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Systemic risk in banking, fire sales, and macroeconomic disasters ☆

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ABSTRACT

We develop a dynamic computational network model of the banking system where fire sales provide the amplification mechanism of financial shocks. Each period a finite number of banks offers a large, but finite, number of loans to households. Banks with excess liquidity also offer loans to other banks with insufficient liquidity. Thus, each period an interbank loan market is endogenously formed. Bank assets are hit by idiosyncratic shocks drawn from a thin tailed distribution. The uneven distribution of shocks across banks implies that each period there are banks that become insolvent. If insolvent banks happen also to be heavily indebted to other banks, their liquidation can trigger other bank failures. We find that the distribution across time of the growth rate of banking assets has a 'fat left tail' that corresponds to rare economic disasters. We also find that the distribution of initial shocks is not a perfect predictor of economic activity; that is some of the uncertainty is endogenous and related to the structure of the interbank network.

"Because of fire sales, risk becomes systemic." (Shleifer and Vishny, 2011)

1. Introduction

In this paper we consider how the banking system can amplify negative shocks to create extreme negative growth rates, which we term macroeconomic "diasasters". Our focus is on the potential for fire sales, triggered by banking insolvencies, to act as an amplification mechanism following initial negative shocks. In a highly connected interbank market, bank failures, together with the effects of fire sales, can create a cascade effect, which can ultimately have a substantial downward effect on aggregate credit and thus growth.

Two prominent periods where such negative macroeconomic events have occurred are the Great Depression of the 1930s and the more recent Great Recession. A common feature of these periods has been the great number of bank insolvencies. Since its

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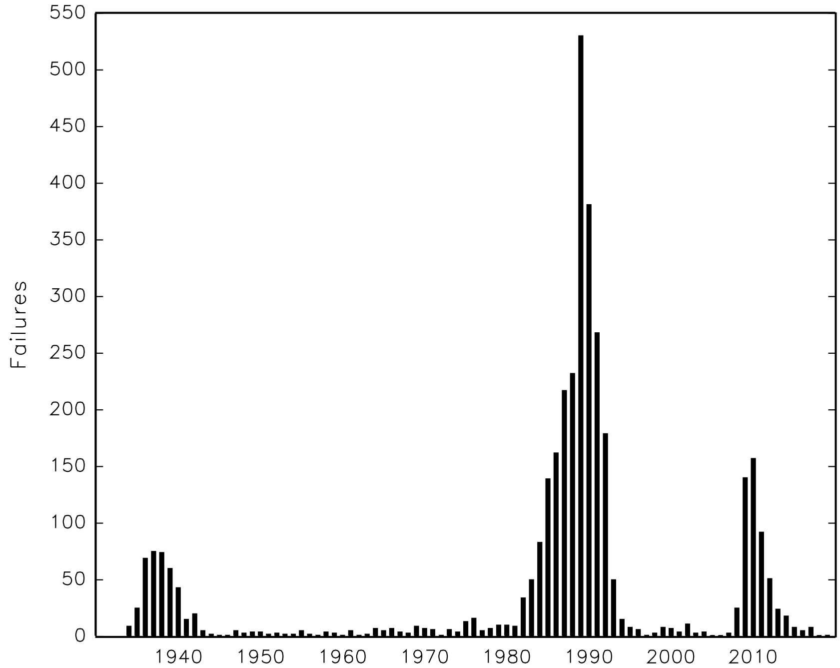

Fig. 1. US bank failures, annual 1934-2019.

Source: Board of Governors of the Federal Reserve System

establishment in 1933, the Federal Deposit Insurance Corporation has been keeping a record of all bank failures in the U.S. banking system. Fig. 1 shows that both macroeconomic disasters were associated with deep banking crises.[1,2]

The recent empirical study by Schularick and Taylor (2012) suggests that there is a causal link that accounts for this association. They find that the level of aggregate credit in the economy is, in general, a good predictor of macroeconomic performance and, in particular, of rare negative macroeconomic events.[3] Fig. 2 shows the time series of total business loans offered by US commercial banks, along with the NBER recession dates. It is clear that economic downturns are associated with a decline in the provision of business loans.

Under the supposition that the portfolio of assets of the banking system is well diversified, the challenge is to identify how shocks on those assets are amplified to eventually have such a detrimental effect on aggregate economic activity.

There are good reasons to believe that the amplification of shocks takes place within the banking system. There is a fast growing literature that applies network theory to the study of systemic risk in financial systems.[4] Banks are interlinked through a complex web of financial contracts such that during periods of distress the inability of one bank to meet its obligations to other banks can cause a cascade of failures. But, as Cifuentes et al. (2005) have demonstrated, the cascades on their own cannot amplify the original shocks. Cascades affect the distribution of aggregate losses but not their size. However, when liquidations also force failed banks to sell their assets at 'fire sales' prices then the original shocks can be amplified. There is a lot of evidence that fire sales played a prominent role during both the Great Depression and the Great Recession. For example, Mitchener and Richardson (2019) provide evidence that demonstrates how fire sales amplified shocks within the interbank network during the Great Depression. The impact of fire sales during the Great Recession has been documented by Adrian and Shin (2010), Brunnermeier (2009), Gorton and Metrick (2012) and Shleifer and Vishny (2011).

In this paper, we develop a dynamic model of the banking system where fire sales provide the amplification mechanism of financial shocks. Each period a finite number of banks offers a large, but finite, number of business loans. Firms approach banks sequentially and choose banks randomly. Banks with excess liquidity also offer loans to other banks with insufficient liquidity. Thus, each period an interbank loan market is endogenously formed. Each period bank assets are hit by idiosyncratic shocks and given the large number of assets and periods these aggregate shocks are normally distributed across time. The uneven distribution of shocks across banks implies that each period there are banks that suffer losses. When losses exceed their equity banks are liquidated. If insolvent banks

[1] Fig. 1 does not account for the large number of bank failures that took place since the onset of the Great Depression but prior to the establishment of the Federal Reserve System in 1933.

[2] Fig. 1 also accounts for the large number of failures during the 1980s Savings and Loans Crisis. Given that those institutions were exclusively specialized in mortgages the crisis did not have a strong impact on the macroeconomy.

[3] See, also, Giglio et al. (2016) for empirical evidence on the impact of systemic risk in financial markets on macroeconomic variables.

[4] See, for example, the reviews by Acemoglu et al. (2017b), Allen and Babus (2009), Bougheas and Kirman (2015), Jackson and Pernoud (2021), and Glasserman and Young (2016).

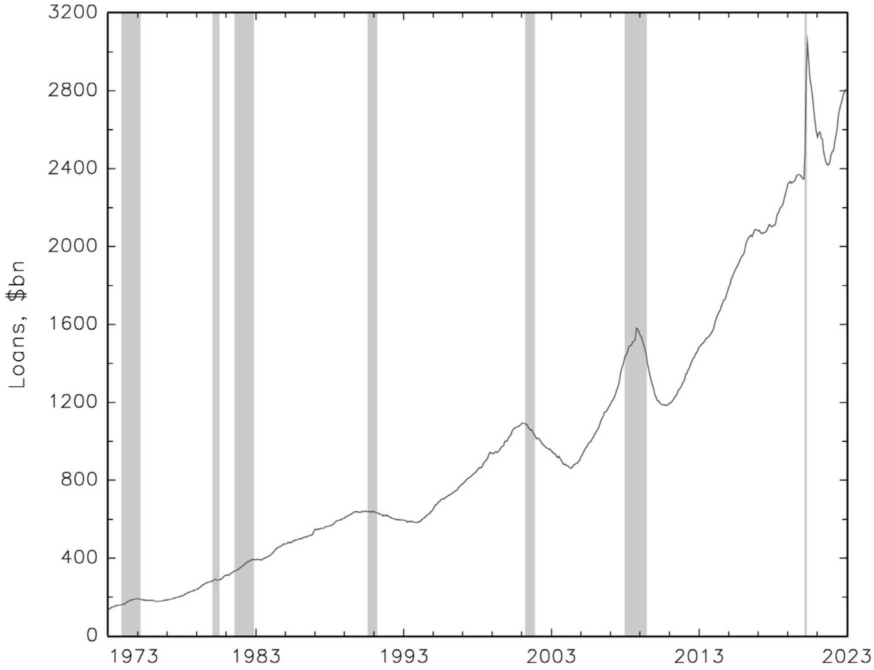

Fig. 2. Total US commercial and industrial bank loans, and NBER recession dates (shaded), monthly 1973M1-2022M12.
Source: Board of Governors of the Federal Reserve System

happen also to be heavily indebted to other banks, their liquidation can trigger other bank failures. The assets of the banking system are proportional to the level of aggregate credit in the economy which, in turn, is a proxy for aggregate economic activity.

We begin by analyzing a simple version of the model without fire sales. We show that this benchmark case behaves like a frictionless Real Business Cycle model where deviations of the growth rate of aggregate economic activity from its trend are asymptotically normally distributed. Thus, this final distribution replicates the distribution of initial aggregate shocks. In this simple version, each period the structure of the interbank network determines only the number of bank failures but shocks are not amplified. Then, we introduce fire sales that amplify shocks and as a result the new distribution has a fat left tail. The amplification is caused by the interaction of fire sales with network effects. As the number of bank insolvencies increase, asset values drop leading to more insolvencies and forcing further liquidations.

Thus, our model predicts an asymmetry between booms and busts that was first studied and empirically verified by Acemoglu and Scott (1997). The tail behavior of post-war US real GDP per capita growth can be seen in Fig. 3, which provides a quantile-quantile (Q-Q) plot of the growth quantiles against those from a standard normal distribution. On comparison with the reference line in Fig. 3, which plots the quantiles of a normal distribution with mean and variance calibrated to that of the growth data, non-normality in the growth rates is apparent, with clear evidence of a fat left tail.

There is a long line of research that exploits variants of the New Keynesian DSGE framework to demonstrate how financial frictions can account for both the persistence of economic shocks and their amplification.[5] However, there is a fundamental difference between the DSGE approach and ours. In the traditional DSGE models there is a one-to-one relationship between the magnitude of the initial shocks and the final impact of aggregate economic activity. Put differently, any information related to some early economic disturbance can be useful in predicting the future state of the economy. In contrast, such information would be a much weaker predictor in network models like the present one where shocks are amplified as they are transmitted along complex network paths. Some of the uncertainty in these network models is endogenous and directly related to the complexity of the structure of the network.

In the penultimate section of the paper we investigate the relationship between this inherent unpredictability implied by our model and the fat left tail in the growth rate distribution. The driving force of our amplification mechanism is directly related to the relationship between the distribution of idiosyncratic shocks across the banking system and the structure of the interbank network. We begin by showing that the fat left tail in the growth rate distribution is matched by a fat right tail in the distribution of total bankruptcies. Then, we show that even without fire sales the number of total bankruptcies is higher than the number of initial bankruptcies caused by the initial shocks. The additional bankruptcies are caused by the interconnectedness of the banking system through the interbank market. When we add fire sales in the model there is a significant increase in additional bankruptcies caused by the above mentioned interaction between fire sales and network effects.

Having shown that when boosted by fire sales the distribution of bank insolvencies has characteristics that match those of the distribution of the growth rate of economic activity, our next step is to understand the exact mechanism by which shocks are amplified

[5] See Brunnermeier et al. (2013) for a comprehensive review of the relevant literature.

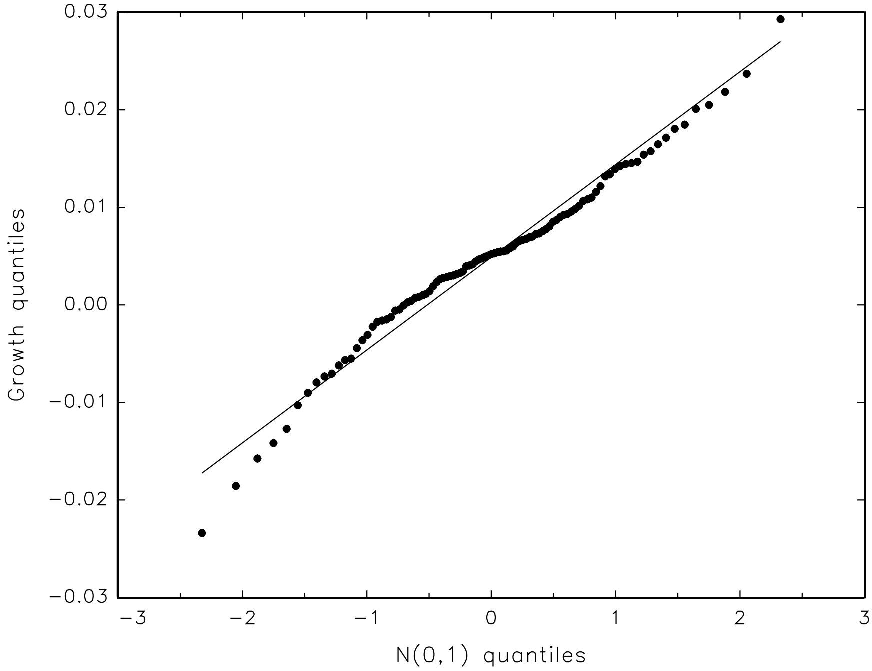

Fig. 3. Q-Q plots: ••• US real GDP per capita growth, quarterly 1947Q2-2022Q4 [Source: Board of Governors of the Federal Reserve System]; —— Normal distribution with mean and variance calibrated to growth data.

and generate the fat left tail. Thus, we concentrate on those periods where the growth rate falls within the 10% left tail of the growth distribution. We have identified two dominant patterns. In a large number of cases, the majority of the bankruptcies were purely caused by a large number of negative initial shocks across banks. In the other cases there were a large number of additional bankruptcies due to contagion caused by network effects. Taking a closer look at those cases we find that almost all these banks that became insolvent due to network effects had offered loans to the two banks hit by the most severe shocks.

The above results are very intuitive. Macroeconomic disasters happen either when a large number of banks are hit by negative shocks or when the banks that fail are heavily indebted to other banks, thus causing a cascade of bank failures. The total number of bank failures is further exaggerated by the trigger of fire sales that depress the value of banking assets leading to new rounds of bank failures. The decline in the value of bank assets will limit the ability of the banking system to offer new loans in later periods. Therefore, it is possible that the overall economy is in a relatively good state but a few banks suffer losses and these are also banks with large financial obligations to the rest of the banking system ('too big to fail'). In this scenario it would be impossible to predict the ensuing macroeconomic downturn from the initial state of the economy.

1.1. *Related literature*

A fast growing number of studies investigate the microeconomic foundations of macroeconomic fluctuations.[6] Their objective is to identify the mechanism by which idiosyncratic shocks in large economies get amplified producing the observed booms and busts at the aggregate level. A large branch of this literature considers the cause of the amplification mechanism to be feedback effects arising from shocks that travel along network links.

One group of papers focuses on production networks with the majority of papers analyzing the distribution of shocks across input-output connections. Theoretical contributions include Acemoglu et al. (2012, 2017a), Anthonisen (2016), Baqaee (2018), Baqaee and Farhi (2019), Bigio and La'O (2020), Nirei (2006) and Taschereau-Dumouchel (2020) while empirical evidence is provided by Acemoglu et al. (2016) and Carvalho et al. (2021).[7] A small group of papers study the impact of financial frictions on production networks. In Bigio and La'O (2016) firms face a pledgeability constraint that limits their ability to finance inputs. In Altinoglu (2021), Battiston et al. (2007) and Delli Gatti et al. (2010), firms are linked through trade credit finance.

Our work belongs to an alternative research program that views financial networks as the place where shocks get amplified. In our model fat tails are caused by fire sales that amplify shocks in the interbank market, thus boosting the number of bank failures which, in turn, lead to the collapse of aggregate credit. The literature on systemic risk in financial networks is vast and above we referred the reader to some comprehensive reviews. Here, we focus on these works that go beyond the study of contagion, that is the

6 Gabaix (2011) is an early and influential paper and Acemoglu et al. (2017a) offers an extensive review of the literature.

7 For an early survey of related work, see Carvalho (2014).

transmission of shocks from one bank to another, to also look at the mechanism by which the shocks are amplified causing the fat tails in the distribution of macro aggregates.

More closely related to our work is a group of papers that explore how the structure of the financial network determines the degree by which fire sales amplify initial shocks. Cifuentes et al. (2005) use simulations to show that the total systemic losses depend, in addition to the structure of the interbank network, on the size of the initial shock, the elasticity of the demand for liquidity and the liquidity of the banking system as a whole.[8] The overall liquidity of the banking system also plays a crucial role in the theoretical work of Caballero and Simsek (2013). They study the equilibrium determination of liquidation prices in a network of banks located on a circle with each bank being uncertain about its exact position on the network. In Gai et al. (2011), the degree of concentration and the overall complexity of the financial system are important indicators for predicting systemic risk. Gai and Kapadia (2010) derive analytical solutions from an interbank network model where they allow for both idiosyncratic and aggregate shocks. They reach a similar conclusion to our work, namely, that various indistinguishable initial shocks can have drastically different aggregate consequences. Where we diverge from these studies is that our banking network is endogenously formed and shocks in our model inflict individual assets rather than having an aggregate impact on a bank's balance sheet.[9]

Our paper also contributes to the branch of the macroeconomic literature that views financial market frictions as the main engine behind the amplification of shocks.[10] The majority of work in this area is carried out within the DSGE framework. Initially, the literature focused on the persistence of shocks but the financial accelerator in Bernanke et al. (1999) provided the original amplification mechanism. There have been some doubts about whether financial frictions in these models can generate sufficiently strong effects at the aggregate level (Ascari et al., 2015; Cordoba and Ripoli, 2004; Kocherlakota, 2000), however, such concerns have been put to rest by a number of studies (Brunnermeier and Sannikov, 2014; Cúrdia et al., 2014; He and Krishnamurthy, 2012; Mendoza, 2010; Quadrini, 2011). Nevertheless, there are some fundamental differences between the DSGE and the network approaches to the study of macroeconomic fluctuations. In DSGE models shocks are amplified by frictions affecting directly either firm or household balance sheets. In network models the main amplification mechanism is generated within the financial system through financial contracts that connect either firms (trade credit) or banks (interbank market). We have argued that for the Great Depression and the Great Recession the latter approach is more plausible. But the two lines of research also reach drastically different conclusions about the predictability of initial shocks for subsequent macroeconomic performance. One objective of the present study is to clarify why this is the case.

Our work is also related to the financial economics literature that examines the macroeconomic impact of fire sales resulting from a drastic drop in aggregate liquidity.[11] Shleifer and Vishny (2011) provide a broad overview of the literature on fire sales but here we concentrate on those works related to the macroeconomy. The majority of this literature focuses on collapses in the values of corporate assets. For example, Kiyotaki and Moore (1997), in a dynamic general equilibrium model, show how feedback effects from changes in the prices of asset values (and, thus, collateral values) affect the ability of firms to raise new capital, which can in turn give rise to endogenous cycles. This feedback effect from collateral values to asset prices is asymmetric during cycles. This is because of non-linearities caused by bankruptcies. Thus, although leverage can cause both bubbles and crises, its effects during downturns are more pronounced as the demand for liquidity is significantly boosted by insolvent firms. Adrian and Shin (2014), Aymanns and Farmer (2015), and Fostel and Geanakoplos (2008) study theoretically the impact of leverage on economic cycles while empirical support for the procyclicality of leverage is offered by Halling et al. (2016).[12]

Lastly, there is also a branch of the literature on fire sales, more closely related to this work, that studies the collapse in the values of banking assets. For example, Diamond and Rajan (2005) show how liquidity shortages and solvency problems interact leading to contagion across the banking system. In Diamond and Rajan (2011) feedback effects are bi-directional. As in the rest of the literature on fire sales, the asset liquidation of those borrowers unable to meet their obligations to the lenders leads to fire sales. However, as prices drop the market for those assets freezes as lenders are unwilling to accept them as collateral, which in turn curtails the provision of new credit putting further pressure on distressed borrowers. In Acharya et al. (2011) banks during downturns are hoarding liquidity so that they can purchase assets of other failing banks at fire sales prices. This choice of liquidity policy is countercyclical leading to low levels of liquidity during booms and too much liquidity during recessions.

We organize the rest of the paper as follows: in Section 2 we present a very simple theoretical model to clarify the role of fire sales in generating fat tails. In Section 3, we build the computational model and analyze the case without fire sales, and in Section 4 we introduce fire sales. In Section 5 we provide some further discussion and extensions on our work, and we offer concluding comments in Section 6.

2. Fire sales and systemic risk: preliminary results

Before we present the more general computational model, we begin by demonstrating the link between fire sales and the distribution of aggregate shocks in a simple version of the Caballero and Simsek (2013) model. The aim of this exercise is to show, firstly, that

[8] Cecchetti et al. (2016) extend Cifuentes et al. (2005) by considering the resilience of alternative network topologies and by allowing for multiple illiquid assets.

[9] See, also, Gurgone et al. (2018) and Van der Hoog and Dawid (2019) who develop full scale agent-based macro models where shocks are amplified through the interactions between banks, embedded in an interbank network, and their clients.

[10] For a comprehensive review, see Brunnermeier et al. (2013).

[11] For some influential earlier work on the relationship between market liquidity and fire sales, see Shleifer and Vishny (1992) and Allen and Gale (1994).

[12] Other papers that study the relationship between fire sales on corporate assets and the macroeconomy include Brunnermeier and Sannikov (2014), Gale and Gottardi (2015) and Lorenzoni (2008).

without fire sales there is no amplification mechanism, and, secondly, that when the liquidation value of assets is inversely related to the number of banks that fail, the distribution of aggregate shocks exhibits a fat left tail.[13] There are M banks located on a circle. A typical bank b_j ($j = 1, ..., M$) has an obligation of B units of the only good in the economy to bank b_{j+1} and, thus, bank b_{j-1} owes the same amount to bank b_j.[14] On the asset side of the balance sheet of each bank there are loans to firms L, while on the liability side there are household deposits D and equity E. Thus, all banks are symmetric. The balance sheet identity implies that $L = D + E$. In what follows, we assume that when a bank is liquidated the claims of depositors have priority over those of other banks. Clearly, the claims of equityholders are residual. In all three cases below we assume that one of the banks, say b_j, has to write off a proportion z of its assets due to the inability of some of the bank's debtors to meet their obligations. Throughout, we make the following two simplifying assumptions: (a) $zL > E$, that is the bank hit by the initial shock becomes insolvent, and (b) that the losses can always be absorbed by the creditor bank without any effect on depositors. It will become clear below that relaxing the second assumption would only mean that depositors will also suffer some losses but will not affect our results concerning the distribution of aggregate losses.

2.1. Case (a): no fire sales

The liquidation of bank b_j at book value prices implies that bank b_{j+1} will have to write off $zL - E$ of its assets (the losses that bank b_j is unable to absorb). If $zL - E < E$ then bank b_{j+1} remains solvent and there are no more liquidations.[15] In contrast, if $zL - E > E$ then bank b_{j+1} also becomes insolvent and bank b_{j+2} will have to write off $zL - 2E$. Induction implies that the number of banks that fail is equal to the smallest integer k such that $(k + 1)E > zL$. Thus, banks b_j, b_{j+1}, ..., b_{j+k-1} fail, while bank b_{j+k} suffers losses but is still solvent.[16] The total losses of the banking system are equal to zL. Thus, as Shleifer and Vishny (2011) suggest, without fire sales there are no systemic losses.

2.2. Case (b): liquidation values are independent of the number of failed banks

Now we introduce fire sales, that is, the liquidation value of assets is below their book value. For the moment, we assume that these liquidation values do not depend on the number of banks that are liquidated. Let the liquidation value of assets be equal to a fraction γ (< 1) of their book value. Then, the liquidation of bank b_j means that the market value of its assets is equal to $\gamma(1 - z)L$, its losses are equal to $(1 - \gamma(1 - z))\, L$ and, thus, bank b_{j+1} will have to write off $(1 - \gamma(1 - z))\, L - E$ of its assets; that is, the losses not absorbed by the equity of bank b_j. If $(1 - \gamma(1 - z))\, L - E < E$ then bank b_{j+1} remains solvent and there are no more liquidations. In contrast, if $(1 - \gamma(1 - z))\, L - E > E$ then bank b_{j+1} also becomes insolvent. In that case, the liquidation value of its assets is equal to γL and bank b_{j+2} will have to write off $(1 - \gamma(1 - z))\, L - 2E + (1 - \gamma)L = (2(1 - \gamma) + \gamma z)\, L - 2E$. The last term on the left-hand side of the inequality is equal to the losses due to the effect of fire sales on the liquidation value of the assets of b_{j+1}. By repeating the process we find that the number of banks that fail is equal to the smallest integer $\hat{k}$ such that $\left(\left(\hat{k} + 1\right)(1 - \gamma) + \gamma z\right) L < \left(\hat{k} + 1\right) E$. To keep the exposition simple we ignore the integer problem. This implies that the equity of bank $b_{j+\hat{k}}$ is just sufficient to absorb the losses and therefore $\hat{k}$ is found by solving the equality $\left(\hat{k}(1 - \gamma) + \gamma z\right) L = \hat{k}E$ and obtaining[17]:

$$\hat{k} = \frac{\gamma z L}{E - (1 - \gamma)L}.$$

It is straightforward to show that $\hat{k}$ is decreasing in γ. The total losses of the banking system are equal to $\left(\hat{k}(1 - \gamma) + \gamma z\right) L$ which is equal to accumulated losses due to the effect of fire sales on the assets of liquidated banks plus the losses due to the initial shock. Given that $\hat{k}$ is linearly increasing in zL the total losses are also linearly increasing in the size of the initial shock.

2.3. Case (c): liquidation values are decreasing in the number of failed banks

In this third case, we allow the fire sales factor γ^* to depend on the number of failed banks, k^*. Let $\gamma^* = f(k^*)$, where $f(0) = 1$ and $f' < 0$. By repeating the same steps as in the case above we find that the number of banks that fail is equal to the smallest integer k^* such that $((k^* + 1)(1 - f(k^*)) + f(k^*)\, z)\, L < (k^* + 1)\, E$. As long as the rate of decrease of $f(k^*)$ is not too high (so that some banks survive) then by ignoring once more the integer problem we can find k^* as the solution of $(k^*(1 - f(k^*)) + f(k^*)\, z)\, L = k^* E$. By totally differentiating we get:

$$\frac{dk}{dz} = \frac{f(k^*)\, L}{E - (1 - f(k^*) + (k - z) f'(k^*))\, L}.$$

[13] In this simple model all banks have the same initial balance sheet, and therefore it does not matter whether liquidation values depend on the number of banks that fail or on the total value of assets offered for sale. In our more general computational model we assume that liquidation values depend on the total supply of such assets.

[14] Notice that $b_{M+1} = b_1$.

[15] Notice that the written expression reflects the symmetry across banks, that is $zL_j - E_j < E_{j+1} \Rightarrow zL - E < E$ since, for all j, $E_j = E_{j+1} = E$ and $L_j = L$.

[16] For simplicity, we have ignored the case where a bank's equity after write-offs is exactly equal to 0.

[17] It must be the case that $E > (1 - \gamma)L$, otherwise the whole banking system would be insolvent.

A necessary condition for the survival of some banks is that the denominator is positive. If this is not the case then the level of equity is not sufficient to cover the liquidation cost alone. Notice that $\frac{d^2k}{dz^2} < 0$; this means that losses due to the initial shock are eventually dominated by the losses due to liquidation costs.

Aggregate losses are given by $(k^*(1 - f(k^*)) + f(k^*)z)L$ and their derivative with respect to the initial shock per unit of assets, z, is given by

$$\left(\left(1 - f\left(k^*\right) - (k - z)f'\left(k^*\right)\right)\frac{dk}{dz} + f\left(k^*\right)\right)L > 0.$$

Differentiating once more with respect to z, we find that, as long as the direct effect dominates the indirect effect on $\frac{dk}{dz}$, aggregate losses are now increasing at an increasing rate with the initial shock, in contrast to linearly increasing losses under Case (b).[18]

In the following two sections we develop a more general dynamic model where each period the network of interbank loans is endogenously formed. In order to understand the exact role that fire sales play in the propagation and amplification of costs we begin by analyzing the model for the case where there are no fire sales and then we proceed in the following section with the analysis of the model with fire sales.

3. The computational model without fire sales

Time is discrete ($t = 1, ..., N$). In each period there are M banks. At the beginning of each period, banks hold reserves on the asset side of their balance sheets. Let V_t^j denote the level of reserves of bank $j \in \{1, ..., M\}$ in period t. On their liability side of their balance sheets banks hold equity E_t^j and household deposits HD_t^j. At the beginning of period t, the balance sheet identity is given by $V_t^j \equiv HD_t^j + E_t^j$. Let $\mathbf{V}_t = \sum_{j=1}^{M} V_t^j$ denote the total amount of reserves in the banking system. Let R_d denote the gross interest rate banks pay on their household deposits.

At the beginning of period $t = 1$, we set the initial conditions as follows. We fix the aggregate level of reserves, $\mathbf{V}_1$, and then we randomly distributed them among the M banks. On the liability side of each bank's balance sheet we set $E_1^j = eV_1^j$ for all $j = 1, ..., M$, with $0 < e < 1$ denoting the common capital requirement ratio, thus $HD_1^j = (1 - e)V_1^j$.[19]

3.1. *The interbank network and systemic risk*

It is convenient to divide each period into four stages. During the first stage banks offer loans to firms and the interbank market network is formed. In the second stage project returns are realized and balance sheets are readjusted. The bankruptcy resolution process takes place in stage 3 and in the final stage all accounts are cleared before the banking system progresses into the next period.

3.1.1. *Stage I: formation of interbank network*

Banks offer loans of unit size to firms. The loan repayment is R_L with probability p and with probability $1 - p$ the loan fails and yields nothing. Loan returns are independently distributed. Given that the number of loans are finite there is aggregate uncertainty at the levels of both the economy and each bank. However, the central limit theorem still applies and given the large number of loans and periods, the distribution of shocks across time is approximately normal.

Loans are financed sequentially. A firm chooses randomly one of the banks, say j, and asks for a loan. If bank j's reserves are larger than or equal to one it offers a loan. On the asset side of bank j's balance sheet 'loans to firms', FL_t^j, increase by one while reserves decrease by one. If bank j's reserves are less than one then bank j chooses randomly one of the other banks, say k, and asks it for a loan. If bank k's reserves are larger than or equal to one then it offers the loan to bank j. Then, the loan will be recorded on the liability side of the balance sheet of bank j as a deposit by bank k, BD_t^{jk}, and on the asset side of the balance sheet of bank k as a loan to bank j, BL_t^{kj}. If bank k's reserves are less than one then bank j will randomly choose another bank. When no other bank can make a loan then the process is completed and no other projects will be financed. Let $\mathbf{BD}_t^j = \sum_{k=1}^{M} BD_t^{jk}$ denote the total amount borrowed by bank j from other banks and $\mathbf{BL}_t^j = \sum_{k=1}^{M} BL_t^{jk}$ denote the total amount of loans offered by bank j to other banks. Clearly, $BD_t^{jj} = BL_t^{jj} = 0$. Moreover, if $BD_t^{jk} = BL_t^{kj} > 0$ then it follows directly that $BD_t^{kj} = BL_t^{jk} = 0$, given that if bank j has already been offered a loan by bank k, if then later bank k runs out of reserves it will not be able to borrow from bank j. Lastly, within the banking system, total lending must always be equal to total borrowing, that is $\sum_{j=1}^{M} \mathbf{BD}_t^j = \sum_{j=1}^{M} \mathbf{BL}_t^j$.

The network formation process described above generates structures dominated by large banks (in terms of initial assets) that, on average, act as creditors to smaller banks. This is because the distribution of initial reserves is widely dispersed across the banking system while loans are randomly allocated to banks. Thus, our networks are dissasortative as big banks are more likely to be connected to smaller banks. As Bargigli et al. (2015) observe dissasortativeness is also related to the core-periphery structures identified in many financial networks.[20]

[18] It also depends on $f''(k^*)$. If the derivative is positive, that is the liquidation costs decrease at a decreasing rate with the number of banks that fail, then its value cannot be too high.

[19] Note that banks are heterogeneous in all aspects except the capital requirement ratio.

[20] Our model has only deposit accepting banks and therefore those with higher initial deposits (large banks) are more likely to become creditors.

Table 1
Interim Bank Balance Sheet.

Assets	Liabilities
$V_t^j \simeq 0$	$R_d H D_t^j$
$R_L F L_t^j$	$R_b \mathbf{BD}_t^j$
$R_b \mathbf{BL}_t^j$	E_t^j

In what follows, we are mainly interested in the behavior over time of the aggregate level of lending to firms which is very closely approximated by the aggregate level of reserves, $\mathbf{V}_t$.[21]

Let R_b denote the gross interbank lending rate that we treat as an exogenous parameter.[22] Table 1 shows the balance sheet of a typical bank after the financing of projects is completed.

Equity is derived as:

$$E_t^j = V_t^j + R_L F L_t^j + R_b \mathbf{BL}_t^j - R_d H D_t^j - R_b \mathbf{BD}_t^j. \tag{1}$$

The interbank market network is represented by the adjacency matrix with a typical element $R_b B D_t^{jk}$.

As our main objective is to understand how shocks are amplified as they are transmitted through the interbank network, we have built a model that abstracts from regulatory issues, such as those related to insufficient liquidity and contagion following insolvencies of financial institutions. With that in mind, we have set the duration of all contracts to a single period and do not allow banks to keep any reserves. One possibility for future work would be to introduce illiquid investments into the model, along with the presence of a central bank. In its role as a financial regulator, the central bank can set reserve and capital requirements, along with other macroprudential policies. Incorporation of such policies into the model would allow analysis of the interplay between illiquidity and insolvency issues, and could provide a useful tool for central banks and financial regulators. In Section 5, we explore the impact of changes in the interbank rate and equity ratio on our numerical results.

3.1.2. *Stage II: realization of loan returns*

When a loan is repaid the bank that offered it receives a repayment R_L which is added to its reserves. In contrast, when the loan is not repaid it is written off. After all loans are either repaid or written off each bank's equity is reevaluated. At this stage $R_L F L_t^j$ is set equal to 0 while reserves have been augmented by the realized loan repayments. We denote this interim level of reserves by $\hat{V}_t^j$. If the realized loan repayments are less than $pR_L F L_t^j$ the bank has made losses on its loans. If these losses exceed its equity, that is $pR_L F L_t^j - \hat{V}_t^j > E_t^j$, then the bank is insolvent. If after the reevaluation of balance sheets all banks still have non-negative equity, that is they are solvent, we move directly to stage IV. If there exists at least one insolvent bank we move to stage III.

3.1.3. *Stage III: bankruptcy resolution*

The bankruptcy resolution process below follows very closely the one in Eisenberg and Noe (2001). The only differences between the two papers are due to minor changes in the balance sheet entries.[23] All insolvent banks, that is those with negative equity, are liquidated. When a bank is liquidated the proceeds will be distributed *pro rata* to all the bank's creditors. In this benchmark case of our model, we assume that the market value of liquidated assets is equal to their book value. Thus, the benchmark case does not allow for fire sales. The bankruptcy resolution process is independent of the order that insolvent banks are liquidated. This is because both the settlement of all accounts, including the contracts in the interbank loan market, and the distribution of liquidation proceeds only take place after the bankruptcy resolution process is completed.[24] Completion of the process implies either that all remaining banks are solvent or that all banks have become insolvent. Eisenberg and Noe (2001) and Acemoglu et al. (2015) have proved that a clearing equilibrium exists and it is generically unique. Below we describe the algorithm of the bankruptcy resolution process.

Choose randomly any of the banks with negative equity, say j. Let $\lambda_j \equiv \frac{R_d H D_t^j}{R_d H D_t^j + R_b \mathbf{BD}_t^j}$ denote the ratio of household deposits to total deposits. The assets of bank j will be distributed to its creditors *pro rata* as follows. The depositors of bank j will receive a fraction λ_j of reserves, V_t^j, and will be allocated a fraction λ_j of the loans that bank j offered to other banks, BL_t^{jk} for all $k \in \{1, \ldots, M\}$, that is the depositors of bank j will now hold deposit accounts at these other banks. Each of the creditor banks to bank j, that is for all

[21] The approximation is because of rounding given that the size of each loan is equal to 1 while reserves take values in the non-negative reals.

[22] Shin (2008) derives equilibrium prices of financial claims when the assets of borrowers include loans offered to third parties. For our computations we have used various values that are compatible with the outside options of each bank.

[23] In this work we have followed Eisenberg and Noe (2001) and have assumed that all creditors are treated equally. As Acemoglu et al. (2015) have shown, the clearing process can easily be modified to allow a class of creditholders (e.g. depositors) to have a priority claim over the bank's assets. There is an ongoing debate over the design of optimal priority rules for banks (for a review of the relevant literature see Bougheas and Kirman, 2016).

[24] Thus, for rounds before the resolution process is terminated creditor banks are allocated a proportion of a failed bank's assets without specifying the exact final values of the liquidation proceeds which are only determined at the termination of the resolution process. This is because if other banks fail in later rounds that have borrowed funds from the bank that is currently liquidated then the assets of the latter bank might be further reduced. Of course, as Jackson and Pernoud (2020) recognize this can only happen if the network has directed cycles.

$k \in \{1, ..., M\}$ such that $BD_t^{jk} > 0$, will receive a fraction $\mu_{jk} \equiv \frac{R_b BD_t^{jk}}{R_d HD_t^j + R_b \mathbf{BD}_t^j}$ of reserves, V_t^j, and will be allocated a fraction μ_{jk} of the loans that bank j offered to other banks, BL_t^{jk} for all k $(1, ..., M)$; where $\sum_{k=1}^{M} \mu_{jk} = 1 - \lambda$.

This will end the bankruptcy procedure for bank j. Given that the creditors of bank j were not fully repaid, it is possible that some other banks that before the liquidation of bank j were solvent now become insolvent; this will be the case if their equity is not sufficiently high to absorb the losses. As long as there are other insolvent banks, one of them will be chosen randomly and the whole process will repeat itself. Otherwise, the bankruptcy resolution process is terminated.

3.1.4. Stage IV: clearing

All remaining banks are solvent. The interbank loan market is settled by the transfer of reserves from debtor to creditor banks. At the end of this process all interbank accounts are set equal to zero. The reserves distributed to households as liquidation proceeds are redeposited in the banking system.

Because of insolvencies, the number of banks will keep declining over time. In order to avoid this from happening each period we recapitalize the system by randomly redistributing aggregate reserves at the end of each period among the M banks. This ensures that our results are independent of the initial numerical choices we made about the number of banks and the initial aggregate reserves. Moreover, in order to be able to compare the results of the benchmark model with those results derived when we introduce fire sales we also increase aggregate reserves, before we redistribute them, by a rate g that is constant over time. The reason for doing so is because fire sales have a negative effect on the growth rate of the economy and without rebalancing reserves the economy will eventually collapse. The growth rate that we have chosen is such that it over-compensates for the losses due to fire sales.

Lastly, we set each bank's equity to a fraction e of its new reserves. Then the banking system enters the following period and it once more goes through the four stages.

3.2. Analysis of the model without fire sales

We are interested in the evolution of aggregate reserves, which in our model is a very close proxy of aggregate lending. Given that in any period loan repayments are identically and independently distributed across the banking system, the total number of successful loan repayments is binomially distributed with parameter p. As the number of loans offered each period is large, the central limit theorem implies that the distribution is approximately normal. In order to determine the end-of-period aggregate reserves we first need to determine the propagation of shocks through the banking system. However, as Cifuentes et al. (2005) have demonstrated, in the absence of fire sales the interbank network does not amplify losses. The structure of the network will affect the number of banks that will fail but not the level of aggregate losses.[25]

Let $\mathbf{Z}_t$ denote the total number of loans that were successfully repaid in period t. The level of aggregate reserves at the beginning of period $t+1$ is given by

$$\mathbf{V}_{t+1} = (1+g) R_L \mathbf{Z}_t$$

which we can write as

$$\mathbf{V}_{t+1} = (1+g)\left(pR_L \mathbf{V}_t + S_t\right)$$

where $S_t = R_L \mathbf{Z}_t - pR_L \mathbf{V}_t$ denotes the period t shock (deviation from expected revenues).

Then, the growth rate of aggregate reserves, and hence aggregate bank lending, is given by:

$$\frac{\mathbf{V}_{t+1} - \mathbf{V}_t}{\mathbf{V}_t} = pR_L(1+g) - 1 + (1+g)pR_L \frac{\mathbf{Z}_t - p\mathbf{V}_t}{p\mathbf{V}_t} = pR_L(1+g) - 1 + (1+g)pR_L s_t, \tag{2}$$

where s_t is the shock normalized by the size of the banking system that would have been obtained in the absence of shocks. Thus, the growth rate of reserves can be decomposed into two parts: $pR_L(1+g) - 1$ is the trend that is comprised of the exogenous growth rate times the return on loans. The second term is equal to the aggregate shock (deviations from trend). Given that project returns are drawn from a binomial distribution and given the large number of projects, the central limit theorem implies that the growth rate of reserves is normally distributed. Without fire sales the aggregate shock can fully account for the growth rate of reserves.[26]

This result is not surprising. In the absence of any external effects, like fire sales, any change in the distribution of shocks across the network of banks will affect the number of bankruptcies (see below) but not aggregate outcomes.[27] Similar results have also been obtained in production networks. In those models, aggregate outcomes depend on the size of sectors hit by shocks and not on the specific network structure of the economy, a result known as 'Hulten's Theorem'.[28]

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[25] When a bank writes off loans the losses will be absorbed firstly by its equityholders, and if their equity is not sufficiently high losses will be absorbed by its depositors and its bank creditors. Eventually, all initial losses will be absorbed by bank equityholders and depositors. However, there is nothing in the network that can amplify the initial shock.

[26] We also considered introducing an externality in the economy, whereby loans in a given period are more likely to be repaid the higher the proportion of loan repayments the period before (success breeds success). The externality introduces persistence in the growth rate. However, in order to maintain our focus purely on the impact of fire sales on the growth rate of economic activity, we do not include the externality in our main analysis.

[27] See also Proposition 2 in Cifuentes et al. (2005) for a derivation of this result in a model very similar to ours.

[28] See Hulten (1978). For a review of similar results within production networks, see Carvalho and Tahbaz-Salehi (2019).

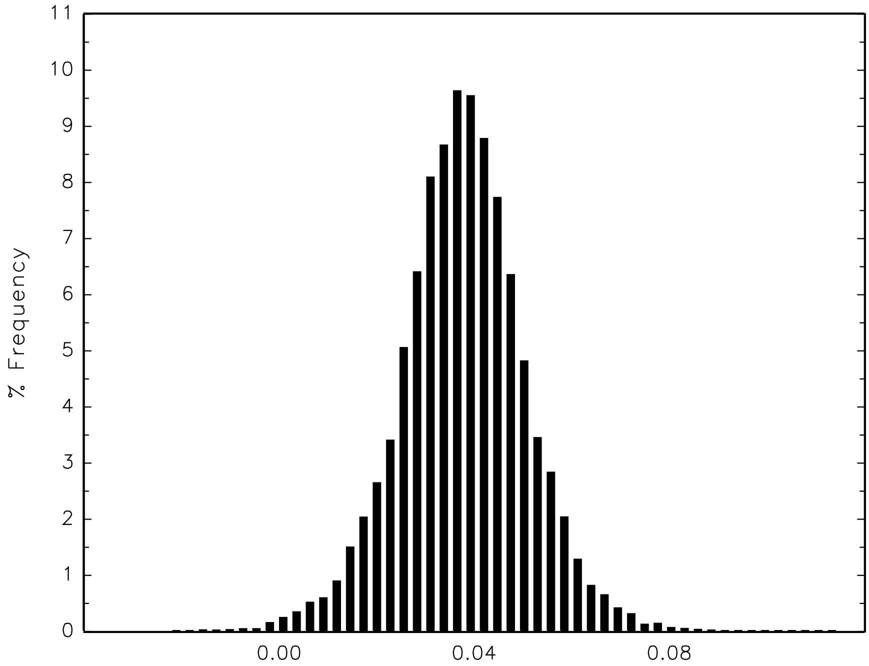


Fig. 4. Output growth without fire sales.

3.3. *Numerical analysis*

The results of the benchmark model are straightforward. However, they will help us to understand the impact of fire sales on the growth rate of this simple economy compared to that caused by network effects. The model is too stylistic to attempt a structural estimation. Nevertheless, the numerical examples highlight the mechanism responsible for the 'fat tails' in the growth distribution of aggregate activity when fire sales are present. Given that one of our aims is to show that shocks on their own cannot explain what happens during rare macroeconomic disasters, we set up the parameters to ensure that there are always some banks failing.

For our numerical analysis we used the following parameters for the benchmark case: $M = 20, V_0 = 1000, p = 0.75, R_L = 1.37, R_d = 1, e = 0.02, R_b = 1.01, g = 0.01$.[29] For each set of parameters we performed 500 Monte Carlo replications with 50 periods each, thus generating 24, 500 growth rates for aggregate reserves. Replications only differ in the initial distribution of deposits among banks. At the beginning of each replication we allocated randomly the initial reserves, V_0, among the M banks. Fig. 4 shows a histogram of the growth rates of aggregate reserves for the benchmark case.

Notice, that the distribution is approximately normal and the average growth rate is equal to 0.03776 which is equal to the trend growth rate derived above. Thus, as we mentioned above, the aggregate shock is a perfect predictor of the deviations of the growth rate of economic activity from its trend.[30]

4. The computational model with endogenous fire sales

In this section, we introduce fire sales into the benchmark model. As Shleifer and Vishny (2011) state in the quote at the beginning of the paper, and as we have also argued in the last section, without fire sales shocks on the interbank network are not sufficient to generate shock amplification. Moreover, fire sales naturally generate an asymmetry between busts and booms that is a typical feature of the behavior of GDP (Acemoglu and Scott, 1997). Typically, fire sales result from the sales of bank assets at prices below book values. In our simple model with single-period loan duration the only assets left on a bank's balance sheet at the time when liquidation takes place are reserves and loans made to other banks. We introduce fire sales on reserves, the idea being that banks hold as reserves liquid assets, such as short-term and long-term government bonds whose value will drop during a crisis. To simplify matters we assume that reserves are comprised of a single asset and we denote its price by P. Thus, in the absence of any insolvent banks $P = 1$. We also denote by $P^{\min} < 1$ the price that will be attained if all banks become insolvent. Then, the inverse demand function for this asset at t is specified as:

[29] The expected return on each bank loan is 2.75% ($p \times R_L$); we considered variations from 2% to 3.5% by varying either p or R_L and the settings used correspond to the midpoint 2.75%. We have set the interbank loan rate to 1% (it has to be lower than the expected return on loans), and the capital requirement ratio to 2%.

[30] The correlation coefficient is 0.99999. Its deviation from unity is due to the 'lumpiness' of projects. A bank can loan out only integer values of funds.

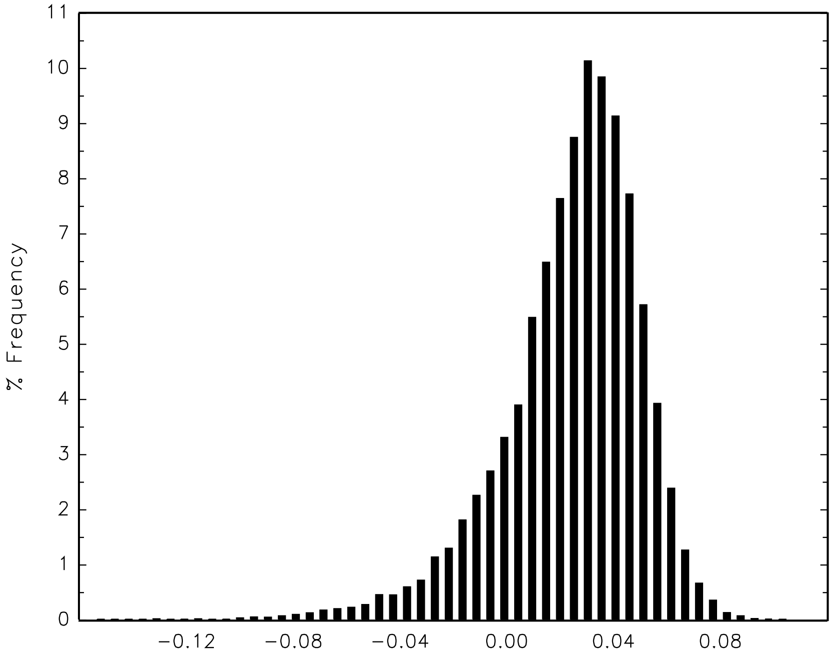


Fig. 5. Output growth with fire sales.

$$P_t = 1 - \frac{\left(1 - P^{\min}\right)\sum_{j\in\{INS\}} V_t^j}{V_t},$$

where $\{INS\}$ denotes the set of insolvent banks. According to this function, the drop in price is proportional to the ratio of the initial value of assets offered for sale to the total value of all assets. Thus, if all banks are solvent, $\sum_{j\in\{INS\}} V_t^j = 0$ and $P = 1$ and if all banks become insolvent, $\sum_{j\in\{INS\}} V_t^j = V_t$ and $P = P^{\min}$.[31]

Jackson and Pernoud (2020) show that with endogenous fire sales there might be multiple clearing equilibria in which case they can be ranked according to the final value of bank assets. We will focus on the 'best' equilibrium, that is, the one with the smallest number of insolvent banks.[32]

The algorithm that we used before for the case without fire sales now has to be adjusted to allow for the effect of insolvencies on equilibrium asset prices. After the initial shocks we clear the system as in Stage IV above. If there are any insolvent banks we use the inverse demand function specified above to calculate a new temporary price for assets. Using the new asset valuations we clear the system once more from the beginning. If there are any further insolvencies we recalculate the price to take into account the sale of assets by the new insolvent banks. We repeat the process until all remaining banks are solvent.

As we remarked in the previous section, because fire sales introduce an asymmetry in the impact of shocks, unless we introduce a compensating growth factor banking system will eventually be reduced to a single bank with a very low volume of reserves. As we noted earlier, to avoid this from happening we allow for reserves to grow exogenously at the rate of growth g.

4.1. Results of the model with fire sales

We again run 500 replications with 50 periods each, with exactly the same parameters as those of the benchmark model, but now we also introduce fire sales with the minimum asset price, $P^{\min} = 0.85$.[33] Fig. 5 shows a histogram of the growth rate of aggregate reserves for the model with fire sales.

[31] Cifuentes et al. (2005) use an exponential inverse demand function while Cecchetti et al. (2016) use a quadratic function. The exponential yields a very large effect on the price even when sales are low while with the quadratic prices keep dropping very fast even as the minimum price is approached. It seems a more realistic function would be the logistic that allows for smoother adjustments at the ends. We have run simulations using all forms and found that the qualitative results are not affected. With that in mind we are presenting the results for the simpler linear form.

[32] See also Cecchetti et al. (2016) who demonstrate the existence of such an equilibrium for an interbank network where the balance sheets of banks are more complex than in this paper. In particular, they allow for priority claims and multiple assets offered for liquidation. Although for their applications they consider simple network structures their existence proof is valid for any type of network structure including the network structures formed in our paper.

[33] We choose the minimum price so that the fat left tail approximately matches what we observe in the data (see below). We can get similar results by changing the functional form of the inverse demand function and at the same time adjusting the value of the minimum price.

Table 2
Aggregate Shocks and Aggregate Growth Rates

Growth Rate	+ Shock	− Shock	0 Shock
Without Fire Sales			
Above Trend	12299	0	0
Below Trend	1	12095	105
With Fire Sales			
Above Trend	7815	0	0
Below Trend	4335	12180	150

Comparing Fig. 5 with Fig. 4 we make two observations. First, the average growth rate has dropped to 0.0242. The drop in the average growth rate relative to the version of the model without fire sales captures the asymmetric impact that fire sales have on the growth of economic activity. The correlation coefficient between the aggregate shock and the growth rate of economic activity is equal to 0.922. In the benchmark case the aggregate shock is a perfect predictor for the deviations of the growth rate of economic activity from its trend but this is not the case anymore. This has important implications for how we understand the causes of macroeconomic disasters. In traditional DSGE models there is a strong correlation between initial shocks and final outcomes. Amplification mechanisms, like the financial accelerator (Bernanke et al., 1999), boost the effects of the shock but do not alter its sign. This is not the case anymore for network economies. What matters is not only the size of the initial shock but also its distribution across the network. Network models seem to suggest that to a great extent macroeconomic unpredictability might be due to endogenous uncertainty due to complex interactions within the financial system.

To get a clearer picture of the effect of fire sales, in Table 2 we show the number of periods where the growth rate of the economy was above or below its trend growth rate of 0.03776 (corresponding to the case without fire sales) conditional on the sign of the aggregate shock.[34]

Notice that for the case without fire sales the results reflect the (almost) perfect correlation between the shock and the growth rate predicted by the model. The reason that we have one case where the growth rate is negative when the shock is positive and also a few cases with negative growth when the shock was zero is because we have unit size loans but reserves are positive real numbers. This rounding effect is insignificant as in all such cases the growth rate was very close to 0. When we introduce fire sales we find that even when the aggregate shock is positive more than a third of the aggregate growth rates are negative. In our model the initial shock depends on the number of loans fully repaid by firms. There were periods when the proportion of loan repayments were sufficiently high that the aggregate shock was positive, however, there were also a small number of banks hit by negative shocks that were liquidated. The ensuing fire sales and network effects dominated the impact of the initial aggregate shock on the growth rate of economic activity.

Our second observation from comparing Figs. 4 and 5 is that the growth rate distribution with fire sales is not symmetric; it has a fat left tail. This is further seen in Fig. 6, which gives a Q-Q plot of the growth quantiles against those from a standard normal distribution, along with a reference line corresponding to quantiles of a normal distribution with mean and variance calibrated to that of the growth rates. The fat left tail is a clear feature of Fig. 6, and a comparison with the Q-Q plot for US growth data in Fig. 3 shows that the left tail behavior of the actual data is mimicked well by the growth outcomes from the model.

At this point it is not clear whether fire sales are wholly responsible for this observation, or if there is also an additional amplification effect due to the interaction of network effects with fire sales. We explore this issue below.

4.1.1. Network effects versus fire sales

In each period after the interbank network is formed the realization of shocks will determine how many banks will be initially liquidated. Then, after clearing and the settlements of all accounts there might be additional liquidations and their number might be further boosted when we introduce fire sales. Fig. 7 shows histograms for the number of bank liquidations before and after the realization of shocks, with and without fire sales.

The top left panel shows the distribution of initial liquidations without fire sales while the top right panel shows the same distribution when we introduce fire sales.[35] The average number of initial liquidations without fire sales is equal to 3.9064 and when we introduce fire sales is equal to 4.7098. The difference is due to a scale effect. As the average bank grows in successive periods they become slightly safer.[36] Therefore, the difference between the two numbers is due to the lower growth rate when we introduce fire sales. The bottom left panel shows the final distribution of liquidations (after clearing) in the absence of fire sales and the average is 4.1502. The increase in liquidations by approximately 6.2% is due to network effects. That is, the inability of some borrowers to meet their obligations forced their creditors into liquidation. The bottom right panel shows the same distribution with fire sales. The

[34] Given that our computations included 500 runs with 50 periods each we have 24500 growth rates.

[35] Keep in mind that only in period 1 of each computation the level and the distribution of reserves will be the same for the benchmark model and for the model with fire sales. In subsequent periods they will differ.

[36] Consider the following example. In our model the equity to total assets of all banks is approximately the same. This implies that for banks to be solvent a minimum percentage of loans needs to be repaid. Suppose that this percentage is equal to 92.3. A bank that has made 10 loans needs all of them to be repaid, a bank that has made 20 loans needs 19 loans to be repaid (95%) while a bank that made 100 loans needs 93 loans to be repaid (93%). Thus, bigger banks can survive slightly bigger shocks.

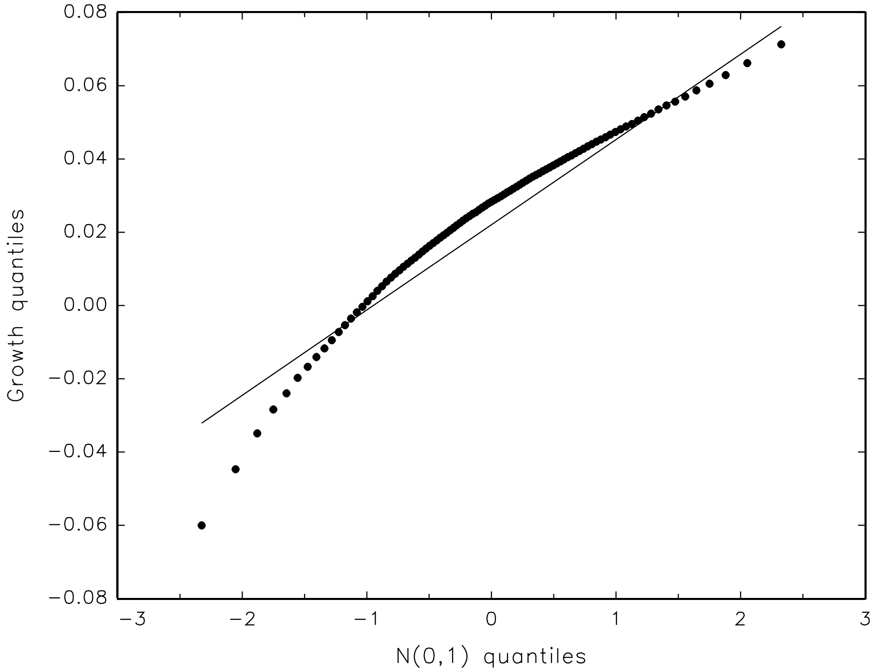


Fig. 6. Q-Q plots: ••• Output growth; —— Normal distribution with mean and variance calibrated to output growth.

average number of liquidations is now equal to 5.4569. This is an increase of approximately 16% and is due to the interaction of network effects with fire sales.

Comparing the shape of the four distributions we find that the one at the bottom right, that corresponds to total bankruptcies in the presence of fire sales, has a more exaggerated right tail. We demonstrate below that this fat right tail is closely related to the fat left tail of the distribution of the growth rate of economic activity.

What we have demonstrated up to this point is that the interaction between the structure of the interbank network and fire sales can provide a potential explanation for the fat left tail in the distribution of the growth rate of economic activity, thus accounting both for the asymmetry in cycles but also for rare but extreme disasters. However, we have not yet fully unraveled the mechanism by which the above interaction works.

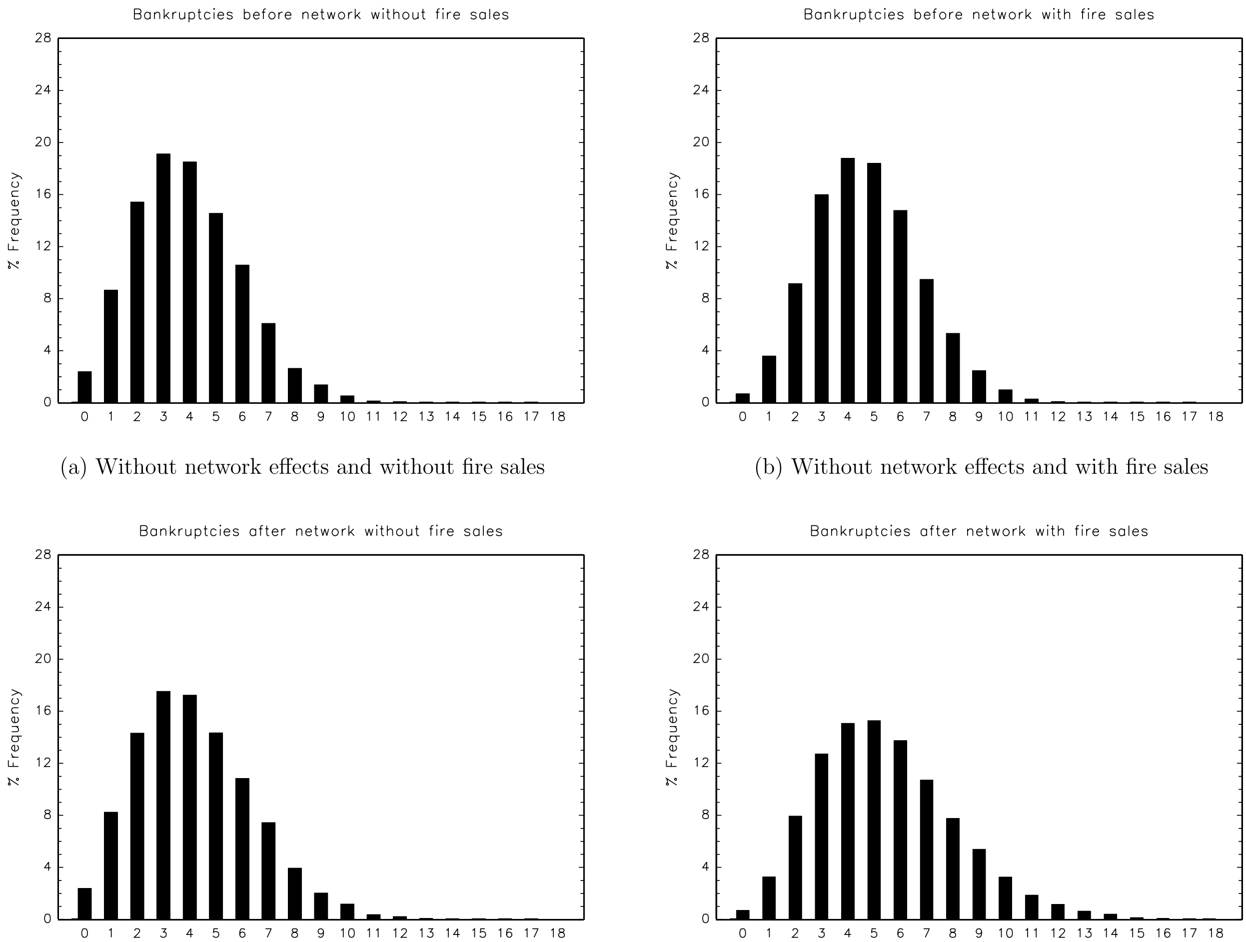
4.1.2. Explaining the fat left tail

Fig. 8 shows the box plots of aggregate output growth by the number of additional bankruptcies brought about through network effects, above and beyond those directly caused by the initial shocks. The values above each box plot provide a count of the periods for which the number of additional bankruptcies was obtained.

There is a clear negative correlation between the number of additional bankruptcies that arise through network effects and the resulting growth rate of aggregate output (measured on the vertical axis). These additional bankruptcies are due to the interconnectedness of the banking system and, as we have observed above, they have been further boosted by fire sales. In our model the banking network is formed randomly and, hence, banks are more likely to have borrowed from multiple lenders. The presence of multiple creditors offers some degree of protection to them as the risks of default by the borrowing bank are spread out. However, as Acemoglu et al. (2015) have shown, when the initial shocks are too large diversification has the opposite effect as such shocks affect a larger number of banks.[37]

One possible explanation is that what happens in the left tail of the growth distribution is that these realizations refer to periods when the banks hit by large negative shocks were heavily indebted to other banks. With this in mind, for each of the 24500 periods we calculated, for each bank, the in-degree and the directed version of eigenvector centrality (Bonacich centrality measure). The in-degree is equal to the number of a bank's creditors while the eigenvector centrality also takes into account indirect exposures (loans made by other banks to creditors). We then multiplied each measure with the corresponding bank shock and averaged the products. High negative values of this index would indicate a period where, on average, heavily indebted banks were hit by negative shocks, while positive values would indicate that, on average, heavily indebted banks were hit by a positive shock. Lastly, we calculated the correlation coefficients between the growth rate and each of the two indices. We would anticipate a positive estimate, that is, negative growth rates are correlated with negative values of the index, but instead we found that the correlations were not significant. These results indicate that network measures might not be the appropriate statistics to use.

[37] See also Battiston et al. (2012) for some related discussion on the effects of diversification.



(a) Without network effects and without fire sales

(b) Without network effects and with fire sales

(c) With network effects and without fire sales

(d) With network effects and with fire sales

Fig. 7. Distribution of number of bankruptcies.

Table 3
Contagion at the Tail

	Worst hit bank	2nd worst hit bank	Both banks
Loans from all liquidated banks	13	13	7
Loans from all but one liquidated banks	13	7	1
Loans from all but two liquidated banks	6	11	0

In order to understand the above results, and also what generates the left fat tail in the distribution of the growth rate of economic activity, we then focused on all those periods where the growth rate is less than −6%. As there were 245 such periods (exactly 10% of the total), this corresponds to the 10% left tail of the distribution. In many periods there was a relatively large number of banks that went into liquidation at the initial stage, that is these banks were hit by large number of negative initial shocks. However, when we concentrate on those 54 periods (about 2%) of the total where the additional bankruptcies were greater than or equal to 6 the picture changes. In Table 3, we show in how many of those periods the two banks hit by the two worst shocks received loans from all, all but one and all but two of those banks that were liquidated because of network effects (additional bankruptcies).

In 33 out of those 54 periods at least one of the two banks was indebted to all banks that were liquidated due to network effects. The worst hit bank received loans from more than two thirds and the second worst hit bank received loans from more than half of those banks that were liquidated because of network effects. Thus, the network effects are present, however, there were also many periods were these network effects were dominated simply by a large number of negative initial shocks.

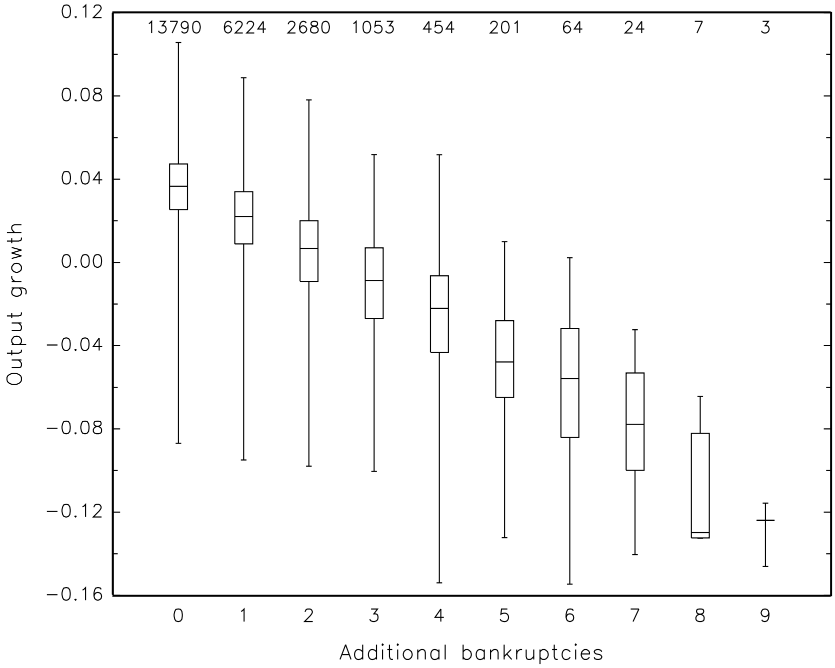


Fig. 8. Box plots of output growth by number of additional bankruptcies due to network effects (values above box plots give counts of periods for which number of additional bankruptcies obtained).

Lastly, we also checked if some of these results can be simply explained by looking at simple network measures. More specifically, we ask if these strong network effects can be explained by changes in the overall structure of the network. We find that this is not the case. For example, the average in-degree for all 25000 networks is 7.21 while for the 54 networks that correspond to the sample examined above the in-degree is surprisingly a bit lower, 7.11. What this implies is that what matters for understanding the left tail of the growth distribution is primarily the distribution of shocks. Moreover, to explain contagion (cascades) what matters is not the network structure alone but the interaction between the distribution of shocks and the connectivity of each bank.

5. Discussion and extensions

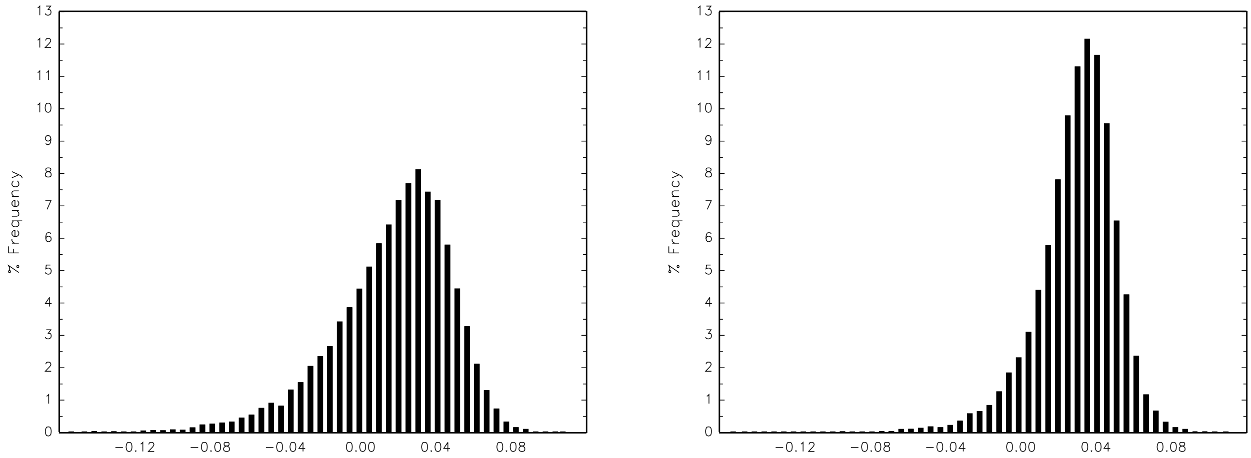
Both the Great Depression of the 1930s and the more recent Great Recession have been precipitated by deep financial crises followed by a large number of bank failures. We suggested that each crisis unfolded as contagion spread across the interbank network, with bank losses exacerbated by drops in the values of bank assets due to fire sales. The curtailment in business loans, not only by those banks that failed but also by banks that survived but had to consolidate their balance sheets, let to the steep declines in economic activity.

One of the policy responses to the Great Depression was the establishment of the Federal Reserve System in 1933. Since then the Federal Reserve Bank (FRB) has played an active role not only by developing regulations that aim to avoid future crises but also by acting as a lender of last resort, ensuring that banks can overcome liquidity problems that could eventually lead to insolvencies. More controversially, the FRB, during the 2009 financial crisis, has also intervened by rescuing financial institutions that either were insolvent or close to becoming so. The main argument in support of such a policy has been that certain banks are 'too-big-to-fail', however, the policy also introduces a moral hazard problem that sows the seeds for future crises (see, for example, Strahan, 2013; Kaufman, 2014).

As we argued in Section 3.1.1 above, in our model large banks are usually the providers of short-term finance to smaller institutions; this is consistent with the core-periphery structure of banking networks. During periods of financial instability, the market for such funds can freeze with many smaller banks getting into trouble as their primary source of funds has been cut off (Diamond and Rajan, 1983). Due to the lack of dynamic interactions, our model is unable to capture such freezes that would undoubtedly magnify the effects of a crisis.

Moreover, as our model does not feature an active central bank we are abstracting from policies related to the resolution of crises. Our focus is on analyzing the impact of shocks that are propagated throughout the interbank network, so we do not include any offsetting impacts that might arise from an active central bank. However, our model does allow us to explore how variations in the equity ratio and the interbank rate - two key banking parameters of the model that are controlled by the FRB - impact the macroeconomy, and in particular the distribution of aggregate growth.

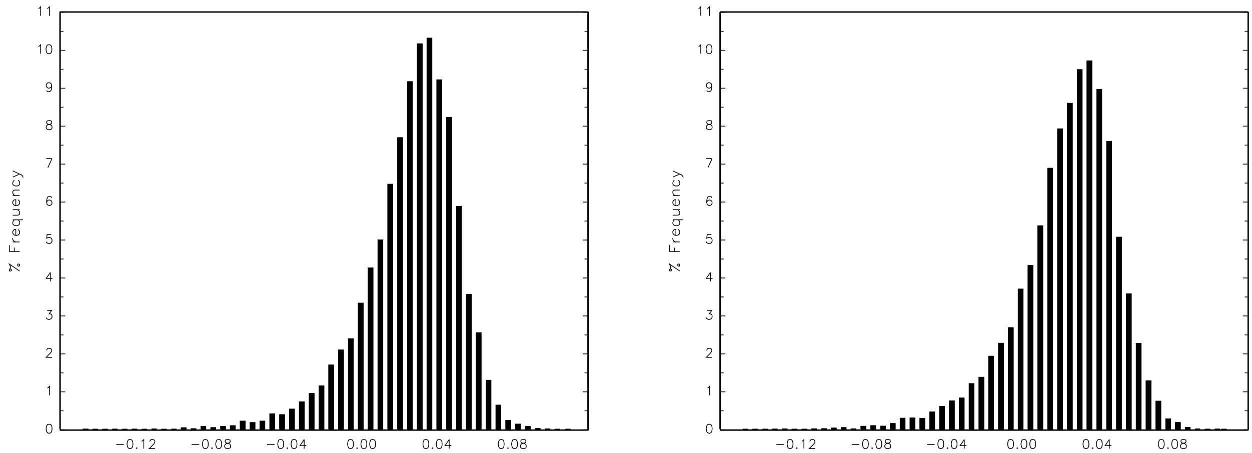
First we consider the influence of the equity ratio setting on our results. This ratio is one macroprudential tool that the FRB can use to affect the stability of the banking system. By increasing that rate and forcing banks to have more 'skin in the game' they



(a) $e = 0.01$

(b) $e = 0.03$

Fig. 9. Output growth with fire sales for different equity ratios



(a) $R_b = 1.005$

(b) $R_b = 1.02$

Fig. 10. Output growth with fire sales for different interbank rates

can reduce the risk of insolvencies. However, changes in this rate can also impact the intermediation services provided by banks. Empirical evidence suggests that higher equity ratios reduce the risks that small banks face during both times of crises and normal times, while improving the performance of large banks during times of crises (Berger and Bouwman, 2013; Laeven et al., 2016). Figs. 9(a) and 9(b) report histograms of the growth rate of aggregate reserves for the model with fire sales (cf. Fig. 5), but with the equity ratio set to $e = 0.01$ and $e = 0.03$, respectively, instead of the benchmark setting $e = 0.02$. Overall the results have the same broad pattern as the benchmark case, with a fat left tail apparent in the distribution. As we might expect, we find that decreasing (increasing) the capitalization of banks has the effect of exaggerating (mitigating) the fat left tail.

We next consider the impact of changes to the interbank rate. The FRB, by setting of the overnight rate (federal funds rate), directly influences all other interest rates, including the interbank rate, thus the interbank rate is effectively another tool that the FRB can use. Changes in the interbank rate would be expected to affect the formation of the interbank network; for example, higher rates might discourage banks from forming links and thus limit the transmission of shocks. However, we are unable to capture such endogenous responses to changes in the FRB's interest rate policy, since in our analysis, the network formation is exogenous. We would therefore expect the setting of the interbank rate in our numerical results to only have modest effects on the distribution of profits across banks, and not to affect the main findings regarding the output growth rates. Figs. 10(a) and 10(b) report histograms of the growth rate of aggregate reserves for the model with fire sales (cf. Fig. 5), with the interbank rate set to $R_b = 1.005$ and $R_b = 1.02$, respectively, instead of the benchmark setting $R_b = 1.01$. We observe the results to be very similar to the benchmark case, with an almost identical fat left tail.

6. Concluding comments

Both the Great Depression of the 1930s and the more recent Global Financial Crisis of 2008 have provided great challenges to traditional macroeconomic explanations of recessions. According to traditional macroeconomic models final outcomes are directly related to the magnitude of initial shocks. These models only provide an amplification mechanism of these initial shocks. However, there are good reasons to believe that some of the macroeconomic uncertainty is endogenous and directly related to the structure of the economy. For example, in the growing literature on production networks shocks are transmitted through the readjustment of prices of intermediate inputs. In Acemoglu et al. (2017a) shocks are transmitted between industries and when relatively large shocks hit well-connected industries their effects on aggregate outcomes can account for the fat tails in the distribution of the growth rate of economic activity. More recently, Acemoglu and Tahbaz-Salehi (2020) examine the transmission of shocks along supply chains at the firm level. That model can also account for the asymmetric impact of shocks given that firms that are hit by negative shocks fail when they are unable to cover their fixed costs.

In this paper, we have offered an alternative explanation for the fat left tail in the distribution of the growth of economic activity. The amplification mechanism also relies on network effects but here the connections represent debt obligations between banks. We have argued that there is sufficient evidence that fire sales played an important role in both crises mentioned above. In our model, tail events take place when banks that have borrowed heavily from other banks are hit by severe shocks. Their inability to meet their obligations means that their creditors have to write off a fraction of their assets. The exact fraction depends on the number of banks that are liquidated, with the corresponding fire sales amplifying and network effects.

In contrast to production network models where all decisions are taken after the realization of shocks, financial decisions in our model are taken before the realization of shocks. This considerably increases the complexity of the environment. Given the above considerations we opted for a computational model and rely on averaging the results of multiple Monte Carlo replications. For checking the robustness of our results, we have also performed a variety of runs with different parameters. However, the greatest disadvantage of the computational approach is that it is subject to the Lucas critique. The model cannot account for any change in the behavior of market participants (banks and firms) to any exogenous policy change. Nevertheless, we can still learn valuable lessons by taking seriously the complex structure of the economy and, thus, account for its inherent unpredictability.

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