The development of young children's mathematical mark making

Janine Davenall, Ann Dowker, Helen J Williams, Catherine Gripton and Sue Gifford share their work on young children's mark-making.

Young children's mathematical mark-making (graphics) can help them along their mathematical journey. Symbols are important in mathematics but these prescribed and accepted means of communicating mathematically can often be opaque for children (Hughes 1986, Rycroft-Smith and Macey 2022) who communicate their mathematical thoughts in a range of other ways including through gesture, talk and mark-making. Making marks with their fingers, chalks, pencils and other tools is a common form of expression for younger children. Whilst these marks are often interpreted as pictures or writing, for the child they can be mathematical. Some examples of children's mark-making are shown in Figure 1. They show a range of responses to a task to record the number of beans under a pot using pen and paper and have been categorised using Hughes' (1986) classifications of idiosyncratic, pictographic, iconic and symbolic.

Mathematical mark-making, where the thinking behind the squiggles and lines is mathematical, can be on all kinds of surfaces such as plastic trays, mirrors, glass, wood and paper as well as in sand or foam or using technology such as mobile phones and tablets. They can show all aspects of mathematics including geometry (such as the route to school) and data (such as the number of children waiting for a turn with the bikes or scooters). These 'mathematical graphics' are as valuable to mathematical learning as mark-making is to writing (Worthington and Carruthers, 2003). It is a way to express and explore mathematical thinking. In their graphics the number of marks can be significant, as well as the shape, position and orientation. What is important is the mathematical meaning they hold for the child. The marks can act as a record to 'hold' and express mathematical thoughts so it is important to observe the process to gain understanding of the mathematics involved. A final record of four balls in a basket may show four lines, for example. The process the child has gone through to record these four marks might have been to write two, then one more and another, as the balls were thrown, so actually reproducing the activity of 2+1+1=4 in the child's mind. See Figure 1.

An apprenticeship approach to using numerals and standard symbols

Over time, the balance shifts from the use of the more informal marks (such as pictographic and iconic in Figure 1) to the more formal (using conventional symbols such as numerals), but the informal is not redundant. For all ages of child, their mark-making can continue to be a useful stepping-stone to learning something new in mathematics, as a helpful way of sense-making as they build new understanding and encounter new problems.

	B. recorded 1 bean (appears to be 'idiosyncratic' as the number and shape of the marks do not seem to relate to the number and shape of the objects
	G. recorded 3 beans (appears to be 'pictographic' as the size and shape of the marks are the same as the objects)
	K. recorded 4 beans (appears to be 'iconic' as there seems to be one-to-one correspondence of objects to marks and the marks are representations rather than pictures of the objects)
	P. recorded 5 beans (appears to be 'symbolic' as the single conventional symbol is used to represent the quantity of objects)

Figure 1: Children's recording of the number of beans under a pot.

Figure 2: How many hens are left when the naughty fox sneaks in and eats some?

Standard symbols (such as numerals and operation symbols) can be difficult for young learners to make sense of (Gifford 1997) but sensitive modelling by adults can encourage children to begin using them in their mathematical graphics. For example, by integrating addition and subtraction symbols into a story or game about adding and removing toys. Figure 2 (Davenall 2015) shows children's graphics where they have captured what is happening in a story when a fox is stealing hens. Here we see the child's understanding of subtraction represented with an emerging understanding of the subtraction symbol. See Figure 2.

With an apprenticeship approach, standard symbols are gradually and meaningfully introduced so the child has a clear rationale or purpose for using them and as an efficient way of representing an operation. In this way,

"… we can help children build meaningful links between the world of written symbols and the world of concrete reality." Hughes (1986: 134)

Through developing children's mathematical graphicacy, children are apprenticed into the way numbers are culturally represented, as well as developing a repertoire of ways of expressing themselves. This is a process that happens over time with the child gaining ownership of the use and understanding of symbols (Gifford 1997). There is a significant leap of understanding involved in confidently making *one* mark, '5', to stand for *five* separate objects and this rests upon a deep and broad range of mathematical experiences. It is therefore important to use graphics and manipulatives alongside standard notation and verbal explanation until children are at least six years of age and not rely on standard notation alone (Dowker 2005). Even older children do not consistently link standard notation to concrete representations (Rycroft-Smith and Macey 2022) and need to continue to use manipulatives and jottings as their understanding of standard notation develops. In order to build this firm foundation of mathematical graphicacy, young children (birth to eight years) need to be offered a range of contexts and invitations to express their mathematical thinking through mark-making.

Supporting children's graphical representations through talk, gesture, play and story contexts

Mathematical graphics offer opportunities for personal sense-making by providing a conceptual link between practical exploration and symbolic representation. The child is not providing a copy of the real world objects but a representation of "their interactions with the object" (Fosnot & Dolk 2001:78). Figure 3. shows a child's representation of a subtraction problem interaction with the hand taking away two cars (Davenall, 2015). The dynamic element is captured through the graphic.

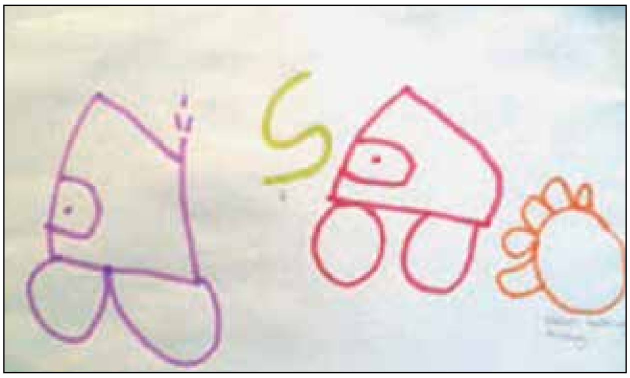

Figure 3. Drawing a hand to represent 'taking away' by a 5-year-old child.

Talking with children about their graphical representations supports children in rehearsing and refining their ideas. Mathematical graphics provide adults with an insight into the child's mathematical understanding if we watch and listen as children create graphics and ask them to interpret them for us. Children, as the experts in their own thinking, are able to help adults to understand their mathematical thinking through their graphics. Talking with children also often leads to the use of gesture and action, as children explain how they combined two quantities, for example. This is particularly powerful where the dramatic action of a story holds significant value for children leading to heightened emotion and energy in the children's explanations. Contexts that are meaningful for the child (and part of their world) are powerful.

Figure 4: Charity shop role play by a 6-year-old child.

Pretend play is a rich context for mathematical mark-making. Finding ways of writing price labels for shop items and till receipts for items sold is a purposeful and meaningful context to write numbers. A six-year-old, in charge of the charity shop role-play area can price items in whole pounds, for example, allowing them to successfully manage the addition of two or three items (Figure 4). Recording a journey or a route, in pretend play or when out and about, allows for representation of position, direction and location. This can provide valuable opportunities for early map-making that involves routes, perspective and scale. Children's recording of their block play is fascinating in showing how they perceive individual and grouped blocks within their construction and how they represent their three-dimensional model in two dimensions. For example, in Figure 5, a four-year-old has represented the three walls of their construction, paying careful attention to the shape of the blocks, the number of layers and the position of the people.

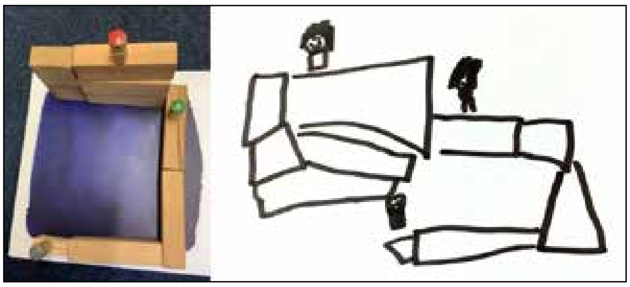

Figure 5: Block play with small figures by a four-year-old child.

A strong foundation for future learning

The depth of understanding that children gain through their mathematical graphicacy provides a strong foundation for future mathematical learning in three ways. Firstly, children build new knowledge by connecting and adapting existing understanding to deep understanding, where children can use and apply knowledge flexibly, providing a secure and stable basis for future learning. Secondly, deep understanding gained through mark making enables children to work autonomously, secure in the knowledge that they can record their process in a way that is meaningful to them. This allows them to check and track back, promoting confidence and resilience. Thirdly, mathematical mark making enables adults to access children's mathematical thinking in that there is a record of thinking that can be shared and discussed.

Where adults value children's early mathematical mark making, children are more confident in showing their working and sharing their thinking. This is a critical mathematical habit for children to carry forward into their future mathematics.

We suggest that an apprenticeship approach to introducing children to standard mathematical symbols could help reduce the likelihood of mathematical anxiety, which is often associated with formal approaches to teaching children to use standard symbols, by helping them to maintain confidence and ownership of their mathematical communication.

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