Title: **Optimal Ultrasonic Sensor Configuration for Plate-Like Structures Using the Value of Information**

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ABSTRACT

Condition-based maintenance is based on reliable and effective structural health monitoring systems able to provide accurate information about the health condition of a structure. Among the factors that influence the accuracy and reliability of such systems, the number and position of sensors stand out as key features. As a rule, the larger the number of sensors, the more and better information can be obtained. However, a practical solution is required to find a healthy balance between the cost of the monitoring system and the adoption of the information that they provide as optimality. To rigorously address such optimization problem and provide an optimal design of the monitoring system, this paper proposes the use of the value of information and the expected information gain. The methodology is illustrated using a stiffened plate-like structure with a bounded damaged area, ultrasonic guided-waves as SHM technique, and different sensor positioning strategies. The results reveal the value of information as a rational index to provide optimal ultrasonic sensor configuration and rank different positioning alternatives.

INTRODUCTION

Accurate and reliable condition-based maintenance relies on optimal sensor configurations of structural health monitoring (SHM) systems. Such configurations seek to find an effective trade-off between cost and information. In general, extremely simple configurations (low number of sensors) may lead to poor structural health information. Oppositely, a relatively high level of information can be obtained if an impractical configuration of sensors (i.e., involving a high number of sensors) is considered. To address such a trade-off, the use of the value of information [1, 2] for the design and optimiza-

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tion of SHM systems and operational decisions has been widely proposed in the literature [3–7]. In this paper, the value of information is proposed in the context of ultrasonic guided-wave based SHM to rigorously obtain the optimal sensor configuration.

The optimal sensor configuration has been partially addressed in the literature for ultrasonic guided-waves. Thus, the area of coverage has been adopted to optimize the sensor positions based on the properties of both sensors and structure [8–11]. Further, several objective functions, e.g. based on the probability of detection, have been proposed and addressed by heuristic algorithms such as genetic algorithms (GA), simulated annealing, or particle swarm optimization [12–15]. However, these methodologies rely on deterministic approaches, which may find certain limitations in the presence of uncertainties. To account for such uncertainties, the optimal sensor configuration has also been addressed using Bayesian approaches in applications other than ultrasonic guided-waves. In these applications, the objective functions are defined in terms of probabilistic metrics of the informativeness obtained from the sensors, such as the Kullback-Leibler (KL) divergence, the Shannon-entropy, or the mutual information [16–20]. The main conclusion drawn from these works is that the more sensors, the more informative SHM configuration, hence showing that an additional criterion is desired to obtain an optimal sensor configuration, i.e., including both position and number of sensors.

In this paper, a methodology based on value of information is proposed for optimal number and position of sensors. First, the optimal sensor placement for damage localization is addressed using the value of information and a greedy algorithm, whereby the sensors are optimally placed stepwise based on [17]. As a result, the expected information gain is obtained for each optimal sensor layout up to an absolute maximum number of sensors. Then, the optimal sensor configuration is obtained by adopting a case specific inverse cost function depending on the number of sensors. A stiffened aluminum plate-like structure with a bounded damage area is used to illustrate the methodology. It is found that the use of geometrically unconstrained sensor configurations are more informative using less number of sensors than linear arrays. This result implies that a flexible sensor system is required to freely place the sensors along the structure. Additionally, factors such as the costs stemming from the electronic components, power consumption, and post-processing computational cost are minimized.

The details of the proposed approach are described in the following section. Next, a case study is presented to illustrate the methdology. And finally, concluding remarks are provided.

METHODOLOGY

The optimization approach based on value of information, which trades-off information and cost, is presented in this section. Note that the optimization is addressed by using an efficient robust Bayesian damage localization approach [21], instead of model-based Bayesian damage identification and quantification methods [22] to reduce computational burden.

Optimal sensor configuration: value of information

The required balance between information and cost is rigorously addressed by us-

ing the value of information. In order for the benefit of measuring data to be quantified, the function $b(n, \boldsymbol{\theta})$ is defined as a function of the sensor configuration n and the model parameters $\boldsymbol{\theta}$. Note that the sensor configuration n entails the definition of the optimal sensor layout of the n sensors. In addition, such benefit function is proposed to be proportional to an inverse of cost $f(n)$ of each sensor configuration n and another function $g(\boldsymbol{\theta})$, which accounts for the information gained by the system, such that $b(n, \boldsymbol{\theta}) \propto f(n)g(\boldsymbol{\theta})$. Note also that $f(n)$ is case specific so it is defined according to manufacturing or maintenance cost law of both the structure and the SHM system.

Next, the concept of maximum prior expected benefit B', which is based on the prior information of the model parameters $p(\boldsymbol{\theta})$ [23], is defined and used to obtain the initial optimal sensor configuration n'_{opt}, as follows [3]:

$$B' = \mathbb{E}_{p(\boldsymbol{\theta})}\left[b(n'_{opt}, \boldsymbol{\theta})\right] \quad n'_{opt} = \arg\max_n \int b(n, \boldsymbol{\theta})p(\boldsymbol{\theta})d\boldsymbol{\theta} \tag{1}$$

Similarly, the maximum posterior expected benefit (PEB) $B''(\mathbf{D})$ [23], which is based on the posterior distribution of the parameters given the data $p(\boldsymbol{\theta}|\mathbf{D})$, is obtained as follows [3]:

$$B''(\mathbf{D}) = \mathbb{E}_{p(\boldsymbol{\theta}|\mathbf{D})}\left[b(n''_{opt}, \boldsymbol{\theta})\right] \quad n''_{opt} = \arg\max_n \int b(n, \boldsymbol{\theta})p(\boldsymbol{\theta}|\mathbf{D})d\boldsymbol{\theta} \tag{2}$$

where the conditioning on $\mathbf{D}$ is to denote that B'' depends on the data obtained by the sensors. Note that $\mathbf{D}$ can be obtained either from preliminary tests or simulations at the design stage, since real data cannot generally be used at this stage. The Bayesian inverse problem (BIP) of damage localization used to obtain both the prior $p(\boldsymbol{\theta})$ and posterior $p(\boldsymbol{\theta}|\mathbf{D})$ distributions, with a particular sensor configuration, is based on a robust hierarchical framework, whose details of implementation can be found in [21]. Then, by subtracting both mathematical expectations evaluated at their optimal sensor configurations n''_{opt} and n'_{opt}, the conditional value of information (CVI) on $\mathbf{D}$ is given by:

$$\text{CVI}(\mathbf{D}) = B''(\mathbf{D}) - B' \tag{3}$$

Note that Equation (3) is defined for only one damage scenario, whereby the sensors acquire the data $\mathbf{D}$. In order to obtain the optimal sensor configuration for a hyper-set of data $\mathfrak{D}$, the maximization of the expectation of the CVI over $\mathfrak{D}$ is required, as follows:

$$\text{VoI} = \mathbb{E}_{p(\mathbf{D})}[\text{CVI}(\mathbf{D})] \tag{4}$$

where VoI denotes to value of information. Note that the computation of the last expectation requires the solution of the optimal sensor configuration for each of the data $\mathbf{D} \in \mathfrak{D}$ and the expensive calculations of the evidence $p(\mathbf{D})$ [23] for each simulated data. A pre-posterior analysis, by using data generated by a model and samples coming from the prior distribution of the model parameters, would be more appropriate at the design level. However, for the purpose of illustrating the methodology, the data are restricted here to one simulated damage scenario, leading to the use of CVI for optimal sensor configuration. Finally, note that the sensor placement approach is based on a greedy algorithm that adds sensors one by one as described in [17], however its position is optimized using GA.

CASE STUDY

The case study proposed here illustrates the methodology using only one set of data, corresponding to one scenario of damage. To this end, a Finite Element (FE) model using Abaqus is created to synthetically simulate the propagation of the guided waves over a stiffened panel such as the one depicted in Figure 1. The damage is simulated using a 2mm × 2mm hole situated at (-4.9, -1.9) cm (considering the origin at the center of the plate) to obtain the scattered guided-waves (i.e., the set of data), whereby the time of flight (ToF) is estimated and the damage localization is addressed. S4R (4-node doubly curved thin or thick shell, reduced integration, hourglass control, finite membrane strains) shell elements [24] with a mesh size of 0.5 mm are used, by which only anti-symmetric modes are captured. The bay is 300mm × 300mm and the stiffeners are made of aluminum of 50mm of height. Both the plate and stiffeners have 2mm thickness and the material properties correspond to the aluminum alloy 2024-T351 with a density of 2780 kg/m^3, a Young's modulus of 73.1 GPa and a Poisson ratio of 0.33. The guided waves are generated using a 5 cycle sine tone burst centered at a frequency of 300 kHz applied in the perpendicular direction at the center of the plate.
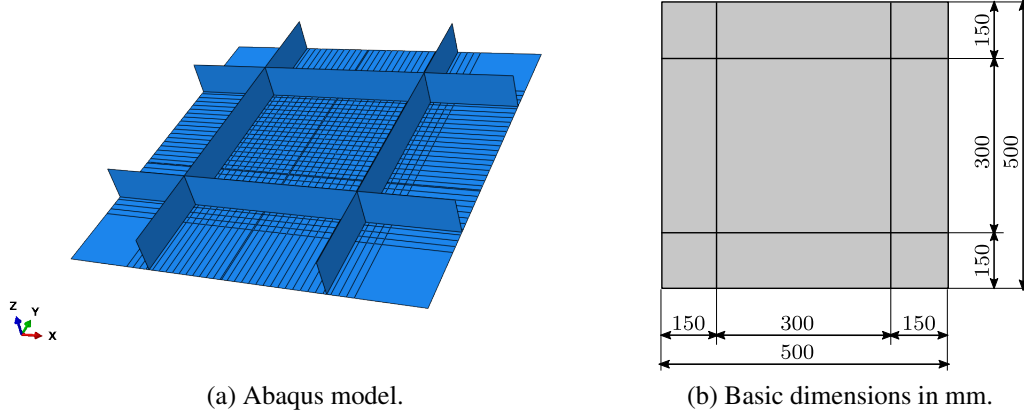

(a) Abaqus model. (b) Basic dimensions in mm.

Figure 1: Abaqus model of the stiffened panel in panel (a) and the corresponding dimensions and position of the perpendicular stiffeners in panel (b).

All the BIPs, managed by the GA, provide the samples of the posterior distribution, whereby the KL divergence and the quality of the sensor distribution is obtained by its value. In addition, the BIPs are addressed by using the asymptotically independent Markov sampling algorithm [25] with a threshold value $\gamma = 1/2$ and $10,000$ samples per annealing level. More details of implementation of this BIP of damage localization are provided in [21].

Three different sensor positioning strategies, i.e., (1) unconstrained open configuration, (2) single linear array configuration, and (3) double linear array configuration, are compared by using the value of information. As observed in Figure 2b, the open configuration results to be the most valuable, followed by the double and single linear array configurations, respectively. Thus, the optimal configurations involve 5 sensors for the open configuration with a $\mathrm{CVI}(\mathbf{D}) = 12.5281$ [bits], 10 sensors for the single array configuration with a $\mathrm{CVI}(\mathbf{D}) = 9.0776$ [bits], and 10 sensors for the double array
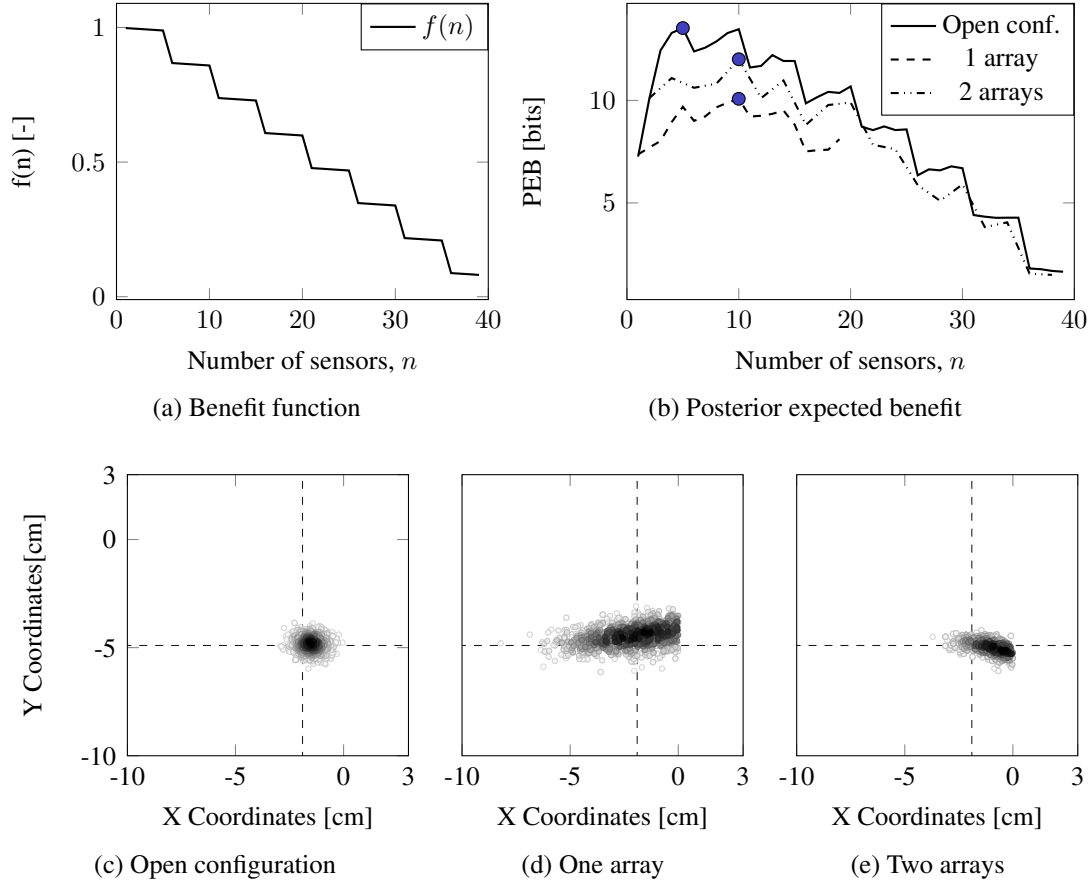
Figure 2: Optimal sensor configurations obtained using the step inverse cost function $f(n)$ shown in (a). In (b), the PEB functions for the different configurations, i.e., open configuration and 1 and 2 array configurations, are depicted. Then, panels (c) to (e) show the resulting damage position reconstruction adopting the optimal sensor layouts using the open configuration, 1 array and 2 array configurations, respectively.

configuration with a $\mathbf{CVI(D)} = 11.0081$ [bits].

The reconstructions of the damage position corresponding to the three sensor configurations are depicted through Figures 2c to 2e. According to these results, the open configuration reconstructs the damage position remarkably accurate, followed by the double array configuration. Finally, the most uncertain damage reconstruction is obtained by the single array configuration. This can be explain since the BIP of damage localization uses an ellipse-based model, whereby the intersections of the ellipses provide the damage position. If those intersections are almost perpendicular, the damage position is accurately reconstructed, as in Figure 2c using the open configuration. However, the intersections of several ellipses close to each other lead to a more uncertain reconstruction of the damage position, as can be observed in Figure 2d using the single array configuration. Finally, Figure 3 depicts the optimal sensor layouts obtained with the three positioning strategies, where both array sensor configurations overlap in the upper zone of the damage.
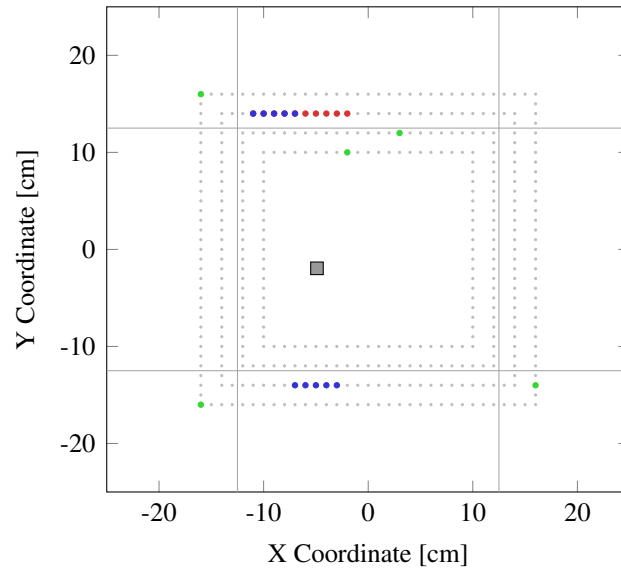
Figure 3: Optimal sensor distributions for the step inverse cost $f(n)$. The colors represents the different sensor positioning strategies as follows: open configuration as green, 1 array configuration as red, 2 array configuration as blue. Gray dots represent the seach space of possible sensor positions. Note that the optimal sensor location of the configuration with 1 array and 2 arrays overlap in the upper zone of the damage which is represented with a dark square.

CONCLUDING REMARKS

A novel framework to obtain the optimal sensor configuration based on the value of information is proposed in this paper. A case study was proposed to illustrate the methodology using synthetically generated ultrasonic data, considering a pre-design stage whereby the data can only be simulated. The value of information has shown efficiency in ranking several sensor positioning strategies, leading to geometrically unconstrained distributions instead of linear array types of sensors layouts. However, the main limitation of this approach is the computational cost required to compute the mathematical expectation over the hyper-set of data. Thus, a future extension of this work would be the application of surrogate modeling techniques to alleviate the computational burden required for calculating both the posterior distribution of the model parameters and the evidence of the model.

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