Linear Power Flow Characterization of DC Power Distribution Systems for More Electric Aircraft Optimization

9- Electrical Systems and Components for Sea, Undersea, Air, and Space Vehicles

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ABSTRACT

The increased utilization of electrical energy in future aviation requires an efficient method to perform load flow for optimizing on-board operating conditions. Load flow models of electrical networks are complex to solve and requires iterative methods to obtain a solution due to the non-linear relationship between voltages and currents. This paper will address the complexity of load flow for DC systems and will present a linear model that does not require to be solved iteratively and can be included in Mixed Integer Linear Programming MILP formulations to allow operational metrics optimization such as loss minimization and reliability maximization while converging to optimality. Load flow results have shown that the error margin is considerably small.

1. INTRODUCTION

The MEA power distribution system allows delivery of electrical energy produced at power sources to the load terminals at a specific voltage level and format (AC or DC). Several network configurations are possible depending on the power conversion requirements of the loads [1]–[4]. In all cases, optimal power flow is essential to increase overall efficiency and reliability [5]–[7] on board. In order to assess optimal power flow in the aircraft electrical distribution, both an efficient optimization formulation and a power flow model are mandatory.

Network design [8] has been extensively used in designing and reconfiguring systems when an operational metric (or metrics) needs to be optimized. Network design uses optimal path formulations to satisfy load demands [9] and it can express a network model with linear balance equations. In the case of power distribution, balance of power is expressed on every node i as a flow balance between incoming P_{ji} and outgoing P_{ij} powers from- or to- all buses j connected to it, as expressed in (1).

$$\sum_j P_{ij} - \sum_j P_{ji} = P_i \qquad \forall i \tag{1}$$

This balance is equal to P_i which, depending on the node type, is 0 for step-buses (non-generation, non-load), generated power $+P_i$, or consumed power $-P_i$. In most cases, node types are known and $\pm P_i$ in the right-hand side of (1) is known a priori. P_{ij} and P_{ji} in (1) can be determined by solving a network design formulation which includes a set of objectives, connectivity rules, and other reliability constraints [10]. However, voltage and currents need to be explicitly introduced in (1) [11] to obtain a complete load flow model that allows determination of losses, efficiencies, and other operational parameters, while maintaining the advantages of solving a linear network problem. Nevertheless, voltages and currents in DC systems have non-linear relationships that could impede the utilization of efficient network design techniques. This paper addresses the problem of introducing a load flow model in a network design formulation that can provide accurate load flow results while exploiting the optimization advantages of a network design for optimal power assessment with Mixed Integer Linear Programming formulations (MILP).

2. POWER FLOW FOR DC SYSTEMS

The voltages and currents in the system's nodes follow the relations in (2), where V is the voltage vector, I is the current vector, $\mathbf{R}$ is the resistance matrix, and $\mathbf{G} = \mathbf{R}^{-1}$ is the conductance matrix. For a specific bus i, voltage can be written as $V_i = \sum_{m=1}^{n} \mathbf{R}_{im} I_m$ or $I_i = \sum_{m=1}^{n} \mathbf{G}_{im} V_m$.

$$V = \mathbf{R}I, \qquad I = \mathbf{G}V \tag{2}$$

Node currents will depend on the different types of connected components [12], which can be modelled as constant conductance g^0, constant current i^0, or constant power p^0 (the knot superscript denotes connection between node and ground). Considering that power in node i is $P_i = V_i I_i$, current I in (2) can be rewritten as [13]:

$$I = G^{\mathbf{0}} \cdot V + \frac{P^0}{V} + I^0 = \mathbf{G}V \tag{3}$$

According to (3), the total current for node i is,

$$I_i = G_i^0 V_i + \frac{P_i^0}{V_i} + I_i^0 = \sum_{j=1}^{n} G_{ij} V_j \tag{4}$$

The total number of nodes (buses) is n. Some nodes are considered as step-buses ($I_i = 0$) while others have devices that perform as constant impedance (G_i^0), constant power (P_i^0), or constant current (I_i^0). Equation (5) allows direct calculation of current, but in order to apply (4) in (1), currents must be expressed as power flows. If current I_i in (4) is multiplied by node voltage V_i, the total node i power can be written as in (5)

$$P_i = G_i^0 V_i V_i + P_i^0 + V_i I_i^0 = V_i \sum_j G_{ij} V_j \tag{5}$$

For DC networks, (5) is the general form of the network balance that can be introduced in (1). For a constant impedance load G_i^0 connected to node i, $P_i = G_i^0 V_i V_i$. If a constant power device is connected, $P_i = \pm P_i^0$ depending if it is a power source or load device. For a step bus, $P_i = 0$ and $V_i \sum_j G_{ij} V_j = 0$. Power sources can be constant voltage nodes in some cases (slack buses). In order to solve (5), a combination of known values for V and I are used. Having solved (5), losses can be calculated by considering connection j to i current to be $I_{ji} = \frac{V_j - V_i}{Z_{ji}} = I_j - I_i$, and its power loss $P_{ji} = V_{ji} I_{ji} = G_{ij} V_{ji}^2$. However, (5) is non-linear due to the bilinear terms involving voltage products $V_i V_j$ and cannot be used directly in (1); these equations are quadratic in V and call for iterative solvers [13]. A picture depicting bilinear terms in Fig. 1 will suggest that in the range [0.9 1.1] (expected operation voltages in MEA), voltage products can be approximated either with McCormick envelopes (MCE), or Piecewise Linear Functions of Two Variables (PWL2). With these approximations, (5) can be introduced in (1) and network design formulations can be used.

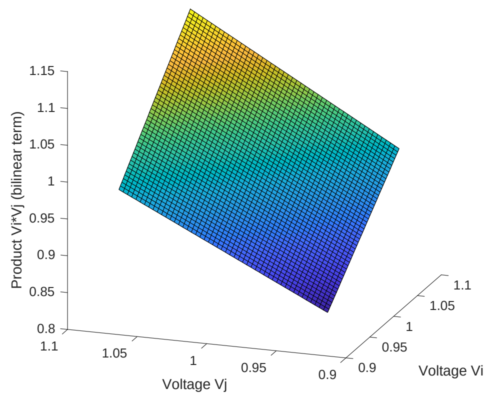

Fig. 1 Bilinear terms in nodal power flow equation for DC systems

McCormick envelopes will provide higher accuracies when the voltage range is shorter, while PWL2 will be more suitable when the voltage range is larger. However, the number of variables is different with both techniques.

3. NETWORK DESIGN MILP OPTIMIZATION

Several assessments can be exercised for optimal power, including:

- Minimum-loss optimization for a given demand, i.e. min $\sum_{i,j} G_{ij} V_{ji}^2$ or min $\sum_{i,j} R_{ij} I_{ji}^2$
- Optimal power flow [14], i.e. minimize cost of power supplied by generators min $\sum_G P_G$
- Optimal transmission switching, i.e. minimize the amount of losses by reconfiguring the system's topology

A network design MILP based optimization is chosen due to its strict convergence to optimality and its flexibility to accommodate design [15] and reconfiguration problems [16]. Let a set of connections A and components N in a network be represented by a graph $\mathcal{G}(A, N)$, where connections y_{ij} have power flow P_{ij} transferred from component i to j. An operational metric of this network can be optimized by defining an optimization objective, i.e. minimization of losses by selecting a number of connections y_{ij} to reconfigure the system, as shown in (6a).

$$\min_{V,I,y} \sum_{(i,j)\in A} \left(y_{ij} + R_{ij} I_{ij}^2\right) \qquad (6a)$$

The constraint of power balance (1) and load flow equations (5) are included in a single equality constraint:

$$\text{subject to} \sum_{j|(i,j)\in A} P_{ij} - \sum_{j|(j,i)\in A} P_{ji} = G_i^0 V_i V_i + P_i^0 + V_i I_i^0 \qquad \forall i \in N \qquad (6b)$$

The selection of connections y_{ij} to reconfigure the system can be performed with [17]:

$$P_{ij} \le G_{ij}\left(V_i - V_j\right) + M\left(1 - y_{ij}\right) \qquad \forall (i,j) \in A \qquad (6c)$$

$$P_{ij} \ge G_{ij}\left(V_i - V_j\right) - M\left(1 - y_{ij}\right) \qquad \forall (i,j) \in A \qquad (6d)$$

Where M is a big value used in MILP to enforce (6c)-(6d) only when y_{ij} is selected. Constraint (6e) limits the power in connections, (6f) defines integrality on y_{ij}, (6g) are the voltage constraints, and (6h) limits currents.

$$P_{MIN} \le P_{ij} \le P_{MAX} \qquad \forall (i,j) \in A \qquad (6e)$$

$$y_{ij} \in \{0,1\} \qquad \forall (i,j) \in A \qquad (6f)$$

$$V_0 = V_{REF} \qquad V_{MIN} \le V_i \le V_{MAX} \qquad \forall i \in N \qquad (6g)$$

$$\left|I_{ij}\right| \le I_{MAX} y_{ij} \qquad \forall (i,j) \in A \qquad (6h)$$

If P^0 in (6b) is zero (no constant power devices), (6b) can be linear if divided by V_i and no further approximations were necessary (some power electronic converters [18] and loads [11] can be modelled as constant current). Nevertheless, approximations are necessary because constant power devices are present in MEA. The aim of (6a)-(6h) is to reconfigure the system (by selecting a group of y_{ij}) such that losses are minimized and a load flow for the DC system is performed within the optimization. Several operating constraints and failure-scenarios can also be included.

4. MILP OPTIMIZATION AND STUDY CASE

The formulation (6a)-(6h) will be used to minimize losses on the MEA electrical network shown in Fig. 1(a) for different operational scenarios. In Fig. 1(b), optimum during normal operation is shown. In the case no load can be shed (all loads are critical), a converter disconnection will have an important impact on losses increase. This is the case shown in Fig.3 (1) where failure in converter C1 led to a considerable increase in generation losses. In the case of G1 failure, the power system is able to switch to G2 and continue to supply critical loads while temporarily disconnecting non-critical ones as shown in Fig. 3 (4); this requires one generator to work at a 0.66 loading capacity compared to normal case in Fig. 1. However, converters will work at 0.50 loading capacity during normal conditions but will be ready to handle all critical loads if a failure in one of them occurs.

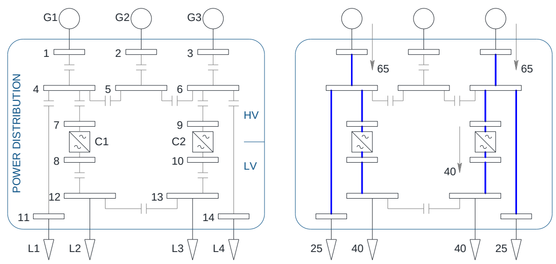

Fig. 2 Power system for MEA (a), all loads are critical; optimum power in normal conditions (b)

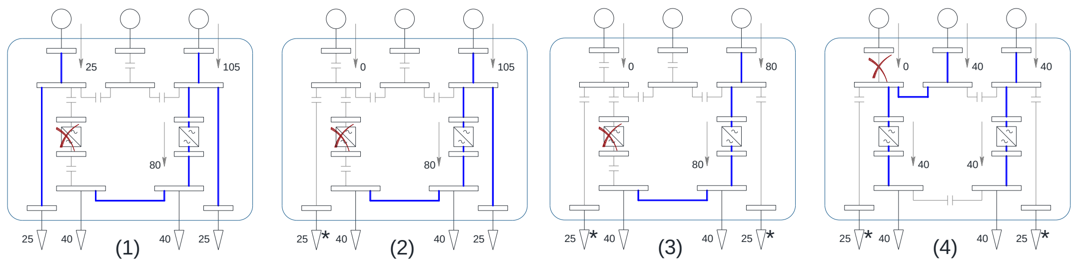

Fig. 3 Optimal power assessment of MEA power system under failure scenario; (1) failure in C1; (2) failure in C1 and shedding of B11; (3) failure in C1 and shedding of loads in B11 and B14; (4) failure in G1 and shedding of loads in B11 and B14. Loads with * can be shed

The losses for configurations in Fig. 2 and 3 are tabulated in Table 1. Note that the difference between non-linear and MILP approximations are not higher than 1% when single McCormick approximation and piecewise linear functions with 21 divisions were used. More accurate approximations can increase the load flow solution accuracy.

TABLE 1: ACTIVE LOSSES FOR THE TOPOLOGY CONFIGURATIONS SHOWN IN *FIG. 2* AND *FIG. 3*.

System study		Active Losses in kW				
		Normal	**1**	**2**	**3**	**4**
DC system	Non-linear (iterative solution)	0.727	1.764	1.728	1.570	0.589
	MILP approximation	0.724	1.759	1.723	1.565	0.586
	Difference (%)	-0.41	-0.28	-0.29	-0.32	-0.51

The optimization approach in (8a)-(8i) can also assist in several design tasks such as conductor selection and component location. These applications will be investigated in the future.

5. CONCLUSION

Non-linearities of DC load flow models can be managed with linear approximations via McCormick envelopes or Piecewise linear functions approximations. The advantage of using such approximations is the flexibility to use network design formulations and MILP techniques to optimize the MEA power distribution system. The load flow results of several topologies optimized with MILP based techniques have been compared to the non-linear solution and the accuracy is less that 1% in all cases. Future investigations will explore accuracy improvements and the use of MILP base network design in other on-board MEA applications.

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