**Letter**

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# Cross Ownership Under Strategic Tax Policy

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**Abstract:** We provide a new reason for the consumer surplus and welfare raising cross ownership. We show that cross ownership reduces the tax rate, and increases consumer surplus and welfare under Cournot and Bertrand competition when the marginal social cost/benefit of public funds is less than unity. We further show that Cournot competition creates higher consumer surplus and welfare compared to Bertrand competition if the marginal social cost/benefit of public funds is less than unity, thus providing a new reason for the Cournot-Bertrand welfare reversal.

**Keywords:** Bertrand; Cournot; cross ownership; strategic tax policy

**JEL Classification:** D43; L10; L13

## 1 Introduction

Passive cross ownership, which refers to a situation where a firm holds non-controlling shares in other firms, has grown significantly in recent decades, and can be found in several industries, such as automobile (Alley 1997), IT (Gilo, Moshe, and Spiegel 2006), telecommunications (Brito, Cabral, and Vasconcelos 2014), banking (Azar, Raina, and Schmalz 2022), airline (Clayton and Jorgensen 2005), and cement industries (Davallou, Soltaninejad, and Tahmasebi 2015). While cross ownership among the rival firms reduces welfare by contracting outputs (Bresnahan and Salop 1986; O'Brien and Salop 2000; Reynolds and Snapp 1986; Shelegia and Spiegel 2012),

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it may increase welfare by reducing production inefficiency under asymmetric costs of production (Fanti 2015; Farrell and Shapiro 1990; Ma, Qin, and Zeng 2024), increasing R & D investments (López and Vives 2019), reducing input prices (Chen, Matsumura, and Zeng 2024; Symeonidis 2008), reallocating outputs through quality choice (Bayona and López 2018), and affecting horizontal product differentiation (Banerjee, Mukherjee, and Poddar 2024; Fang, Huang, and Zeng 2024).

We provide a new reason for the consumer surplus and welfare raising cross ownership in the presence of strategic tax/subsidy policies. It is well known that governments may use tax/subsidy policies to improve welfare by reducing the distortion of the imperfectly competitive product market (Hamilton 1999; Myles 1996). However, the literature on cross ownership did not pay much attention to the effects of strategic tax/subsidy policies. We fill this gap in the literature.

We show that a higher percentage of cross ownership increases the subsidy rate (or decreases the tax rate), and increases consumer surplus and welfare if the marginal social cost/benefit of public funds is less than unity.[1] We first show this result under symmetric cross ownership and Cournot competition. We then show that the result holds under asymmetric cross ownership, and Bertrand competition.

The reason for our result is as follows. For a given tax/subsidy rate, a higher percentage of cross ownership tends to reduce consumer surplus and welfare by increasing collusive behavior in the product market. On the other hand, a higher percentage of cross ownership tends to increase consumer surplus and welfare by increasing the subsidy rate (or decreasing the tax rate). The second effect can dominate the first effect and a higher percentage of cross ownership can increase consumer surplus and welfare if the marginal social cost/benefit of public funds is less than unity.

Using a specific demand function, we also show that consumer surplus and welfare can be higher under Cournot competition compared to Bertrand competition if the marginal social cost/benefit of public funds is less than unity. Thus, we contribute to the literature on Cournot-Bertrand welfare comparison (see, Arya, Mittendorf, and Sappington 2008; Mukherjee 2011; Singh and Vives 1984).

Liu, Mukherjee, and Wang (2015) showed consumer surplus and welfare raising horizontal merger in a Cournot oligopoly under pollution and strategic tax policy. In contrast, there is no pollution in our paper, and our result holds for partial cooperation between the firms due to cross ownership. Further, we show our results

**1** The marginal social cost/benefit of public funds can be greater (less) than unity, suggesting that the tax/subsidy revenues are more (less) valuable to the governments compared to the firms and the consumers. It can be greater than unity for the distributional reasons but can be less than unity if the government maximizes a political support function that is a weighted average of welfare and political contributions, which is equal to consumer surplus and profit. See, e.g., Neary and Leahy (2004), for this discussion.

under both Cournot and Bertrand competition, and also contribute to the literature on Cournot-Bertrand welfare comparison.

Cheng, Wu, and Zeng (2024) showed that cross ownership might benefit the consumers under strategic tax policy provided the firms have asymmetric costs. Unlike that paper, we show our results under symmetric costs. The marginal social cost/benefit of public funds is important for our results. We also show the welfare implications and Cournot-Bertrand welfare reversal.

Fershtman and Judd (1987) showed that incentive delegation by the owners to the managers increases (decreases) welfare under Cournot (Bertrand) competition by encouraging the managers to produce more (less) outputs compared to no incentive delegation. In contrast, there is no incentive delegation in our paper but the owners of firms hold non-controlling shares in the rival firms. Cross ownership can increase consumer surplus and welfare under both Cournot and Bertrand competition by reducing the tax rates.

Hamilton and Requate (2004) consider the "third country" model of Brander and Spencer (1985) with domestic polluting input markets and vertical contracts between the domestic input suppliers and domestic final goods producers. Vertical contracts in their paper eliminate the need for export promotion policies to improve welfare of the countries. In contrast, there are no vertical contracts, no pollution, and no competition between different governments in our paper. In our analysis, cross ownership between the final goods producers reduce the tax rate to improve welfare.

The remainder of the paper is organized as follows. Section 2 describes the model under Cournot competition and shows the results. Section 3 shows the results under Bertrand competition. Section 4 provides Cournot-Bertrand welfare comparison. Section 5 concludes. The case of asymmetric cross ownership under Cournot competition is in the Online Appendix.

## 2 The Model: Cournot Competition

Assume that firms 1 and 2 produce horizontally differentiated products and compete in quantities. Each firm holds $\alpha \in [0, 0.5]$ fraction of shares in the rival firm.

Consider the utility function of a representative consumer as $U = U(q_1, q_2) + \xi$, where $q_1$ and $q_2$ are the outputs of firms 1 and 2 respectively and $\xi$ is the numéraire good. The utility maximization gives the inverse demand functions for firms 1 and 2 as $P_1(q_1, q_2)$ and $P_2(q_1, q_2)$ respectively. Assume $\frac{\partial P_1}{\partial q_1} = \frac{\partial P_2}{\partial q_2} < 0$ and $\frac{\partial P_1}{\partial q_2} = \frac{\partial P_2}{\partial q_1} < 0$ (i.e., the price of a firm's product decreases with higher outputs of that firm and the rival firm and the relationship is symmetric for both firms), $\frac{\partial^2 P_1}{\partial q_1^2} = \frac{\partial^2 P_2}{\partial q_2^2} \leq 0$ (i.e., the demand curves are concave), $\frac{\partial^2 P_1}{\partial q_1 \partial q_2} = \frac{\partial^2 P_2}{\partial q_2 \partial q_1} \leq 0$ (i.e., higher output of the rival firm either decreases or does not affect the slope of a firm's demand function),[2] $\left|\frac{\partial P_1}{\partial q_2}\right| = \left|\frac{\partial P_2}{\partial q_1}\right| \le \left|\frac{\partial P_1}{\partial q_1}\right| = \left|\frac{\partial P_2}{\partial q_2}\right|$ and $\left|\frac{\partial^2 P_1}{\partial q_1 \partial q_2}\right| = \left|\frac{\partial^2 P_2}{\partial q_2 \partial q_1}\right| \le \left|\frac{\partial^2 P_1}{\partial q_1^2}\right| = \left|\frac{\partial^2 P_2}{\partial q_2^2}\right|$ (i.e., the own effects are stronger than the cross effects). Assume that both firms have the same constant marginal cost of production, $c$.

We consider the following game. Given the cross ownership, the government determines the tax rate $t$ (subsidy rate if $t$ is negative) in the first stage to maximize social welfare. In the second stage, firms choose their outputs simultaneously and the profits are realized. We solve the game through backward induction.

In the second stage, firms 1 and 2 maximize the following expressions to determine $q_1$ and $q_2$ respectively:

$$\pi_1 = (1-\alpha)\left[P_1(q_1, q_2) - c - t\right]q_1 + \alpha\left[P_2(q_1, q_2) - c - t\right]q_2 \tag{1}$$

$$\pi_2 = \alpha\left[P_1(q_1, q_2) - c - t\right]q_1 + (1-\alpha)\left[P_2(q_1, q_2) - c - t\right]q_2. \tag{2}$$

The equilibrium outputs are determined by differentiating (1) and (2) with respect to $q_1$ and $q_2$ respectively and solving the first order conditions. However, given our symmetric structure, the symmetric equilibrium outputs, $q$, can be found from the following first order condition that is created by differentiating (1) by $q_1$, and using the conditions $q_1 = q_2 = q$ and $\frac{\partial P_1}{\partial q_2} = \frac{\partial P_2}{\partial q_1}$:

$$F\left(q(t,\alpha), t, \alpha\right) = (1-\alpha)\left[P_1 - c - t + q\frac{\partial P_1}{\partial q_1}\right] + \alpha q \frac{\partial P_1}{\partial q_2} = 0, \tag{3}$$

where $\left(P_1 - c - t\right) > 0$ from the firms' profit maximization problem. Using the implicit function theorem, we get the following result.

**Lemma 1:** $\frac{\partial q}{\partial \alpha} = -\frac{-(P_1-c-t)+q\left(\frac{\partial P_1}{\partial q_2} - \frac{\partial P_1}{\partial q_1}\right)}{\Omega} < 0$ *and* $\frac{\partial q}{\partial t} = \frac{1-\alpha}{\Omega} < 0,$ *where* $\Omega = 2(1-\alpha)\frac{\partial P_1}{\partial q_1} + \frac{\partial P_1}{\partial q_2} + q\left[(1-\alpha)\frac{\partial^2 P_1}{\partial q_1^2} + \alpha\frac{\partial^2 P_1}{\partial q_2^2} + \frac{\partial^2 P_1}{\partial q_1 \partial q_2}\right] < 0.$

*Proof:* We get from (3),

$$\frac{\partial q}{\partial \alpha} = -\frac{\frac{\partial F}{\partial \alpha}}{\frac{\partial F}{\partial q}} = -\frac{-\left(P_1 - c - t\right) + q\left(\frac{\partial P_1}{\partial q_2} - \frac{\partial P_1}{\partial q_1}\right)}{\Omega} < 0,$$

2 The condition $\frac{\partial^2 P_1}{\partial q_1 \partial q_2} = \frac{\partial^2 P_2}{\partial q_2 \partial q_1} \le 0$ is sufficient for strategic substitutability, i.e., for $\frac{\partial^2 \pi_i}{\partial q_i \partial q_j} < 0$, $i, j = 1, 2, i \ne j$, since, e.g., $\frac{\partial^2 \pi_1}{\partial q_1 \partial q_2} = (1-\alpha)\left(\frac{\partial P_1}{\partial q_2} + q_1\frac{\partial^2 P_1}{\partial q_1 \partial q_2}\right) + \alpha\left(\frac{\partial P_2}{\partial q_1} + q_2\frac{\partial^2 P_2}{\partial q_2 \partial q_1}\right) < 0$ for $\frac{\partial^2 P_1}{\partial q_1 \partial q_2} = \frac{\partial^2 P_2}{\partial q_2 \partial q_1} \le 0$.

and

$$\frac{\partial q}{\partial t} = -\frac{\frac{\partial F}{\partial t}}{\frac{\partial F}{\partial q}} = \frac{1-\alpha}{\Omega} < 0.$$

■

The results shown in Lemma 1, i.e., a higher marginal cost as well as a higher percentage of cross ownership reduce the equilibrium outputs, are known. However, these results indicate that a higher percentage of cross ownership may increase consumer surplus and welfare if it reduces the tax rate. We will show below the required condition for this to happen.

Now look at the first stage where the government sets the tax rate. With the symmetric equilibrium outputs, we can write the utility function as $U = u(q) + \xi$, where $u(q) = U(q, q)$. Hence, with the symmetric equilibrium outputs, welfare is $W = u - 2cq + 2(\lambda - 1)tq$, where following Neary and Leahy (2004) and Liu, Mukherjee, and Wang (2015), $\lambda$ represents the marginal social cost/benefit of public funds, and it can be higher or lower than unity due to the reasons mentioned in the introduction.

The welfare maximizing tax rate is determined from

$$\frac{\partial W}{\partial t} = G(t(\alpha), \alpha) = 2(\lambda - 1)q - \left[2(c + t - t\lambda) - \frac{\partial u}{\partial q}\right]\frac{\partial q}{\partial t} = 0. \tag{4}$$

Given the symmetric equilibrium outputs, we get $\frac{\partial u}{\partial q} = 2\frac{\partial U}{\partial q} = 2P_1 = 2P_2$ from the utility maximization problem. Hence, the expression (4) can be written as:

$$\frac{\partial W}{\partial t} = G(t(\alpha), \alpha) = 2(\lambda - 1)q + 2(P_1 - c - t + t\lambda)\frac{\partial q}{\partial t} = 0. \tag{5}$$

It follows from (5) that $t < 0$ for $\lambda \leq 1$, but $t$ can be positive for $\lambda > 1$.

We assume that the second order condition for welfare maximization holds, i.e.,

$$\frac{\partial^2 W}{\partial t^2} = 2\frac{\partial q}{\partial t}\left[2(\lambda - 1) + \frac{\partial P_1}{\partial q}\frac{\partial q}{\partial t}\right] + 2(P_1 - c - t + t\lambda)\frac{\partial^2 q}{\partial t^2} = \Upsilon < 0. \tag{6}$$

We assume that $\frac{\partial^2 q}{\partial t^2}$ is small so that $\frac{\partial q}{\partial t} + t\frac{\partial^2 q}{\partial t^2} < 0$. Hence, we get $\Upsilon < 0$ for $\lambda > \frac{\frac{\partial q}{\partial t}\left(2 - \frac{\partial P_1}{\partial q}\frac{\partial q}{\partial t}\right) - (P_1 - c - t)\frac{\partial^2 q}{\partial t^2}}{2\frac{\partial q}{\partial t} + t\frac{\partial^2 q}{\partial t^2}} = \lambda^C$, which is assumed to hold.

Using the implicit function theorem, we get from (5)

$$\frac{\partial t}{\partial \alpha} = -\frac{\frac{\partial G}{\partial \alpha}}{\frac{\partial G}{\partial t}} = -\frac{2\left[\left(\lambda - 1 + \frac{\partial q}{\partial t}\frac{\partial P_1}{\partial q}\right)\frac{\partial q}{\partial \alpha} + (P_1 - c - t + t\lambda)\frac{\partial^2 q}{\partial t \partial \alpha}\right]}{\Upsilon}, \tag{7}$$

where $\frac{\partial G}{\partial t} = \Upsilon < 0$ due to the second order condition of welfare maximization.

The above discussion gives the following result immediately.

**Lemma 2:** *A higher percentage of cross ownership reduces the equilibrium tax rate, i.e.,* $\frac{\partial t}{\partial \alpha} < 0$ *for* $\frac{\partial G}{\partial \alpha} < 0$.[3]

The above result suggests that a higher percentage of cross ownership reduces the tax rate, i.e., $\frac{\partial t}{\partial \alpha} < 0$, if its direct impact on the marginal effect of taxation on welfare is negative, i.e., $\frac{\partial G}{\partial \alpha} < 0$. Since a higher percentage of cross ownership reduces the output, i.e., $\frac{\partial q}{\partial \alpha} < 0$, it reduces the tax rate, i.e., $\frac{\partial t}{\partial \alpha} < 0$, for $\lambda > \lambda^C$ if $\frac{\partial^2 q}{\partial t^2}$ and $\frac{\partial^2 q}{\partial t \partial \alpha}$ are small, regardless of the signs of $\frac{\partial^2 q}{\partial t^2}$ and $\frac{\partial^2 q}{\partial t \partial \alpha}$.

We consider $\frac{\partial t}{\partial \alpha} < 0$ in the following analysis. Since a higher percentage of cross ownership reduces the equilibrium output for a given tax rate and also reduces the equilibrium tax rate, it is intuitive that it increases the total profits of the firms by reducing competition in the product market and reducing the marginal costs of production. Hence, cross ownership is profitable whenever it reduces the equilibrium tax rate.

Now consider the effects of a higher percentage of cross ownership on the equilibrium consumer surplus and welfare. First, look at the effects on consumer surplus. The equilibrium consumer surplus is $CS^C = u\big(q(t(\alpha), \alpha)\big) - 2P_1\big(q(t(\alpha), \alpha)\big)$. We get

$$\frac{\partial CS^C}{\partial \alpha} = -2q\frac{\partial P_1}{\partial q}\left(\frac{\partial q}{\partial \alpha} + \frac{\partial t}{\partial \alpha}\frac{\partial q}{\partial t}\right). \tag{8}$$

Since $\frac{\partial P_1}{\partial q} < 0$, the following result follows immediately from (8).

**Proposition 1:** *If* $\left(\frac{\partial q}{\partial \alpha} + \frac{\partial t}{\partial \alpha}\frac{\partial q}{\partial t}\right) > 0$, *a higher percentage of cross ownership increases consumer surplus.*

As mentioned in the introduction, a higher percentage of cross ownership tends to reduce consumer surplus by reducing the equilibrium output, i.e., due to $\frac{\partial q}{\partial \alpha} < 0$, but it tends to increase consumer surplus by reducing the tax rate, which helps to increase the output, i.e., due to $\frac{\partial t}{\partial \alpha} < 0$ and $\frac{\partial t}{\partial \alpha}\frac{\partial q}{\partial t} > 0$. Hence, a higher percentage of cross ownership helps to increase consumer surplus if the second effect is stronger than the first effect, thus creating $\left(\frac{\partial q}{\partial \alpha} + \frac{\partial t}{\partial \alpha}\frac{\partial q}{\partial t}\right) > 0$.

3 If $\lambda = \lambda^C$, we get $\left(\lambda - 1 + \frac{\partial q}{\partial t}\frac{\partial P_1}{\partial q}\right) = \frac{\frac{\partial q}{\partial t}\frac{\partial P_1}{\partial q}\left(\frac{\partial q}{\partial t} + t\frac{\partial^2 q}{\partial t^2}\right) - (P_1 - c)\frac{\partial^2 q}{\partial t^2}}{2\frac{\partial q}{\partial t} + t\frac{\partial^2 q}{\partial t^2}} > 0$ for small $\frac{\partial^2 q}{\partial t^2}$. Hence, $\frac{\partial G}{\partial \alpha} < 0$ if $\frac{\partial^2 q}{\partial t^2}$ and $\frac{\partial^2 q}{\partial t \partial \alpha}$ are small.

Now consider the effect on welfare. The equilibrium welfare is $W^C = u\big(q(t(\alpha),\alpha)\big) - 2cq(t(\alpha),\alpha) + \big(\lambda - 1\big)2t(\alpha)q(t(\alpha),\alpha)$. We get

$$\frac{\partial W^C}{\partial \alpha} = 2q\big(\lambda-1\big)\frac{\partial t}{\partial \alpha} + 2\big(P_1 - c - t + t\lambda\big)\left(\frac{\partial q}{\partial \alpha} + \frac{\partial t}{\partial \alpha}\frac{\partial q}{\partial t}\right). \tag{9}$$

Using (5), we get from (9), $\frac{\partial W^C}{\partial \alpha} = -\frac{2q(\lambda-1)\frac{\partial q}{\partial \alpha}}{\frac{\partial q}{\partial t}}$. Since $\frac{\partial q}{\partial \alpha} < 0$ and $\frac{\partial q}{\partial t} < 0$, we get $\frac{\partial W^C}{\partial \alpha} > (<)0$ for $\lambda < (>)1$. Hence, the following result is immediate.

**Proposition 2:** *Cross ownership increases (decreases) welfare if $\lambda < (>)1$.*

The above result shows that cross ownership increases welfare if $\lambda < 1$, and it happens since cross ownership helps to increase the output by reducing the tax rate.

The roles of the strategic tax policy captured by $\frac{\partial t}{\partial \alpha}$ are clear from (8) and (9). With no strategic tax policy, i.e., if the government does not adjust the tax rate depending on the extent of cross ownership, we get $\frac{\partial t}{\partial \alpha} = 0$. In this situation, both $\frac{\partial CS^C}{\partial \alpha} < 0$ and $\frac{\partial W^C}{\partial \alpha} < 0$.

We have shown the above results under symmetric cross ownership. We show in the Online Appendix that the consumer surplus and welfare raising cross ownership can occur even under asymmetric cross ownership. We show it by considering a situation where only one firm holds non-controlling shares in the other firm.

# 3 Bertrand Competition

Now we consider a game similar to Section 2 with the exception that competition in the product market is characterised by Bertrand competition. Since the procedure is similar to that of in Section 2, we will skip the mathematical details and will mention the condition for the consumer surplus and welfare raising cross ownership.

Assume that the demand functions for firms 1 and 2 are respectively $Q_1\big(p_1, p_2\big)$ and $Q_2\big(p_1, p_2\big)$ with $\frac{\partial Q_i}{\partial p_i} < 0$ and $\frac{\partial Q_i}{\partial p_j} < 0$, $i, j = 1, 2, i \neq j$. Firms 1 and 2 maximize $\pi_1 = (1-\alpha)\big(p_1 - c - t\big)Q_1 + \alpha\big(p_2 - c - t\big)Q_2$ and $\pi_2 = \alpha\big(p_1 - c - t\big)Q_1 + (1-\alpha)\big(p_2 - c - t\big)Q_2$ respectively to determine their prices. Under a symmetric cross ownership, we will get the symmetric prices, $p$, and outputs, $Q$, in equilibrium, with $\frac{\partial Q}{\partial p} < 0$.

Following the procedure of Section 2, we can find from the first order conditions of profit maximization that $\frac{\partial p}{\partial \alpha} > 0, \frac{\partial p}{\partial t} > 0$. We can get from the first order condition of welfare maximization the condition for $\frac{\partial t}{\partial \alpha} < 0$. It is intuitive that cross ownership is profitable with $\frac{\partial p}{\partial \alpha} > 0$ and $\frac{\partial t}{\partial \alpha} < 0$.

We find $\frac{\partial CS^B}{\partial \alpha} > 0$ for $-2Q\left(\frac{\partial p}{\partial \alpha} + \frac{\partial t}{\partial \alpha}\frac{\partial p}{\partial t}\right) > 0$, which happens for $\frac{\partial t}{\partial \alpha} < 0$ and $\left(\frac{\partial p}{\partial \alpha} + \frac{\partial t}{\partial \alpha}\frac{\partial p}{\partial t}\right) < 0$, i.e., if the effect of a higher percentage of cross ownership due to a lower tax dominates the effect of a higher percentage of cross ownership due to a higher price. Hence, the basic reason for the consumer surplus raising cross ownership under Bertrand competition is similar to that of under Cournot competition.

We find $\frac{\partial W^B}{\partial \alpha} > 0$ if $2(\lambda - 1)Q\frac{\partial t}{\partial \alpha} + 2(p - c - t + t\lambda)\frac{\partial Q}{\partial p}\left(\frac{\partial p}{\partial \alpha} + \frac{\partial t}{\partial \alpha}\frac{\partial p}{\partial t}\right) > 0$. With $\frac{\partial t}{\partial \alpha} < 0$ and $\left(\frac{\partial p}{\partial \alpha} + \frac{\partial t}{\partial \alpha}\frac{\partial p}{\partial t}\right) < 0$, we get $\frac{\partial W^B}{\partial \alpha} > 0$ for $\lambda < 1$.

Hence, the results for the consumer surplus and welfare raising symmetric cross ownership are the same under Bertrand and Cournot competition. Like Cournot competition, the effects of the strategic tax policy, capturing $\frac{\partial t}{\partial \alpha} < 0$, is responsible for our results.

## 4 Welfare Comparison Under Cournot and Bertrand Competition

Since a general welfare comparison is difficult, we use a specific demand function to compare welfare under Cournot and Bertrand competition. Assume that firm 1 and firm 2 face the demand functions $P_1 = 1 - q_1 - \gamma q_2$ and $P_2 = 1 - q_2 - \gamma q_1$ respectively, where $\gamma \in [0, 1]$ is the degree of product differentiation. The products are homogeneous (isolated) for $\gamma = 1$ ($\gamma = 0$). To avoid Bertrand paradox and to consider competition between the firms, here we consider $\gamma \in (0, 1)$.

Given these demand functions, we can derive the following result.

**Proposition 3:** *Consider $\lambda^C < \lambda$. We get $(t^C - t^B) < 0$, and the equilibrium consumer surplus and welfare are higher under Cournot (Bertrand) competition for $\lambda^C < \lambda < 1(1 < \lambda)$.*

*Proof:* We get $\frac{\partial^2 W}{\partial t^2} < 0$ for $\frac{3-3\alpha+\gamma+\alpha\gamma}{4-4\alpha+2\gamma} = \lambda^C < \lambda$ $\left(\frac{3-3\alpha-2\gamma+\alpha\gamma}{4-4\alpha-2\gamma} = \lambda^B < \lambda\right)$ under Cournot (Bertrand) competition. It can be found that $\lambda^B < \lambda^C$. So, consider $\lambda^C < \lambda$.

We get $(t^C - t^B) = -\frac{(1-c)(1-2\alpha)\gamma^2\lambda}{\psi\omega} < 0$ for $\lambda^C < \lambda$,

$$\left(CS^C - CS^B\right) = \frac{4(1-2\alpha)(1-c)^2\gamma^2(1-\lambda)\lambda^2\sigma}{(1+\gamma)\psi^2\omega^2} > (<)0 \quad \text{for} \quad \lambda^C < \lambda < 1(1 < \lambda),$$

$$\left(W^C - W^B\right) = \frac{2(1-2\alpha)(1-c)^2\gamma^2\left(1-\lambda\right)\lambda^2}{(1+\gamma)\psi\omega} > (<)0 \quad \text{for} \quad \lambda^C < \lambda < 1\left(1 < \lambda\right),$$

since

$$\psi = 2\gamma\left(1-\lambda\right) + 4\lambda + \alpha\left(3-\gamma-4\lambda\right) - 3 > 0,$$

$$\omega = 2(2+\gamma)\lambda + \alpha\left(3-\gamma-4\lambda\right) - 3 - \gamma > 0,$$

and

$$\begin{aligned}\sigma = &\left(4\lambda - \alpha^2(1+\gamma)\left(3-\gamma-4\lambda\right) - 3 - \gamma\left(1-\gamma-2\lambda+\gamma\lambda\right)\right.\\ &\left.+\alpha\left(6-8\lambda+\gamma\left(3-\gamma-6\lambda\right)\right)\right) > 0.\end{aligned}$$

■

The government provides lower subsidy (or imposes higher tax) under Bertrand competition compared to Cournot competition. This happens since competition is fierce under Bertrand competition, and therefore, the need for the subsidy to tackle the product market distortion is lower under Bertrand competition compared to Cournot competition. Further, we get $\left(t^C - t^B\right)$ reduces (or equivalently the difference in subsidies increases) with a lower $\lambda$. Hence, a significantly higher subsidy under Cournot competition compared to Bertrand competition makes the consumer surplus and welfare higher under Cournot competition compared to Bertrand competition for $\lambda^C < \lambda < 1$.

# 5 Conclusions

Although cross ownership creates output contraction and makes the policy makers concerned about its adverse welfare effects, previous research found several factors which might help to improve welfare in the presence of cross ownership. We provide a new reason for the consumer surplus and welfare raising cross ownership.

We show that a higher degree of cross ownership increases consumer surplus and welfare under strategic tax/subsidy policy if the marginal social cost/benefit of public funds is less than unity, which may occur if the government maximizes a political support function that is a weighted average of welfare and political contributions. Hence, under certain conditions, the policy makers may prefer to encourage cross ownership in the presence of tax/subsidy policies to benefit the consumers and the society.

Our results hold under both Cournot and Bertrand competition, suggesting that the policy makers' preference for cross ownership may not be affected by the type of competition. However, our analysis suggests that when cross ownership benefits the consumers and the society, the policy makers would prefer Cournot competition over Bertrand competition, i.e., they would prefer a product market characterized by less intense competition.

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