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## Uncertainty analysis of an augmented industrial robot

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### Abstract

Industrial robots are often used in additive manufacturing (AM) environments to automate the process of creating complex parts and structures. One example of an industrial robot used in AM is for wire and arc additive manufacturing. This process involves using an industrial robot to deposit metal wire onto a workpiece while simultaneously melting it with an electric arc. This allows for the creation of large-scale metal parts with high precision and efficiency. Moreover, industrial robots can be used to move a printing nozzle along a predefined path to create a three-dimensional (3D) object. This method is commonly used to produce plastic parts and is often faster and more cost-effective than traditional manufacturing methods. However, industrial robots can sometimes struggle with achieving the required positional accuracy for AM tasks. To overcome this challenge, a 3D move stage is added to the robot's end effector, and it is then possible to fine tune its pose in closed loop using optical metrological devices, such as laser trackers and laser interferometers. These devices benefit from high levels of accuracy, which make it possible to use them in a feedback loop to increase the overall accuracy of the industrial robots and the moving stage. However, adding the move stage to the industrial robot changes the system's geometry. In this paper, an overall uncertainty analysis for the augmented robot is performed. The forward kinematics of the overall industrial robot and the move stage are obtained. The probability distribution of position depends on industrial robot joint angle values and move stage displacement values. As a part of uncertainty analysis, the probability density function associated with a sample industrial robot joint angle values and a move stage displacement value is calculated.

Uncertainty analysis; industrial robots; linear stages; surface; laser tracker.

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### 1. Introduction

In an industry 4.0 environment, there exist an increasing demand for automated factory elements including industrial robots. As a result of the ever-increasing utilisation of industrial robots in a factory floor, their precision control becomes more important. Pick and place applications, drilling and wire and arc additive manufacturing [1] are example applications which are handled by an industrial robot. Increasing the precision of industrial robots may contribute to high quality outcomes. Zero offset for encoder settings [2], robot geometrical irregularities, manufacturing tolerances associated with robot dimensions, and environmental factors during robot utilisation may result in uncertainty in the robot positioning [3]. Modification of an industrial robot geometry using augmented industrial robot may contribute to a more precise industrial robot. In this work in order to increase the robot's positional accuracy, we added an industrial move stage. We show that the repeatability of the industrial manipulator added to the industrial robot is five times better than the main industrial robot. Using this approach, we will be able to position the industrial robot end effector very close to the desired position and then fine tuning will be performed using the more accurate 3D move stage mounted at the end of the industrial robot. Feedback provided by a laser tracker as a non-contact optical metrology equipment is used by the control algorithm to compensate for positional error.

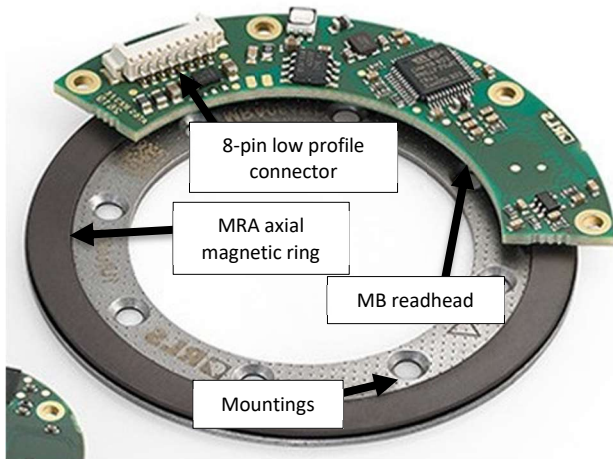
Adding the 3D move stage at the end of the industrial robot modifies its overall geometry. Hence, it is required to perform an uncertainty analysis on the overall augmented system. There exist different sources of uncertainty associated with industrial robot positioning, these will include uncertainty in the

estimation of the Denavit–Hartenberg parameters, irregularities in the robot's geometry and its assembly, and sensor uncertainties [4]. The added 3D move stage uses a lead screw to convert its rotational movement to a linear one which may cause uncertainty in its positioning. The stepper motor used in the structure of a 3D move stage is the second source of uncertainty in a 3D move stage.

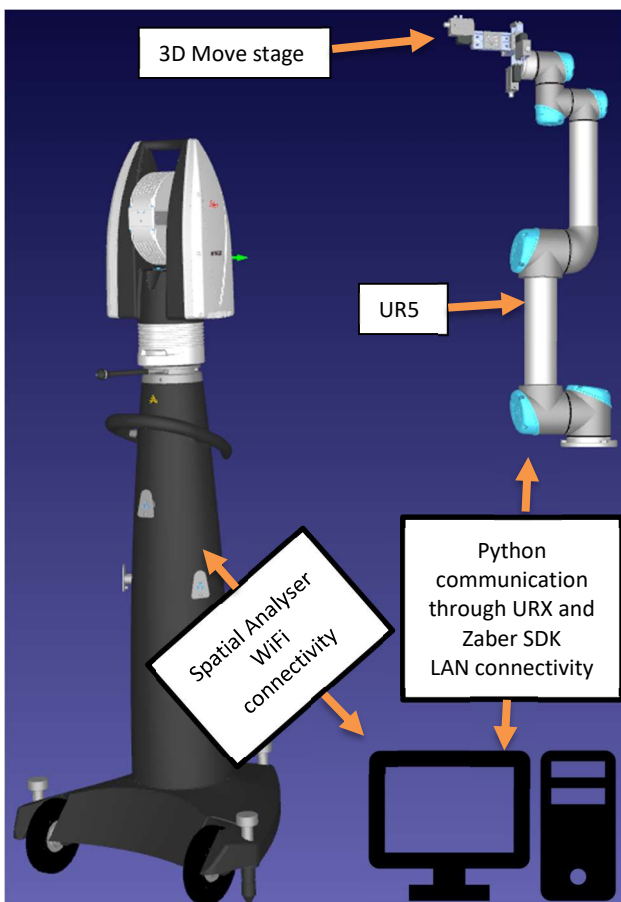
In this study the uncertainties associated with the augmented industrial robot 3D position caused by its joint angle encoders and the displacements of the 3D move stage are considered and analysed. To perform such an analysis, it is required to perform an uncertainty analysis on the robot joint angle sensor. The uncertainty associated with joint encoders are propagated to the industrial robot position and orientation through the forward kinematics (FK) function. Using the law of the unconscious statistician [5], it is possible to obtain the probability density function (PDF) for the position and orientation of the industrial robot due to uncertainty in the industrial robot joint angles. Such an analysis provides useful information about the industrial robot operation and is beneficial to increase its positioning accuracy. It is further possible to control the probability density function associated with industrial robot position and improve it for more consistent positioning [6].

### 2. Overall system setup

The experimental setup is composed of an industrial robot, in this case a UR5, a 3D move stage, and a laser tracker (see Fig. 1). UR5 is responsible for large movements as it benefits from large workspace and longer robotic links. However, the 3D move stage with its links is capable of fine tuning the overall position of the



**Figure 1.** RLS' AksIM series magnetic rotary encoder for UR5 joint angle measurement



**Figure 2.** The overall experiment setup

end-effector improving its precision. The duty of the laser tracker is to validate the overall positional uncertainty. The repeatability of UR5 is 0.1 mm [7], and the repeatability of the move stage mounted on the UR5 is 20  $\mu\text{m}$  [3]. While UR5 is composed of six links, the 3D move stage is composed of three linear move stages.

### 2.1. UR5

UR5 is a six degrees-of-freedom (6 DoF) collaborative robot capable of working in a close proximity with human without any cage or extra safety measures. The joints of this robot benefit from encoders and motor current sensors. Using these sensors, it is possible to measure the position and speed of each joint as well as motor currents. This robot benefits from teaching

pendant and safety button. The position repeatability of this robot is 0.1 mm, and its load capacity is 5 Kg. The PC interface used for industrial robot is under robot operating system (ROS) and Python programming language.

One of the major sources of uncertainty for UR5 is its encoder (see Fig. 2). UR5 uses RLS' AksIM series magnetic rotary encoder [8], supplied by Renishaw's associate company RLS. The encoder is mounted on the output side of gearbox which makes it to measure the actual rotating angle of the joint. Moreover, since this encoder is an absolute one, no batteries are required to maintain its joint angles during its turnoff period. This encoder features excellent dirt immunity with a IP64 rating. Light weight, low energy consumption and multiple built-in self-monitoring functions are among the specification of this encoder which makes it an appropriate choice for measuring UR5 joint angles. The resolution of this encoder is up to 20 bits and its accuracy up to  $\pm 0.1^\circ$ .

### 2.2. XY-linear stage

High precision, light weight XY-linear stages used in this project are manufactured by Zaber® namely: X-XY-LSM050A. This product is highly reliable one which is intended for critical medical, marine [9], aviation, 3D printing [10, 11], and military applications. Three linear stages can be easily mounted perpendicularly forming a 3D motion stage. The power supply of these linear stages is (24-48) VDC and their maximum load capacity is 250 N. Each stage benefits from a two-phase stepper motor. The rating motor currents is 600 mA/phase, and a precision lead screw converts their rotational movement to a linear one. It also benefits from a rotary quadrature encoder with resolution equal to 800 states/rev. The micro-step sizes for this linear stage are equal to 0.047625  $\mu\text{m}$ , its accuracy is 15  $\mu\text{m}$ , and its repeatability is 3  $\mu\text{m}$ . Furthermore, the highest speed of the stage is 104 mm/s, its highest trust is 55 N, and it benefits from the maximum load capacity of 250 N. The communication interface between the PC and each linear stage is provided by a RS232 connection and the communication protocol is Zaber ASCII or Zaber binary. The maximum communication baud rate of these motion stages is 115200 bps, and a RS232/USB converter is provided within the linear stage to provide its connectivity with the PC. To sense the home position for the linear stage, a magnetic hall sensor is used. This product is controllable from PC either by using Zaber Console software, or Zaber Launcher software. Zaber motion libraries are also available under Python 3, C#, C++, JavaScript, Java, and MATLAB. Zaber linear stage may also be controlled using Arduino with the software library through Zaber website (see <https://www.zaber.com/software>). The data connection and the power connection for the first device are provided separately and for the next stages, a Daisy chain connection provides power as well as data to control in a network fashion.

### 2.3. Laser tracker

The laser tracker system which is used in this experiment is AT960-MR. This laser tracker is a portable dynamic 6DOF laser measurement system manufactured by Hexagon metrology GMBH, Wetzlar. This type of measurement system is a widely used measurement one to precisely inspect critical distances, locations and surfaces [12] (see Figure 3). This metrology equipment benefits from a single class II laser source, and it complies with IEC certified IP54. This contactless metrology equipment benefits from Wi-Fi connectivity and can collect data @10Hz. It is however required to use a target retroreflector as the measurement target mounted on the augmented UR5 and move stage end effector. The target for the laser tracker is a precision Leica 1.5" red ring reflector detectable through the laser tracker at the maximum distance of 60 meters. In this work,

the laser tracker is used within about 2.8m from the robot base. It also benefits from an internal meteorstation which measures environmental factors such as temperature, pressure, and humidity. Using these environmental measurements compensations are done within the laser tracker controller to obtain precision measurements.

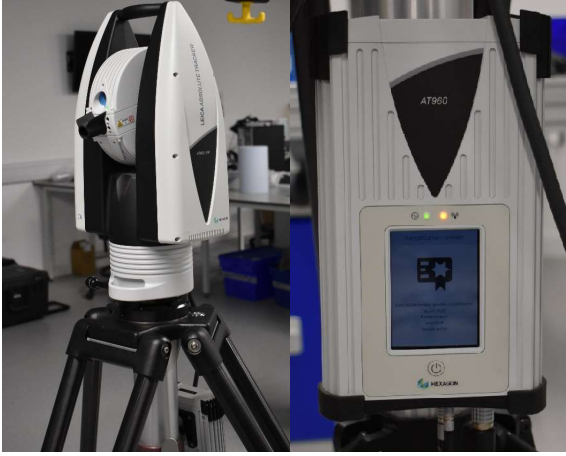


Figure 3. Laser tracker and its controller

### 3. Forward kinematics of the augmented industrial robots

Forward kinematic model of the overall system is the nonlinear mathematical relationship which relates the joint angles of UR5, and the displacements of the 3D move stage to the final position and orientation of the industrial robot. The link transformation matrix from the link  $i - 1$  to the link  $i$  using the Denavit-Hartenberg (DH) parameters of the industrial robot is given as [13, 14]

$${}_{i-1}T_1(q_i) = \begin{bmatrix} cq_i & -ca_i sq_i & sa_i sq_i & a_i cq_i \\ sq_i & ca_i cq_i & -sa_i cq_i & a_i sq_i \\ 0 & sa_i & ca_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where  $q_i, i = 1, \dots, 6$  represent the joint angle  $i$ ,  $a_i, i = 1, \dots, 6$ ,  $d_i, i = 1, \dots, 6$  present other DH parameters of robot. Furthermore,  $cq_i$ ,  $sq_i$ ,  $ca_i$ , and  $sa_i, i = 1, \dots, 6$  represent  $\cos(q_i)$ ,  $\sin(q_i)$ ,  $\cos\alpha_i$ , and  $\sin(\alpha_i), i = 1, \dots, 6$ , respectively. The link transformation matrix corresponding to the 3D move stage ignoring the miss-perpendicular fixtures is obtained as follows

$${}^0_1T_2(x) = \begin{bmatrix} 0 & 0 & 0 & x \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (2)$$

$${}^1_2T_2(y) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & y \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$${}^2_1T_2(z) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

The overall transformation matrix for the augmented system is obtained as the multiplication of individual transformations.

Table 1 The DH parameters of the 6DOF robot

Link	$q$	$d$	$a$	$\alpha$
1	$q_1$	$d_1$	$0$	$\pi/2$
2	$q_2$	$0$	$a_2$	$0$

3	$q_3$	$0$	$a_3$	$0$
4	$q_4$	$d_4$	$0$	$\pi/2$
5	$q_5$	$d_5$	$0$	$-\pi/2$
6	$q_6$	$d_6$	$0$	$0$

The matrixes of the links of UR5 and the links of the move stage are given by the following matrix

$$T = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$= {}^0_1T_1(q_1) {}^1_2T_1(q_2) {}^2_3T_1(q_3) {}^3_4T_1(q_4) {}^4_5T_1(q_5) {}^5_6T_1(q_6) {}^6_0T_2(x) {}^0_1T_2(y).$$

The sensitivity analysis is a valid tool to assess the sensitivity of industrial robot FK with respect to the industrial robot joint angles and move stage displacements. The sensitivity of industrial robot FK is obtained as follows

$$\begin{aligned} \Delta T &= \frac{\partial {}^0_1T_1(q_1)}{\partial q_1} {}^6_1T_1(q_2, q_3, q_4, q_5, q_6) {}^3_0T_2(x, y, z) \Delta q_1 \\ &+ {}^0_1T_1(q_1) \frac{\partial {}^1_2T_1(q_2)}{\partial q_2} {}^6_2T_1(q_3, q_4, q_5, q_6) {}^3_2T_2(x, y, z) \Delta q_2 \\ &+ {}^0_1T_1(q_1, q_2) \frac{\partial {}^2_3T_1(q_3)}{\partial q_3} {}^6_3T_1(q_4, q_5, q_6) {}^3_2T_2(x, y, z) \Delta q_3 \\ &+ {}^0_1T_1(q_1, q_2, q_3) \frac{\partial {}^3_4T_1(q_4)}{\partial q_4} {}^6_4T_1(q_5, q_6) {}^3_2T_2(x, y, z) \Delta q_4 \\ &+ {}^0_1T_1(q_1, q_2, q_3, q_4) \frac{\partial {}^4_5T_1(q_5)}{\partial q_5} {}^6_5T_1(q_6) {}^3_2T_2(x, y, z) \Delta q_5 \\ &+ {}^0_1T_1(q_1, q_2, q_3, q_4, q_5) \frac{\partial {}^5_6T_1(q_6)}{\partial q_6} {}^3_0T_2(x, y, z) \Delta q_6 \\ &+ {}^6_0T_1(q_1, q_2, q_3, q_4, q_5, q_6) \frac{\partial {}^0_1T_2(x)}{\partial x} {}^3_2T_2(y, z) \Delta x \\ &+ {}^6_0T_1(q_1, q_2, q_3, q_4, q_5, q_6) {}^1_0T_2(x) \frac{\partial {}^2_1T_2(y)}{\partial y} {}^3_2T_2(z) \Delta y \\ &+ {}^6_0T_1(q_1, q_2, q_3, q_4, q_5, q_6) {}^2_0T_2(x, y) \frac{\partial {}^3_2T_2(z)}{\partial z} \Delta z. \end{aligned}$$

The overall uncertainty associated with the UR5 FK considering Gaussian PDF for joint angle values and move stage displacement is obtained as follows

$$(\Delta T)^2 = \sum_{i=1}^6 \left( \frac{\partial T}{\partial q_i} \Delta q_i \right)^2 + \left( \frac{\partial T}{\partial x} \Delta x \right)^2 + \left( \frac{\partial T}{\partial y} \Delta y \right)^2 + \left( \frac{\partial T}{\partial z} \Delta z \right)^2. \quad (6)$$

Industrial robot forward kinematics is a nonlinear function acting on industrial robot joint angles to obtain the position and the orientation of its end effector. Industrial robot joint angles are considered as random variables. To obtain the PDF associated with industrial robot positions and orientations due to the uncertainty in a single joint angle, it is required to obtain the expected value of this nonlinear function.

**Theorem 1 [5].** (Expected value theorem, version of a single variable) let  $X$  denote a random variable with pdf  $f_X(x)$  and let  $G(X)$  denote a real-valued function defined on the domain  $X$ . Then the expected value of the random variable  $Z = G(X)$  is

$$E[G(X)] = \sum_x G(x) f_X(x) \quad (7)$$

in discrete case. The expected value theorem or the law of the unconscious statistician for the case when the output variable  $Z$  is a function of two random variables with joint cumulative distribution function of  $f_{XY}(x, y)$  is as follows.

**Theorem 2 [5].** (Expected value theorem, version of a two variables) let  $X$  and  $Y$  denote two random variables with joint pdf of  $F_Z(z)$  and let  $G(x, y)$  denote a real-valued function defined on the domain  $X$ , and  $Y$ . Then the expected value of the random variable  $Z = G(x, y)$  is

$$E[G(x, y)] = \sum_{(x,y)} G(x, y) f_X(x, y) \quad (8)$$

in discrete case.

Discrete version may be used to approximate the probability density function.

$$f_Z(z) = \sum_{\{(x,y):G(x,y)=z\}} G(x, y) f_X(x, y) \quad (9)$$

#### 4. Results and next step

Statistical analysis of the RLS' AksIM series magnetic rotary encoder which is used to measure UR5 joint angles is the performed in the next step. The results of the experiments along with the results provided by the manufacturer forms the required material to perform the uncertainty analysis of UR5 position and orientation. Using the law of the unconscious statistician, it would be possible to perform statistical analysis to obtain the probability density function associated with UR5 position and orientation. This statistical analysis is then validated using the laser tracker. Moreover, the probability density function associated with industrial robot position is controlled. The subject of probability density function control strategy is to design a controller to regulate the shape of the system output probability density function to a desired one [6]. Using this approach, it is possible to have a more consistent industrial robot position.

**Table 2.** Statistical properties of industrial robot position in different dimensions

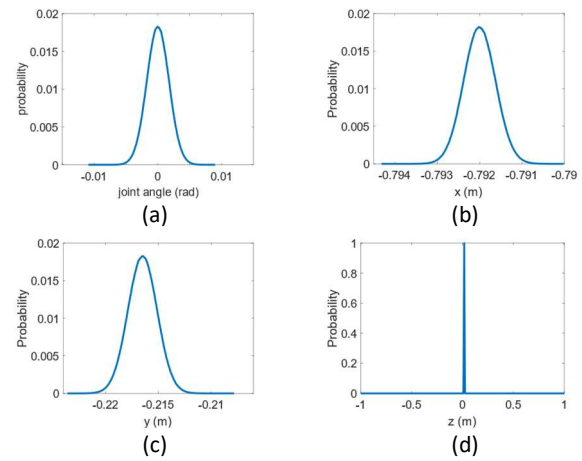
	Mean	Standard deviation	Skewness
X-dimension	-0.7920	3.7784e-04	0.0201
Y-dimension	-0.2165	0.0014	5.1763e-04
Z-dimension	0.0195	1.8041e-16	1

##### 4.1. Initial results

To provide some initial results for the analysis of the probability density function of the industrial robot position considering a Gaussian PDF for the first joint angle with standard deviation equal to  $0.1^\circ$ , the UR5 joint angle values studied is equal to zero and the displacements considered for the move stage is 2.5 cm. this approach can be generalized for all UR5 joint angle values and move stage displacements. The corresponding PDFs are analysed in discrete domain. The PDF of the first joint angle is given in Figure 4.a and the PDF for the industrial robot positions in x, y, and z dimensions are given in Figure 4.b, 4.c, and 4.d. The statistical properties of the position in x, y, z-dimensions are given in table 2. As can be seen from the table, in x, and y dimensions, the skewness values are between -0.5 and 0.5 which means that the distributions are approximately symmetrical. However, in z-dimension, the skewness value is between 0.5, and 1 which means that the distribution in this dimension is moderately skewed.

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**Figure 4.** a) PDF of joint angle 1 b) PDF of position in x-dimension c) PDF of position in y-dimension d) PDF of position in z-dimension

#### References

- [1] Z. Hu, L. Hua, X. Qin, M. Ni, Z. Liu, and C. Liang, "Region-based path planning method with all horizontal welding position for robotic curved layer wire and arc additive manufacturing," *Robotics and Computer-Integrated Manufacturing*, vol. 74, p. 102286, 2022.
- [2] H. Chen, T. Fuhlbrigge, S. Choi, J. Wang, and X. Li, "Practical industrial robot zero offset calibration," in *2008 IEEE International Conference on Automation Science and Engineering*, 2008: IEEE, pp. 516-521.
- [3] M. A. Khanesar, M. Yan, M. Isa, S. Piano, M. A. Ayoubi, and D. T. Branson, "Enhancing Positional Accuracy of the XY-Linear Stage Using Laser Tracker Feedback and IT2FLS," *Machines*, vol. 11, no. 4, p. 497, 2023.
- [4] G. Gao, G. Sun, J. Na, Y. Guo, and X. Wu, "Structural parameter identification for 6 DOF industrial robots," *Mechanical Systems and Signal Processing*, vol. 113, pp. 145-155, 2018.
- [5] B. Flury, *A first course in multivariate statistics*. Springer Science & Business Media, 2013.
- [6] M. Ren, Q. Zhang, and J. Zhang, "An introductory survey of probability density function control," *Systems Science & Control Engineering*, vol. 7, no. 1, pp. 158-170, 2019.
- [7] M. A. Khanesar, M. Yan, W. P. Syam, S. Piano, R. K. Leach, and D. T. Branson, "A Neural Network Separation Approach for the Inclusion of Static Friction in Nonlinear Static Models of Industrial Robots," *IEEE/ASME Transactions on Mechatronics*, 2023.
- [8] *Off-Axis Rotary Absolute Magnetic Encoder*, 2021. [Online]. Available: <https://www.renishaw.com/resourcecentre/en/details/Data-sheet-AksIM-rotary-absolute-magnetic-encoder--129111>.
- [9] J. E. Dusek, M. S. Triantafyllou, and J. H. Lang, "Piezoresistive foam sensor arrays for marine applications," *Sensors and Actuators A: Physical*, vol. 248, pp. 173-183, 2016.
- [10] J. Castellanos-Ramos, R. Navas-González, I. Fernández, and F. Vidal-Verdú, "Insights into the mechanical behaviour of a layered flexible tactile sensor," *Sensors*, vol. 15, no. 10, pp. 25433-25462, 2015.
- [11] C. B. Arrington, D. A. Rau, C. B. Williams, and T. E. Long, "UV-assisted direct ink write printing of fully aromatic Poly (amide imide) s: Elucidating the influence of an acrylic scaffold," *Polymer*, vol. 212, p. 123306, 2021.
- [12] S. Kyle, "Operational features of the Leica laser tracker," 1999.
- [13] K. Kufieta, "Force estimation in robotic manipulators: Modeling, simulation and experiments," *Department of Engineering Cybernetics NTNU Norwegian University of Science and Technology*, 2014.
- [14] J.-D. Sun, G.-Z. Cao, W.-B. Li, Y.-X. Liang, and S.-D. Huang, "Analytical inverse kinematic solution using the DH method for a 6-DOF robot," in *2017 14th international conference on ubiquitous robots and ambient intelligence (URAI)*, 2017: IEEE, pp. 714-716.