1	Effects of embedded walls on tunnelling-induced sandy ground displacements:
2	a numerical investigation
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11	Abstract
12	Urban tunnelling projects pose significant risks to the integrity of nearby structures due to ground
13	movements induced by the excavation process. Embedded walls are commonly employed as a
14	protective measure to mitigate these adverse effects. This paper presents a comprehensive
15	numerical investigation into the effects of embedded walls on tunnelling-induced ground
16	displacements, aiming to provide insights and recommendations for optimal embedded wall design.
17	The study assesses the impact of varying embedded wall length and horizontal distance from the
18	tunnel on soil settlement and horizontal displacements. Results demonstrate the complex interplay
19	between embedded wall length, horizontal distance, and ground movement patterns, and the highly
20	non-linear influence of key parameters on embedded wall efficiency (i.e. its ability to reduce
21	settlements). A preliminary design chart is proposed to guide engineers in determining the
22	appropriate horizontal location and depth of embedded walls to effectively reduce tunnelling-

- 23 induced ground displacements. The findings contribute to a better understanding of embedded wall
- 24 performance in the context of tunnelling and provide valuable guidance for the practical design and
- 25 implementation of protective measures in urban areas.
- 26 Keywords: Diaphragm & in situ walls; Ground movements; Numerical modelling; Soil/structure
- 27 interaction; Tunnels & Tunnelling
- 28

29 Notation

С	cover depth	$U_{\rm x}$	horizontal displacement
$d_{ m w}$	embedded wall horizontal distance to the	U_{z}	settlement
	tunnel		
D_{50}	sand average diameter	$U_{ m z,bw}$	soil settlement behind the embedded
			wall
$D_{\rm t}$	tunnel diameter	$U_{ m z,ref}$	greenfield settlement at embedded wall
			location
$e_{\rm c0}$	maximum void ratio at zero pressure	V _{l,t}	tunnel volume loss
$e_{\rm d0}$	minimum void ratio at zero pressure	x	horizontal distance
e_{i0}	critical void ratio at zero pressure	Ζ	depth
$e_{\rm max}$	maximum void ratio of sand	Zt	tunnel axis depth
e_{\min}	minimum void ratio of sand	α	exponent for hypoplastic model
EI	bending stiffness of the embedded wall	β	exponent for hypoplastic model
hs	granular hardness	φ'c	critical state friction angle
Id	relative density of sand	$\eta^{ u}_{bw}$	settlement efficiency
K_0	coefficient of horizontal pressure	η^h_{bw}	horizontal efficiency
$L_{\rm w}$	length of the embedded wall	η^s_{bw}	slope efficiency
М	bending moment	$ ho_{ m s}$	soil density
n	exponent for hypoplastic model	$ ho_{ m w}$	density of the embedded wall
t	embedded wall thickness		

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32 1 Introduction

Tunnelling in urban areas inevitably leads to stress relief and movements in the surrounding ground, which in turn may have detrimental effects on nearby structures. To prevent damage to buildings from tunnelling, various protective measures have been adopted to inhibit tunnelling-induced ground movements. The parameters and specifics of the chosen countermeasure require meticulous planning and design to safeguard buildings from potential adverse effects of tunnelling whilst ensuring cost-effective tunnel construction.

39 Protective measures can be categorised into three groups: in-tunnel measures, ground 40 treatment measures, and structural measures (Harris, 2001). Among these, structural measures are 41 commonly used to mitigate tunnelling-induced ground movements and structural deformation when 42 tunnels are excavated near buildings. This approach typically involves the construction of an 43 intervening row of piles or an embedded wall. In practice, structural measures are popular as they 44 provide an immediate and efficient control method. Ledesma and Alonso (2017) presented a case 45 study in which a row of reinforced concrete piles was used to protect sensitive buildings from 46 tunnelling-induced damage. Monitoring results indicated that soil displacements were considerably 47 lower than the predicted values at the design stage. Similarly, Di Mariano et al. (2007) described the use of a row of bored cast-in-situ piles to safeguard several seven-storey residential buildings 48 49 from tunnelling-induced displacements. The observed results revealed that the settlement curve 50 shape was altered substantially, and the building damage was decreased from category 3 to 0 (from 51 moderate to negligible based on Burland (2001)). Gens et al. (2005) documented the use of 52 micropile 'screens' to mitigate the effects of tunnelling on a primary urban motorway. Interestingly,

the final ground settlements were caused primarily by the construction of the micropiles themselves whereas tunnelling-induced settlements were minimal. Losacco and Viggiani (2019) also demonstrated the ability of an embedded barrier of bored piles to protect historic monuments from the construction of Line C of the Rome underground.

57 To provide guidance for practice, it is desirable to identify the optimal embedded wall 58 geometric and material parameters to minimise tunnelling-induced ground movements. Bilotta 59 (2008) and Bilotta and Stallebrass (2009) described a centrifuge testing and complementary numerical modelling programme exploring the influence of embedded diaphragm wall length on 60 61 settlements caused by nearby tunnelling in clay; it was concluded that the embedded wall's effectiveness was highly dependent on its length. Similarly, Song and Marshall (2021) used 62 63 geotechnical centrifuge test results to show that a 'deep' embedded wall significantly reduced 64 uneven pile settlements and structural distortions, whereas a 'shallow' embedded wall offered 65 minimal benefits. A further numerical study by Song et al. (2023) identified an optimal depth of an 66 embedded wall for reducing piled building damage due to tunnelling. However, these studies only 67 considered a single case where the building and embedded wall were located very close to the 68 tunnel. Bilotta and Russo (2011) used numerical analyses to highlight the pile spacing ratio as one of the key geometric parameters for a row of piles to mitigate ground displacements in stiff clay. A 69 70 recent study by Rampello et al. (2019) confirmed that the soil-structure interface roughness also 71 plays a role: a barrier with a smoother surface tends to achieve higher efficiency. Interestingly, 72 Rampello et al. (2019) also showed that a shorter barrier farther away from the tunnel might have 73 a similar reduction effect on the settlement compared to a deeper barrier placed closer to the tunnel,

highlighting the complex dependence of results on the horizontal proximity of the embedded wall to the tunnel. Employing a simplified elastic continuum-based two-stage model, Franza et al. (2021) compared the efficiency of "close location" and "far location" pile walls in reducing soil settlements caused by tunnelling and noted that the pile wall efficiency was underpredicted for large tunnellinginduced ground displacements (compared against field data and numerical results), which are of utmost practical interest.

80 To summarise, previous studies have primarily focused on the use of a line of piles or an 81 embedded wall to mitigate tunnelling-induced damage in clay, while the scenario in sandy ground 82 has received less attention. Furthermore, although results have demonstrated the significance of 83 embedded wall length on its effectiveness to reduce tunnelling-induced settlement, the role of the 84 horizontal distance of the embedded wall remains an open question. Consequently, no 85 comprehensive guidance currently exists for optimising embedded wall parameters when 86 employing structural measures to minimise tunnelling-induced ground movement, particularly in 87 sand.

To address this gap, this paper presents results from a numerical study on the use of embedded walls to reduce ground displacements caused by tunnelling in sand. The advanced constitutive model known as the hypoplastic model (von Wolffersdorff, 1996) was used to simulate the soil behaviour and calibrated using data from element and centrifuge testing. A "wished-in-place" embedded wall is modelled, with outcomes providing novel insights regarding the role of the embedded wall length and its horizontal distance from the tunnel in reducing tunnelling-induced ground movements in sandy ground. First, the structural response of the embedded walls due to 95 tunnelling is presented. This is followed by an examination of surface and subsurface soil 96 displacements to elucidate the effects of an embedded wall on the ground movement mechanisms. 97 Lastly, the effectiveness of the embedded walls is assessed using an efficiency parameter, followed 98 by the suggestion of a preliminary design chart in determining the embedded wall length and 99 location; practical implications of the results for embedded walls are also briefly discussed.

100 **2 Finite element model**

101 **2.1 Problem definition**

102 Figure 1 illustrates the scenario considered in this study: a tunnel with a diameter (D_t) of 7.2 m and 103 a cover depth (C) of 12.96 m (resulting in a tunnel axis depth, z_t , of 16.56 m) is constructed in dry 104 sand near an existing embedded wall. These dimensions are adopted to enable comparisons of the 105 numerical results with the experimental data presented in Song and Marshall (2021); the 106 simulations presented here are representative of the same full-scale (prototype) scenario. To 107 examine the effects of the embedded wall's length and location in reducing tunnelling-induced 108 ground movements, the results from 21 numerical models are considered (see Table 1): 20 tunnel-109 wall interaction tests (divided into three groups based on the horizontal distance of the embedded 110 wall to the tunnel) and one greenfield test for reference. The embedded wall length (L_w) varied from 11.76 m to 23.76 m ($L_w/z_t = 0.71$ to 1.43), and the horizontal distance, d_w , ranged from 4.4 m to 111 112 11.6 m ($d_w/D_t = 0.61$ to 1.61); all embedded walls had the same thickness of 0.8 m. A singular 113 shallow tunnelling scenario was considered because, in practice, the construction of a shallow tunnel has the greatest potential to impact neighbouring structures. A default volume loss $V_{l,t}=3\%$ 114 was adopted unless otherwise specified; this relatively high value was chosen considering that 115

- 116 protective measures are usually adopted when a high tunnel volume loss or large ground movements
- 117 are expected.



- Figure 1. Illustration of the problem and parameter definition
 - $L_{\rm w}/z_{\rm t}$ $d_{\rm w}/D_{\rm t}$ No $L_{\rm w}$: m d_{w} : m Group 1 Greenfield: reference test -2 11.76 4.4 0.71 0.61 3 14.16 4.4 0.86 0.61 4 16.56 4.4 1.00 0.61 5 18.96 4.4 1.14 0.61 Group A 6 19.44 4.4 1.17 0.61 7 20.16 4.4 1.22 0.61 8 1.29 21.36 4.4 0.61 9 23.76 4.4 1.43 0.61 10 11.76 8.0 0.71 1.11 11 14.16 0.86 1.11 8.0 12 16.56 8.0 1.001.11 Group B 13 18.96 8.0 1.14 1.11 14 21.36 8.0 1.29 1.11 15 23.76 8.0 1.43 1.11 16 11.76 11.6 0.71 1.61 17 14.16 11.6 0.86 1.61 18 16.56 1.00 11.6 1.61 Group C 19 18.96 1.61 11.6 1.14 20 21.36 11.6 1.29 1.61 21 23.76 11.6 1.43 1.61

Table 1 Overview of numerical parametric study

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123 **2.2 Element mesh and boundary conditions**

124 The finite element (FE) analysis software ABAQUS (Hibbitt, 2002) was adopted for this study. 125 Figure 2 shows an example FE mesh consisting of 85,507 nodes and 73,840 elements which was 126 adopted on the basis of a mesh sensitivity study. The model's width, depth, and thickness (out-of-127 plane) are 7.78 Dt, 4.24 Dt and 1.67 Dt, respectively to match the prototype conditions of the Song 128 and Marshall (2021) centrifuge tests. Eight-node brick elements (C3D8) were used to simulate the 129 soil, while 'incompatible' eight-node brick elements (C3D8I - an improved version of the C3D8-130 element) were employed for embedded walls and the tunnel boundary. The bottom boundary of the 131 computational domain was fixed and a vertical roller boundary was applied to all sides of the soil mesh; no constraints were applied to the soil surface. 132







135 Figure 2. Typical finite element mesh showing mesh discretisation near tunnel and imposed displacement

boundary conditions during the analysis ($L_w/z_t = 1.00$, $d_w/D_t = 0.61$; test No.4 in Table 1).

138 **2.3 Constitutive model and model parameters**

139 The soil constitutive model known as the hypoplastic model proposed by von Wolffersdorff (1996) 140 was employed to simulate the behaviour of the sand. It incorporates pressure- and density-141 dependency of soil behaviour, and can provide reliable predictions of various sand behaviours, e.g. 142 volumetric change due to shearing and small strain stiffness degradation (Herle and Gudehus, 1999). 143 The hypoplastic model has been extensively used in the study of various soil-structure interaction 144 problems and has been shown to provide predictions that are in good agreement with experimental 145 data (Wang et al., 2022; Khalajzadeh et al., 2023; Song et al., 2023). The basic hypoplastic model 146 has eight parameters: critical state friction angle φ'_{c} , granular hardness h_{s} , fitting parameter n, 147 minimum/maximum/critical void ratios at zero pressure $e_{d0}/e_{i0}/e_{c0}$, and α ; β , which govern the soil's 148 stiffness. These parameters were calibrated based on data from oedometer and drained triaxial 149 compression tests, as explained in more detail in Figures S1 and S2 in the Supplemental Materials 150 (modified from Song and Marshall (2020a)).

151 To enable comparison with the centrifuge test data documented in Song and Marshall (2021), the 152 same fine-grained silica sand ('Leighton Buzzard Fraction E') was simulated in the numerical 153 models. This sand has an average diameter of $D_{50} = 0.14$ mm, and maximum (e_{max}) and minimum 154 (emin) void ratios of 1.01 and 0.61, respectively (Zhao 2008, Lanzano et al. 2016). A relative density 155 (I_d) of 90% was considered, corresponding to a density ρ_s of 1603 kg/m³, again to match the 156 conditions of the centrifuge experiments. Table 2 summarises the model parameters used for the 157 present study. The model embedded wall and tunnel are linear elastic with a Young's modulus of 70 GPa and a density ρ_w of 2700 kg/m³ (consistent with Song and Marshall (2021)). Soil-wall and 158

159	soil-tunnel interfaces were simulated using a Coulomb friction law with a friction coefficient of tan
160	$(\varphi'_c) = 0.62$, assuming a perfectly rough soil-structure interface. An absolute elastic slip distance
161	(1.5 mm) was used to define the tangential behaviour of the soil-wall interface, based on centrifuge
162	pile jacking tests (Song and Marshall, 2020b). Note that, whilst the material parameters were
163	selected to match specific experimental conditions, the outcomes from this study, in terms of the
164	efficiency of the embedded wall, should also be applicable to a wider range of conditions.

166 **Table 2.** Hypoplastic model parameters adopted for Leighton Buzzard Fraction E sand, after Song and



Parameter		Source
Critical state friction angle, $\varphi'_{c}(^{\circ})$	32	Heap test
Granular hardness, h_s (MPa)	1969	Oedometer test
Exponent, <i>n</i>	0.447	Oedometer test
Minimum void ratio at zero pressure, e_{d0}	0.624	Herle and Gudehus (1999)
Critical void ratio at zero pressure, e_{i0}	1.16	Oedometer test
Maximum void ratio at zero pressure, e_{c0}	1.392	Herle and Gudehus (1999)
Exponent, α	0.08	Triaxial test, $I_d = 90\%$
Exponent, β	1.5	Triaxial test, $I_d = 90\%$

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169 **2.4 Numerical modelling procedure**

Tunnel construction is simulated using the displacement control method described in Song and Marshall (2020a), assuming maximum displacement at the tunnel crown and minimum deformation at the tunnel invert (see Figure 2). Firstly, a predetermined initial stress condition was imposed on the soil mesh, calculated based on the soil's self-weight and K_0 =0.5. The gravity within the model

was subsequently increased from 1g to 80g, followed by the application of a geostatic step. Soil elements within the tunnel and embedded walls were then eliminated, and the tunnel lining, embedded wall and soil-structure interfaces were activated in-place. Lastly, a non-uniform displacement profile was imposed on the tunnel boundary elements to emulate the effects of tunnel excavation. This displacement-control method for tunnelling has been extensively used in both centrifuge tests (Boonsiri and Takemura, 2015; Song and Marshall, 2021) and numerical modelling (Boldini et al., 2018; Fu et al., 2018).

181 **2.5 Numerical model validation**

182 To validate the present numerical modelling approach, greenfield tunnelling-induced surface soil 183 displacements corresponding to a tunnel volume loss of $V_{1,t}=3.0\%$ obtained from both numerical 184 (this paper) and centrifuge (Song and Marshall, 2021) models are compared in Figures 3(a) 185 (normalised settlement U_z/D_t) and 3(b) (normalised absolute value of horizontal displacement 186 $|U_x|/D_t$). Downward vertical displacements and horizontal displacements toward the right are 187 defined as positive. Note that all displacements from the numerical model are those caused by 188 tunnel volume loss (i.e., displacements due to geostatic stresses and embedded wall activation were 189 subtracted). The numerical model predictions compare very well to the measured trends in terms of 190 both displacement magnitude and contour distribution, which highlights the reliability of the 191 numerical model.



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193Figure 3 Comparison between FE model predictions and centrifuge test measurements of surface194displacements in greenfield tests at $V_{1,t}=3.0\%$: (a) normalised vertical settlements, U_z/D_t ; (b) absolute value195of normalised horizontal displacements, $|U_x|/D_t$ [embedded wall locations from Table 1 shown for196information].

198 **3 Results**

199 **3.1 Embedded wall response to tunnelling**

Figure 4 presents depth-wise profiles of horizontal displacements of the embedded walls in the three groups, i.e., $d_w/D_t = 0.61$, 1.11 and 1.61 (see Table 1), along with the greenfield horizontal displacements at corresponding locations. For $d_w/D_t = 0.61$, the embedded wall rotates in a clockwise manner about a depth of approximately $z/z_t = 0.5$, causing positive horizontal displacements above $z/z_t = 0.5$ which are in the opposite direction to greenfield displacements; this response is explained in detail later. For embedded walls longer than $L_w/z_t > 1.2$, the base of the embedded wall is embedded in soil that experiences minimal horizontal displacements from the 207 tunnel, with horizontal embedded wall displacement decreasing towards zero as embedded wall





displacements for various embedded wall embedment depths and for embedded wall locations of (a)

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$$d_w/D_t=0.61$$
; (b) $d_w/D_t=1.11$; (c) $d_w/D_t=1.61$; $V_{1,t}=3.0\%$.

215 Figure 4(b) illustrates that, for the $d_w/D_t = 1.11$ group, a rigid body motion accompanied by a clockwise rotation (greater horizontal displacement at the toe than at the top) of the embedded wall 216 217 was observed for the shortest embedded wall, resulting in a negative horizontal displacement across 218 the entire embedded wall. As the embedded wall length is increased, the toe of the embedded wall 219 is progressively embedded into soil that experiences minimal horizontal displacements from 220 tunnelling. The surrounding soil constrains the movement of the embedded wall toe, leading to a 221 reduction in the embedded wall's horizontal displacement. Interestingly, the embedded wall 222 response does not follow expectations based on the greenfield displacements at this location, which 223 would suggest that, as the toe of the embedded wall is embedded in more stationary soil, the embedded wall should bend/rotate in an anti-clockwise manner. Instead, the embedded wall 224 225 rotates/bends in a clockwise direction, similar to the results in the $d_w/D_t = 0.61$ group; this response 226 is examined further later in the paper with the use of contour plots of soil displacements. As 227 expected, the embedded walls in the $d_w/D_t = 1.61$ group showed negligible displacements at the toe 228 of the embedded wall and negative (towards the left) horizontal displacements at the upper section 229 of the embedded wall; the longer the embedded wall, the smaller the displacements.

In practice, the bending strength is important in the design of a embedded wall. Figure 5 illustrates the variation of normalised bending moments (*Mt/EI*, where *M* is the bending moment, *t* is the thickness of the embedded wall, *EI* is the bending stiffness of the embedded wall) with depth along the embedded walls in all numerical models. Figure 5(a) reveals that, for the embedded wall situated near the tunnel ($d_w/D_t = 0.61$), the bending moment profiles are highly dependent on the embedded wall length. For a shorter embedded wall ($L_w/z_t < 1.0$), the embedded wall predominantly displays a single deformation mode, with the negative (hogging away from the tunnel) maximum bending moment located at the embedded wall's mid-depth position (normalised depth $z/z_t \approx 0.5$), and bending moment increases with embedded wall length. As the embedded wall length increases, the embedded wall's deformation mode transitions from hogging only to a combined sagging (toward the tunnel) and hogging deformed shape, with the maximum bending moment dominated by the sagging component and located close to the tunnel axis ($z/z_t \approx 0.9$ -1.0).



Figure 5 FE calculations of the depth-wise profiles of normalised embedded wall bending moments for various embedded wall embedment depths and for embedded wall locations of (a) $d_w/D_t=0.61$; (b)

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$$d_w/D_t=1.11;$$
 (c) $d_w/D_t=1.61; V_{1,t}=3.0\%$

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250 For the $d_w/D_t = 1.11$ group, there is a significant decrease in the maximum bending moments 251 compared to the d_w/D_t =0.61 results, and the deformation mode of the embedded wall is 252 characterised by both sagging and hogging zones (see Figure 5(b)). For the shorter embedded walls $(L_w/z_t = 0.71, 0.86)$, the bending moments are negligible; for the relatively long embedded walls, 253 254 the maximum bending moments are located at $z/z_t \approx 0.6$. Lastly, the response of the embedded walls at a horizontal distance of $d_w/D_t = 1.61$ are considered in Figure 5(c). The results indicate that, 255 256 unlike the deformation modes of the other two scenarios, the embedded walls in Figure 5(c)257 primarily exhibit a singular deformation mode (sagging away from the tunnel). An increase in the 258 embedded depth of the embedded wall is shown to have minor effects on the bending moment when 259 $L_{\rm w}/z_{\rm t} > 1.0.$

To gain further insight into the response of the embedded walls to tunnel excavation, Figure 6 presents, according to normalised embedded wall depth (L_w/z_t), the maximum value of the normalised bending moment (Figure 6(a)) and the corresponding normalised depth-wise location of this bending moment (z/L_w ; Figure 6(b)). From Figure 6(a), the influence of the embedded wall depth on the maximum bending moment is highly dependent on its horizontal proximity to the tunnel: a very close embedded wall shows a variable response with an initial reduction and then a substantial increase in bending moment as the embedded wall is embedded into soil less affected by tunnelling. For the cases of larger tunnel-wall offsets ($d_w/D_t = 1.11$ and 1.61), there is a gradual increase in the maximum bending moment with embedded wall length. Consequently, the embedded wall strength is most critical when placed close to the tunnel.



Figure 6 FE calculations of the influence of embedded wall location on the variation of (a) the maximum normalised embedded wall bending moments and (b) their corresponding depth-wise location with normalised embedded wall depth; $V_{l,t}$ =3.0%.

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Figure 6(b) shows that the normalised depth of the maximum bending moments tend to fluctuate around the middle of the embedded wall ($z/L_w \approx 0.5$), indicating that the central part of the embedded wall tends to be the most vulnerable section. However, for embedded walls positioned at $d_w/D_t = 0.61$, a longer embedded wall ($L_w/z_t > 1.2$) tends to have a deeper ($z/L_w \approx 0.7$) normalised depth of the maximum bending moment, which also corresponds to the highest magnitude of bending moment from Figure 6(a). This location ($z/L_w \approx 0.7$), approximately at the tunnel axis depth, should also be considered in the design stage for a close embedded wall.

282 **3.2 Surface soil displacements**

To examine the influence of the embedded wall on tunnelling-induced surface displacements, 283 284 Figures 7 and 8 display the normalised settlements (U_z/D_t) and horizontal displacements (U_x/D_t) at 285 the soil surface with and without the presence of an embedded wall. Figure 7 demonstrates that the 286 presence of a embedded wall significantly modifies the surface settlement profile, with the 287 alteration being more pronounced for embedded walls situated closer to the tunnel. For the d_w/D_t 288 =0.61 group, Figure 7(a) shows that the embedded walls substantially reduce the exterior (i.e. on 289 the far side of the embedded wall) soil settlements but also contribute to larger interior (on the 290 tunnel-side of the embedded wall) maximum settlements relative to the corresponding greenfield values. Generally, the longer the embedded wall, the more effective it is at reducing settlements on 291 292 the exterior side of the embedded wall. Note that, in this investigation where a displacement control 293 method was adopted for tunnel volume loss, the embedded wall does not influence the ground loss 294 magnitude or pattern around the tunnel (i.e. wall-tunnel interactions are not simulated); the same 295 "input" of ground loss is imposed on each model and, as such, decreases in settlements on one side 296 of the embedded wall are balanced by increases on the other side. For a pressure/stress control 297 tunnel simulation method, the presence of the embedded wall (particularly for the close embedded 298 wall) may alter the magnitude and shape of ground loss around the tunnel, thus affecting soil 299 displacements and embedded wall response; this aspect was not considered in this study and could 300 be the focus of future work.



308 Similar findings can be deduced for the $d_w/D_t = 1.11$ and 1.61 results in Figures 7(b) and 7(c),

309 where the settlement of the soil on the exterior side of the embedded wall was again notably reduced

310 by the embedded wall. Furthermore, the impact of the embedded wall on the maximum interior soil 311 settlements (compared against the greenfield test) is determined by the horizontal distance of the 312 embedded wall to the tunnel: the farther the embedded wall, the less the impact.

313 In practice, to assess tunnelling induced building damage, a staged risk assessment procedure 314 proposed by Mair et al. (1996) is often adopted. A typical threshold of settlement in the preliminary 315 assessment stage is 10 mm (corresponding to 0.139% for U_z/D_t), which is also included in Figure 316 7. First, it is noted that the settlement of the zone at $V_{1,t}=3.0\%$ within $x/D_t=\pm 2.55$ exceeds the 317 threshold of 10 mm, indicating the potential necessity of adopting protective measures within this 318 zone. For close embedded walls ($d_w/D_t = 0.61$), it is shown that an embedded wall longer than L_w/z_t = 1.29 can reduce the settlement to this threshold immediately adjacent to the embedded wall, 319 whereas for $L_w/z_t = 1.00$ and 0.86, settlements reach this threshold at $x/D_t = 1.11$ and 1.61, 320 321 respectively. This indicates that the closer the embedded wall is to the tunnel, the longer the 322 embedded wall is required to be to achieve a certain threshold of settlement, consistent with the 323 results reported by Rampello et al. (2019).

The impact of the embedded wall on soil horizontal displacements is examined in Figure 8. For the embedded wall situated at $d_w/D_t = 0.61$, although the embedded wall is effective in minimising exterior soil displacements, there is again a substantial increase in the interior soil displacements compared to greenfield; the longer the embedded wall, the more pronounced the effect, as illustrated in Figure 8(a). Intriguingly, when the embedded wall is located at $d_w/D_t = 1.11$ (Figure 8(b)), there is a decrease in both the interior (left side of the embedded wall) and exterior (right side of the embedded wall) displacements compared against the close embedded wall 331 scenario, and exterior soil displacements are nearly reduced to zero. With a further increase in the 332 horizontal distance to d_w/D_t =1.61, the embedded wall is less effective in minimising exterior 333 horizontal soil displacements, resulting in marginally smaller (but in the same direction) values than 334 the corresponding greenfield case.



337 Figure 8 FE calculations of the influence of embedded wall depth on normalised soil surface horizontal

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displacements for embedded wall locations of: (a) $d_w/D_t=0.61$; (b) $d_w/D_t=1.11$; (c) $d_w/D_t=1.61$; $V_{1,t}=3.0\%$.

340 3.3 Subsurface soil displacement contours

341 To gain further insights into the settlement mechanisms, this section considers contours of soil 342 settlements (Figure 9(a)) and horizontal displacements (Figure 9(b)) for selected tests; the 343 corresponding greenfield contours were provided in Figure 3. Figure 9(a) shows that the embedded 344 walls significantly affect the spatial distribution of the soil settlements. Interestingly, the close 345 embedded wall ($d_w/D_t = 0.61$) is not as effective as the embedded walls further away ($d_w/D_t = 1.11$ or 1.16) in reducing the exterior settlements, and a larger settlement zone (compared to the 346 347 greenfield scenario) is formed between the tunnel crown towards the interior side of the embedded 348 wall.





Figure 9 FE contours of soil displacements: (a) settlements U_z/D_t ; (b) horizontal displacements U_x/D_t .

355 Figure 9(b) shows that, compared against the greenfield scenario in Figure 3, the close 356 embedded wall ($d_w/D_t = 0.61$) altered the direction of the horizontal displacements of the soil 357 adjacent to the exterior side of the embedded wall above a depth of about $z/z_t=0.5$, causing the soil in this area to move away from the tunnel (contrary to greenfield); this matches with the observed 358 359 rotation of the embedded wall in Figure 4(a), where the movement of the toe of the embedded wall 360 causes a clockwise rotation of the embedded wall, acting to push the soil near the surface away 361 from the tunnel on the exterior side of the embedded wall. The somewhat counter-intuitive 362 rotational behaviour of the middle embedded wall ($d_w/D_t = 1.11$) presented in Figure 4(b) can be 363 explained here. For the shallower middle embedded walls, a similar mechanism to that of the close 364 embedded wall is observed, with the toe of the embedded wall being pulled towards the tunnel,

365 causing a clockwise rotation of the embedded wall. For the deeper middle embedded walls, where 366 the toe becomes embedded in more stationary ground, Figure 9(b) shows that, nearer the surface, 367 the zone of maximum positive horizontal displacements shifts to the right; this is coincident with a 368 tilting (to the right) of the settlement mechanism above the tunnel in Figure 9(a). These deeper 369 embedded walls prevent ground loss from propagating to the right of the embedded wall and, to 370 compensate for this (for the adopted displacement control method for tunnel ground loss), ground 371 movements are increased on the left side of the embedded wall, causing the observed increase in 372 the size of the area of soil experiencing displacements to the right, and the resulting clockwise 373 rotation/bending of the embedded wall seen in Figure 4(b). For the furthest location from the tunnel 374 $(d_w/D_t = 1.61)$, the embedded wall has a minimal effect on the spatial distribution of the soil 375 horizontal displacements where only the horizontal displacements of the near-surface soil on the 376 exterior side of the embedded wall are decreased, and the longer embedded wall tends to have a 377 greater impact.

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379 **3.4 Effectiveness of the embedded wall**

Bilotta (2008) defined a dimensionless factor η_{bw}^{ν} for quantifying the impact of an embedded wall on ground movements due to tunnelling as follows:

382 $\eta_{bw}^{v} = \frac{U_{z,ref} - U_{z,bw}}{U_{z,ref}} \tag{1}$

where $U_{z,bw}$ represents the vertical displacement of the surface soil immediately behind the embedded wall, and $U_{z,ref}$ denotes the reference settlement of the soil at the same location in the greenfield scenario, as shown in Figure 1; an efficiency of 0 indicates no benefit of the embedded 386 wall. Note that the alternative integral efficiency defined by Rampello et al. (2019) may also be387 used; for the data obtained in this study, the two methods provided consistent results.

388 It has been suggested that for structures with discontinuous foundations, horizontal soil 389 displacements caused by tunnelling are also a key design concern (Goh and Mair, 2014). Recent 390 centrifuge studies (Xu et al., 2020) have shown that for bare framed structures, the distortion of the 391 frame caused by tunnelling is dominated by shear deformation, which is closely related to the 392 ground slope S. The slope $S = \delta U_z/\delta x$ can be computed as the ratio of the differential settlement at 393 two points on the foundation edges ($\delta U_z = U_{z,A}$ - $U_{z,B}$, see Figure 1) to the horizontal distance 394 $(\delta x = |x_A - x_B|)$ separating the two points (with anti-clockwise rotation defined as positive). Consequently, similar dimensional factors are proposed to assess the effect of the embedded wall 395 at reducing the horizontal displacement (η_{bw}^h) and the maximum slope (η_{bw}^s) of the surface soil 396 397 behind the embedded wall. These parameters can be calculated using equations analogous to Equation (1). In this study, in calculating η_{bw}^s , it was assumed that δx is infinitely small, hence the 398 399 tangential slope of the settlement curve at the point of interest was used.

Figure 10(a) – (d) presents numerically derived values of η_{bw}^{v} based on soil settlements just behind the embedded walls, η_{bw}^{v} at an offset of x/D_t =1.61 for different embedded wall locations (i.e. based on soil settlements at x/D_t =1.61), η_{bw}^{h} , and η_{bw}^{s} , respectively, and their dependence on the normalised embedded wall depth L_w/z_t , all at $V_{1,t}$ =3.0%. Typical thresholds of settlement (10 mm) and slope (1/500) suggested by Mair et al. (1996) for practical use in preliminary damage assessments were used to determine threshold efficiency parameters at different embedded wall locations and are also included in Figure 10(a), (b) and (d). Furthermore, efficiency results from the 407 following cases are also included in Figure 10(a): field testing in sandy ground from Nikiforova 408 and Vnukov (2012), numerical modelling for a scenario with sandy silt and silty sand underlain by 409 sandy gravel and stiff clay from Rampello et al. (2019), and 2D Elastic modelling for undrained 410 soil conditions from Ledesma and Alonso (2017). Due to the differences in soil properties, 411 modelling techniques and volume losses between studies, a direct comparison between the data 412 from the literature and the numerical results obtained here should be avoided; the intention is to 413 allow a more qualitative comparison and to give some context to the outcomes obtained in this 414 study.

For the close embedded walls ($d_w/D_t = 0.61$ in Figure 10(a)), the variation of η_{bw}^{ν} with L_w/z_t is 415 highly nonlinear, with η_{bw}^{ν} increasing sharply after $L_w/z_t > 1.17$ and stabilising at ≈ 0.8 after 416 $L_w/z_t=1.29$. In contrast, η_{bw}^{ν} increases monotonically with embedded wall length for $d_w/D_t=1.11$ and 417 418 1.61, consistent with Rampello et al. (2019). When considering the thresholds for preliminary risk 419 assessment, only the embedded walls above $L_w/z_t=1.29$ achieve the necessary efficiency of 0.776 420 for $d_w/D_t = 0.61$, whereas for $d_w/D_t = 1.11$ the critical embedded wall length to achieve the necessary 421 efficiency of 0.653 is $L_w/z_t=1.00$, and for $d_w/D_t=1.61$ the critical embedded wall length to achieve 422 the necessary efficiency of 0.340 is reached at about $L_w/z_t=0.9$.

Practically, the shallowest embedded wall is desirable to reduce costs. Considering an example where a structure is located near the position of $x/D_t = 1.61$ for the parameters adopted here, where the greenfield settlement exceeds the settlement threshold (see Figure 7(a)), Figure 10(b) demonstrates that the embedded wall at $d_w/D_t = 1.11$ is likely the best option. Whilst the closest embedded wall ($d_w/D_t = 0.61$) meets the settlement threshold criteria even for short embedded walls, 428 a conservative efficiency is not reached until embedded wall length exceeds about $L_w/z_t=1.22$. The 429 embedded wall at $d_w/D_t=1.11$ achieves the highest settlement efficiency at the location $x/D_t=1.61$



430 for a reasonable embedded wall depth of $\sim L_w/z_t=1.0$.

Figure 10 Efficiency parameters and their dependence on embedded wall depth and location: (a) soil settlement η_{bw}^{v} just behind embedded walls (including data from Nikiforova and Vnukov (2012), Rampello et al. (2019) and Ledesma and Alonso (2017)), (b) soil settlement η_{bw}^{v} at

438 $x/D_t=1.61$, (c) horizontal displacement of the soil η_{bw}^h just behind the embedded walls and (d) 439 surface slope of the soil η_{bw}^s just behind the embedded wall; $V_{l,t}=3.0\%$.

440

Figure 10(a) also shows that, despite the embedded wall efficiency values being obtained for different soil properties, embedded wall locations and methodologies, there is good agreement between data points from literature sources, with slightly higher settlement efficiencies for sandy ground obtained in this study and in Nikiforova and Vnukov (2012). The results further confirm the dominant role of embedded wall length L_w/z_t in reducing tunnelling induced ground displacements, consistent with Bilotta (2008), Bilotta and Stallebrass (2009), and Franza et al. (2021).

In contrast to settlements, Figure 10(c) shows that η_{bw}^h is more sensitive to the horizontal location of the embedded wall. Interestingly, the presence of the close embedded wall alters the direction of the horizontal displacement of the soil (η_{bw}^h is greater than 1), whereas the embedded walls at $d_w/D_t = 1.11$ achieve an efficiency of $\eta_{bw}^h \approx 1$ for all embedded wall lengths. Note that an efficiency of 1.4 is equivalent in terms of displacement magnitude to 0.6, but in the opposite direction, hence the embedded walls at $d_w/D_t = 0.61$ and 1.61 perform about the same in terms of horizontal displacements.

Finally, Figure 10(d) shows that the embedded walls are effective in reducing the slope of the soil surface, with efficiency value of η_{bw}^{s} for all embedded walls greater than 0.7, which is sufficiently safe for all locations (Mair et al., 1996).

- 457 4 Discussion
- 458 The present numerical results indicate that both the length and horizontal distance of the embedded

459 wall from the tunnel can significantly influence ground displacements caused by tunnelling. Their 460 effects on ground movements are highly nonlinear, as demonstrated in Figure 10. This poses 461 significant challenges for engineers when attempting to use embedded walls to protect structures 462 from tunnelling-induced damage.

463 For embedded walls situated very close to the tunnel (e.g. $d_w/D_t = 0.61$), Figure 11(a) highlights 464 the nonlinear relationships between embedded wall settlement, tunnel volume loss and embedded wall length, with a maximum settlement of 0.86% D_t (and associated drop in η_{bw}^{v} in Figure 10(a)) 465 466 when $L_w/z_t = 1.14$. These results indicate that when the toe of the embedded wall is located very 467 close to the tunnel, the settlement of the embedded wall becomes significant (larger than the 468 corresponding greenfield surface displacement), and, in turn, the embedded wall's ability to reduce 469 tunnelling induced soil displacements diminishes, as also pointed out by Ledesma and Alonso 470 (2017). Conversely, for the embedded walls positioned at $d_w/D_t = 1.11$ and 1.61 (Figures 11(b)-(c)), 471 their settlement generally decreases monotonically with their length, resulting in a continuous increase of the efficiency parameter η_{bw}^{ν} (see Figures 10 (a)), consistent with Rampello et al. (2019). 472 Moreover, for η_{bw}^{h} and η_{bw}^{s} , the intermediate embedded wall location ($d_{w}/D_{t}=1.11$) is optimal, and 473 474 its bending moment and horizontal displacement are considerably smaller than the embedded wall 475 situated very close to the tunnel (see Figures 4-6).



Figure 11 FE calculations of the influence of embedded wall depth on the variation of normalised

482 embedded wall settlements with tunnel volume loss for embedded wall locations of: (a) $d_w/D_t=0.61$; (b)

 $d_w/D_t=1.11;$ (c) $d_w/D_t=1.61$

485 The numerical outputs were used to develop a preliminary design chart for determining the 486 optimal horizontal location and depth of an embedded wall to reduce tunnelling-induced ground 487 displacements, as shown in Figure 12. The chart is based on a relatively high tunnel volume loss $V_{l,t}=3.0\%$; at lower values of tunnel volume loss, efficiency values tend to be higher (see η_{bw}^{ν} values 488 489 at $V_{1,t}=1.0\%$ in Figure S3), however the outcomes are consistent with the design chart in Figure 12. 490 For close embedded walls, caution should be exercised when selecting the depth of the 491 embedded wall, as an embedded wall with its toe close to the tunnel tends to settle significantly, 492 thereby reducing the embedded wall's efficiency. For this case, an embedded wall length of $L_w/z_t > 1.3$ is recommended, as it can reduce ground settlement with an efficiency value $\eta_{bw}^{\nu} \approx 0.8$ 493 (below the preliminary assessment threshold from Mair et al. (1996)). 494

495 Embedded walls positioned in the region $0.9 < d_w/D_t < 1.2$ are likely the optimal solution, as 496 they can reduce soil settlements and surface slope with higher efficiency than embedded walls 497 closer and farther from the tunnel, whilst also significantly restricting horizontal soil displacements. 498 To support this conclusion, efficiency values are plotted against normalised wall distance from the 499 tunnel in Figure S4 of the Supplemental Materials, including results from additional models not 500 discussed above $(L_w/z_t=1.14 \approx 1.1; d_w/D_t=0.86, 1.20, 1.30 \text{ and } 1.40)$; the data demonstrate that, in 501 the region 0.9 $d_w/D_t < 1.2$, an embedded wall with $L_w/z_t > 1.1$ can reduce soil settlement with an efficiency $\eta_{bw}^{\nu} > 0.7$ (sufficient for a preliminary damage assessment). Finally, embedded wall 502 lengths of $L_w/z_t \ge 1.0$ at $d_w/D_t \ge 1.5$ are likely to generate satisfactory efficiencies to meet 503 504 preliminary damage assessment criteria, however for this embedded wall depth, an embedded wall 505 placed at an intermediate location will likely provide greater efficiencies, and will more effectively

506 reduce horizontal displacements.



507

508 Figure 12 Design chart to inform an optimal choice of the depth and horizontal distance of an embedded 509 wall (based on relatively high tunnel volume loss $V_{l,t}$ =3.0%; effects of wall construction are not included) 510

511 Limitations arising from some of the assumptions adopted for the numerical modelling should 512 be noted. First, the embedded walls were assumed to be "wished-in-place" and thus not causing 513 any ground movements during construction of the embedded walls. A recent numerical study by 514 Rampello et al. (2019) suggested that the wall installation (excluding construction) causes minimal 515 ground displacements. In reality, however, the displacements caused by embedded wall 516 construction may not be negligible, and in some cases, may account for the majority of the final 517 surface settlement, as documented by Gens et al. (2005). Furthermore, as a consequence of the wished-in-place walls adopted here, the wall installation effects on the tunnel-wall interactions were 518 519 not captured within the numerical modelling outcomes, however these are believed to play a 520 secondary role in the overall interaction problem (future work may consider this aspect). Second,

521 this paper assumed a perfectly rough soil-structure interface (a friction coefficient of tan (φ'_c) = 0.62) for the soil-embedded wall interface. This provides a conservative outcome in Figure 12 as 522 523 previous studies (centrifuge tests by Bilotta (2008) and numerical modelling by Rampello et al., 524 2019) have shown that a smoother soil-wall interface results in higher local efficiencies due to lower 525 downwards shear stresses applied by the wall to the soil. Finally, the tunnelling process was 526 simulated numerically using a displacement control method; the adoption of a pressure control 527 method may result in slightly different local efficiency values and design chart; again this could be 528 an area of further study.

529

530 5 Conclusions

This paper presented a comprehensive numerical investigation into the effects of embedded walls on tunnelling-induced ground displacements, providing valuable insights into the optimal design of embedded walls for urban tunnelling projects. The study has shown that the efficiency of embedded walls in reducing tunnelling-induced ground displacements is influenced by both embedded wall length and horizontal distance from the tunnel, with the relationship between these factors being nonlinear and highly inter-dependent.

The findings reveal that embedded walls located at a horizontal distance of $0.9 < d_w/D_t < 1.2$ from the tunnel are likely to offer the best balance between settlement reduction, horizontal displacement restriction, and structural efficiency. Additionally, an embedded wall length of $L_w/z_t >$ 1.1 is suggested as the optimal depth at this location, resulting in an efficiency >0.7 which, based on the current study, was sufficient for mitigating the potential structural damage due to tunnelling. However, for embedded walls situated extremely close to the tunnel ($d_w/D_t < 0.7$), the depth selection requires careful consideration to avoid excessive settlement and decreased efficiency, and an embedded wall length of $L_w/z_t > 1.3$ is suggested.

A design chart was presented to serve as a tool for engineers to determine preliminary embedded wall (horizontal) locations and depths in the context of tunnelling projects. Whilst the findings contribute to a better understanding of embedded wall performance, it is important to acknowledge that site-specific conditions and soil properties may affect the results. Furthermore, limitations arising from the simplification of the numerical modelling should be also noted. Therefore, further investigation and validation through physical experiments or additional numerical analyses are recommended to enhance the general applicability of the design chart.

552

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638 Supplemental Materials

To obtain the parameter values in the hypoplastic model, Song and Marshall (2020a) describes a detailed approach. The critical state friction angle φ'_{c} was obtained based on five heap tests. Herle and Gudehus (1999) suggested that the initial void ratio in a proportional compression test (i.e. an oedometer test) with very loose sand can be considered as an appropriate estimate of e_{c0} . Herle and Gudehus (1999) defined granular hardness h_{s} as:

$$h_s = 3p'\left(\frac{ne_p}{c_c}\right) \tag{S1}$$

645 where $C_c = \Delta e / \Delta \ln p'$ is the tangent compression index.

646 Using data from an oedometer test and considering two values of C_c at different magnitudes 647 of mean effective stress p' (i.e. two points on an $e - \ln(p')$ curve; indicated below by subscripts 1 648 and 2), Equation (S1) can be used to obtain an expression for n:

649
$$n = \frac{\ln(e_1 C_{c2}/e_2 C_{c1})}{\ln(p_2/p_1)}$$
(S2)

650 Using the above approach on data obtained from an oedometer test on Fraction E sand (see 651 Figure S1), values of $e_{c0}=1.16$, $h_s=1969$ MPa and n=0.447 were obtained.

652 The value of minimum void ratio at zero pressure e_{d0} can be determined using (Herle and 653 Gudehus, 1999):

654
$$e_{d0} = e_d \exp\left[\left(3p'/h_s\right)^n\right]$$
(S3)

where the value of e_d mainly depends on the coefficient of uniformity C_u and grain shape. Youd (1973) measured the value of e_d using a simple shear test under a vertical pressure of 96 kPa, corresponding to $p' \approx 55$ kPa (based on assumption of K_0 =0.4), and proposed a diagram relating e_d to grain angularity and C_u . Based on the diagram of Youd (1973), a value of e_d =0.615 was obtained. By substituting this value into Equation (S3) (along with previously determined parameter values), a value of $e_{d0}=0.624$ was obtained. As suggested by Herle and Gudehus (1999), the value of maximum void ratio at zero pressure e_{i0} can be approximated as $e_{i0} \approx e_{c0} \times 1.2$, therefore, the critical void ratio at zero pressure e_{i0} was determined as 1.392.



663

Figure S1 Oedometer test data and numerical results for Leighton Buzzard Fraction E sand (modified from
 Song and Marshall, 2020a)

Two drained triaxial compression tests were conducted at an effective confining pressure of 200 kPa using the same sand, with the same relative density of $I_d = 90\%$. To calibrate the parameters α and β , a single 3D brick element (C3D8) numerical analysis was conducted in ABAQUS to replicate the triaxial tests. Figure S2 shows that numerical simulation results using α =0.065 and β =0.3 compare well against both the triaxial volumetric data and the non-linear stress–strain response of the soil.



Figure S2 Drained triaxial test data for Leighton Buzzard Fraction E sand and numerical (FEA) results
using hypoplastic model (effective confining stress = 200kPa): (a) Deviator stress; (b) Volumetric strain;
modified from Song and Marshall (2020a)

To compare the efficiency η_{bw}^{ν} values at a low tunnel volume loss with those obtained at a high tunnel volume loss, the settlement efficiency η_{bw}^{ν} values at $V_{l,t}=1.0\%$ for all embedded walls are presented in Figure S3. Results show that η_{bw}^{ν} values at low $V_{l,t}=1.0\%$ tend to be higher than those at $V_{l,t}=3.0\%$.





684 **Figure S3** Efficiency parameter η_{bw}^{ν} with embedded wall depth and location at $V_{l,t}=1.0\%$.

To support the design chart presented in Figure 12, results from an additional four models at $d_w/D_t = 0.86, 1.20, 1.30$ and 1.40 are considered; the length of all models is $L_w/z_t=1.14 \approx 1.1$. The resulting efficiency values are plotted against normalised wall distance from the tunnel in Figure S4. Results shows that, in the region $0.9 < d_w/D_t < 1.2$, an embedded wall with $L_w/z_t > 1.1$ can reduce soil settlement with an efficiency $\eta_{bw}^v > 0.7$.



692 Figure S4 Efficiency parameter η_{bw}^{ν} with horizontal distance d_w/D_t to the tunnel for walls with

 $L_{\rm w}/z_{\rm t}=1.14\approx 1.1.$