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The role of cognitive and applied executive function skills in learning rational number knowledge

Camilla Gilmore [a,*], Emine Simsek [a], Joanne Eaves [b], Lucy Cragg [b]

[a] Centre for Mathematical Cognition, Loughborough University, Loughborough, UK
[b] School of Psychology, University of Nottingham, Nottingham, UK

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ABSTRACT

Executive functions are associated with concurrent and future mathematics achievement, however, we know less about how they are involved in learning new mathematics material. We investigated the contribution of executive functions to learning new mathematical material, specifically rational number knowledge, in a standard classroom situation. We measured rational number knowledge as well as cognitive and applied executive functions prior to 8- to 9-year-old children's first introduction to symbolic rational numbers. Rational number knowledge was measured again 6 and 20 months later. Latent growth curve models revealed that rational number knowledge at Time 1 was predominantly predicted by cognitive measures of executive function while growth in rational number knowledge was predominantly predicted by applied measures. These findings demonstrate that, to understand the role of executive functions in classroom learning, we must consider not only an individual's executive function capacity, but also how well they can recruit this in applied settings.

Educational relevance statement: Executive functions are the set of skills that allow us to control our thoughts and behaviour. We investigated the role of executive function skills in learning about rational numbers in mathematics lessons. We found that executive function skills were related both to children's performance of mathematical procedures as well as how well they could learn new procedures over time. This suggests that one reason why children learn at different rates is differences in their executive function skills. Therefore, it may be beneficial for teachers to consider the executive function demands of classroom activities.

1. The importance of mathematics

Mathematics is essential for everyday activities such as managing money, planning schedules, and preparing meals. It is also important in more formal settings, including school and the workplace (OECD, 2013). Good mathematical skills are associated with higher rates of employment, better medical and financial decision making, and a better quality of life (Gerardi et al., 2013; Parsons & Bynner, 2005; Reyna & Brainerd, 2007; Skagerlund et al., 2018). However, many individuals struggle to achieve the mathematical skills they need to succeed in everyday life (Department for Education, 2019). Mathematics is a hierarchical domain where new knowledge builds on existing understanding, therefore the foundational building blocks are crucial to developing more advanced skills. Understanding the learning that takes place to acquire these foundational skills is key to improve mathematics outcomes for all learners.

Mathematical skills are underpinned by a range of domain-specific and domain-general processes. Executive functions (EFs), the processes that control our thoughts and actions (Diamond, 2013), are important for success in mathematics (e.g. Coolen et al., 2021; Peng et al., 2016; Van der Ven et al., 2012). Many existing studies investigating EFs in mathematics have focussed on mathematics achievement; performance of skills and knowledge that has already been mastered. However, executive functions may be potentially even more important in the process of learning mathematics (Cragg et al., 2017). To investigate this, performance of specific mathematical skills needs to be measured before and after individuals are taught a new topic, yet few studies of this kind have been conducted. In this study, we investigate the contribution of EFs to the learning of new mathematical material, specifically rational number knowledge, i.e. an understanding of fractions and decimals. Below we first consider why rational number knowledge is a valuable domain to test these questions before reviewing previous research on the involvement of executive functions for performance in this topic. Finally, we discuss what learning studies can

* Corresponding author at: Centre for Mathematical Cognition, Loughborough University, Epinal Way, Loughborough, Leicestershire LE11 3TU, UK.
E-mail address: c.gilmore@lboro.ac.uk (C. Gilmore).

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contribute to our understanding of the role of executive functions, over and above studies of children's performance.

2. Rational number knowledge

A key development during primary school mathematics occurs when children move on from whole number reasoning and arithmetic to using fractions and decimals. It has been suggested that rational numbers hold a "central position" in mathematical development (Siegler et al., 2012), because they represent the first opportunity that individuals have to learn that whole number properties are not true of all numbers (e.g. multiplication does not always result in a larger number). Indeed, rational number knowledge in primary school predicts both later high school mathematics achievement (Siegler et al., 2012), and growth in mathematics achievement (Bailey et al., 2012), over and above the influence of whole number mathematics.

There are two features of rational numbers that may pose difficulties for children. First, rational numbers don't behave in the same way as whole numbers. For example, simple heuristics such as "multiplication makes more" no longer apply (e.g. 4 × ½ results in a number smaller than 4). Consequently, difficulties with rational numbers can stem from children inappropriately applying reasoning, biases, or procedures from whole number knowledge (often referred to as the whole number bias or natural number bias, Ni & Zhou, 2005). Whole number magnitude information can interfere when children compare fraction magnitudes, for example, leading children to conclude that 4/7 is bigger than 4/5 because 7 is bigger than 5 (Van Hoof et al., 2015). Similarly, children often inappropriately apply the whole number heuristic that longer numbers are larger (e.g. 125 is larger than 45) to decimal numbers (i.e. concluding that 0.125 is larger than 0.45; Vamvakoussi & Vosniadou, 2004). Children also have difficulty performing arithmetic with fractions, and often inappropriately apply strategies from whole number arithmetic or confuse the procedures for addition/subtraction with those for multiplication/division (Siegler & Pyke, 2013). Overcoming established knowledge and procedures is known to rely on executive functions, in particular inhibition, in order to suppress this previously relevant, but currently inappropriate, information (e.g. Rossi et al., 2019).

Second, unlike whole numbers there are multiple ways to represent the same rational number (e.g. 'half' as 0.5, 1/2, 2/4, 50 %), as well as multiple ways of conceptualising them: as a part-whole, a ratio, a magnitude, or an operation (e.g. "what is half of…"). Coordinating and switching between these different representations and meanings can be particularly challenging (Moss, 2005). Switching between representations is a hallmark of executive functions, requiring the ability to hold multiple representations in mind as well as to suppress one representation in order to work with another (see Cragg & Chevalier, 2009 for a review).

3. Executive functions and rational number knowledge

Executive functions may play a particularly important role when learning and operating with rational numbers. A number of studies have assessed the contribution of a range of cognitive variables, including executive functions, on multiple measures of rational number knowledge including fraction concepts, fraction procedures, fraction arithmetic, and fraction estimation with cross-sectional or longitudinal data (Hansen et al., 2015; Hecht et al., 2003; Jordan et al., 2013; Stelzer et al., 2021; Vukovic et al., 2014; Ye et al., 2016). In sum, they found that working memory was associated with fraction outcomes, but often operated indirectly by building general numerical competencies such as calculation skill (e.g. multiplication and division skills) and magnitude reasoning (e.g. whole number line estimation), which in turn helped fraction performance.

Other studies have specifically investigated the role of inhibition in rational number knowledge (Avgerinou & Tolmie, 2020; Fitzsimmons et al., 2020; Gómez et al., 2015). Gómez et al. (2015) found that children's accuracy scores on a fraction comparison task were significantly associated with their inhibition scores on a numerical Stroop task, although this relationship was mediated by general mathematics achievement. Furthermore, Fitzsimmons et al. (2020) did not find any evidence that inhibition skills helped adults perform a rational number line estimation task. Finally, Avgerinou and Tolmie (2020) found that performance on a non-numerical Stroop task was associated with 8–10 year olds' speed on a magnitude comparison task involving fractions and decimals, but only when presented under a high cognitive load (i.e. when rational numbers were presented with additional, unnecessary information such as pictures). Inhibition might therefore have a nuanced relationship with rational number knowledge, where it is needed more in cognitively demanding conditions. This might be particularly relevant for learning situations because building and integrating new knowledge is particularly cognitively demanding, and classroom environments contain multiple sources of information that need filtering.

In addition to standard cognitive measures of executive function, several studies found that teacher ratings of children's attention and behaviour were a unique predictor of performance on different fraction tasks alongside working memory (Hansen et al., 2015; Hecht et al., 2003; Jordan et al., 2013; Vukovic et al., 2014; Ye et al., 2016). This highlights the importance of considering observed behaviour in the classroom in addition to performance on cognitive tests. Teacher ratings provide insight into how well children can control their attention and behaviour in busy classroom situations. However, questionnaire ratings of executive function behaviour often show weak correlations with standard cognitive measures (Nin et al., 2022; Saunders et al., 2018). Moreover, previous research suggests that these two types of measures contribute differently to academic achievement (Gerst et al., 2017; Ten Eycke & Dewey, 2016). This suggests that these types of measures may index different aspects of children's executive function and behavioural control. It is plausible that these different aspects may relate to different learning outcomes. Consequently, to understand the role of domain-general skills in children's mathematics learning we need to consider measures of both cognitive and behavioural aspects of executive function skills.

4. The role of executive functions in learning

There is now a substantial body of literature on the relationship between executive functions and concurrent, as well as future, mathematics achievement. We believe that while these studies provide valuable information on the role of executive functions, they do not provide specific evidence of the importance of executive function for children's *learning*. Many longitudinal designs capture general or specific measures of children's performance at different timepoints, but aren't typically designed to focus on material currently being taught in the classroom. Children's ability to understand and remember new information, procedures and concepts is related to their executive functions (Bascandziev et al., 2016, 2018; Grenell & Carlson, 2021; Miller et al., 2016), and this is likely to also play out in a classroom situation where they're being taught new material. Longitudinal designs are unlikely to be sensitive enough to isolate this specific mechanism.

We are unaware of any research that has specifically investigated the role of executive functions in learning new mathematical material in classroom settings. The closest research that has been carried out is tutoring studies that track changes in students' knowledge across a relatively short timeframe during which they receive individual or small group instruction (Fuchs et al., 2005, 2013; Powell et al., 2009, 2017; Powell & Fuchs, 2010; Supekar et al., 2013). Many of these studies were concerned with neural predictors (Iuculano et al., 2011; Jolles et al., 2016; Supekar et al., 2013) and the severity or type of pre-existing mathematics difficulties (Powell et al., 2009). However, a small number have investigated the role of general cognitive skills (e.g. executive functions, IQ, reading, attention) as well (Fuchs et al., 2005, 2013;

Iuculano et al., 2015; Supekar et al., 2013). For example, Fuchs et al. (2005, 2013) tracked the performance of 6–7-year-olds at risk of mathematics difficulties as they underwent small group tutoring (3 sessions per week for 16 weeks) to improve simple arithmetic. They found that working memory and teachers' ratings of attention predicted performance on mathematics measures at the end of the period of tutoring, indicating that children's executive functions do have an impact on their ability to learn new mathematical material. A similar study (Powell et al., 2017) compared the influence of child-level predictors on responsiveness to calculation and word problem tutoring in at-risk 7–8-year-olds. They found that children with greater working memory benefitted more from calculation training, but not word problem training, compared to children with lower working memory capacity. This suggests that good executive functions may help children to compensate for poor mathematics skills. While these studies of small group and individual tutoring in at-risk populations indicate a potential role for executive functions in mathematical learning, it remains unclear whether these findings generalise to all learners in a standard classroom situation. This is what we aimed to investigate in the current study.

5. The current study

This study compared the role of cognitive and behavioural aspects of executive function skills in learning new mathematical material in classroom settings, using rational number learning as an example. We included two types of executive function tasks and measures: The first were standard measures of cognitive executive function processes (visuospatial working memory, inhibition, shifting); the second were measures of applied executive function skills and behaviours (following instructions, teacher ratings, classroom behaviours). Based on previous research we predicted that all three cognitive executive function processes would be related to performance of mathematics. We also predicted that cognitive vs. applied measures would contribute differently to children's learning of new mathematical material over time. We followed a single cohort of 8–9-year-olds over the course of 20 months, at the start of which they received classroom instruction on rational numbers. Children came from the same school, which provides some assurance that the children experienced similar educational environments and teaching practices. We used structural equation modelling to assess whether executive functions explain individual differences in growth of rational number knowledge across three timepoints.

6. Method

6.1. Participants

Eighty-eight 8–9-year-olds ($M = 8.58$, $SD = 0.29$, 52 female) took part. All children attended a suburban primary school in a predominantly White British, average socio-economic status neighbourhood. Parents provided informed consent and children provided assent to take part. Ethical approval for the study was obtained from the Loughborough University Ethics Review Sub-Committee.

All parents provided consent and therefore we included the entire Year 4 cohort (3 classes). No children were excluded from the study. All children were following the same curriculum and thus the sample may include children with special education needs. The children were all in the same educational environment and were taught using the same scheme of work to the same schedule. Schools in the UK are required to follow the National Curriculum, however this provides only a high-level description of topics and does not give detailed descriptions of precisely what should be taught, how and when. To achieve our aim of closely associating the study to instruction of a particular topic required all children to be in the same educational environment and thus would not be possible across different schools or year groups. We collaborated with teachers in the design of the study, including: identifying a suitable mathematical topic (rational numbers), inclusion of the following instructions task, the practicalities of timing of assessment points in relation to classroom instruction.

6.2. Design and procedure

The study had three testing time-points (Time 1, Time 2, Time 3), determined in collaboration with the class teachers. Baseline testing at Time 1 took place immediately before teachers delivered the first block of focused teaching about fractions and decimals. Teachers returned to this topic to recap and further develop children's knowledge periodically between Time 1 and Time 2 (6 months later). Time 2 testing took place immediately following one of these recap weeks. Time 3 testing took place 14 months after Time 2. Although material about fractions and decimals had been taught during this period, Time 3 testing did not closely follow a period of studying this material. The National Curriculum followed by the children included: recognise, write, compare and order unit and non-unit fractions (symbolically and using diagrams), addition and subtraction of fractions with the same denominators, recognise and write decimals with tenths and hundredths, compare numbers with the same number of decimal places, solve simple problems involving these fractions and decimals.

At Time 1 each participant was tested individually in a one-hour session in a quiet room away from the classroom. This session included a rational number knowledge test, four measures of cognitive executive function processes (based on Cragg et al., 2017), and one measure of applied executive function skills: the following instructions task (based on Yang et al., 2014). The WIAT-II Numerical Operations subtest and a rational number estimation task were completed in a separate whole-class session. At Time 2 and Time 3 the rational number knowledge test was administered in a ten-minute individual session away from the classroom and the rational number estimation task was assessed in a whole-class session. Two further measures of applied executive function skills were collected: At Time 1, teachers were asked to rate children's executive function skills using the Behavior Rating Inventory of Executive Function (BRIEF; Gioia et al., 2000), and observations of each child's attention and behaviour in the classroom were recorded during the first block of focused teaching (based on Blatchford et al., 2011).

6.3. Tasks

6.3.1. Mathematics tasks

6.3.1.1. Rational number knowledge. Children completed a worksheet assessing their knowledge of decimals and fractions. Each question was read aloud by the researcher and repeated once if requested. Twelve items assessed knowledge of fractions and 12 items assessed knowledge of decimals. The questions were designed to reflect the curriculum content for fractions in Year 4 at that time. Skills assessed included: (i) the ability to flexibly switch between types of representations (for example, writing verbally presented fractions/decimals, matching symbolic fractions to concrete objects, matching pictures to written symbols), (ii) understanding of fraction and decimal magnitude (comparing and ordering) and (iii) simple computation (addition or subtraction). The questions included verbal word problems, written arithmetic, pictures and 3D objects and there were a variety of response types, including written responses, multiple choice, ordering and drawing. The measure used was total correct score (max = 24) and a higher score reflected better performance. Omega reliability for the Time 1 data was 0.82.

6.3.1.2. Rational number estimation. Each child was given a booklet of 10 number lines on A4 paper. Each trial was on a separate page with a 25 cm line placed in the centre of the page, with '0' above the left end of the line and '1' above the right end of the line. Children were asked to mark the position of a fraction or decimal (written in the centre at the top of the page). Fraction items (1/19, 1/7, 1/4, 3/8, 1/2, 4/7, 2/3, 7/9, 5/6, 12/13) and decimal items (0.05, 0.1, 0.25, 0.4, 0.5, 0.57, 0.67, 0.78, 0.8, 0.9) were matched for approximate magnitude. Children completed all the fraction items in one block and all the decimal items in another block with items pseudo-randomised within each block. Block order was counterbalanced across participants. The measure of performance was percent absolute error (the average absolute distance between the actual and estimated positions of numbers on the line, divided by the length of the line and multiplied by 100) and a lower score reflected better performance. Omega reliability for the Time 1 data was 0.59.

6.3.1.3. Arithmetic. The WIAT-II^{UK} (Wecshler Individual Achievement Test; Wechsler, 2005) Numerical Operations subtest was administered. This is a paper-and-pencil test that measures the ability to solve written calculations and simple equations. The total number of correct items (starting from item 7 with credit given for the earlier items) was used as the outcome measure and a higher score reflected better performance. Reliability for this age group is 0.91 (Wechsler, 2005).

6.3.2. Cognitive executive function processes

6.3.2.1. Visuospatial working memory (VSWM). We used the VSWM measure from Cragg et al. (2017). A series of 3 × 3 grids of black squares were presented with symbols on three of the squares. The children had to click on the odd-one-out symbol as quickly as they could. Following a series of grids (determined by the span length) the children were asked to recall the position of the odd-one-out symbol on each grid, in the correct order. The children first completed a practice block with one trial containing a span length of one and two trials with a span length of two. The practice trials could be repeated if necessary. For the test trials there were three trials at each span length, beginning with a span length of two. If the children responded correctly to at least one of the trials at each span length they continued to the next span length, up to a maximum of nine. The measure used was the total number of locations correctly recalled and a higher score reflected better performance.

6.3.2.2. Non-numerical inhibition. We used the non-numerical inhibition measure from Cragg et al. (2017). On each trial two animal images were presented on either side of the screen, one large animal (e.g. an elephant or giraffe) and one small animal (e.g. a ladybird or frog). One of the animal images was presented with an area on screen four times larger than the other image. On congruent trials (50 %) the animal that was larger in real life was also the larger image on the screen, and on incongruent trials (50 %) the animal that was smaller in real life was the larger image on the screen. The children were asked to press a button on the keyboard that corresponded to the side of the screen of the larger animal in real life.

The task included a practice block of eight trials and two test blocks of 48 trials. At the start of the task the children were shown the animal images individually and asked whether the animal was large or small in real life. All children completed this without problem, indicating that they had the necessary real-world knowledge to perform the task. Median reaction times for correct trials were calculated for the congruent and incongruent trials. The measure of inhibition was the difference in reaction time for congruent and incongruent trials. Smaller differences indicated better performance.

6.3.2.3. Numerical inhibition. We used the numerical inhibition measure from Cragg et al. (2017). On each trial, children were shown two arrays of white dots on opposite sides of a black screen, created using the Gebuis and Reynvoet (2011) method to control continuous quantities. The ratio between the number of dots ranged from 0.5 to 0.8 and the number of dots in each array ranged from 5 to 28. On congruent trials (50 %) the more numerous array had larger dots and the array encompassed a larger area. On incongruent trials (50 %) the more numerous array had smaller dots and the array encompassed a smaller area. The children were asked to press a button on the keyboard that corresponded to the side that the larger array was on. Two example trials were completed first and feedback was provided. The children then completed six practice trials and two blocks of 20 test trials. Accuracy was calculated for the congruent and incongruent trials separately. The measure of inhibition was the difference in accuracy for congruent and incongruent trials. Smaller differences indicated better performance. Omega reliability based on accuracy was 0.56 for congruent trials and 0.62 for incongruent trials.

6.3.2.4. Shifting. Children completed the Animal Sorting subtest from the NEPSY-II (Korkman et al., 2007). The test contains 8 cards coloured blue or yellow which contain pictures of animals (e.g., a cat, 2 fish, an elephant, etc.) within a scene. The cards can be sorted in different ways, for example blue vs. yellow cards, pictures with sun vs. pictures with rain. Following an example, the children were given up to 360 s to sort the cards in as many different ways as they could. If children stated they had finished, or if 120 s elapsed without a response, the test was discontinued. A raw score of the total number of permissible sorts was calculated. A higher score indicated better performance. Reliability for raw scores for this age group is 0.7 (Korkman et al., 2007).

6.3.3. Applied executive function skills

6.3.3.1. Following instructions task. We used an adapted version of the following instructions task from Yang et al. (2014). Six different coloured objects, including pencils, rubbers and rulers were laid out in front of a set of six different coloured locations, including gift bags, folders and boxes. The children wore earphones and heard pre-recorded auditory instructions about what to do with these objects, for example, "Spin the black pencil", "Put the pink rubber in the blue bag." There were eight trials, ranging from lists of two instructions up to five, presented in the same pseudorandomised order for all participants. The arrangement of both objects and locations differed on each trial. The experimenter recorded when the participants first moved to follow the instructions, and whether they were accurate in following the whole list. Accuracy scores for completing the whole set was used as the measure. A higher score reflects better performance. Omega reliability was 0.79.

6.3.3.2. Teacher ratings of executive function. Teachers were asked to complete the Behavior Rating Inventory of Executive Function (BRIEF; Gioia et al., 2000) for each child. This includes 86 items designed to measure everyday behaviours associated with different areas of executive function in children aged 5 to 18 years. We used the raw score for the Global Executive Composite where a lower score reflects better everyday executive function. Internal reliability of the Global Executive Composite is 0.98 (Gioia et al., 2000).

6.3.3.3. Classroom observations of attention and behaviour. Each child was observed for between four and fifteen 10 s periods (mean = 11.1). For the majority of children ($n = 74$) these observations were across two lessons on different days. For each 10 s observation period their behaviour was coded as one of three on-task or four off-task behaviours (see Blatchford et al., 2011 for details). Following piloting we used a simplified version of their coding with the following categories: on-task independently, on-task with teacher, on-task with other children, off-task independently passive, off-task independently active, off-task actively with other children, off-task with teacher. All observations were completed by the same trained observer and a time signal (audible only to the researcher) was used to mark 10 s intervals. The measure used was the overall proportion of time on-task. A higher score reflects more time on-task. At the end of the study we also asked teachers to rate

Table 1
Descriptive statistics of the outcome measures at three time points and the time-invariant predictors.

	M	*SD*	Min.	Max.	Skew.	Kurt.
Outcome measures						
Rational Number Knowledge 1	7.26	4.32	1.00	21.00	1.22	1.56
Rational Number Knowledge 2	10.93	5.96	0.00	24.00	0.60	−0.64
Rational Number Knowledge 3	14.29	5.76	0.00	24.00	−0.20	−0.89
Rational Number Estimation 1	25.11	8.27	3.19	40.64	−0.89	0.66
Rational Number Estimation 2	22.54	10.28	2.31	51.19	−0.04	−0.37
Rational Number Estimation 3	18.47	10.17	1.74	41.88	0.09	−0.66
Predictors						
Arithmetic	16.09	4.14	9.00	30.00	1.03	1.47
VSWM	26.70	13.27	3.00	64.00	0.20	−0.33
Non-numerical inhibition	0.13	0.08	−0.05	0.35	0.28	0.51
Numerical inhibition	0.31	0.19	−0.20	0.85	2.81	15.89
Shifting	4.25	2.10	0.00	9.00	−0.11	−0.50
Following instructions	0.73	0.27	0.00	1.00	−0.65	−0.55
BRIEF	105.19	39.29	63.00	214	1.32	0.83
Classroom observations	0.76	0.19	0.27	1.00	−0.05	−0.83

Note. VSWM = Visuospatial Working Memory. Min./Max. = Observed minimum/maximum. Skew = Skewness, Kurt = Kurtosis. $n = 88$.

each child's behaviour on a 5-item scale measuring children's attention and behaviour in class (items: Follows rules and instructions; listens attentively; completes work on time; works autonomously; works and play co-operatively with other children). The correlation between proportion of time on-task from our classroom observations and overall teacher ratings was $r_s = 0.41$, $p < .001$).

6.4. Analysis plan

We first conducted descriptive statistics and correlations among variables, followed by a repeated-measures ANOVA of the outcome measures to explore the pattern of growth. The data were then analysed with Latent Growth Curve Modelling (LGM), using Mplus (Version 8.1; Muthén & Muthén, 1998). We modelled growth trajectories for both the rational number knowledge and estimation tasks and explored the degree to which cognitive executive function processes, applied executive function skills and prior arithmetic skills predicted initial performance and growth over the three time points. First, the development of performance on the rational number and estimation tasks was investigated in two separate univariate latent growth curve models (unconditional models). Next, models including the predictor variables were fit to the data for each outcome measure (conditional models). Models were evaluated based on overall model fit indices (Browne & Cudeck, 1992; Byrne, 2012; Hu & Bentler, 1999; see Table 3).

7. Results

7.1. Descriptive statistics and preliminary analyses

Table 1 presents the descriptive statistics and Table 2 presents the correlations for all measures. The measures were all within the acceptable limits of skewness (< 3) and kurtosis (<4; Kline, 2011), except for the numerical inhibition measure. This was due to one participant who did not fall within 3 standard deviation of the mean and to correct for this we excluded their numerical inhibition measure from the analyses.

To examine children's average development, we performed two repeated measures ANOVAs, one for rational number knowledge and one for rational number estimation, with Time (three time points) as the within-subjects factor (see Fig. 1). For rational number knowledge there was a significant main effect of Time, $F(2, 170) = 133.84$, $p < .001$, $\eta_p^2 = 0.61$. Tests of polynomials indicated a significant linear effect, $F(1, 85) = 422.88$, $p < .001$, $\eta_p^2 = 0.76$, and a non-significant quadratic effect of Time, $F(1, 85) = 0.91$, $p = .663$, $\eta_p^2 = 0.002$. For rational number estimation there was also a significant main effect of Time, $F(2, 168) = 21.520$, $p < .001$, $\eta_p^2 = 0.61$. Tests of polynomials indicated a significant linear effect, $F(1, 84) = 36.988$, $p < .001$, $\eta_p^2 = 0.31$, and a non-significant quadratic effect of Time, $F(1, 84) = 1.005$, $p = .319$, $\eta_p^2 = 0.01$.

7.2. Growth curve models

7.2.1. Rational number knowledge

First, we built an unconditional growth model (without predictors) to identify an appropriate growth curve that would accurately depict development on the individual level. Based on the above ANOVA results, we initially hypothesised linear growth across time. This model included initial status (i.e. intercept) and growth (i.e. slope) latent factors. The factor loadings for the intercept were fixed to 1. For the slope factor, the first factor loading (the outcome in the first measurement point) was fixed to 0 to represent initial status and the other two factor loadings were fixed to 0.6 and 2, respectively according to the unequal intervals between time points. This model was not identified due to negative residual variance values for Time 1 and Time 3. As both values for the negative residual variance were small and insignificant, we fixed them to 0 and reran the model. The linear growth model did not demonstrate good fit based on the RMSEA (0.242), CFI (0.904), TLI (0.904) and SRMR (0.085) fit indices.

As Fig. 1 demonstrates that growth was not perfectly linear, we ran a non-linear latent growth model where the slope factor loadings for the outcome at Time 3 was freely estimated. This model demonstrated good fit to the data and was accepted as our final model (Table 3). The estimated factor loading for Time 3 was 1.143 ($SE = 1.116$, $p < .001$). The significant and positive mean of the slope factor ($M = 6.090$, $p < .001$) indicated a substantial gain in rational number knowledge over the period of 20 months. There was significant variance in the intercept and the slope ($V_{intercept} = 21.563$, $p < .001$, and $V_{slope} = 13.238$, $p < .05$, respectively), indicating variability in the rate of children's learning. The negative correlation between the intercept and the slope, $r = -0.135$, was not significant ($p = .931$), indicating that there is no significant relationship between a child's performance at Time 1 and learning over time.

Having identified the best-fitting unconditional model, we subsequently ran two conditional models by including predictors. In the first conditional model, arithmetic skill, VSWM, non-numerical inhibition, numerical inhibition, shifting, following instructions, BRIEF, and classroom observation were used to predict the intercept and slope growth factors. A second conditional model was run without arithmetic skill as a predictor.

Both models fit the data well (see Table 3 for the fit indices). The standardised regression coefficients are reported in Table 4 for model 1 and Table 5 for model 2. In conditional model 1 only arithmetic skill ($\beta = 0.576$) was a significant predictor of rational number knowledge at

Table 2
Correlations among the study variables.

	2	3	4	5	6	7	8	9	10	11	12	13	14
1. RNK 1	0.779**	0.736**	−0.604**	−0.697**	−0.572**	0.724**	0.425**	−0.175	−0.136	0.290**	0.214*	−0.426**	0.307**
2. RNK 2		0.750**	−0.623**	−0.695**	−0.682**	0.683**	0.427**	−0.110	−0.032	0.322**	0.333**	−0.456**	0.430**
3. RNK 3			−0.517**	−0.679**	−0.685**	0.658**	0.428**	−0.257**	−0.121	0.294**	0.292**	−0.521**	0.384**
4. RNE 1				0.599**	0.437**	−0.608**	−0.387**	0.004	0.000	−0.326**	−0.048	0.203*	−0.341**
5. RNE 2					0.582**	−0.566**	−0.298**	0.213*	0.079	−0.204*	−0.100	0.412**	−0.262**
6. RNE 3						−0.437**	−0.319**	0.181*	−0.010	−0.328**	−0.154	0.395**	−0.196*
7. Arithmetic							0.380**	−0.292**	−0.011	0.358**	0.146	−0.464**	0.301**
8. VSWM								0.046	−0.075	0.286**	0.185*	−0.312**	0.343**
9. Non-numerical inhibition									−0.026	0.108	0.079	0.039	0.116
10. Numerical inhibition										−0.051	−0.175	−0.165	0.079
11. Shifting											0.247*	−0.308**	0.317**
12. Following instructions												−0.029	0.133
13. BRIEF													−0.313**
14. Classroom observations													1

Note. RNK = Rational Number Knowledge. RNE = Rational Number Estimation. VSWM = Visuospatial Working Memory.
* $p < .05$.
** $p < .01$ (1-tailed).

Time 1, accounting for 58 % of the variance in the intercept factor. Following instructions ($\beta = 0.319$) and BRIEF ($\beta = -0.315$) were significant predictors of growth, accounting for 34 % of the variance in the slope factor. In conditional model 2, VSWM ($\beta = 0.264$), non-numerical inhibition ($\beta = -0.239$) and BRIEF ($\beta = -0.288$) were significant predictors of Time 1 performance, accounting for 38 % of the variance. Following instructions ($\beta = 0.296$) and BRIEF ($\beta = -0.288$) were again significant predictors of growth, accounting for 31 % of the variance in model 2. In both models, initial status correlated negatively with the slope factor, (model 1: $r = -0.55$, $p < .001$; model 2: $r = -0.48$, $p < .001$).

7.2.2. Rational number estimation

Similarly to rational number knowledge, first, we built an unconditional linear growth model for rational number estimation without predictors, using the factor loadings to 0, 0.6 and 2 for the outcomes measured at three time points. This model demonstrated good fit to the data (see Table 3) and we accepted this as our final unconditional model. There was significant variance in both the intercept and slope ($V_{intercept} = 54.663$, $p < .001$, and $V_{slope} = 20.741$, $p < .001$, respectively). The initial status correlated negatively with the slope factor, $r = -0.27$, $p = .05$.

We subsequently ran two conditional models by including predictors. In the first conditional model, arithmetic skill, VSWM, non-numerical inhibition, numerical inhibition, shifting, following instructions, BRIEF, and classroom observation were used to predict the intercept and slope growth factors. A second conditional model was run without arithmetic skill as a predictor. Neither Conditional Model 1 nor Model 2 demonstrated good fit (see Table 3 for the fit indices). Based on the modification indices, we tried several things to improve model fit, e.g. (i) allowing the BRIEF measure to covary with the outcome at Time 1 and (ii) adding covariance terms between the outcomes and BRIEF measure and adding some constraints (i.e. equal residual variances of the growth factors and equal effects on each time point). However, the fit deteriorated every time. As we were not able to address the exact cause of the misfit, we were not able to adjust and test the conditional

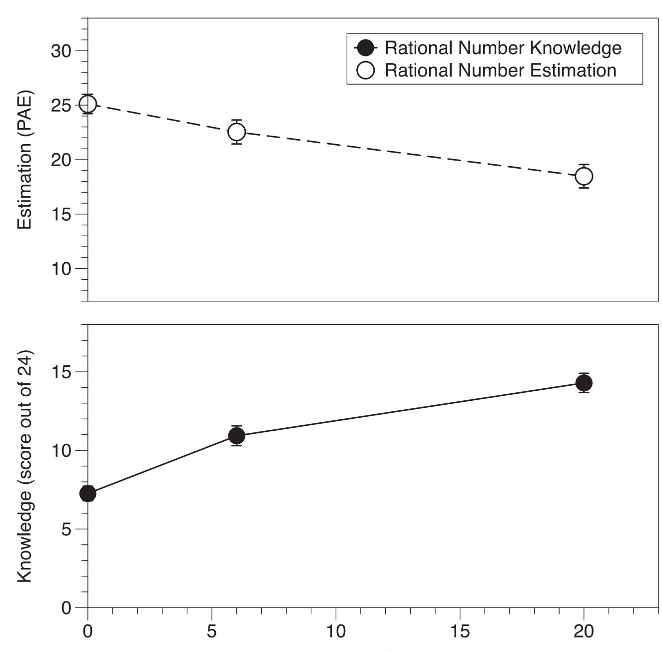

Fig. 1. Average development of rational number knowledge and rational number estimation across the three time points. (Knowledge: higher scores mean better performance, estimation: lower scores mean better performance.)

Table 3
Fit indices on the unconditional and conditional (i.e. with predictors) latent growth models (LGMs) and the corresponding fit criteria.

	χ^2	*df*	χ^2/df	RMSEA	CFI	TLI	SRMR
Rational Number Knowledge							
Unconditional	0.904	1	0.904	0.000	1.000	1.0002	0.038
Conditional Model 1	12.351	9	1.372	0.067	0.986	0.959	0.046
Conditional Model 2	9.561	8	1.195	0.048	0.993	0.978	0.044
Rational Number Estimation							
Unconditional	0.615	2	0.308	0.000	1.000	1.029	0.031
Conditional Model 1	20.077	9	2.231	0.122	0.919	0.785	0.056
Conditional Model 2	19.829	9	2.203	0.120	0.900	0.734	0.053
Fit Criteria							
			$0 \leq \chi2/df \leq 2$	< 0.05	≥ 0.95	≥ 0.95	$0 \leq$ SRMR ≤ 0.05

Note. χ^2 = chi-square value; *df* = degrees of freedom; CFI = Comparative Fit Index; TLI = Tucker–Lewis Index; RMSEA = Root Mean Square Error Approximation; SRMR = Standardised Root Mean Square Residual.

Table 4
Standardised regression coefficients from Conditional Model 1 for intercept and slope growth factors of rational number knowledge.

	Intercept (Initial Status)		Slope (Growth)	
	β	*SE*	*β*	*SE*
Arithmetic	0.576**	0.084	−0.076	0.162
VSWM	0.131	0.081	0.100	0.141
Non-numerical inhibition	−0.066	0.079	−0.225	0.135
Numerical inhibition	−0.133	0.075	−0.020	0.132
Shifting	−0.011	0.082	−0.067	0.143
Following Instructions	0.070	0.075	0.319*	0.137
BRIEF	−0.127	0.084	−0.315*	0.144
Classroom observations	0.057	0.081	−0.228	0.143

Note. VSWM = Visuospatial Working Memory.
* $p < .05$.
** $p < .01$.

Table 5
Standardised regression coefficients from Conditional Model 2 for intercept and slope growth factors of rational number knowledge.

	Intercept (Initial Status)		Slope (Growth)	
	β	*SE*	*β*	*SE*
VSWM	0.264**	0.094	0.070	0.137
Non-numerical inhibition	−0.239**	0.087	−0.220	0.125
Numerical inhibition	−0.154	0.089	−0.024	0.129
Shifting	0.098	0.096	−0.074	0.136
Following Instructions	0.116	0.091	0.296*	0.139
BRIEF	−0.288**	0.095	−0.288*	0.135
Classroom observations	0.119	0.097	0.214	0.140

Note. VSWM = Visuospatial Working Memory.
* $p < .05$.
** $p < .01$.

models further, and therefore did not interpret estimation scores any further.

8. Discussion

In this study we examined the role of executive functions in rational number learning across three time points. We included two types of executive function tasks and measures; standard measures of *cognitive executive function processes* which capture children's capacity to control their attention and behaviour in a focused situation, and broader measures of children's *applied executive function skills*, which capture the extent to which children control their attention and behaviour in a real-world environment. Our findings extend the previous literature on the relationship between executive function skills and rational number knowledge (Hansen et al., 2015; Hecht et al., 2003; Jordan et al., 2013; Stelzer et al., 2021; Vukovic et al., 2014; Ye et al., 2016) by separating out contributions to performance and learning. We found that applied executive function skills consistently predicted rational number learning. In contrast, children's cognitive executive function processes were related to their performance of rational number knowledge at a single point in time, but only when arithmetic skills were not included in the model. Although performance on the rational number estimation task improved over time, the conditional models did not demonstrate a good fit. This may reflect, in part, lower internal reliability for the measure of rational number estimation than rational number knowledge. We therefore base our interpretations on the rational number knowledge measure.

We ran two conditional models to explore predictors of rational number learning, one including arithmetic skill in addition to the cognitive and applied executive function measures, and one without. When arithmetic skill was included in the model it was the only predictor of rational number knowledge at Time 1. In the second model, without arithmetic skill, Time 1 performance was associated with visuospatial working memory, non-numerical inhibition and the BRIEF. Given the well-established strong relationship between executive function and arithmetic (e.g. Cragg et al., 2017; Peng et al., 2016) and the strong relationship between arithmetic and rational number knowledge (e.g. Jordan et al., 2013; Vukovic et al., 2014), it is not surprising that arithmetic was the strongest predictor of concurrent rational number knowledge. This is consistent with previous research into predictors of rational number knowledge and demonstrates the importance of prior arithmetic skills for initial levels of rational number knowledge, prior to focused instruction on this topic.

In line with previous research (e.g. Jordan et al., 2013; Stelzer et al., 2021) we found shared variance between executive function and domain-specific mathematics predictors, such that working memory and inhibition were the only significant predictors of concurrent rational number knowledge when arithmetic was not included in the model. This pattern of results supports framework models of mathematics whereby executive function skills feed into performance on specific components of mathematics, which in turn support overall mathematics achievement (Cragg et al., 2017; Geary, 2004; Gilmore, 2023).

The conditional model produces two different measures, the intercept and slope, which tap into different aspects of learning and performance. Performance at Time 1 (intercept) captures both prior learning as well as the processes that are recruited when the task is performed. These processes include holding the question in mind, translating between representations (pictures, words and digits) and suppressing whole number knowledge, all while mentally calculating the answer. Time 1 scores were not at floor level (M = 7.26 out of 24), indicating that children did have some existing rational number knowledge prior to the first formal block of instruction. Growth (slope) on the other hand, captures how well children have acquired new knowledge and skills over time. This is driven by how well they can connect new and existing knowledge, increase the efficiency of procedures and pay attention in the classroom. There was significant variability in both performance at Time 1 and growth over time, indicating the wide range of starting points and learning rates within a single classroom.

Growth in rational number knowledge was predominantly predicted by applied executive function skills, namely performance on the following instructions task and teacher ratings on the BRIEF. This was consistent for both Model 1 (including arithmetic) and Model 2 (without arithmetic). This is in keeping with the notion that learning is driven by children's attention and behaviour in the classroom as well as how well they can harness their executive function skills. It is logical that these classroom behavioural skills are important predictors of mathematics success. For example, children who focus and stay 'on task' are more likely to process information and experience a better quality and quantity of practice to help them learn new concepts and procedures compared to children who are distracted and do not engage with material. There may also be an interaction between a child's attention and behaviour and their level of prior knowledge. One recent study (Geary et al., 2021) found that 12–13-year-olds with poor in-class attention but good prior knowledge made less progress in mathematics in one year than those with good in-class attention and weaker prior knowledge. Thus, classroom attention is a key skill for mathematics success, as it may partially compensate for difficulties with other skills (e.g. knowledge).

Perhaps surprisingly, performance on our cognitive visuospatial working memory measure was only weakly associated with growth. This appears to be at odds with previous studies showing that working memory predicts growth in mathematics achievement over time (Ribner, 2020). However, a strong predictor of growth was performance on the following instructions task, which has been used as a measure of children's ability to apply their working memory (Holmes et al., 2009). One interpretation of this is that cognitive tests of working memory measure the maximal capacity of what you can do in a controlled experimental setting, whereas applied measures such as the following instructions task, indicate how well you are able to recruit that capacity in a less controlled situation, more similar to everyday life. It is the raw capacity that may be important when actively processing mathematical information, as reflected by the relationship between working memory and performance at a single time point (Table 2). However, learning depends on how likely you are to recruit that capacity in busy demanding situations. Hence the applied working memory measure (following instructions) was related to learning over time.

Teacher's ratings of children's everyday executive functions using the BRIEF were associated with learning over time in both models and Time 1 performance when arithmetic was not included in the model. Teachers' judgements of a child's executive function are likely to reflect both children's executive function capacity as well as how likely they are to apply it, and may also reflect the contribution of prior learning to performance at a single timepoint. When arithmetic was included in the model, inhibition was not related to either Time 1 performance or growth. This likely reflects shared variance given the previously-established relationship between inhibition and arithmetic (e.g. Cragg et al., 2017; Megías et al., 2015). For model 2, not including arithmetic, non-numerical inhibition had a comparable association with both Time 1 performance and learning over time, although this did not reach significance for growth. This finding supports previous studies indicating that inhibition supports rational number knowledge (Avgerinou & Tolmie, 2020; Gómez et al., 2015), most likely by helping to suppress established, but interfering, whole-number knowledge and procedures (e.g. Rossi et al., 2019). However, it suggests that inhibition may also play a wider role in the classroom by helping children to filter out irrelevant information and focus on what they are learning.

8.1. Strengths and limitations

This study is one of the first to investigate the processes involved in the learning of new mathematical material in classroom settings rather than changes in mathematics achievement or specific mathematical skills over time. This allows us to separate learning and performance more clearly. Moreover, our design was developed in collaboration with teachers who helped us to choose a topic where they felt there was variation in children's rates of learning, determine the best timepoints to capture that learning and identify applied executive functions skills that they viewed as most important for learning.

However, the study also had important limitations. Due to working collaboratively with teachers and to ensure that our measurement points would coincide with periods of instruction (which would differ from school-to-school) our study was run in a single school with all Year 4 children taking part. However, there is no reason to think that the relationships we observed between executive function skills and mathematical outcomes would be unique to this school. Importantly however, the involvement of a single school limited our sample to 88 children. This is a small sample for latent growth modelling, particularly given the non-linear growth, although this is within some published recommendations (e.g. Curran et al., 2010; Shi et al., 2021) We attempted to mitigate this by testing only theoretically-driven models and by the relative simplicity of our model (only three measurement points). We also note that we have no issues such as missing data or non-normality. However, our findings should be considered tentative until replicated with a larger, pre-registered study.

Future research should also include multiple measures for each executive function process or skill in order to control for task-specific measurement error, as well as a wider range of cognitive measures. This should include measures of intelligence in order to more accurately specify the contribution of executive function skills. This would reveal how much of the shared variance between executive function skills and rational number knowledge is also shared with intelligence vs. that which is unique. Previous work has indicated substantial shared variance between measure of intelligence and working memory (Ackerman et al., 2005), but not necessarily inhibition and shifting (Benedek et al., 2014; Friedman et al., 2006). Consequently, unpicking this relationship would more precisely specify the contribution of executive function skills.

Finally, our design involved assessing executive function skills at the first timepoint and only assessing mathematics learning at subsequent time points. Consequently, we are not able to pinpoint bidirectional relationships between executive functions and mathematics (e.g., Coolen et al., 2021). It would be valuable to combine multiple assessments of executive functions with the current learning-focused design used here. Further research with a more comprehensive, and therefore potentially less noisy, measure of rational number estimation may allow growth models for this outcome to be run. This would allow insight into whether children's magnitude conceptions of rational number develop similarly to their knowledge of rational numbers and are influenced by similar cognitive skills.

8.2. Conclusion

In conclusion, our study adds to the literature implicating a role for executive functions in mathematics. We go beyond previous findings to demonstrate a role for executive function, in particular working memory and inhibitory control, in both the performance and learning of mathematics. Crucially, we found that it was the application of executive function skills that predicted learning, rather than more standard cognitive executive function processes that are typically studied in relation to mathematics. This highlights that standard lab-based measures of executive function processing capacity do not tell the whole story and we also need to consider whether or not children are able to apply these processes in their everyday life.

CRediT authorship contribution statement

Camilla Gilmore: Conceptualization, Investigation, Methodology, Writing – original draft. **Emine Simsek:** Formal analysis, Writing – review & editing. **Joanne Eaves:** Writing – original draft. **Lucy Cragg:** Conceptualization, Investigation, Methodology, Writing – original draft.

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