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STRUCTURAL ANOMALY DETECTION BASED ON PROBABILISTIC METRIC DISTANCE OF TRANSMISSIBILITY FUNCTIONS

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ABSTRACT

Transmissibility function (TF) has been extensively used as damage-sensitive features in structural condition assessment. Based on the theoretical findings of circularly-symmetric complex Gaussian ratio distribution for transmissibility, the study proposes a new data-driven damage detection algorithm by accommodating multiple uncertainties of frequency responses. Based on the analytical probability density function of TFs of the healthy and of different possibly damage scenarios, a probabilistic metric is calculated as a damage index to identify the dissimilarity between the probability distributions of TFs under different states, which allows the automatic identification of structural anomaly. Numerical studies are carried out to verify the effectiveness and accuracy of the proposed methodology.

Keywords: Transmissibility function, Uncertainty quantification, Damage detection, Probabilistic metric, Operational variation

1. INTRODUCTION

The anticipated service life of engineering structures decreases due to hostile-loading environments and lack of timely maintenance. Assessing the structural condition of engineering structures have attracted widespread attention [1,2]. Output-only vibration testing without resorting to expensive excitation setups are particularly attractive as it can realize real-time safety monitoring for the structures without interrupting their normal operations. Therefore, considerable efforts have been spent in extracting damage features from ambient vibration measurements [3].

Over the past few years, transmissibility function (TF) as a mathematical representation of the output-to-output relationship has proven to be an attractive tool in SHM due to its unique properties [4]. Without measuring the input or assuming a specific input model, TF-based damage detection has aroused considerable interest due to its advantages [5]. For example, there is no need to undertake a

modal identification, while it is not necessary to have any analytic or numerical model of the structure [6].

Motivated by the fact that structural damage will cause dissimilarity of the probability distributions of frequency responses, and Symmetric Kullback-Leibler (SKL) Divergence is able to measure the degree of difference between two probability distributions [7], the study proposes a new statistical data-driven damage detection algorithm by accommodating multiple uncertainties of transmissibility measurements. It has been strictly proved that TFs follow circularly-symmetric complex Gaussian ratio distribution [8,9]. As a result, SKL divergence-based damage index is calculated to detect the dissimilarity between two probability distributions of TFs corresponding to different states, which allows the automatic identification of anomaly. Numerical studies are carried out to verify the effectiveness of the proposed methodology.

2. THEORETICAL BACKGROUND

2.1. Analytical probabilistic model of transmissibility functions

For a linear system subjected to a stationary excitation, discrete measurements $\boldsymbol{y}(t) = \{y_1(t), y_2(t), \cdots, y_{n_o}(t)\}$ are available for n_o measured dofs, and its corresponding FFT coefficients are denoted by $\boldsymbol{Y}(\omega_k) = \{Y_1(\omega_k), Y_2(\omega_k), \cdots, Y_{n_o}(\omega_k)\}$. The sampling frequency is assumed to be $\frac{1}{\Delta t}$. A TF $T_{i,o}^{(k)}$ is defined as the ratio of FFT of an arbitrary response $Y_i(\omega_k)$ to a reference response $Y_o(\omega_k)$:

$$T_{i,o}^{(k)} = \frac{Y_i(\omega_k)}{Y_o(\omega_k)} \tag{1}$$

Using the new theorem on multivariate circularly-symmetric complex Gaussian ratio distribution [8], the probability density function (PDF) of multiple TFs are given by [9]:

$$p_{\boldsymbol{T}_k}(\boldsymbol{\tau}_k) = \frac{(n_o - 1)!}{\pi^{(n_o-1)}|det\,\boldsymbol{\Lambda}_k|(\boldsymbol{\zeta}_k^* \boldsymbol{\Lambda}_k^{-1} \boldsymbol{\zeta}_k)^{n_o}} \tag{2}$$

where $\zeta_k = \left[1, \boldsymbol{\tau}_k^T\right]^T = \left[1, \tau_{1,o}^{(k)}, \tau_{2,o}^{(k)}, \cdots, \tau_{(n_o-1),o}^{(k)}\right]^T$ denotes the value of random vector $\{1, T_k\} = \left\{1, T_{1,o}^{(k)}, T_{2,o}^{(k)}, \cdots, T_{i,o}^{(k)}, \cdots, T_{(n_o-1),o}^{(k)}\right\}$; $\tau_{i,o}^{(k)}$ denotes the value of the random variable $T_{i,o}^{(k)}$; $\boldsymbol{\Lambda}_k$ denotes the covariance matrix of FFT coefficients vector $\boldsymbol{Y}(\omega_k)$. It is worth mentioning here that, in this work, all 'k' shown in the bracket, in the subscript or in the superscript denote frequency f_k.

2.2. Symmetric KL divergence

KL divergence is a measure of how one probability distribution is different from the reference probability distribution. In probability theory, KL divergence is also known as information divergence, information gain, or relative entropy. Applications include characterizing the relative (Shannon) entropy in information systems, randomness in continuous time-series, and information gain when comparing statistical models of inference. For two continuous PDFs $p_1(x)$ and $p_2(x)$, KL divergence is defined to be the integral given by [7]

$$D_{KL}\big(p_1(x) \| p_2(x)\big) = \int_X p_1(x)\, log \frac{p_1(x)}{p_2(x)} dx \tag{3}$$

The KL divergence from $p_1(x)$ to $p_2(x)$ is generally not the same as that from $p_2(x)$ to $p_1(x)$. It is not a true "distance" measure between the two distributions since it is not symmetric, i.e. $D_{KL}\big(p_1(x) \| p_2(x)\big) \neq D_{KL}\big(p_2(x) \| p_1(x)\big)$, and does not satisfy the triangle inequality. Therefore, the

symmetric KL (SKL) divergence between $p_1(x)$ and $p_2(x)$ will be employed in this study to formulate damage index to make sure it satisfy the definition of 'distance'. SKL divergence is defined as [7]

$$SKL\big(p_2(x); p_1(x)\big) = \int_X [p_1(x) - p_2(x)]\, log \frac{p_1(x)}{p_2(x)} dx \tag{4}$$

The above equation represents a non-negative measure between two PDFs.

2.3. Damage index based on SKL divergence

The TF between $i - th$ and $j - th$ DOFs for the healthy state and damaged state are denoted by $T_{ij}^h(\omega_k)$ and $T_{ij}^d(\omega_k)$, while their PDFs are denote by $p_{ij}^h(x)$ and $p_{ij}^d(x)$, respectively. The divergence of $p_{ij}^h(x)$ from $p_{ij}^d(x)$ relate to the idea that the distribution of damaged state will deviate the distribution of healthy state. Therefore, one can achieve a SKL divergence $SKL_{ij}(\omega_k)$ at frequency ω_k between the possibly damaged state $T_{ij}^d(\omega_k)$ and the healthy state $T_{ij}^h(\omega_k)$:

$$SKL_{ij}(\omega_k) = \int_X \big[p_{ij}^h(x) - p_{ij}^d(x)\big]\, log \frac{p_{ij}^h(x)}{p_{ij}^d(x)} dx \tag{5}$$

As a result, the damageindicator DI can be obtained by incorporating $SKL_{ij}(\omega_k)$ corresponding to different measurements at different frequency points:

$$DI = \sum_{k=1}^{N_\omega} \sum_{\substack{j=1 \\ j \neq i}}^{n_o} \sum_{i=1}^{n_o} SKL_{ij}\,(\omega_k) \tag{6}$$

where n_ω denotes the number of frequency ordinates; where n_o denotes the number of measurements.

3. CASE STUDY

The theory proposed in the above will be verified by a numerical example. This section details the processing of synthesis data based on simulated response of a 10-storey shear building that forms the cornerstone of the work. The stiffness at each floor is assumed to be $1.50 \times 10^6 N/m$, while mass at each floor is assumed to be $1000kg$. Classical Rayleigh damping is assumed. To simulate the responses of the structure, the structure is excited by ambient input modeled as Gaussian white noise at the 5-th dof, while the prediction error is also assumed to be Gaussian white noise.

Table 1. damage scenarios

Scenarios	Damage Element	Damage extent
1	5	10%
2	5	30%
3	5	30%
	10	20%
4	4	20%
	5	30%
	10	20%

Table 2. DI values for different damage scenarios

Scenarios		DI values
Undamaged	0	151.499
Damaged	1	201.883
	2	598.411
	3	852.798
	4	1114.588

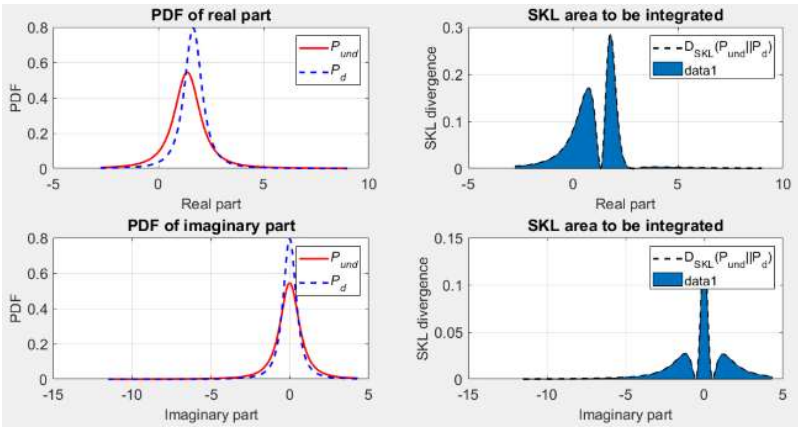

Figure 1. PDF and SKL of $T_{1,2}^{(k)}$ at $\omega_k = 27.72\pi\ rad/s$ corresponding to healthy and damaged states

In addition to the undamaged structure, four damage patterns are studied as a part of the problem. The damage patterns are defined in **Table** 1. 500 samples are generated for the healthy state, which serves as the baseline of calculating SKL divergence. The TF of the first sensor and the second sensor $T_{1,2}^{(k)}$ at $\omega_k = 27.72\pi$ rad/s is observed in detail. The PDF of $T_{1,2}^{(k)}$ corresponding to the healthy state and the second damage state are shown in **Figure** 1. The damage indicator of the TFs acquired from the data sets corresponding to measurements of different states are computed using Eq.(6). The DI incorporating SKL of different transmissibility functions at diffenent frequencies are shown in **Table** 2. As is seen from **Table** 2, the DI values are proportional to the damage level, indicating that the disimilarity of TFs different states increases with the level of damage. Therefore, the proposed damage index can reflect the damage extent, and SKL can be employed as a reasonable damage index for structural anomaly detection.

4. CONCLUSIONS

Condition assessment for infrastructure protection and health monitoring of structures has been a subject of strong research within the engineering community. The difficulty of achieving controlled input has led to the development of new output-only SHM approaches. TF has been extensively used as damage-sensitive features in structural condition assessment. Based on the theoretical findings of circularly-symmetric complex normal ratio distribution for scalar TFs, the study proposes a new statistical data-driven damage detection algorithm by accommodating multiple uncertainties of frequency responses, which has rarely been considered in the literature available. Based on the PDF of TFs of the healthy and of different possibly damage scenarios, symmetric KL divergence is calculated as an index to quantify the dissimilarity between PDFs of TFs under different damage conditions, which can reflect the structural anomaly properly. A numerical case study is employed to prove that the proposed method is able to diagnose the existence of damage and assess the overall health condition of a structure using ambient vibration testing.

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REFERENCES

[1] Worden, K., Farrar, C.R., Manson, G., & Park, G. (2007). The fundamental axioms of structural health monitoring. Proc. R. Soc. A, 463, 1639-64.

[2] Chronopoulos, D., Droz, C., Apalowo, R., Ichchou, M., & Yan, W. J. (2017). Accurate structural identification for layered composite structures, through a wave and finite element scheme. Composite Structures, 182, 566-578.

[3] Brownjohn, J. M. (2007). Structural health monitoring of civil infrastructure. Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences, 365 (1851), 589-622.

[4] Yan, W. J., Zhao, M. Y., Sun, Q., & Ren, W. X. (2019). Transmissibility-based system identification for structural health Monitoring: Fundamentals, approaches, and applications. Mechanical Systems and Signal Processing, 117, 453-482.

[5] Simon, C., & Arnaud, D. (2013). Damage localization using transmissibility functions: A critical review. Mechanical Systems and Signal Processing, 38 (2), 569-584.

[6] Maia, N.M.M., Almeida, R., Urgueira, A.P.V., & Sampaio, R.P.C. (2011). Damage detection and quantification using transmissibility. Mechanical Systems and Signal Processing, 25 (7), 2475-2483.

[7] Goldberger, J., Gordon, S., & Greenspan, H. (2003, October). An efficient image similarity measure based on approximations of KL-divergence between two Gaussian mixtures. [C]//null. IEEE, 2003: 487

[8] Yan, W.J., & Ren, W.X. (2016). Circularly-symmetric complex normal ratio distribution for scalar transmissibility function. Part I: theory. Mechanical Systems and Signal Processing, 80, 58-77.

[9] Yan, W.J., & Ren, W.X. (2016). Circularly-symmetric complex Normal ratio distribution for scalar transmissibility functions. Part II: application. Mechanical Systems and Signal Processing, 80, 78-98.