(\varphi), \\
& \mu a_{FH}(\varphi) q_{FH}(\varphi) - t_{FH}^f(\varphi) \geq l_{FH}(q_{FH}(\varphi), \varphi).
\end{aligned}
$$

In order to induce firms to self-select into exports, there is no longer a reason for Home to create a Harberger triangle, as previously illustrated in Figure 1, since it now can simply offer a fixed fee that brings total profits, inclusive of the fee, to zero at the unconstrained level of imports,

$$t_{FH}^f(\varphi) = [\mu a_{FH}(\varphi) q_{FH}^u(\varphi) - l_{FH}(q_{FH}^u(\varphi), \varphi)].$$

This again leads to positive discrimination in favor of the least profitable foreign firms, but this now takes the form of lower fixed fees for these firms, as formally established in Online Appendix E.

Uniform Tariffs. In our baseline analysis as well as in the previous extensions we have characterized optimal trade policy under the assumption that the home government is not only free to discriminate between firms from different countries by using trade taxes, but also unlimited in its ability to discriminate between firms from the same country. In practice, informational or legal constraints may place some constraints on this type of taxation. Here, we turn to the other polar case in which the previous constraints are so extreme that Home's government must set uniform taxes: $t_{HF}(\varphi) = \bar{t}_{HF}$, $t_{HH}(\varphi) = \bar{t}_{HH}$, $s_{HF}(\varphi) = \bar{s}_{HF}$, and $s_{HH}(\varphi) = \bar{s}_{HH}$ for all φ.

Online Appendix F shows that optimal uniform taxes satisfy

$$\frac{(1 + \bar{t}_{FH}^*)/(1 + \bar{t}_{HH}^*)}{(1 + \bar{s}_{HF}^*)/(1 + \bar{s}_{HH}^*)} = 1 + \frac{1 + \sigma \kappa_F^* x_{FF}^*}{(\sigma - 1) x_{FF}^*}, \tag{22}$$

where x_{FF}^* denotes the share of expenditure on local goods by Foreign and κ_F^* denotes the elasticity, with respect to Q_{FH}, of the marginal rate of transformation between exports and local goods in Foreign. Equation (22) is a strict generalization of the results of Gros (1987), obtained in the case of homogeneous firms and no fixed exporting costs, for which $\kappa_F^* = 0$, and the results of Demidova and Rodríguez-Clare (2009) and Felbermayr, Jung and Larch (2013), obtained in the case of Pareto distribution of firm-level productivity

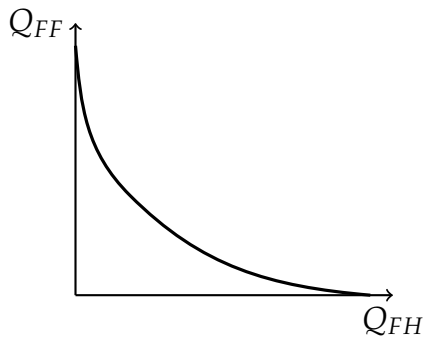


Figure 3: Firm Heterogeneity, Selection into Exports, and Aggregate Nonconvexities

and constant fixed exporting costs.[20]

Compared to prior parametric examples, our general analysis isolates aggregate non-convexities as the key economic channel through which the self-selection of heterogeneous firms into exports tends to lower the overall level of trade protection. Whenever selection is active, in the sense that there is positive density of foreign firms indifferent between selling and not selling in at least one destination $j = H, F$, κ_F^* is strictly negative and the level of trade protection is lower than what it would have been if $\kappa_F^* = 0$.

Like in our baseline analysis, selection does not affect the general rationale for trade taxes: prices depend on quantities, which consumers and firms do not internalize. But compared to a neoclassical environment, it creates a new force that tends to flip the sign of the relationship between prices and quantities. Since Foreign's aggregate production set is not convex, as illustrated in Figure 3, a government may lower the price of its imports by raising their volumes and inducing more foreign firms to become exporters, a force towards subsidizing imports.[21]

[20] In this case we would have $\kappa_F^* = -\frac{\nu-(\sigma-1)}{[\sigma\nu-(\sigma-1)]x_{FF}^*} < 0$, with $\nu > \sigma - 1$ the shape parameter of the Pareto distribution, as discussed in Costinot, Rodríguez-Clare and Werning (2016).

[21] To isolate this mechanism, it is convenient to focus on a generalized version of our baseline analysis where domestic and foreign varieties belong to two different CES nests, as in Costinot, Rodríguez-Clare and Werning (2016). In this case, equation (22) generalizes to

$$\frac{(1+\bar{t}_{FH}^*)/(1+\bar{t}_{HH}^*)}{(1+\bar{s}_{HF}^*)/(1+\bar{s}_{HH}^*)} = 1 + \frac{1+\kappa_F^*\epsilon_F^* x_{FF}^*}{(\epsilon_F^*-1)x_{FF}^*},$$

where ϵ_F^* denotes the upper-level elasticity of substitution between the domestic and foreign nests, which is independent of κ_F^*. When foreign preferences are linear, $\epsilon_F^* \to \infty$, the optimal level of trade protection converges to $\kappa_F^* < 0$. In this limit case, the last neoclassical force calling for an import tariff, diminishing marginal rates of substitution in Foreign, is eliminated. More generally, an import subsidy is optimal if and only if non-convexities on the supply-side dominate convexities on the demand-side, $\kappa_F^* < -1/(\epsilon_F^* x_{FF}^*)$.

6.3 Nash Taxes

We now assume that both governments are strategic and simultaneously set their taxes, taking the taxes of the other government as given. In Online Appendix G, we show that our primal approach can be extended to study the Nash equilibrium of such games. For each country, the idea is again to start by solving a fictitious planner's problem, now taking the vector of taxes in the other country as given, and then to characterize the taxes that decentralize the solution to this planner's problem. By construction, such taxes are best-responses to the taxes in the other country.

In our baseline analysis, we have shown that Home has incentives to impose discriminatory tariffs, which induce a sub-optimal allocation of foreign labor in terms of foreign welfare. Now that Foreign can respond to Home's policy, a natural conjecture is that it will seek to undo the induced reallocation of labor towards its least profitable firms, perhaps leading to a Nash allocation that remains the same as in the world's first best. As Online Appendix G formally demonstrates, this is not so. Just like in our baseline analysis, taxes at a Nash equilibrium are such that: (i) domestic taxes are uniform across all domestic producers; (ii) export taxes are uniform across all exporters; and (iii) import taxes are uniform across Foreign's most profitable exporters and strictly increasing with profitability across its least profitable ones.

In fact, Online Appendix G establishes that imposing uniform export taxes or subsidies is a dominant strategy for both countries. From the point of view of Home's government, discriminatory tariffs in Foreign act as heterogeneous demand shifters. Instead of receiving export prices proportional to $(q_{HF}(\varphi))^{-1/\sigma}$, it now receives export prices proportional to $(q_{HF}(\varphi))^{-1/\sigma}/(1+t_{HF}(\varphi))$. This is no different than the situation faced by Home's exporters who also face inverse demand curves given by $(q_{HF}(\varphi))^{-1/\sigma}/(1+t_{HF}(\varphi))$. As a result, regardless of whether Foreign is passive or not, Home has no incentives to discriminate among its exporters. And for the same reason, Foreign has no incentives to undo Home's import subsidies on its least profitable exporters.

7 Concluding Remarks

In this paper we have studied the policy implications of firms' selection into exports in the context of a canonical model of trade with firm heterogeneity à la Melitz (2003). A number of robust and novel conclusions have emerged from our analysis.

First, from the point of view of a country that unilaterally imposes ad-valorem trade taxes to maximize domestic welfare, it is optimal to subsidize the imports of the least prof-

itable foreign firms within each industry. When firms only export if they can sell enough, an increase in imported quantities lowers the prices of the goods sold by firms that would have not selected into exports otherwise, a form of terms-of-trade manipulation. Compared to a standard neoclassical environment with diminishing marginal returns, however, import prices decrease rather than increase with import quantities. This creates a motive for import subsidies rather than taxes on the least profitable foreign firms.

Second, from a unilateral standpoint, it is also optimal to impose export taxes or subsidies that are uniform across all domestic firms within each industry. Unlike domestic consumers that ignore that buying more from foreign firms may lower the domestic prices that they face, domestic firms fully internalize the benefit of exporting or not, up to general equilibrium considerations. Hence, there is no reason for a government to discriminate across domestic exporters. While financial and information frictions may provide a rationale for subsidizing small domestic firms, a canonical model of trade with firm heterogeneity à la Melitz (2003) does not provide any.

Our findings hold regardless of whether the rest of the world is passive or strategic. At a Nash equilibrium, countries still have incentives to subsidize the imports of the least profitable foreign firms as well as no incentives to subsidize or tax the exports of the least profitable domestic firms. Since discriminatory ad-valorem taxes create Pareto inefficiencies, our analysis suggests that rather than expanding the exports of less profitable firms, an optimal trade agreement may want to restrict them. The general logic, again, is that selection creates a motive for terms-of-trade manipulation, but one with a flipped sign, hence the opposite recommendation for the design of trade agreements.

We view our analysis based on Melitz (2003) as a natural environment in which to study the policy implications of firms' selection into exports. There are, of course, other ways to rationalize this empirical phenomenon, with potentially different policy implications. In particular, Melitz (2003) assumes that fixed exporting costs are paid by foreign exporters. In practice, domestic importers may instead decide whether or not to pay the fixed costs associated with the imports of additional varieties, as in Gopinath and Neiman (2014), Halpern, Koren and Szeidl (2015), Antras, Fort and Tintelnot (2017), and Blaum, Lelarge and Peters (2018). Intuitively, if domestic importers already internalize the fixed costs of importing, a government may no longer want to discriminate across foreign exporters. Empirically, we know fairly little about whether fixed costs are paid by exporters, importers, or both. Our analysis suggests that such considerations may be critical for the design of trade policy.

A similar observation applies to our assumptions about the set of available tax instruments. In our baseline analysis, we have assumed that governments have full knowledge

of the economic environment and access to a full set of ad-valorem taxes, as one would assume when studying optimal trade taxes in a neoclassical environment. In Section 6.2, we have already considered an alternative environment where governments do not have firm-level information, which forces them to impose uniform taxes across all firms. In future work, it would be interesting to explore intermediate cases, in the spirit of Mirrlees (1971), where governments may impose heterogeneous taxes across firms, but those may only depend on the value of a firm's sales.

In these and other alternative environments, we hope that the methods presented in this paper will prove useful as well.

References

Antras, Pol, Teresa C. Fort, and Felix Tintelnot, "The Margins of Global Sourcing: Theory and Evidence from U.S. Firms," *American Economic Review*, 2017, *107* (9), 2514–2564.

Arkolakis, Costas, Arnaud Costinot, and Andres Rodríguez-Clare, "New Trade Models, Same Old Gains?," *American Economic Review*, 2012, *102* (1), 94–130.

Bagwell, Kyle and Robert W. Staiger, *The Economics of the World Trading System*, MIT press, 2002.

_ **and** _ , "The economics of trade agreements in the linear Cournot delocation model," *Journal of International Economics*, 2012, *88*, 32–46.

_ **and** _ , "Profit shifting and trade agreements in imperfectly competitive markets," *International Economic Review*, 2012, *53* (4), 1067–1104.

_ **and** _ , "Delocation and trade agreements in imperfectly competitive markets," *Research in Economics*, 2015, *69*, 132–156.

_ **and Seung Lee**, "Trade policy under monopolistic competition with firm selection," *mimeo Stanford*, 2018.

_ **and** _ , "Trade policy under monopolistic competition with heterogeneous firms and quasi-linear preferences," *mimeo Stanford*, 2018.

Baldwin, Robert E., "Equilibrium in International Trade: A Diagrammatic Analysis," *Quarterly Journal of Economics*, 1948, *62* (5), 748–762.

Bernard, Andew B., Jonathan Eaton, J. Bradford Jensen, and Samuel Kortum, "Plants and Productivity in International Trade," *American Economic Review*, 2003, *93* (4), 1268–1290.

Bernard, Andrew B. and J. Bradford Jensen, "Exceptional Exporter Performance: Cause, Effect, or Both," *Journal of International Economics*, 1999, *47*, 1–25.

Bernard, Andrew, Bradford Jensen, Steve Redding, and Peter J. Schott, "FIrms in International Trade," *Journal of Economic Perspectives*, 2007, *21* (3), 105–130.

Bhagwati, Jagdish N., *The Generalized Theory of Distortions and Welfare*, Vol. Trade, Balance of Payments, and Growth: Papers in International Economics in Honor of Charles P. Kindleberger, Amsterdam: North-Holland, 1971.

Blaum, Joaquin, Claire Lelarge, and Michael Peters, "The Gains from Input Trade with Heterogeneous Importers," *American Economic Journal: Macroeconomics*, 2018, *10* (4), 77–127.

Bombardini, Matilde, "Firm heterogeneity and lobby participation," *Journal of International Economics*, 2008, *75* (2), 329–48.

Campolmi, Alessia, Harald Fadinger, and Chiara Forlati, "Trade and Domestic Policies in Models of Monopolistic Competition," *CEP Discussion paper 13219*, 2018.

Costinot, Arnaud and Ivan Werning, "Lerner Symmetry: A Modern Treatment," *American Economic Review: Insights*, 2019, *1* (1), 13–26.

_ **, Andres Rodríguez-Clare, and Ivan Werning**, "Micro to Macro: Optimal Trade Policy with Firm-Level Heterogeneity," *NBER working paper 21989*, 2016.

_ **, Dave Donaldson, Jonathan Vogel, and Ivan Werning**, "Comparative Advantage and Optimal Trade Policy," *Quarterly Journal of Economics*, 2015, *130* (2), 659–702.

_ **, Guido Lorenzoni, and Ivan Werning**, "A Theory of Capital Controls as Dynamic Terms-of-Trade Manipulation," *Journal of Political Economy*, 2014, *122* (1), 77–128.

Demidova, Svetlana, "Trade Policies, Firm Heterogeneity, and Variable Markups," *Journal of International Economics*, 2017, *108*, 260–273.

_ **and Andres Rodríguez-Clare**, "Trade policy under firm-level heterogeneity in a small economy," *Journal of International Economics*, 2009, *78* (1), 100–112.

Dhingra, Swati and John Morrow, "Monopolistic Competition and Optimum Product Diversity under Firm Heterogeneity," *Journal of Political Economy*, 2019, *127* (1), 196–232.

Dixit, Avinash, "Tax policy in open economies," in A. J. Auerbach and M. Feldstein, eds., *Handbook of Public Economics*, Vol. 1 of *Handbook of Public Economics*, Elsevier, 1985, chapter 6, pp. 313–374.

Dixit, Avinash K. and Joseph E. Stiglitz, "Monopolistic Competition and Optimum Product Diversity," *The American Economic Review*, 1977, *67* (3), 297–308.

Eaton, Jonathan and Samuel Kortum, "Technology, Geography and Trade," *Econometrica*, 2002, *70* (5), 1741–1779.

_ , _ , **and Francis Kramarz**, "An Anatomy of International Trade: Evidence from French Firms," *Econometrica*, 2011, *79* (5), 1453–1498.

Everett, Hugh, "Generalized Lagrange Multiplier Methods for Solving Problems of Optimal Allocation of Resources," *Operations Research*, 1963, *11* (3), 399–417.

Felbermayr, Gabriel J., Benjamin Jung, and Mario Larch, "Optimal Tariffs, Retaliation and the Welfare Loss from Tariff Wars in the Melitz Model," *Journal of International Economics*, 2013, *89* (1), 13–25.

Gopinath, Gita and Brent Neiman, "Trade Adjustment and Productivity in Large Crises," *American Economic Review*, 2014, *104* (3), 793–831.

_ **, Elhanan Helpman, and Kenneth Rogoff, eds**, *Handbook of International Economics*, Vol. 4, Elsevier, 2014.

Gros, Daniel, "A Note on the Optimal Tariff, Retaliation and the Welfare Loss from Tariff Wars in a Framework with Intra-Industry Trade," *Journal of International Economics*, 1987, *23*, 357–367.

Haaland, Jan I. and Anthony J. Venables, "Optimal Trade Policy with Monopolistic Competition and Heterogeneous Firms," *Journal of International Economics*, 2016, *102*, 85–95.

Halpern, Laszlo, Miklos Koren, and Adam Szeidl, "Imported Inputs and Productivity," *American Economic Review*, 2015, *105* (12), 3660–3703.

Helpman, Elhanan and Paul R. Krugman, *Trade Policy and Market Structure*, Cambridge, Massachussetts: MIT Press, 1989.

Johnson, Harry G., "Optimum Tariffs and Retaliation," *Review of Economic Studies*, 1953, *21* (2), 142–153.

Kim, In Song, "Political Cleavages within Industry: Firm-level Lobbying for Trade Liberalization," *American Political Science Review*, 2017, *111* (1), 1–20.

Krugman, Paul, "Scale Economies, Product Differentiation, and the Pattern of Trade," *The American Economic Review*, 1980, *70* (5), 950–959.

Leontief, Wassily, "Note on the Pure Theory of Transfer," in "Explorations in Economics," Taussig Fetschrift volume, 1936.

Luenberger, David G., *Optimization by Vector Space Methods*, John Wiley and Sons, Inc, 1969.

Maggi, Giovanni, "International Trade Agreements," 2014, *4*.

Melitz, Marc and Stephen Redding, *Heterogeneous Firms and Trade*, Vol. 4 of *Handbook of International Economics*, Elsevier, 2014.

Melitz, Marc J., "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 2003, *71* (6), 1695–1725.

Mill, J.S, *Essays on Some Unsettled Questions of Political Economy*, London: Parker, 1844.

Mirrlees, James A., "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies*, 1971, *38* (2), 175–208.

Nocco, Antonella, Gianmarco I. P. Ottaviano, and Matteo Salto, "Monopolistic Competition and Optimum Product Selection," *American Economic Review, Papers and Proceedings*, 2014, *104* (5), 304–309.

_ , _ , and _ , "Geography, Competition, and Optimal Multilateral Trade Policy," *Journal of International Economics*, 2019, *120*, 145–161.

Ossa, Ralph, "A "New Trade" Theory of GATT/WTO Negotiations," *Journal of Political Economy*, 2011, *119* (1), 112–152.

Romer, Paul, "New Goods, Old Theory, and the Welfare of Trade Restrictions," *Journal of Development Economics*, 1994, *43* (1), 5–38.

Samuelson, Paul A., "The Transfer Problem and Transport Costs: The Terms of Trade When Impediments Are Absent," *Economic Journal*, 1952, *62*, 278–304.

Spence, Michael, "Product Selection, Fixed Costs, and Monopolistic Competition," *The Review of Economic Studies*, 1976, *43* (2), 217–235.

Torrens, R., *The Budget: On Commercial Policy and Colonial Policy*, London: Smith, Elder, 1844.

A Proofs of Section 3

In Sections 4.1-4.3, we have described the solution to Home's relaxed planning problem:

$$\max_{\{\mathbf{q_{ij}},Q_{ij}\}_{i,j=H,F},\mathbf{p_{FF}},\mathbf{p_{FH}},P_{FF},P_{HF}} U_H(Q_{HH},Q_{FH}) \tag{A.1a}$$

subject to:

$$Q_{ij}^{1/\mu} \leq \int_\Phi N_i(q_{ij}(\varphi))^{1/\mu} dG_i(\varphi), \text{ for } i=H \text{ or } j=H, \tag{A.1b}$$

$$q_{FF}(\varphi) = \begin{cases} \bar{q}_{FF}(\varphi) & \text{if } \mu a_{FF}(\varphi)\bar{q}_{FF}(\varphi) \geq l_{FF}(\bar{q}_{FF}(\varphi),\varphi), \\ 0 & \text{otherwise,} \end{cases} \tag{A.1c}$$

$$P_{FF}^{1-\sigma} = \int_\Phi N_F(p_{FF}(\varphi))^{1-\sigma} dG_F(\varphi), \tag{A.1d}$$

$$p_{Fj}(\varphi) = \begin{cases} \bar{p}_{Fj}(\varphi) & \text{if } \mu a_{Fj}(\varphi)q_{Fj}(\varphi) \geq l_{Fj}(q_{Fj}(\varphi),\varphi), \\ \infty & \text{otherwise,} \end{cases} \text{ for } j=H,F, \tag{A.1e}$$

$$\Pi_F = N_F \sum_{j=H,F} \int_\Phi [\mu a_{Fj}(\varphi)q_{Fj}(\varphi) - l_{Fj}(q_{Fj}(\varphi),\varphi)] dG_F(\varphi), \tag{A.1f}$$

$$Q_{HF},Q_{FF} \in \arg\max_{\tilde{Q}_{HF},\tilde{Q}_{FF}} \{U_F(\tilde{Q}_{HF},\tilde{Q}_{FF}) | \textstyle\sum_{i=H,F} P_{iF}\tilde{Q}_{iF} = w_F L_F + \Pi_F\}, \tag{A.1g}$$

$$L_i = N_i \sum_{j=H,F} \int_\Phi l_{ij}(q_{ij}(\varphi),\varphi) dG_i(\varphi), \text{ for } i=H,F. \tag{A.1h}$$

We now provide the formal arguments used to characterize this solution.

A.1 Home's Production Possibility Frontier (Section 3.1)

This appendix discusses some technical details behind our analysis and characterization of optimal policy. Our approach in Section 4.1 was to derive necessary conditions for optimality, appealing to global Lagrangian necessity theorems. This appendix clarifies how we can invoke such results, despite the apparent non convexity of the problems.

The minimization problem in 4.1 is not a convex optimization problem. We first convexify this problem by allowing randomization: instead of specifying a single quantity $q(\varphi)$ for each blueprint φ, we relax this problem and let the planner choose a distribution over q conditional on φ. Formally, for each φ there is a CDF over q_{Hj} given by $M_{Hj}(q;\varphi)$. Letting $\mathbf{M_{Hj}} \equiv \{M_{Hj}(q;\varphi)\}$, the inner planning problem becomes

$$L_H(Q_{HH},Q_{HF}) \equiv \min_{\mathbf{M_{HH}},\mathbf{M_{HF}}\in\mathcal{M}} N_H \left(\sum_{j=H,F} \int_\Phi \int_{[0,\infty)} l_{Hj}(q,\varphi) dM_{Hj}(q;\varphi) dG_H(\varphi) \right)$$
$$N_H \int_\Phi \int_{[0,\infty)} q^{1/\mu} dM_{Hj}(q;\varphi) dG_H(\varphi) \geq Q_{Hj}^{1/\mu}, \text{ for } j=H,F,$$

where $\mathcal{M}$ is the set of all families of CDFs.

Note that $\mathcal{M}$ is a convex subset of a vector space. As stated, the above planning problem is linear and, thus, convex in $\mathbf{M_{HH}}$ and $\mathbf{M_{HF}}$.We have relaxed the equality to an inequality constraint to ensure that there exists an interior point, i.e. an $\mathbf{M_{Hj}}$ such that the constraint holds with strict inequality. Thus, we can apply a Lagrangian necessity theorem such as Theorem 1, p. 217 from Luenberger (1969). This guarantees that there exists $\lambda_{Hj} \geq 0$ such that any solution to the above problem must also minimize

$$N_H \left(\sum_{j=H,F} \int_\Phi \int_{[0,\infty)} l_{Hj}(q,\varphi) dM_{Hj}(q;\varphi) dG_H(\varphi) \right)$$
$$+ \sum_{j=H,F} \lambda_{Hj} \left(Q_{Hj}^{1/\mu} - N_H \int_\Phi \int_{[0,\infty)} q^{1/\mu} dM_{Hj}(q;\varphi) dG_H(\varphi) \right)$$

over $\mathbf{M_{HH}}, \mathbf{M_{HF}} \in \mathcal{M}$.

Next, we argue that this minimization must be attained without randomization. In other words, it can be described by two functions $q_{Hj}(\varphi)$ for $j = H, F$. This follows from the following two observations: (i) any $\mathbf{M_{Hj}} \in \mathcal{M}$ is dominated by $\hat{\mathbf{M}}_{\mathbf{Hj}} \in \mathcal{M}$ that assigns probability one to the set of points where $l_{Hj}(q;\varphi) - \lambda_{Hj} q^{1/\mu}$ is minimized; and (ii) the set of minimizers of $l_{Hj}(q;\varphi) - \lambda_{Hj} q^{1/\mu}$ is almost everywhere unique. To verify (ii), note that from the characterization in Section 4.1, $l_{Hj}(q;\varphi) - \lambda_{Hj} q^{1/\mu}$ has multiple minimizers only when

$$(\mu - 1)(\mu/\lambda_{Hj})^{-\sigma} (a_{Hj}(\varphi))^{1-\sigma} = f_{Hj}(\varphi).$$

Since we have assumed that for any $f_{Hj} > 0$ the distribution over a_{Hj} is smooth, this condition can only hold on a set with probability zero.

At this point, we have established that the solution to the relaxed minimization problem, with randomization, needs to minimize the associated Lagrangian and that the solution to the Lagrangian problem does not involve randomization. This implies that any solution to the original minimization problem in 4.1, without randomization, must also minimize

$$N_H \sum_{j=H,F} \int_\Phi \left(l_{Hj}(q_{Hj}(\varphi),\varphi) - \lambda_{Hj} (q_{Hj}(\varphi))^{1/\mu} \right) dG_H(\varphi).$$

Finally, note that for the solution of the Lagrangian problem to satisfy (9b), λ_{Hj} must also be non-zero, as stated in the main text.

A.2 Foreign's Offer Curve (Section 3.2)

The full problem of maximizing Home's imports, Q_{FH}, conditional on its aggregate exports, Q_{HF}, subject to Foreign's equilibrium conditions, i.e, conditions (1) and (4) for $i = F$ and $j = F$ and (2),

(5), (6) for $i = F$, and (3) for $j = F$, is given by

$$Q_{FH}^{1/\mu}(Q_{HF}) \equiv \max_{\mathbf{q_{FF}},\mathbf{q_{FH}},\mathbf{p_{FF}},\mathbf{p_{FH}},P_{HF},P_{FF},Q_{FF}} \int_{\Phi} N_F q_{FH}^{1/\mu}(\varphi) dG_F(\varphi)) \tag{A.2a}$$

$$q_{FF}(\varphi) = \begin{cases} \bar{q}_{FF}(\varphi) & \text{if } \mu a_{FF}(\varphi)\bar{q}_{FF}(\varphi) \geq l_{FF}(\bar{q}_{FF}(\varphi),\varphi), \\ 0 & \text{otherwise,} \end{cases} \tag{A.2b}$$

$$P_{FF}^{1-\sigma} = \int_{\Phi} N_F [p_{FF}(\varphi)]^{1-\sigma} dG_F(\varphi), \tag{A.2c}$$

$$p_{Fj}(\varphi) = \begin{cases} \bar{p}_{Fj}(\varphi) & \text{if } \mu a_{Fj}(\varphi) q_{Fj}(\varphi) \geq l_{Fj}(q_{Fj}(\varphi),\varphi), \\ \infty & \text{otherwise,} \end{cases} \quad \text{for } j = H, F, \tag{A.2d}$$

$$Q_{HF}, Q_{FF} \in \arg\max_{\tilde{Q}_{HF},\tilde{Q}_{FF}} \{U_F(\tilde{Q}_{HF},\tilde{Q}_{FF}) | \textstyle\sum_{i=H,F} P_{iF}\tilde{Q}_{iF} = L_F + \Pi_F\}, \tag{A.2e}$$

$$\Pi_F = N_F \textstyle\sum_{j=H,F} \int_{\Phi} [\mu a_{Fj}(\varphi) q_{Fj}(\varphi) - l_{Fj}(q_{Fj}(\varphi),\varphi)] dG_F(\varphi), \tag{A.2f}$$

$$L_F = N_F \textstyle\sum_{j=H,F} \int_{\Phi} l_{Fj}(q_{Fj}(\varphi),\varphi) dG_F(\varphi). \tag{A.2g}$$

Constraints (A.2b)-(A.2d) can be used to solve for the local micro quantities and prices in Foreign, as a function of Q_{FF},

$$q_{FF}(\varphi|Q_{FF}) = \begin{cases} \bar{q}_{FF}(\varphi|Q_{FF}) & \text{, if } \mu a_{FF}(\varphi)\bar{q}_{FF}(\varphi|Q_{FF}) \geq l_{FF}(\bar{q}_{FF}(\varphi|Q_{FF}),\varphi), \\ 0 & \text{, otherwise;} \end{cases} \tag{A.3}$$

$$p_{FF}(\varphi|Q_{FF}) = \begin{cases} \mu a_{FF}(\varphi) & \text{, if } \mu a_{FF}(\varphi) q_{FF}(\varphi|Q_{FF}) \geq l_{Fj}(q_{FF}(\varphi|Q_{FF}),\varphi), \\ \infty & \text{, otherwise;} \end{cases} \tag{A.4}$$

$$P_{FF}(Q_{FF}) = \left(\int_{\Phi} N_F (p_{FF}(\varphi|Q_{FF}))^{1-\sigma} dG_F(\varphi) \right)^{1/(1-\sigma)}, \tag{A.5}$$

with $\bar{q}_{FF}(\varphi|Q_{FF}) \equiv [\mu a_{FF}(\varphi)/P_{FF}(Q_{FF})]^{-\sigma} Q_{FF}$. Total employment associated with the local sales of foreign firms, $L_{FF}(Q_{FF})$, is then given by

$$L_{FF}(Q_{FF}) \equiv N_F [\int_{\Phi} l_{FF}(q_{FF}(\varphi|Q_{FF}),\varphi) dG_F(\varphi)]. \tag{A.6}$$

In turn, constraint (A.2e) can be used to solve for Home's export price as a function of Q_{HF} and Q_{FF}. The necessary first order conditions for utility maximization in Foreign imply

$$P_{HF}(Q_{HF}, Q_{FF}) = P_{FF}(Q_{FF}) MRS_F(Q_{HF}, Q_{FF}), \tag{A.7}$$

where $MRS_F(Q_{HF}, Q_{FF}) \equiv (\partial U_F/\partial Q_{HF})/(\partial U_F/\partial Q_{FF})$ is the marginal rate of substitution in Foreign. Combining the previous equation with Foreign's budget constraint, as well as constraints

(A.2f) and (A.2g), we can rearrange constraint (A.2e) more compactly as

$$N_F \int [\mu a_{FH}(\varphi) q_{FH}(\varphi)] dG_F(\varphi) = P_{FF}(Q_{FF}) MRS_F(Q_{HF}, Q_{FF}) Q_{HF}.$$

Substituting the previous expressions into problem A.2, we get that Home's optimal import quantities, $\mathbf{q_{FH}}$, as well as the level of local output in Foreign, Q_{FF}, must solve (11).

We can derive necessary conditions for optimality of $\mathbf{q_{FH}}$, appealing to global Lagrangian necessity theorems, as we did in Section 4.1. Consider the subproblem that takes Q_{HF} and Q_{FF} as given and maximizes over $\mathbf{q_{FH}}$. Specifically, allowing for randomization we can rewrite the problem as a choice over CDF $M_{FH}(q;\varphi)$. The problem then becomes

$$\begin{aligned}
&\max_{\mathbf{M_{FH}} \in \mathcal{M}_{FH}} \int_\Phi \int_{[0,\infty)} N_F q^{1/\mu} dM_{FH}(q;\varphi)\, dG_F(\varphi) \\
&N_F \int_\Phi \int_{[0,\infty)} \mu a_{FH}(\varphi) q\, dM_{FH}(q;\varphi)\, dG_F(\varphi) = P_{FF}(Q_{FF}) MRS_F(Q_{HF}, Q_{FF}) Q_{HF}, \\
&L_F = L_{FF}(Q_{FF}) + N_F \int_\Phi \int_{[0,\infty)} l_{FH}(q,\varphi)\, dM_{FH}(q;\varphi)\, dG_F(\varphi),
\end{aligned}$$

where

$$\mathcal{M}_{FH} = \{\mathbf{M_{FH}} \in \mathcal{M} : \forall \varphi \in \Phi, \int_{\{q:\mu a_{FH}(\varphi) q - l_{FH}(q,\varphi) < 0\}} dM_{FH}(q;\varphi) = 0\}$$

is the set of probability distributions that ensures positive profits with probability one for all firms. As stated, this problem is linear and, thus, convex. It features two equality constraints. Invoking the Lagrangian necessity theorem given by Theorem 1, p. 217 from Luenberger (1969) extended in Exercise 8.8.7 (p. 236), there exist multipliers λ_T and λ_L for the equality constraints so that any solution must also maximize the Lagrangian

$$\int_\Phi \int_{[0,\infty)} [q^{1/\mu} - \lambda_T \mu a_{FH}(\varphi) q - \lambda_L l_{FH}(q,\varphi)] dM_{FH}(q;\varphi)\, dG_F(\varphi)$$

over $\mathbf{M_{FH}} \in \mathcal{M}_{FH}$. As before, since the objective is linear in $\mathbf{M_{FH}}$ it follows that we can focus on a "bang bang" solution that puts full weight on any point q for each φ that minimizes

$$q^{1/\mu} - \lambda_T \mu a_{FH}(\varphi) q - \lambda_L l_{FH}(q,\varphi)$$

over the set of q satisfying $\mu a_{FH}(\varphi) q \geq l_{FH}(q,\varphi)$. By virtue of the analysis carried out in Section 4.2, the solution to this problem is unique almost everywhere. This follows since indifference obtains only if $f_{FH}(\varphi) > 0$ and for at most two values of $\theta_{FH}(\varphi)$, namely $\theta_{FH}(\varphi) = \lambda_L^{1/\sigma} (\chi_{FH})^{1/\mu}$, if $\lambda_T < 0$, or $\theta_{FH}(\varphi) = \max\{0, \lambda_L + \lambda_T\}$, otherwise. But under our assumption that for any $f_{FH} > 0$ the distribution over $a_{FH}(\varphi)$ is smooth, it follows that indifference happens with probability zero. As in the first micro problem, this implies that any solution to the second micro problem without randomization is also a solution to the relaxed version with randomization, and so maximizes the

corresponding Lagrangian.

Once optimal quantities, $q_{FH}(\varphi|Q_{HF}, Q_{FF})$, have been solved for, optimal import prices are given by

$$p_{FH}(\varphi|Q_{HF}, Q_{FF}) = \begin{cases} \mu a_{FH}(\varphi) & \text{, if } \mu a_{FH}(\varphi) q_{FH}(\varphi|Q_{HF}, Q_{FF}) \geq l_{FH}(q_{FH}(\varphi|Q_{HF}, Q_{FF}), \varphi), \\ \infty & \text{, otherwise.} \end{cases} \tag{A.8}$$

Finally, the optimal level of local output in Foreign, $Q_{FF}(Q_{HF})$, is then given by the solution to the outer problem

$$Q_{FF}(Q_{HF}) \in \text{argmax}_{Q_{FF} \in \Omega_F} \int_{\Phi} N_F q_{FF}^{1/\mu}(\varphi|Q_{HF}, Q_{FF}) dG_F(\varphi)) \tag{A.9}$$

with Ω_F the set of Q_{FF} for which a solution to the inner maximization problem exists.

A.3 First-Order Conditions of the Macro Problem (Section 3.3)

At an interior solution to the macro problem (14), the necessary first-order conditions are given by

$$\begin{aligned} U_{HH}^* &= \Lambda_H L_{HH}^*, \\ U_{FH}^* &= \Lambda_T, \\ \Lambda_T Q_{FH}'(Q_{HF}^*) &= \Lambda_H L_{HF}^*, \end{aligned}$$

where $U_{iH}^* \equiv \partial U_H / \partial Q_{iH}$ denotes the marginal utility at home of the aggregate good from country $i = H, F$; $L_{Hj}^* \equiv \partial L_H / \partial Q_{Hj}$ denotes the marginal cost of producing and delivering one unit of the home good in country $j = H, F$; and Λ_T and Λ_H are the Lagrange multipliers associated with constraints (14b) and (14c). After eliminating the Lagrange multipliers, we obtain

$$\frac{U_{HH}^*}{U_{FH}^*} = \frac{L_{HH}^* Q_{FH}'(Q_{HF}^*)}{L_{HF}^*}. \tag{A.10}$$

To conclude, note that at a solution to (11), constraint (11b) implies

$$P_{HF}(Q_{HF}^*) Q_{HF}^* = \tilde{P}_{FH}(Q_{HF}^*, Q_{FH}^*) Q_{FH}^*.$$

Combining this expression with equation (A.10), we finally get

$$\frac{U_{HH}^*}{U_{FH}^*} = \frac{L_{HH}^*}{L_{HF}^*} \frac{P_{HF}(Q_{HF}^*)}{\tilde{P}_{FH}(Q_{HF}^*, Q_{FH}^*)} \frac{Q_{HF}^* Q_{FH}'(Q_{HF}^*)}{Q_{FH}(Q_{HF}^*)}.$$

Equation (15) follows from this equation and the definitions of $MRS_H^* \equiv U_{HH}^*/U_{FH}^*$, $MRT_H^* \equiv L_{HH}^*/L_{HF}^*$, $P^* \equiv P_{HF}(Q_{HF}^*)/\tilde{P}_{FH}(Q_{HF}^*, Q_{FH}^*)$, and $\eta^* \equiv d\ln Q_{FH}/d\ln Q_{HF}$.

B Proofs of Section 4

Let $(\{\mathbf{q_{ij}^*}, Q_{ij}^*\}_{i,j=H,F}, \mathbf{p_{FF}^*}, \mathbf{p_{FH}^*}, P_{FF}^*, P_{HF}^*)$ denote a solution to Home's relaxed planning problem. In the main text, we have already described some of these variables. Before establishing Lemmas 4 and 5, we provide a complete characterization of this solution. The three macro quantities, $(Q_{HH}^*, Q_{HF}^*, Q_{FH}^*)$, are given by the solution to (14). Conditional on Q_{HH}^* and Q_{HF}^*, the domestic micro quantities, $\mathbf{q_{HH}^*} = \{q_{HH}(\varphi|Q_{HH}^*, Q_{HF}^*)\}$ and $\mathbf{q_{HF}^*} = \{q_{HF}(\varphi|Q_{HH}^*, Q_{HF}^*)\}$, are given by equation (10). Conditional on Q_{HF}^*, the local output in Foreign, $Q_{FF}^* = Q_{FF}(Q_{HF}^*)$, is given by condition (A.9), whereas the foreign micro quantities, $\mathbf{q_{FH}^*} = \{q_{FH}(\varphi|Q_{HF}^*)\}$ and $\mathbf{q_{FF}^*} = \{q_{FF}(\varphi|Q_{FF}(Q_{HF}^*))\}$, are given by conditions (13) and (A.3). Finally, the prices of foreign varieties, $\mathbf{p_{FF}^*} = \{p_{FF}(\varphi|Q_{FF}(Q_{HF}^*))\}$ and $\mathbf{p_{FH}^*} = \{p_{FH}(\varphi|Q_{FF}(Q_{HF}^*))\}$, are given by equations (A.4) and (A.8), whereas the aggregate price indices, $P_{HF}^* = P_{HF}(Q_{HF}^*, Q_{FF}(Q_{HF}^*))$ and $P_{FF}^* = P_{FF}(Q_{FF}(Q_{HF}^*))$, are given by equations (A.5) and (A.7). We also let $\tilde{P}_{FH}^* = \tilde{P}_{FH}(Q_{HF}^*, Q_{FH}^*)$ denote the average cost of imports at home. Note that $\tilde{P}_{FH}^*$ differs from the import price index faced by Home consumers in the decentralized equilibrium, P_{FH}, which is inclusive of taxes.

B.1 Lemma 4

Proof of Lemma 4. First, consider the marginal rate of substitution, $MRS_j^* \equiv U_{Hj}^*/U_{Fj}^*$, in country $j = H, F$ at a solution to Home's relaxed planning problem. In Foreign, the necessary first order conditions for utility maximization imply $MRS_F^* = P_{HF}^*/P_{FF}^*$. Combining this expression with equation (A.5), we obtain

$$MRS_F^* = \frac{P_{HF}^*}{(\int_\Phi N_F(p_{FF}^*(\varphi))^{1-\sigma} dG_F(\varphi))^{1/(1-\sigma)}}. \tag{B.1}$$

At home, we already know from equation (15) that

$$MRS_H^* = \eta^* MRT_H^*(P_{HF}^*/\tilde{P}_{FH}^*).$$

By the Envelope Theorem, we also know that

$$MRT_H^* = (\lambda_{HH}/\lambda_{HF})(Q_{HH}^*/Q_{HF}^*)^{-1/\sigma}.$$

From equations (9b) and (10), we also know that the Lagrange multipliers satisfy

$$\lambda_{Hj} = [N_H \int_{\Phi_{Hj}} (\mu a_{Hj}(\varphi))^{1-\sigma} dG_H(\varphi)]^{1/(1-\sigma)} (Q_{Hj}^*)^{1/\sigma}.$$

Combining the two previous expressions, we get

$$MRT_H^* = \frac{(\int_{\Phi_{HH}} (a_{HH}(\varphi))^{1-\sigma} dG_H(\varphi))^{1/(1-\sigma)}}{(\int_{\Phi_{HF}} (a_{HF}(\varphi))^{1-\sigma} dG_H(\varphi))^{1/(1-\sigma)}}, \tag{B.2}$$

and in turn,

$$MRS_H^* = \frac{\eta^* (\int_{\Phi_{HH}} (a_{HH}(\varphi))^{1-\sigma} dG_H(\varphi))^{1/(1-\sigma)} P_{HF}^*}{(\int_{\Phi_{HF}} (a_{HF}(\varphi))^{1-\sigma} dG_H(\varphi))^{1/(1-\sigma)} \tilde{P}_{FH}^*}. \tag{B.3}$$

Next, consider a decentralized equilibrium with taxes that implements a solution to the relaxed planning problem. The marginal rate of substitution for each of the two countries is determined by conditions (2)-(4). Using the fact that the set of varieties available for consumption in the decentralized equilibrium must be the same as in the solution to the relaxed planning problem, we obtain

$$MRS_F^* = \frac{(\int_{\Phi_{HF}} N_H (\mu w_H a_{HF}(\varphi)/(1+s_{HF}^*(\varphi)))^{1-\sigma} dG_H(\varphi))^{1/(1-\sigma)}}{(\int_{\Phi} N_F (p_{FF}^*(\varphi))^{1-\sigma} dG_F(\varphi))^{1/(1-\sigma)}}, \tag{B.4}$$

$$MRS_H^* = \frac{(\int_{\Phi_{HH}} N_H ((1+t_{HH}^*(\varphi)) \mu w_H a_{HH}(\varphi)/(1+s_{HH}^*(\varphi)))^{1-\sigma} dG_H(\varphi))^{1/(1-\sigma)}}{(\int_{\Phi_{FH}} N_F ((1+t_{FH}^*(\varphi)) \mu a_{FH}(\varphi))^{1-\sigma} dG_F(\varphi))^{1/(1-\sigma)}}. \tag{B.5}$$

Combining equations (B.1), (B.3), (B.4), and (B.5) with the micro-level taxes in Lemmas 1-3, we get

$$\frac{(1+t_{FH}^*)/(1+t_{HH}^*)}{(1+s_{HF}^*)/(1+s_{HH}^*)} = \frac{\tilde{P}_{FH}^*}{\eta^* (\int_{\Phi_{FH}} N_F (\min\{1, \frac{\theta_{FH}(\varphi)}{\chi_{FH}}\} \mu a_{FH}(\varphi))^{1-\sigma} dG_F(\varphi))^{1/(1-\sigma)}}. \tag{B.6}$$

By definition of $\tilde{P}_{FH}^*$, we know that

$$\tilde{P}_{FH}^* Q_{FH}^* = \int_{\Phi} N_F \mu a_{FH}(\varphi) q_{FH}^*(\varphi) dG_F(\varphi).$$

Together with equation (13), this implies

$$\frac{(\mu \chi_{FH})^{\sigma} \tilde{P}_{FH}^* Q_{FH}^*}{N_F \mu} = \int_{\Phi_{FH}^u} (a_{FH}(\varphi))^{1-\sigma} dG_F(\varphi) + \int_{\Phi_{FH}^c} \left((\frac{\theta_{FH}(\varphi)}{\chi_{FH}})^{\mu} a_{FH}(\varphi) \right)^{1-\sigma} dG_F(\varphi).$$

At a solution to Home's relaxed planning problem, constraint (14b) must be satisfied with equality. Otherwise, Home could raise its imports, Q_{FH}, and hence the utility of its representative agent. Using equations (13) and (14b), one can also check that

$$\frac{(\mu \chi_{FH})^{\sigma-1} (Q_{FH}^*)^{1/\mu}}{N_F} = \int_{\Phi_{FH}^u} (a_{FH}(\varphi))^{1-\sigma} dG_F(\varphi) + \int_{\Phi_{FH}^c} (\frac{\theta_{FH}(\varphi)}{\chi_{FH}} a_{FH}(\varphi))^{1-\sigma} dG_F(\varphi).$$

Combining the two previous expressions, we then obtain

$$\frac{\tilde{P}^*_{FH}}{(N_F)^{1/(1-\sigma)}\mu} = \frac{\int_{\Phi^u_{FH}}(a_{FH}(\varphi))^{1-\sigma}dG_F(\varphi) + \int_{\Phi^c_{FH}}((\frac{\theta_{FH}(\varphi)}{\chi_{FH}})^{\mu}a_{FH}(\varphi))^{1-\sigma}dG_F(\varphi)}{(\int_{\Phi^u_{FH}}(a_{FH}(\varphi))^{1-\sigma}dG_F(\varphi) + \int_{\Phi^c_{FH}}(\frac{\theta_{FH}(\varphi)}{\chi_{FH}}a_{FH}(\varphi))^{1-\sigma}dG_F(\varphi))^{\sigma/(\sigma-1)}}. \tag{B.7}$$

Substituting into equation (B.6) and using the definition of Φ^u_{FH} and Φ^c_{FH} we get equation (21). □

B.2 Lemma 5

Proof of Lemma 5. In order to show the existence of a decentralized equilibrium that implements the desired allocation, we follow a guess and verify strategy. Consider: (i) quantities such that

$$q_{ij}(\varphi) = q^*_{ij}(\varphi), \tag{B.8}$$

$$Q_{ij} = Q^*_{ij}; \tag{B.9}$$

(ii) aggregate profits such that

$$\Pi_i = N_i \sum_{j=H,F} \int_{\Phi} [\mu w_i a_{ij}(\varphi) q_{ij}(\varphi) - w_i l_{ij}(q_{ij}(\varphi), \varphi)] dG_i(\varphi) \text{ for all } i; \tag{B.10}$$

(iii) wages such that

$$w_H = P^*_{HF}/(\mu L^*_{HF}), \tag{B.11}$$

$$w_F = 1; \tag{B.12}$$

(iv) goods prices such that

$$p_{Hj}(\varphi) = \begin{cases} \bar{p}_{Hj}(\varphi) & \text{, if } \mu a_{Hj}(\varphi) q_{Hj}(\varphi) \geq l_{Hj}(q_{Hj}(\varphi), \varphi), \\ \infty & \text{, otherwise,} \end{cases} \tag{B.13}$$

$$p_{Fj}(\varphi) = p^*_{Fj}(\varphi), \tag{B.14}$$

and

$$P^{1-\sigma}_{HH} = \int_{\Phi} N_H [(1 + t_{HH}(\varphi)) p_{HH}(\varphi)]^{1-\sigma} dG_H(\varphi), \tag{B.15}$$

$$P_{HF} = P^*_{HF}, \tag{B.16}$$

$$P_{FH} = \tilde{P}^*_{FH}/\eta^*, \tag{B.17}$$

$$P_{FF} = P^*_{FF}; \tag{B.18}$$

(v) taxes such that

$$s_{Hj}(\varphi) = s^*_{Hj}, \text{ for all } \varphi \text{ and for } j = H, F, \tag{B.19}$$

$$t_{HH}(\varphi) = t^*_{HH}, \text{ for all } \varphi, \tag{B.20}$$

$$t_{FH}(\varphi) = t^*_{FH}(\varphi), \text{ if } \varphi \in \Phi_{FH}, \tag{B.21}$$

$$t_{FH}(\varphi) \geq t^*_{FH}, \text{ otherwise}, \tag{B.22}$$

with $s^*_{Hj} = 0$ for $j = H, F$, $t^*_{HH} = 0$, $t^*_{FH}(\varphi)$ given by equation (20), and t^*_{FH} given by equation (21); and (vi) a lump-sum transfer such that

$$T_H = \sum_{j=H,F}[\int_\Phi N_j t_{jH}(\varphi) p_{jH}(\varphi) q_{jH}(\varphi) dG_j(\varphi) - \int_\Phi N_H s_{Hj}(\varphi) p_{Hj}(\varphi) q_{Hj}(\varphi) dG_H(\varphi)]. \tag{B.23}$$

We now check that the previous allocation and prices satisfy the equilibrium conditions (1)-(7).

First, consider conditions (5) and (7). Since they are equivalent to equations (B.10) and (B.23), respectively, they trivially satisfied by construction.

Second, consider condition (2). For goods that are produced by home firms, they are equivalent to equations (B.13). So, it is again trivially satisfied. For goods that are produced by foreign firms, condition (2) derives from equations (A.4), (A.8), (B.8), and (B.14).

Third, consider condition (4). For goods locally sold by home firms, it directly derives from condition (B.15). For goods exported by home firms, one can use the same argument as in the proof of Lemma 4 to show that

$$\begin{aligned} L^*_{HF} &= \lambda_{HF}(Q^*_{HF})^{-1/\sigma}/\mu, \\ \lambda_{HF} &= [N_H \int_{\Phi_{HF}} (\mu a_{HF}(\varphi))^{1-\sigma} dG_H(\varphi)]^{1/(1-\sigma)} (Q^*_{HF})^{1/\sigma}, \end{aligned} \tag{B.24}$$

which imply

$$L^*_{HF} = [N_H \int_{\Phi_{HF}} (\mu a_{HF}(\varphi))^{1-\sigma} dG_H(\varphi)]^{1/(1-\sigma)}/\mu. \tag{B.25}$$

Combining the previous expression with equation (B.11), we get

$$P^*_{HF} = [N_H \int_{\Phi_{HF}} (\mu w_H a_{HF}(\varphi))^{1-\sigma} dG_H(\varphi)]^{1/(1-\sigma)}.$$

Condition (4) then derives from the previous equation and equations (B.13) and (B.19). Next, consider goods locally sold by foreign firms. For those, condition (4) derives from equations (A.4), (A.5), (B.8), (B.14), and (B.18). Finally, for goods exported by foreign firms, we already know from equation (B.6) that

$$\frac{(1+t^*_{FH})/(1+t^*_{HH})}{(1+s^*_{HF})/(1+s^*_{HH})} = \frac{\tilde{P}^*_{FH}}{\eta^*(\int_{\Phi_{FH}} N_F(\min\{1, \frac{\theta_{FH}(\varphi)}{\chi_{FH}}\}\mu a_{FH}(\varphi))^{1-\sigma} dG_F(\varphi))^{1/(1-\sigma)}}.$$

Combining the previous expression with equations (20), (B.21), (B.22), and using the fact that

$s^*_{Hj} = 0$ for $j = H, F$ and $t^*_{HH} = 0$, we then get

$$\left(\int_{\Phi_{FH}} N_F((1 + t_{FH}(\varphi))\mu a_{FH}(\varphi))^{1-\sigma} dG_F(\varphi)\right)^{1/(1-\sigma)} = \tilde{P}^*_{FH}/\eta^*.$$

Condition (4) derives from the previous expression and equations (A.8), (B.14), and (B.17).

Fourth, consider condition (1). For goods locally sold by foreign firms, condition (1) directly derives from equations (A.3), (B.8), (B.9), and (B.18). For goods exported by home firms, note that by equations (B.8), (B.11), (B.13), (B.9), and (B.19) with $s^*_{HF} = 0$, condition (1) holds if

$$(\mu a_{HF}(\varphi)/\lambda_{HF})^{-\sigma} = [P^*_{HF} a_{HF}(\varphi)/(L^*_{HF} P_{HF})]^{-\sigma} Q^*_{HF}.$$

Since the previous equation follows from equations (B.24) and (B.16), condition (1) must hold for goods exported by home firms. We can use a similar logic to analyze micro-level quantities sold at Home. Given equations (B.8), (B.11), (B.13), (B.9), and (B.20) with $t^*_{HH} = 0$, condition (1) holds for goods locally sold by home firms if

$$(\mu a_{HH}(\varphi)/\lambda_{HH})^{-\sigma} = (P^*_{HF} a_{HH}(\varphi)/(L^*_{HF} P_{HH}))^{-\sigma} Q^*_{HH}. \tag{B.26}$$

Using the same argument as in the proof of Lemma 4, one can also show that

$$L^*_{HH} = \lambda_{HH} (Q^*_{HH})^{-1/\sigma}/\mu.$$

Hence, condition (B.26) is equivalent to

$$P^*_{HF}/P_{HH} = L^*_{HF}/L^*_{HH}, \tag{B.27}$$

which follows from equations (B.2), (B.13), (B.15), (B.16), (B.19), (B.20), as well as the fact that condition (4) holds for goods exported by home firms. Lastly, consider goods exported by foreign firms. Given equations (B.8), (B.9), (B.12), (B.14), (B.21), and (B.22), condition (1) holds if

$$\begin{aligned}
(\mu \chi_{FH} a_{FH}(\varphi))^{-\sigma} &= [(1 + t^*_{FH})\mu a_{FH}(\varphi)/P_{FH}]^{-\sigma} Q^*_{FH}, \text{ if } \varphi \in \Phi^u_{FH}, \\
f_{FH}(\varphi)/((\mu - 1)a_{FH}(\varphi)) &= [(1 + t^*_{FH})(\theta_{FH}(\varphi)/\chi_{FH})\mu a_{FH}(\varphi)/P_{FH}]^{-\sigma} Q^*_{FH}, \text{ if } \varphi \in \Phi^c_{FH},
\end{aligned}$$

Given the definition of $\theta_{FH}(\varphi)$, both conditions reduce to

$$1/\chi_{FH} = (Q^*_{FH})^{1/\sigma} P_{FH}/(1 + t^*_{FH}). \tag{B.28}$$

Using equations (13) and (14b), one can again use the same strategy as in the proof of Lemma 4 to

show that

$$1/\chi_{FH} = (Q^*_{FH})^{1/\sigma}\left(\int_{\Phi_{FH}} N_F(\mu(\min\{1, \frac{\theta_{FH}(\varphi)}{\chi_{FH}}\})a_{FH}(\varphi))^{1-\sigma} dG_F(\varphi)\right)^{1/(1-\sigma)}.$$

Since condition (4) holds for goods exported by foreign firms, we also know from equations (B.10), (B.21), and (B.22) that

$$P_{FH} = (1+t^*_{FH})\left(\int_{\Phi_{FH}} N_F(\mu(\min\{1, \frac{\theta_{FH}(\varphi)}{\chi_{FH}}\})a_{FH}(\varphi))^{1-\sigma} dG_F(\varphi)\right)^{1/(1-\sigma)}.$$

Equation (B.28) derives from the two previous observations. Hence, condition (1) must also hold for goods exported by foreign firms.

Fifth, consider the labor market condition (6). Abroad, this condition derives from equations (11c), (A.6), (B.8), and (B.9). At home, constraint (14c) implies

$$L_H(Q^*_{HH}, Q^*_{HF}) = L_H. \tag{B.29}$$

Condition (6) then derives from the definition of $L_H(Q_{HH}, Q_{HF})$ and equations (B.8), (B.9), and (B.29).

Finally, consider condition (3). Abroad, it is trivially satisfied by construction. At Home, we know from equation (15) that at the desired allocation

$$U^*_{FH}/U^*_{HH} = \frac{1}{\eta^*}((L^*_{HF}\tilde{P}^*_{FH})/(L^*_{HH}P^*_{HF})). \tag{B.30}$$

Equations (B.27) and (B.30) imply

$$U^*_{FH}/U^*_{HH} = \frac{1}{\eta^*}(\tilde{P}^*_{FH}/P_{HH}).$$

By equation (B.17), we then get

$$U^*_{FH}/U^*_{HH} = P_{FH}/P_{HH}. \tag{B.31}$$

At the desired allocation, constraint (14b) also implies

$$\tilde{P}^*_{FH}Q^*_{FH} = P^*_{HF}Q^*_{HF},$$

and in turn, using equation (B.9),

$$P_{HH}Q_{HH} + \tilde{P}^*_{FH}Q_{FH} = P_{HH}Q_{HH} + P^*_{HF}Q_{HF}. \tag{B.32}$$

Since conditions (1) and (4) hold for goods sold by home firms at home and abroad, we know that

$$P_{HH}Q_{HH} = N_H \int_\Phi (1 + t_{HH}(\varphi)) p_{HH}(\varphi) q_{HH}(\varphi) dG_H(\varphi),$$
$$P_{HF}Q_{HF} = N_H \int_\Phi p_{HF}(\varphi) q_{HF}(\varphi) dG_H(\varphi).$$

Combining this observation with equations (B.13), (B.16), (B.19), and (B.20) we get

$$P_{HH}Q_{HH} + P^*_{HF}Q_{HF} = N_H w_H \Big(\int_\Phi \mu a_{HH}(\varphi) q_{HH}(\varphi) dG_H(\varphi) + \int_\Phi \mu a_{HF}(\varphi) q_{HF}(\varphi) dG_H(\varphi) \Big).$$

Since condition (5) holds at home, this can be rearranged as

$$P_{HH}Q_{HH} + P^*_{HF}Q_{HF} = N_H w_H \sum\nolimits_{j=H,F} \int_\Phi l_{Hj}(q_{Hj}(\varphi), \varphi) dG_H(\varphi) + \Pi_H.$$

Since condition (6) also holds, we then get

$$P_{HH}Q_{HH} + P^*_{HF}Q_{HF} = w_H L_H + \Pi_H.$$

Combining this expression with equation (B.32), we obtain

$$P_{HH}Q_{HH} + \tilde{P}^*_{FH}Q_{FH} = w_H L_H + \Pi_H. \tag{B.33}$$

Since conditions (1) and (4) hold for goods sold by foreign firms at home, we must have

$$P_{FH}Q_{FH} = N_F \int_\Phi (1 + t_{FH}(\varphi)) p_{FH}(\varphi) q_{FH}(\varphi) dG_H(\varphi),$$

which, using equation (B.14), leads to

$$P_{FH}Q_{FH} = N_F \int_\Phi \mu (1 + t_{FH}(\varphi)) a_{FH}(\varphi) q_{FH}(\varphi) dG_F(\varphi). \tag{B.34}$$

From the definition of $\tilde{P}^*_{FH}$ as well as equations (B.9), we also know that

$$\tilde{P}^*_{FH}Q_{FH} = N_F \int_\Phi \mu a_{FH}(\varphi) q_{FH}(\varphi) dG_F(\varphi). \tag{B.35}$$

Combining equation (B.23) with equations (B.33), (B.34), and (B.35), we finally obtain

$$P_{HH}Q_{HH} + P_{FH}Q_{FH} = w_H L_H + \Pi_H + T_H. \tag{B.36}$$

Condition (3) at home derives from equations (B.31) and (B.36). □