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Welfare reducing vertical integration in a bilateral monopoly under Nash bargaining

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Abstract

We consider a bilateral monopoly where a linear input price is determined by Nash bargaining. We show, with an increasing marginal cost of input production, that vertical integration reduces consumer surplus and welfare compared with bilateral monopoly if the bargaining power of the input supplier is low. This result is important for competition policies as it questions the common wisdom suggesting vertical integration increases welfare by eliminating the problem of double marginalization. Overproduction under bilateral monopoly compared with vertical integration is the reason for our result. Interestingly, consumer surplus and welfare can be higher under a linear input price compared with a two-part tariff input price.

KEYWORDS

bilateral monopoly, convex cost, overproduction, vertical integration, welfare

1 | INTRODUCTION

We consider a bilateral monopoly between an input supplier and a final good producer. Common wisdom suggests that vertical integration helps increase welfare compared with bilateral monopoly with a linear input price by eliminating the double marginalization problem (see, e.g., Arya et al., 2008; Colangelo, 1995; Economides, 1999; Häckner, 2003; Ordover et al., 1990; Salinger, 1988). In a bilateral monopoly with Nash bargaining for a linear input price, we show that this conclusion may not hold true if the marginal cost (MC) of input production is increasing. Although we are considering a setup with an input supplier and a final good producer, our analysis will also be valid for a bilateral monopoly between a manufacturer and a retailer that sells the product of the manufacturer.

Considering an industry with increasing MC of input production and Nash bargaining for a linear input price, we show that if the bargaining power of the input supplier is sufficiently low, the output under bilateral monopoly is higher than that of under vertical integration. Overproduction under bilateral monopoly compared with vertical integration creates the trade-off between a higher consumer surplus and lower total profits of the input supplier and the final good producer under the former than the latter. As a result, vertical integration may reduce welfare compared with a bilateral monopoly. It also follows from our analysis that a bilateral monopoly can achieve the level of output under vertical integration for a positive bargaining power of the input supplier.

Thus, our paper provides a new reason for the consumer surplus and welfare reducing vertical integration, which is important for competition policies. Our result is due to the rising MC of input production that is different from the factors mentioned in the extant literature for the welfare reducing vertical integration, such as vertical foreclosure (see Rey & Tirole, 2007; Riordan, 1998, for surveys of this literature), investments (Allain et al., 2016; Gilbert, 2006), and product differentiation (Chen, 2016; Matsushima, 2009; Zanchettin & Mukherjee, 2017).

We consider a generalized Nash bargaining model of input price determination, which is used in the literature to show the implications of vertical integration in comparison to a bilateral monopoly (e.g., Collard-Wexler et al., 2019; Horn & Wolinsky, 1988)[1] and in many other contexts (e.g., Aghadadashli et al., 2016; Björnerstedt & Stennek, 2007; Crawford et al., 2018; Davidson, 1988; Dobson & Waterson, 2007; Grout, 1984; Inderst & Montez, 2019; Iozzi & Valletti, 2014). As mentioned by Collard-Wexler et al. (2019), the Nash bargaining model "has become a workhorse bargaining model in applied analyses of bilateral oligopoly." Although the extant literature on vertical integration used the Nash bargaining solution to determine the input price in the context of a bilateral monopoly, to the best of our knowledge, that literature did not consider increasing the MC of input production, which is observed in many industries, as mentioned below.

Banker et al. (1994) show that decreasing returns may prevail in the software industry. The typical two-digit industries in the US (Basu & Fernald, 1997), and the three-digit manufacturing industries in Singapore (Kee, 2002) appear to have decreasing returns to scale. Using four-digit Standard Industrial Classification (SIC) data, Nguyen and Reznek (1991) show that small firms in certain industries, such as Women's, Misses', and Juniors' dresses, exhibit decreasing returns to scale. Many of these industries also experience vertical integration. For example, looking at the creator-publisher relationship in the book and software industries, which can benefit from digital distribution, MacInnes et al. (2004) find that software tends to be more vertically integrated than books. Lee (2013) shows the implications of vertically integrated software in the US video game industry. Richardson (1996) shows vertical integration in the fashion apparel

[1]Horn and Wolinsky (1988) considered vertical integration with a Nash bargained input price under a bilateral monopoly where the firms have equal bargaining power.

manufacturing. Gertner and Stillman (2001) analyze the strategy of vertically integrated retailers in the apparel industry compared with nonintegrated vendors. Tucker and Wilder (1977) use the SIC data to analyze the trend in vertical integration for the four-digit manufacturing industries.

We show that vertical integration may have adverse implications for the consumers and the society under a rising MC input production. However, bargaining over only the input price is important for our results. If there is bargaining over both the input price and the quantity of input, the equilibrium final good output under bilateral monopoly will be equal to the vertically integrated output (Dasgupta & Devadoss, 2002). In this situation, there will be no under or overproduction with bilateral monopoly compared with vertical integration.

Our paper can be related to Tintner (1939) and Fellner (1947), which following Bowley (1928) and Hicks (1935), formalized the analysis for bilateral monopoly that is widely used in the textbooks. These analyses suggest that if there is a bilateral monopoly bargaining for a linear input price, its output cannot be higher than the output of the vertically integrated firm; that is, the above-mentioned overproduction cannot occur.

We show that the above-mentioned result may not hold true in a Nash bargaining model of linear input price determination. The textbook view of a bilateral monopoly assumes that even if the final good producer determines the input price, the input supplier determines the amount of inputs by equating the MC of input production to the input price determined by the final good producer. Hence, the *final good producer faces a positively sloped supply curve for the input when the MC of input production is increasing*. As a result, in the textbook view, overproduction cannot occur since the input price there cannot be lower than the MC of input production at the equilibrium output. In contrast, in the Nash bargaining model of input price determination, the input supplier and the final good producer bargain for the input price, and after that, the final good producer purchases inputs as per its demand at the bargained input price. In other words, here, the final good producer faces an *infinitely elastic input supply at the bargained input price*. As a result, in our analysis, overproduction can occur since the input price here can be lower than the MC of input production at the equilibrium output. Hence, one needs to be careful about the input supplier's output determination process when considering the effects of a bilateral monopoly.

Spindler (1974) is an earlier critique of the textbook view of bilateral monopoly provided by Bowley (1928). It suggests that the bargained price would correspond to the joint profit-maximizing output of the input supplier and the final goods producer. Hence, it did not address the problem of overproduction discussed in our analysis.

It is shown in the literature that the presence of convex costs may often give rise to counterintuitive results in oligopoly. For example, Fuess and Loewenstein (1991) and Marjit and Mishra (2020) show that a steeper convex cost may increase the profits of the firms. Einy et al. (2002) show that a firm with superior information about market demand and cost may receive lower profit compared with a firm with inferior information about market demand and cost when there are convex costs (see their Example in p. 155). Amir (2003) shows that an increase in the number of firms may increase the total profits of the firms in the presence of convex costs (see Section 3 and Proposition 4 in his paper). Mukherjee (2023) shows that cross ownership in a duopoly final goods market can be unprofitable in the presence of convex costs when a monopolist input supplier determines the input prices for the final goods producers. In this paper, we provide some new counterintuitive results under convex cost in the context of vertical integration in a bilateral monopoly with Nash bargaining.

The remainder of the paper is organized as follows. Section 2 describes the model and derives the results. Section 3 concludes.

2 | THE MODEL AND THE RESULTS

Consider a bilateral monopoly with an input supplier, called firm U, and a final good producer, called firm D. Firm U produces a critical input that firm D uses to produce the final good. We assume for simplicity that one unit of input is required to produce one unit of the final good. Assume that if firm U produces q amount of input (which is also equal to the amount of the final good), the cost of input production is $C(q)$, with $C_q > 0$ and $C_{qq} > 0$, where the subscripts show the variables of successive differentiation. Thus, we assume a convex cost for input production.

Assume that the inverse market demand function for the final good is $P = P(q)$, with $P_q < 0$, where P and q represent the price and quantity of the final good, respectively.

We will consider the following two situations for our analysis:

(1) Vertical integration between firms U and D, where the vertically integrated firm produces both the input and the final good to maximize the total profits.
(2) Bilateral monopoly where firm U produces input and firm D produces the final good. A linear input price is determined through a generalized Nash bargaining between firms U and D, where the bargaining power of firm U is α and that of firm D is $(1 - \alpha)$, with $\alpha \in [0, 1]$. Firm U (D) has full bargaining power for $\alpha = 1$ ($\alpha = 0$).

2.1 | Vertical integration

The vertically integrated firm determines the amount of final good to maximize the joint profits of firms U and D. Hence, the vertically integrated firm maximizes the following expression:

$$\underset{q}{Max}\, P(q)q - C(q). \tag{1}$$

The equilibrium output is determined by the following first-order condition:

$$P^v + q^v P_q{}^v - C_q{}^v = 0, \tag{2}$$

where the superscript v stands for vertical integration.

We consider interior solution. Hence, we assume that the second-order condition for maximization holds, that is, $2P_q{}^v + qP_{qq}{}^v - C_{qq}{}^v < 0$.

2.2 | Bilateral monopoly

We consider the following game under bilateral monopoly. At stage 1, firms U and D bargain for the linear input price, w. At stage 2, firm D determines the amount of outputs to be produced and purchases the amount of inputs accordingly at the bargained input price. Then the profits are realized. We solve the game through backward induction.

Given the input price w, firm D maximizes $\underset{q}{Max}\,(P - w)q$. The equilibrium output of firm D is determined by the following first-order condition:

$$H(q(w), w) = P^b + q^b {P_q}^b - w = 0, \tag{3}$$

where the superscript b stands for bilateral monopoly.

We consider interior solution. Hence, we assume that the second-order condition holds, that is, $2{P_q}^b + q{P_{qq}}^b < 0$.

We get this by using the implicit function theorem

$$q_w^{b*} = \frac{-H_w}{H_q} = \frac{1}{2{P_q}^b + q{P_{qq}}^b} < 0. \tag{4}$$

Hence, if the input price, w, increases, it decreases the equilibrium output of firm D.

The equilibrium profits of firms D and U are, respectively, $\pi^D = (P(q^{b*}) - w)q^{b*} = (P^{b*} - w)q^{b*}$ and $\pi^U = wq^{b*} - C(q^{b*}) = wq^{b*} - C^{b*}$.

The input price is determined by maximizing $\underset{w}{Max}\; Z = \underset{w}{Max}\,(\pi^U)^\alpha(\pi^D)^{1-\alpha}$, since the disagreement profits of both firms are zero. The first-order condition for the above maximization is given by

$$Z_w = F(w(\alpha), \alpha) = \frac{\alpha\left(q^{b*} + \left(w - C_q^{b*}\right)q_w^{b*}\right)}{wq^{b*} - C^{b*}} - \frac{(1-\alpha)\left(q^{b*} - \left(P^{b*} - w + q^{b*}P_q^{b*}\right)q_w^{b*}\right)}{(P^{b*} - w)q^{b*}}$$
$$= 0$$

or

$$\alpha\pi^D\left(q^{b*} + \pi^u_{q^{b*}} q_w^{b*}\right) - (1-\alpha)\pi^U q^{b*} = 0, \tag{5}$$

which gives the equilibrium input price. The second-order condition for maximization is assumed to hold, that is, $Z_{ww} = F_w < 0$.[2]

Proposition 1. *(i) If firm U has full bargaining power, that is,* $\alpha = 1$, *the equilibrium input price under bilateral monopoly is above the corresponding MC of input production.*

(ii) If firm D has full bargaining power, that is, $\alpha = 0$, *the equilibrium input price under bilateral monopoly is below the corresponding MC of input production.*

(iii) There exists a threshold bargaining power α^* *such that for* $\alpha \in [0, \alpha^*)$ $(\alpha \in (\alpha^*, 1])$, *the equilibrium input price under bilateral monopoly is lower (higher) than the MC of input*

[2] $\underset{w}{Max}\; Z$ is subject to $\pi^U \geq 0$ and $\pi^D \geq 0$. Since we work under (3), it satisfies $\pi^D \geq 0$. Further, it follows from Proposition 1 that the minimum equilibrium input price $w^{b*} = \frac{C^{b*}}{q^{b*}}$ is at $\alpha = 0$, satisfying $\pi^U \geq 0$.

production at the equilibrium output under vertical integration and the equilibrium output under bilateral monopoly is higher (lower) than that of under vertical integration.
Proof. *See the appendix.* □

Proposition 1(iii) shows over(under)production under bilateral monopoly compared with vertical integration, that is, $q^{b*} > (<) q^{v*}$, for $\alpha \in [0, \alpha^*)$ ($\alpha \in (\alpha^*, 1]$).[3]

If the average cost (AC) of input production is constant, we have the AC of input production equals to the MC of input production. Hence, it follows from Proposition 1(ii) that the equilibrium input price cannot be lower than the corresponding MC of input production, implying that overproduction cannot occur in this situation. Further, Proposition 1 suggests that the firms produce the vertically integrated output under bilateral monopoly at α^*, and this does not happen under constant AC of input production.

Since vertical integration maximizes the total profits of firms U and D, the following result follows immediately from Proposition 1.

Corollary 1. *Bilateral monopoly reduces the total profits of firms* U *and* D *compared with vertical integration by creating over(under)production for* $\alpha \in [0, \alpha^*)$ ($\alpha \in (\alpha^*, 1]$).

While underproduction under bilateral monopoly compared with vertical integration is well known, overproduction under bilateral monopoly is a new result shown above. Overproduction happens in our structure since firm D can use as much input as it wants at the Nash bargained linear input price. In other words, the final good producer faces an infinitely elastic supply of inputs at the bargained input price. Hence, if the bargaining power of firm U is low, it creates an equilibrium input price that is lower than the MC of input production at the vertically integrated output level. As a result, firm D's profit-maximizing output at this low input price goes above the vertically integrated output level.

Since consumer surplus is positively related to final good production, it is immediately apparent that overproduction under bilateral monopoly creates higher consumer surplus than vertical integration.

Hence, overproduction under bilateral monopoly creates a trade-off on welfare. On the one hand, it tends to increase welfare under bilateral monopoly compared with vertical integration by increasing consumer surplus. On the other hand, it tends to decrease welfare under bilateral monopoly compared with vertical integration by reducing the total profits of firms U and D.

It is intuitive that the first effect dominates the second effect to create higher welfare under bilateral monopoly compared with vertical integration if overproduction under bilateral monopoly takes the equilibrium final good production towards the welfare maximization output (that eliminates the deadweight loss of monopoly) but does not cross it. In other words, a higher output under a bilateral monopoly increases welfare compared with vertical integration by reducing the deadweight loss of monopoly if the output under a bilateral monopoly is lower than the welfare maximizing output. However, if the output under a bilateral monopoly is higher than the welfare maximizing output, overproduction under a bilateral monopoly may reduce welfare under a bilateral monopoly compared with vertical integration.

Formally, welfare in our analysis is $W = \int_0^q P(q)\,dq - C(q)$. Hence, $W_q = P - C_q$ and $W_{qq} = P_q - C_{qq} < 0$, implying that the welfare function is concave and the welfare maximizing

[3]At α^*, the levels of production under bilateral monopoly and vertical integration would be the same.

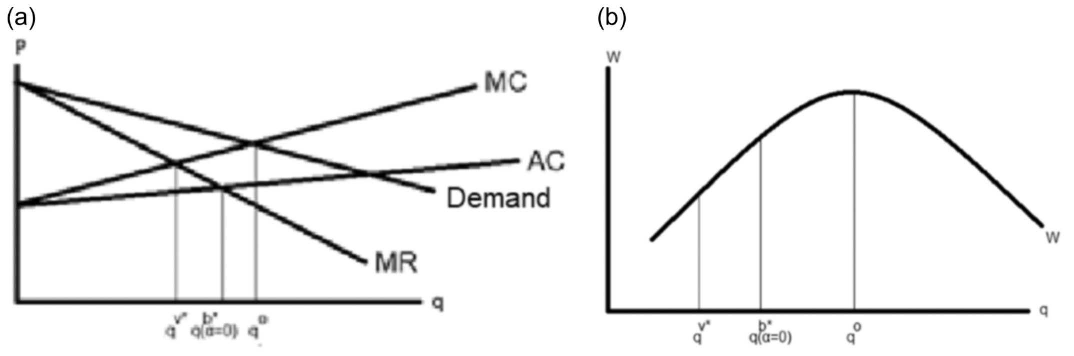

FIGURE 1 Overproduction: lower welfare under vertical integration. (a) outputs, (b) welfare implication. AC, average cost; MC, marginal cost; MR, marginal revenue; W, welfare.

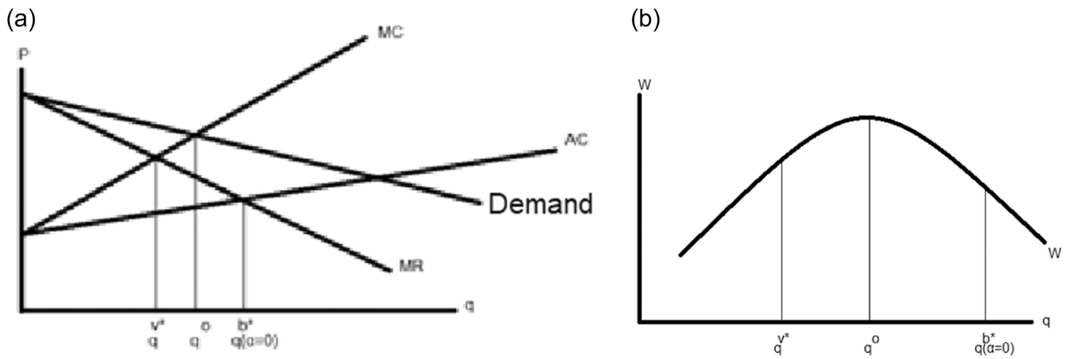

FIGURE 2 Overproduction: higher welfare under vertical integration. (a) outputs, (b) welfare implication. AC, average cost; MC, marginal cost; MR, marginal revenue; W, welfare.

output, denoted by q^o, is given by $P = C_q$. Since the equilibrium output under vertical integration, q^{v*}, is less than q^o,[4] an output between q^{v*} and q^o will increase welfare compared with vertical integration. Hence, bilateral monopoly will always create higher welfare compared with vertical integration if $q^{v*} < q^{b*}(\alpha = 0) < q^o$, where $q^{b*}(\alpha = 0)$ is the equilibrium output under bilateral monopoly for $\alpha = 0$.[5] However, if $q^{v*} < q^o < q^{b*}(\alpha = 0)$, overproduction under bilateral monopoly may not create higher welfare compared with vertical integration in this situation.[6]

Figures 1 and 2 show overproduction under bilateral monopoly and the corresponding welfare implications for $\alpha = 0$. In panel (a) of both figures, we draw the demand curve (Demand) for the final good, the marginal revenue curve for the final good (MR), the MC curve

[4]This happens since we can say by using (2) that $W_q = P - C_q$ at q^{v*} is $W_q = -qP_q > 0$.

[5]Note that the output under bilateral monopoly is maximum at $\alpha = 0$ since it creates a minimum input price.

[6]Evaluating $W_q = P - C_q$ at $q^{b*}(\alpha = 0)$, we get $W_q = P - C_q = \frac{C}{q} - C_q - qP_q$, since $P = w - qP_q$ from (3) and $w = \frac{C}{q}$ at $q^{b*}(\alpha = 0)$. If $\left(\frac{C}{q} - C_q\right) = 0$, that is, the cost function is linear, we get $W_q > 0$, implying $q^{b*}(\alpha = 0) < q^o$ in this situation. If the cost function is sufficiently convex so that $\left(\frac{C}{q} - C_q\right)$ is sufficiently negative to create $\left(\frac{C}{q} - C_q\right) < qP_q$, we get $W_q < 0$ and, therefore, $q^o < q^{b*}(\alpha = 0)$ in this situation.

for input production, and the AC curve for input production, as linear, for simplicity. In panel (b) of both figures, the curve W shows welfare. Hence, in both figures, the equilibrium outputs under vertical integration are created by MR = MC, the equilibrium outputs under bilateral monopoly are created by MR = AC (since we are looking at $\alpha = 0$), and the equilibrium welfare maximizing outputs are created by Demand = MC.

Figure 1 shows overproduction under bilateral monopoly creating higher welfare compared with vertical integration. Figure 2 shows the possibility of lower welfare under a bilateral monopoly compared with vertical integration, even if a bilateral monopoly creates overproduction.

The following proposition summarizes the above discussions.

Proposition 2. *If there is overproduction under bilateral monopoly compared with vertical integration, vertical integration reduces consumer surplus and vertical integration always reduces welfare if the output under bilateral monopoly is not higher than the welfare maximizing output.*

3 | CONCLUSION

In a bilateral monopoly with a Nash bargained linear input price and increasing MC of input production, we show that vertical integration may reduce consumer surplus and welfare compared with a bilateral monopoly. This happens when the bargaining power of the input supplier is sufficiently low, leading to a lower input price that creates overproduction under bilateral monopoly compared with vertical integration. This is in contrast to the extant literature with constant AC of input production, where overproduction does not occur under a bilateral monopoly.

Our analysis also shows that there is a positive bargaining power at which the bilateral monopoly produces the vertically integrated output, implying that the minimum bargaining power of the upstream firm is not desirable for creating this output. This is in contrast to the extant literature with constant AC of input production, where the minimum bargaining power of the input supplier creates the vertically integrated output.

We have compared bilateral monopoly with vertical integration. A related issue is to see the effects of a higher bargaining power on the final good producer. In our analysis, a low bargaining power of the input supplier creates higher output under bilateral monopoly compared with vertical integration. Hence, if the bargaining power of the input supplier is low, a further increase in the bargaining power of the final good producer may reduce welfare by reducing the total profits of the firms, but it always increases consumer surplus, as it always increases final good production.

We contribute to the literature on linear input pricing and the double marginalization problem. It can be verified easily that over or underproduction does not occur if we have considered a two-part tariff input price, since the firms choose the per-unit input price to achieve the vertically integrated output level and share the surplus through the fixed-fee. However, the fixed-fee can be positive or negative depending on the bargaining power. An interesting implication of this result is that consumer surplus and welfare under a linear input price can be higher than that of a two-part tariff input price if the output under a bilateral monopoly with a linear input price is higher than the output under vertical integration.

We have considered interior solutions for our analysis, which require a downward-sloping MR curve under a bilateral monopoly. If we consider a demand function creating an upward-sloping MR curve, we can get an interior solution under vertical integration, but we will get a corner solution under bilateral monopoly since the final good producer will be able to increase its profit by producing up to the demand at the bargained input price. Hence, overproduction under bilateral monopoly occurs in this situation as long as the bargained input price is lower than the MR.

Finally, we have considered a bilateral monopoly for our analysis. However, our results will go through under bilateral oligopoly where input suppliers specific to the final good producers and the respective final good producers bargain for the input prices under no vertical integration, and in the case of vertical integration, the respective pairs of input suppliers and final good producers integrate.[7]

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DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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[7]The details of this analysis can be found by the authors upon request.

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APPENDIX

Proof of Proposition 1. (i) If $\alpha = 1$, we get $\pi^D\left(q^{b*} + \pi^u_{q^{b*}} q^{b*}_w\right) = 0$, which implies $q^{b*} + \pi^u_{q^{b*}} q^{b*}_w = 0$ or $w^{b*} = C^{b*}_q - \frac{q^{b*}}{q^{b*}_w} > C^{b*}_q$.

(ii) If $\alpha = 0$, we get $\pi^U q^{b*} = 0$, implying $w^{b*} q^{b*} - C^{b*} = 0$ or $w^{b*} = \frac{C^{b*}}{q^{b*}} < C^{b*}_q$, since the MC is higher than the AC for convex cost functions.

(iii) Using (5) and (3), we get

$$F_\alpha = \frac{q^{b*} + \left(w^{b*} - C^{b*}_q\right) q^{b*}_w}{q^{b*} w^{b*} - C^{b*}} + \frac{1}{(P^{b*} - w^{b*})} = \frac{1}{\alpha (P^{b*} - w^{b*})} = \frac{1}{-\alpha q^{b*} P^{b*}_q} > 0.$$

Hence, using the implicit function theorem, we get $w_\alpha = -\frac{F_\alpha}{F_w} > 0$, since $F_w < 0$. Since w^{b*} is continuous, increasing in $\alpha \in [0, 1]$, and greater (lower) than C^{b*}_q at $\alpha = 1$ ($\alpha = 0$), there will be an $\alpha = \alpha^*$ so that $w^{b*} = C^{b*}_q$ at α^*. However, it follows from (2) and (3) that the equilibrium output under bilateral monopoly will be equal to that of under vertical integration if $w = C_q$. Hence, if $\alpha = \alpha^*$, the equilibrium output under bilateral monopoly will be equal to that of under vertical integration, and the equilibrium input price under bilateral monopoly will be equal to the MC at the equilibrium output under vertical integration. It is then immediate that if $\alpha \in [0, \alpha^*)$ ($\alpha \in (\alpha^*, 1]$), the equilibrium input price under bilateral monopoly is lower (higher) than the MC at the equilibrium output under vertical integration, and the equilibrium output under bilateral monopoly is higher (lower) than that of under vertical integration. □