Labor Responses, Regulation and Business Churn

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Abstract

We develop a model of sluggish firm entry to explain short-run labor responses to technology shocks. We show that the labor response to technology and its persistence depend on the degree of returns to labor and the rate of firm entry. Existing empirical results support our theory based on short-run labor responses across US industries. We derive closed-form transition paths that show the result occurs because labor adjusts instantaneously whilst firms are sluggish, and closed-form eigenvalues show that stricter entry regulation results in slower convergence to steady state. Finally we show that our theoretical results hold in a quantitative model with capital accumulation and interest rate dynamics.

Keywords: Deregulation, Dynamic entry, Endogenous entry costs, short-run labor responses

JEL Codes: D25, E20, L11, O33

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The short-run response of labor hours to technology shocks has been widely debated in macroeconomics. Empirical studies, such as Chang and Hong 2006, show that labor responses to technology shocks differ across U.S. manufacturing industries. Using 4-digit manufacturing sector data, Chang and Hong show that while some industries exhibit a temporary reduction in employment in response to a permanent increase in technology, many more industries exhibit a short-run increase in both employment and hours per worker. However, the theory underlying these responses is not fully understood. In this paper, we identify a novel mechanism based on dynamic firm entry to explain short-run labor responses and subsequent persistence. Cross-industry data supports our theory. Additionally, we show that persistence of labor responses depends on firm sluggishness, which regulation affects through endogenous entry costs.

Our mechanism focuses on endogenous variation in labor per firm, which occurs when firm creation is sluggish but labor adjusts instantaneously. Endogenous variation in labor per firm is important for aggregate labor responses except in the special case of a constant marginal product of labor (MPL) in the firm's production function. For example, if a positive technology shock increases hours, but the stock of firms is fixed, hours per firm increase. With short-run increasing MPL, the rise in hours per firm increases MPL, increases wages and increases hours. Subsequent firm entry decreases hours per firm, decreases MPL, decreases wage, and decreases labor to its long-run level.[1] This channel is typically overlooked because either labor per firm is fixed (due to instantaneous free entry) or the MPL is constant.[2]

We develop a DGE small open economy (SOE) model in continuous time extended to include dynamic firm entry. Output is produced with labour, and there is an internationally traded bond with world interest rates equal to the household discount rate. Hence the household perfectly smooths utility, so consumption dynamics do not play a role, which allows a closed-form analysis of firm dynamics. Households can invest in new firms by paying an endogenous entry cost. Once operational, firms compete under monopolistic competition and pay a fixed overhead cost each period. The restriction to one state variable (number of firms) keeps eigenvalues tractable, so we can study persistence and short-run versus long-run effects analytically.

To model dynamic entry we assume that the entry costs depend on the flow of entry due to a congestion effect caused by red tape (Datta and Dixon 2002). As a result, output per firm and operating profits vary in the short run, whilst in the long run firms fully adjust so that there is a free-entry, zero-profit steady-state. In steady state average

[1] With decreasing MPL, the signs are reversed.

[2] In principle, other mechanisms that cause variation in employment at the firm-level could cause similar effects. We focus on the sluggishness of firm entry, but equally slow aggregate labor adjustment could also affect employment at the firm level – providing the adjustment of firms is not exactly proportional to the adjustment of labor, such that labor per firm does not vary, which is the case in zero-profit, free-entry models.

firm size is independent of technology. The speed of convergence captured in the stable eigenvalue depends on the flow of firm creation, which in turn depends on the level of regulation in an economy. We characterize deregulation as cut in red tape, which causes less congestion in the entry process decreasing the endogenous sunk entry cost and speeding-up business churn. Our model is parsimonious in order to derive general analytic results and provides testable implications consistent with empirical literature.

We also consider a quantitative RBC model with capital and a variable interest rate, keeping sluggish firm entry and allowing for variation in the slope of the marginal cost curve. We find very similar results to our simple SOE model, which shows that the simplifying assumptions we make for an analytical solution are not necessary for our results to hold in larger quantitative models.

Related Literature: Recent literature in macroeconomics has focused on the importance of firm entry dynamics for business cycle fluctuations. Bilbiie, Ghironi, and Melitz 2012 (BGM) developed a popular model of sluggish entry based on a *fixed* entry cost and a time-to-build lag in discrete time. We extend the idea of sluggish entry adjustment to a continuous-time model of *endogenous* entry costs. This has the benefit of allowing for a tractable analysis and offers a new angle to study deregulation. The endogenous entry cost creates an intertemporal zero-profit condition that equates the cost of entry in each instant to the net present value of incumbency. This causes the number of firms to gradually adjust to its long-run value. Entry costs are endogenous because they depend on the number of entering firms. Lewis 2009 shows that these so-called entry *congestion effects* are important for macroeconomic propagation in empirical work, and recent theoretical papers also include this mechanism (Bergin and Lin 2012).

Cantore, Ferroni, and Leon-Ledesma 2017 (Fig. 1, p.70) provide empirical evidence that, at the aggregate level, short-run responses of labor to technology have reversed over the past century in the US from decreasing to increasing, and that the deviation now persists for longer. We show that increased persistence can occur because of slower business churn caused by higher entry regulation. Our analysis contributes a novel angle to existing studies of entry regulation. Most literature focuses on the effect of decreasing fixed entry costs. This determines the stock of firms operating in the long-run which has implications for static allocations (Barseghyan and DiCecio 2011). However, we analyze deregulation of endogenous entry costs that affect the speed at which firms transition to arbitrage profits, and therefore determine the persistence of aggregate variables. Cacciatore and Fiori 2016 explore deregulation in goods and labor markets. They find that reforms are beneficial in the long run, but can have short-run recessionary effects. Similarly to our work, they include sluggish firm adjustment, but they also have sluggish labor adjustment due to search frictions.

Our paper contributes to the debate on short-run labor responses to productivity shocks. Gali 1999 found negative short-run labor responses to technology shocks which contradicted contemporary RBC theory.[3] The result was disputed by Christiano, Eichenbaum, and Vigfusson 2003 and has ignited a long-literature rationalising these opposing results. Many papers attempt to establish empirical facts for different industries and different countries (e.g. Ko and Kwon 2015), and a smaller literature provides theoretical justifications. Theoretical papers typically extend an RBC model and analyze its ability to match empirical labor responses for different model calibrations. Rebei 2014 compares six model extensions, and finds that the Francis and Ramey 2005 model of habits in consumption and investment adjustment costs performs best. Cantore, Ferroni, and Leon-Ledesma 2017 focus on variations in the capital-labor substitution parameter. They argue that this varies due to biased technical change. Mandelman and Zanetti 2014 introduce labor market search and matching frictions based on Blanchard and Gali 2010. Their initial modification cannot replicate negative labor responses, but an extension to cyclical hiring costs performs well. Relative to existing theoretical literature our mechanism is highly tractable. We analyze a deeply micro-founded parameter that is present in all of these models, and can contribute positively or negatively to short-run labor responses under the conditions we explain. From the empirical work on short-run labor responses, Basu, Fernald, and Kimball 2006 and Chang and Hong 2006 are closely related to our model predictions. Basu, Fernald, and Kimball 2006 estimate a returns-to-scale parameter which proxies for our labor returns parameter. They show that in US manufacturing industries (durable and non-durable) returns to labor (hour per worker) are increasing, whereas in non-manufacturing returns are decreasing. Their goal is to re-estimate the driving technology process and observe aggregate responses. Chang and Hong 2006 adopt their regression technique in order to study more granular 4-digit industry responses – they find significant differences across industries. As a by-product they estimate our parameter of interest at this level which allows us to match to industry short-run labor responses and verify our model predictions.

Outline: Section 1 presents the model; Section 2 summarizes equilibrium; Section 3 solves the steady-state and model dynamics; Section 4 analyzes labor responses and empirical relevance; Section 5 shows that deregulation speeds-up convergence; Section 6 performs a quantitative exercise.

[3] Gali 1999 estimates an SVAR on US data which shows that hours worked fall while labor productivity rises after a positive permanent shock to technology.

1 Model

1.1 Household

There is a small open economy, with a world capital market interest rate r equal to the discount rate ρ of the Ramsey household:

$$r = \rho \tag{1}$$

The representative household has King, Plosser, and Rebelo 1988 preferences

$$U(C, H) = \ln C - \frac{H^{1+\eta}}{1+\eta} \tag{2}$$

U denotes the the period utility function which is concave. It is increasing in consumption C and decreasing in labor hours H.[4] The parameter $\eta \in (0, \infty)$ is inverse Frisch elasticity of labor supply to wages. The household earns income from three sources: supplying labor at wage w, receiving interest income from foreign bonds rB and receiving profit income Π from owning firms. The household treats profit income as given. The household can spend income on consumption or foreign bond investment $\dot{B}$. Therefore the household solves:

$$\max_{C,H} \int_0^\infty U(C, H) e^{-\rho t} dt$$

subject to

$$\dot{B} = rB + wH + \Pi - C \tag{3}$$

$$B(0) = B_0 \tag{4}$$

where (4) is the initial condition on wealth and (1) holds. In addition to (3) and (4), the optimal solutions satisfy

$$\dot{\lambda} = 0 \implies \lambda = \bar{\lambda} \tag{5}$$

$$\bar{C} = \frac{1}{\bar{\lambda}} \tag{6}$$

$$H = (\lambda w)^{\frac{1}{\eta}}, \quad \eta \in (0, \infty) \tag{7}$$

$$\lim_{t\to\infty} \lambda B e^{-rt} = 0 \tag{8}$$

[4] Additive separability $U_{CH} = 0$ is sufficient for our results to hold when there are increasing marginal costs (decreasing returns to labor). But we require KPR preferences for the decreasing and constant marginal cost cases.

where we use bar notation for variables that are constant over time. (7) is the intratemporal labor supply condition. Given wage, labor supply H is increasing in the marginal utility of consumption λ. A high λ means low consumption and vice versa.[5] The assumptions of perfect capital markets and additively separable utility simplify dynamics to distill those arising from firm entry, which will affect wage.[6] To ensure the private agent satisfies the intertemporal budget constraint, the transversality condition (8) must hold.

1.2 Firms

1.2.1 Firm Production

The aggregate consumption good C is either imported or produced domestically by a perfectly competitive industry. The final goods production technology has constant returns and employs intermediate inputs which are monopolistically supplied. The aggregate price level is P. There is a continuum of possible intermediate products y_i for $i \in [0, \infty)$ with price p_i. At instant t, there is a range of active products $N(t) < \infty$ so that $i \in [0, N(t))$ are active and available, whilst $i > N(t)$ are inactive and not produced.

Final Good Producer's Problem: The final good producer solves

$$\max_{y_i} \quad PY - \int_0^N p_i y_i \, di$$

subject to

$$Y \equiv N^{1-\frac{\theta}{\theta-1}} \left[\int_0^N y_i^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \tag{9}$$

where $\theta > 1$ is the elasticity of substitution between products. (9) is the aggregate technology which relates total domestic output Y to inputs y_i. The first-order conditions for the final goods producer give input-demand for each available product i

$$y_i = \left(\frac{p_i}{P}\right)^{-\theta} \frac{Y}{N} \tag{10}$$

This is the well-known constant elasticity form. The corresponding price elasticity of demand $\varepsilon_{py} \equiv \frac{dp_i}{dy_i} \frac{p_i}{y_i}$ is $\varepsilon_{py} = -\frac{1}{\theta}$. Combining (9) with (10), yields the aggregate price

[5] See supplementary Appendix B for full derivation of first-order conditions. If $r \neq \rho$ then no interior steady state exists. The trajectory of consumption will then be either increasing $r > \rho$ or decreasing $r < \rho$ through time. This 'knife-edge' condition is a widely-discussed model closing device (Oxborrow and Turnovsky 2017; Uribe and Schmitt-Grohé 2017). Under perfect foresight, this will cause steady-state to depend on initial conditions.

[6] Additive separability $u_{CH} = 0$ creates the simple relationship between consumption and marginal utility of consumption. Perfect international capital markets $\rho = r$ imply the household can completely smooth its consumption so $\dot{\lambda} = 0 \implies \lambda = \bar{\lambda}$. In combination they imply the marginal utility of consumption is unchanging over time.

index that is consistent with zero-profits in the perfectly competitive final goods sector:

$$P \equiv N^{-\left(1-\frac{\theta}{\theta-1}\right)} \left(\int_0^N p_i^{1-\theta} \, di \right)^{\frac{1}{1-\theta}} \tag{11}$$

Intermediate Good Producer's Problem: There is a continuum of potential firms, and each firm can produce one product. At time t, firm $i \in [0, N(t))$ has labor demand h_i to supply output y_i using the technology:

$$y_i = \begin{cases} Ah_i^{\nu} - \phi, & \text{if } Ah_i^{\nu} > \phi, \\ 0 & \text{else.} \end{cases} \tag{12}$$

The parameter $\phi \geq 0$ is a fixed overhead cost denominated in output terms.[7] A is a technology parameter. The parameter $\nu > 0$ captures whether an increase in labor increases or decreases the marginal product of labor (MPL) – i.e. the slope of the MPL – it represents whether an extra unit of labor will increase or decrease the efficiency at which labor is employed at the firm. When $\nu < 1$ there are decreasing returns to labor (MPL slope is negative); $\nu = 1$ implies constant returns (MPL slope is flat); $\nu > 1$ implies increasing returns (MPL slope is positive).

An individual firm maximizes profit, where w is the real wage, by solving

$$\max_h \pi_i = p_i y_i - Pwh_i$$

subject to its production function (12) and the demand function (10).[8] The factor market is perfectly competitive meaning we assume that labor is homogeneous so the firm is a price-taker in the input market. This yields the following optimizing choice of labor input

$$w = \frac{p_i}{P} \frac{\nu}{\mu} Ah_i^{\nu-1} \tag{13}$$

Where $\mu \equiv \frac{\theta}{\theta-1} \in [1, \infty)$ is the markup of price over marginal cost. When products are perfectly substitutable the markup tends to unity $\lim_{\theta \to \infty} \mu = 1$. If firms have U-shaped average cost curves, which is one of the cases we study, a perfect competition equilibrium will exist with $\mu = 1$. Given (13), a firm's maximized profit is:

$$\pi_i^{\max} = p_i \left[\left(1 - \frac{\nu}{\mu} \right) Ah_i^{\nu} - \phi \right]$$

If marginal cost is decreasing $\nu > 1$, the solution to (13) might not maximize profits.

[7] See supplementary Appendix B for a discussion of this production function with a fixed cost and non-constant marginal costs.

[8] We solve the firm profit maximization problem in supplementary Appendix B and show the second-order condition for profit maximization holds under our assumptions.

We assume the degree of increasing returns to labor is bounded above by the degree of monopoly power. This ensures the second-order condition for profit maximization holds.[9]

Lemma 1. *$\mu \geq \nu$ is a sufficient condition for (13) to maximize profits.*[10]

This implies that although the marginal cost curve can be downward sloping, the slope must be shallower than the downward sloping marginal revenue curve to ensure they intersect.

1.2.2 Firm Entry

What determines the number of firms operating at each instant t? We develop a congestion effects model of firm entry which equates a time-varying cost of entry to the net present value of a firm. A partial equilibrium version of the model is presented in Datta and Dixon 2002. Entry and exit are symmetric in the sense that the two channels do not operate independently – there is either entry following a positive shock or exit following a negative shock.[11] We shall focus on positive shocks and firm entry. At time t there is a flow cost of entry $q(t)$ which increases in net entry $E(t)$.

$$E(t) \equiv \dot{N} \tag{14}$$

$$q(t) = \gamma E(t) \tag{15}$$

The sensitivity to congestion parameter $\gamma \in (0, \infty)$ represents red tape or regulation in firm creation. Filing papers or gaining accreditation makes start-ups more sensitive to flows of entry as regulator's workflows become more congested (i.e. a queuing cost). Aggregating across all entry in a period gives a quadratic firm entry adjustment cost function

$$\mathcal{C}(E) \equiv \int_0^E q \, dE = \frac{\gamma}{2} E^2 = \frac{q^2}{2\gamma} \tag{16}$$

$\mathcal{C}(E)$ is a non-negative, convex function of the rate of entry. With zero entry, the aggregate cost and marginal cost of firm creation is zero $\mathcal{C}(0) = \mathcal{C}_E(0) = 0$. The interpretation of modelling the aggregate sunk cost as an adjustment cost is that firm creation and destruction, whether positive (net entry) or negative (net exit), generates resource costs.

[9] In supplementary appendix B we provide a (weaker) necessary and sufficient condition for profit-maximization. However it turns out this is redundant as $\mu \geq \nu$ is *necessary* and sufficient for steady state existence.

[10] Hornstein 1993; Devereux, Head, and Lapham 1996 and Kim 2004 provide similar conditions in instantaneous-entry, zero-profit models with returns to scale.

[11] Symmetry implies that in the case of a negative shock and net exit the entry cost becomes an exit fee (severance payments or dismantling fee). Firms wishing to leave the market must pay a cost $-q > 0$, where $q < 0$ so the double-negative makes the exit cost positive. This means in bad economic times incumbent firms may have an incentive to delay their exit to a later date when the severance fees are lower. This symmetry is not essential as we show in our quantitative exercise which has an exogenous death rate.

The flow of entry in each instant is determined by an *arbitrage condition* that equates the return on bonds (opportunity cost of entry) with the return on setting up a new firm. It is a differential equation in q, which determines the entry flow by (15).[12]

$$\frac{\pi}{q} + \frac{\dot{q}}{q} = r \tag{17}$$

$$N(0) = N_0 \tag{18}$$

In equilibrium operating profits π depend on N which will make this a nonlinear second-order differential equation in N.[13] The first left-hand side term is the number of firms per dollar $(1/q)$ times the flow operating profits (dividends) the firm will make if it sets up. The second term reflects the change in the cost of entry. If $\dot{q}/q > 0$, then it means that the cost of entry is increasing, so that there is a capital gain associated with entry at time t if $\dot{q}/q < 0$ it means entry is becoming cheaper, thus discouraging immediate entry. The sunk cost $q(t)$ represents the net present value of incumbency: it is the present value of profits earned if you are an incumbent at time t.[14] This arises since the entrants are indifferent between entering and staying out. When $q < 0$, the present value of profits is negative: in equilibrium this is equal to the cost of exit.

In steady state, we have $E = q = 0$, so that the entry model implies the zero-profit condition. Entry costs only arise on convergence to steady state.

2 Equilibrium

Firms have identical production functions (12) so we can impose a symmetric equilibrium:

$$\forall i \in [0, N(t)) : \quad h_i = h, \ y_i = y, \ p_i = p$$

Under symmetry (11) implies $P = p$ and as we set the aggregate consumption good as the numeraire $P = p = 1$. Under symmetry, (9) implies

$$Y = Ny \tag{19}$$

[12]The arbitrage equation can be written in a way directly analogous to the user cost of capital $\pi = q\left(r - \frac{\dot{q}}{q}\right)$ in capital adjustment cost models.

[13]Note that our entry model has the standard models as limiting cases: when $\gamma = 0$, we have instantaneous free entry so that (17) becomes $\pi = 0$ and there are zero profits each instant. If $\gamma \to +\infty$, then changes in N become very costly and N moves little if at all which approximates the case of a fixed number of firms.

[14]This is because of the free-entry assumption that sunk costs equal the net present value of the firm. See Stokey 2008 for a general discussion.

substituting in (12), the aggregate production function is

$$Y(N,H) = AH^{\nu}N^{1-\nu} - N\phi \tag{20}$$

Symmetry and perfectly competitive factor markets (homogeneous labor) imply labor is divided equally among firms

$$H = Nh \tag{21}$$

Therefore labor demand (13) relates wage to aggregate variables by

$$w = \frac{\nu}{\mu}AH^{\nu-1}N^{1-\nu} \tag{22}$$

Operating profits at their maximum are

$$\pi = \left(1 - \frac{\nu}{\mu}\right)Ah^{\nu} - \phi = \left(1 - \frac{\nu}{\mu}\right)AH^{\nu}N^{-\nu} - \phi \tag{23}$$

This rearranges to give profit-maximizing firm size and equivalently the relationship between aggregate output, number of firms and operating profit

$$y = \frac{\mu\pi + \nu\phi}{\mu - \nu}, \qquad Y = N\left(\frac{\mu\pi + \nu\phi}{\mu - \nu}\right) \tag{24}$$

In general equilibrium the household budget constraint becomes the aggregate accounting identity by substituting out aggregate profits. Aggregate profits returned to the representative household are total operating profits (dividends) less total entry costs $\Pi \equiv N\pi - \mathcal{C}(E)$ and under symmetry $N\pi = N(y - wh) = Y - wH$. Hence by subsitution (3) yields the goods market clearing condition:

$$Y + rB = C + \mathcal{C}(E) + \dot{B} \tag{25}$$

Definition 1. A decentralised equilibrium is defined by paths $t \in [0, \infty)$ of bonds $\{B(t)\}$, factor price $\{w(t)\}$, factor demands $\{H(t)\}$, firms' operating decisions $\{y(t)\}$, measures of the stock of firms and entry, $\{N(t), E(t)\}$, and consumption $\{C(t)\}$, given initial conditions (4) and (18), such that

(i) consumers choose $\{C(t), H(t)\}$ optimally according to (6) and (7) given factor prices, and bonds $\{B(t)\}$ satisfy the transversality condition (8);

(ii) incumbent firms choose $\{h(t)\}$ and consequently $\{y(t)\}$ to satisfy (13) which maximizes operating profits given factor price

(iii) entry and the number of firms $\{E(t), N(t)\}$ equate the net present value of incumbency to the entry cost through the arbitrage condition (17)

(iv) wage $\{w(t)\}$ clears the labor market by equating labor supply (7) and demand (22)

(v) the goods market clears (25)

(vi) aggregate output and inputs are divided equally among firms following (19) and (21).

2.1 General Equilibrium Existence

A sufficient condition for equilibrium existence is that there are decreasing or constant returns to labor $\nu \leq 1$. However, in the case when there are increasing returns to labor $1 < \nu$ equilibrium may not exist.

Proposition 1 (General Equilibrium Existence)**.** *A necessary and sufficient condition for equilibrium existence is*

$$\nu < \min\left[\mu, 1+\eta\right] \tag{26}$$

Proof. Combine profit existence Lemma 1 and labor market existence Lemma 2. □

The condition ensures equilibrium in the goods market and labor market respectively. The $\nu < \mu$ condition ensures when marginal cost is downward sloping it still intersects the downward sloping marginal revenue curve, at a positive level of output. It is necessary and sufficient to prevent the zero-profit output level being negative. Additionally it is sufficient for the second-order profit maximization condition to hold. The second condition ensures that when labor demand is upward sloping it still intersects the upward-sloping labor supply curve. We discuss this below.

2.2 Labor Market Equilibrium

In labor market equilibrium labor supply (7) equals labor demand (22), giving:

$$H(\bar{\lambda}, N) = \left(N^{1-\nu}\bar{\lambda}\frac{\nu A}{\mu}\right)^{\frac{1}{1-\nu+\eta}}, \quad 1-\nu+\eta > 0 \tag{27}$$

Lemma 2 (Labor Market Equilibrium Existence)**.** *To ensure that the labor market condition is well-defined $\nu - 1 < \eta$*

The restriction $\nu < 1+\eta$ ensures that labor demand and supply intersect. The labor supply curve slope is $\frac{dw}{dH}\frac{H}{w} = \eta$, and the labor demand curve slope is $\frac{dw}{dH}\frac{H}{w} = \nu - 1$. In our model labor demand is upward sloping if returns to labor are increasing $\nu > 1$. This condition ensures that when labor demand is upward sloping, it is less steep than the upward sloping labor supply curve, which ensures they intersect. Equation (27) shows that the number of firms affects labor providing $\nu \neq 1$.

Proposition 2 (Equilibrium Labor-Entry Elasticity)**.** *The labor elasticity to number of firms is*

$$\varepsilon_{HN} \equiv \frac{dH}{dN}\frac{N}{H} = \frac{1-\nu}{1-\nu+\eta} \tag{28}$$

Therefore, the response of hours to firm entry depends on ν*:*

$$\varepsilon_{HN} \gtreqless 0 \iff \nu \lesseqgtr 1, \quad \textit{where } \nu \in (0, \infty)$$

Proof. Take the derivative of (27). □

This result captures the importance of entry for labor responses when there are non-constant returns to labor.[15] In the status quo case of constant returns to labor, entry does not affect labor. This is because N does not play a role in the aggregate labor demand condition. However, when ν is able to diverge from unity entry affects the aggregate demand condition through the MPL and consequently the wage. With decreasing returns to labor at the firm level, entry decreases labor per firm, which increases its MPL at any individual firm and in turn increases the real wage and hence labor supply. When there are increasing returns to labor $\nu > 1$ at the firm level, an extra firm dividing labor across more units, decreases the efficiency at which it is employed (MPL) and consequently decreases wage and labor in general equilibrium.

3 Model Solution

3.1 Reduced-form Equilibrium

The equilibrium conditions reduce to a five-dimensional system $\{\lambda, N, q, B, H\}$ with four differential equations and one static equation. The static intratemporal condition (27) implies $H(\lambda, N)$, so the system can be reduced to four differential equations in four unknowns, and since the consumption differential equation implies consumption is constant $\lambda(t) = \bar{\lambda}$, we have three dynamic equations in N, q, B:

$$\dot{N} = \frac{q}{\gamma} \tag{29a}$$

$$\dot{q} = rq - \pi(N, H(\bar{\lambda}, N)) \tag{29b}$$

$$\dot{B} = rB + Y(N, H(\bar{\lambda}, N)) - \mathcal{C}(q) - \bar{C}(\bar{\lambda}) \tag{29c}$$

[15]See supplementary Appendix B for a discussion of bounds on the labor elasticity to entry.

where the endogenous functions $\mathcal{C}$, $\bar{C}$, Y are specified in (5), (16),(20) and substituting (27) into (23) gives

$$\pi(N, H(\bar{\lambda}, N)) = \left(\frac{A^{1+\eta}(\nu\lambda)^{\nu}}{\mu^{1+\eta}N^{\eta\nu}}\right)^{\frac{1}{1-\nu+\eta}}(\mu-\nu)-\phi \tag{30}$$

Accompanying the differential equations in system (29) there are three boundary conditions: the household transversality (8); the initial condition on bonds (4); the initial condition on number of firms (18). Industry dynamics (N, q) form an independent, two-dimensional, subsystem of the three-dimensional system, where bonds are determined through (29c) alone. Therefore we shall solve recursively: first solving the industry dynamics subsystem for $N(t), q(t)$, then solve for bonds $B(t)$ based on these solutions.

3.2 Steady-state

Steady state is non-standard because there are three steady state conditions $\dot{N} = \dot{q} = \dot{B} = 0$ but four unknowns $\bar{\lambda}, q, N, B$.[16] In order to get an extra equation to solve this system for steady state, first we find a solution to the dynamic system for its timepaths of $N(t, \bar{\lambda}), q(t, \bar{\lambda}), B(t, \bar{\lambda})$ conditional on knowing one steady-state variable $\bar{\lambda}$. Second we use the limit of the bond solution and transversality to acquire an extra steady state condition, allowing us to solve for steady state. It is this procedure which causes steady state to depend on initial conditions N_0, B_0, so-called path dependency or hysteresis.[17]

We use a tilde to denote a steady state variable. The $\dot{N} = 0$ differential equation immediately implies that steady-state sunk costs are zero, which equivalently implies the net present value of a firm in steady state is zero:

$$\tilde{q} = 0 \tag{31}$$

This leaves two steady-state conditions $\dot{q} = \dot{B} = 0$ in three unknowns $\tilde{N}, \bar{\lambda}, \tilde{B}$. Through the arbitrage condition (29b), zero sunk costs (31) imply operating profits are zero

$$\tilde{\pi} = 0 \tag{32}$$

The zero profit condition determines labor per firm (or aggregate labor as a linear function of number of firms $\tilde{H}(\tilde{N})$)

$$\tilde{h} = \left(\frac{\mu\phi}{A(\mu-\nu)}\right)^{\frac{1}{\nu}} \tag{33}$$

[16] This occurs because the consumption differential equations is always in steady-state ($\dot{\lambda} = 0$) due to perfect consumption smoothing from $r = \rho$ which implies consumption is fixed $\lambda = \bar{\lambda}$, but it does not relate to other variables in the system.

[17] An implication of this feature is that temporary shocks may have permanent effects.

Labor per firm determines output per firm and wage[18]

$$\tilde{y} = \frac{\nu\phi}{\mu - \nu} \tag{34}$$

$$\tilde{w} = \left(\frac{A}{\mu}\right)^{\frac{1}{\nu}} \nu \left(\frac{\phi}{\mu - \nu}\right)^{1-\frac{1}{\nu}} \tag{35}$$

With $\tilde{h}$ and $\tilde{w}$ determined by the free entry arbitrage condition $\tilde{\pi} = 0$, then the labor market equilibrium condition (27) determines the number of firms as a function of the consumption index, and therefore labor as a function of consumption index:

$$\tilde{N}(\bar{\lambda}) = \frac{(\bar{\lambda}\tilde{w})^{\frac{1}{\eta}}}{\tilde{h}} \tag{36}$$

$$\tilde{H}(\bar{\lambda}) = (\bar{\lambda}\tilde{w})^{\frac{1}{\eta}} \tag{37}$$

In order to find $\bar{\lambda}$, we are left with one steady-state condition $\dot{B} = 0$ that we have not used: the output market clearing condition (steady-state bond accumulation equation).

$$\bar{C}(\bar{\lambda}) - \tilde{w}\tilde{H}(\bar{\lambda}) - r\tilde{B} = 0 \tag{38}$$

This is an excess demand function for the steady state in terms of the price of marginal utility $\bar{\lambda}$. The term $C(\bar{\lambda})$ represents expenditure and is decreasing in $\bar{\lambda}$. The term $wH(\bar{\lambda}) + r\tilde{B}$ represents income and are increasing in $\bar{\lambda}$. By the intermediate value theorem, this implies that there exists a $\bar{\lambda} > 0$ such that the economy is at the steady state equilibrium given $\tilde{B}$.

In this section we partly defined steady-state $\{\tilde{N}, \bar{\lambda}, \tilde{B}\}$ for the primitive variables of the dynamical system $N, \bar{\lambda}, B$, given steady-state bonds $\tilde{B}$. We gave $\tilde{N}(\bar{\lambda})$ analytically in (36), then used (38) to prove a steady-state $\bar{\lambda}$ must exist given $\tilde{B}$. In the next section, we derive solutions for dynamics which provide an additional steady-state condition $\tilde{B}(\bar{\lambda})$ that teamed with (38) and (36) can solve for $\bar{\lambda}$ by expressing (38) entirely in $\bar{\lambda}$ terms

$$\frac{1}{\bar{\lambda}} - \tilde{w}^{1+\frac{1}{\eta}}\bar{\lambda} - r\tilde{B}(\bar{\lambda}) = 0$$

3.3 Linearized system

The analysis of the steady state was conditional on the level of steady state bonds $\tilde{B}$. However to determine $\tilde{B}$ we need to know the path taken to equilibrium. The dynamics of the system will be analyzed by linearizing around the steady state.[19] Where the 3×3

[18]Since zero profits imply $0 = \tilde{y} - \tilde{w}\tilde{h}$ then steady-state wage is equivalent to labor productivity $\tilde{w} = \frac{\tilde{y}}{\tilde{h}}$.

[19]We provide a full derivation in Appendix A.1.

matrix is the Jacobian $\mathbf{J}$, the linearized system is

$$\begin{bmatrix} \dot{N} \\ \dot{q} \\ \dot{B} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\gamma} & 0 \\ \frac{1}{\tilde{N}(\bar{\lambda})}\frac{\nu\eta\phi}{1-\nu+\eta} & r & 0 \\ \tilde{\Omega} & 0 & r \end{bmatrix} \begin{bmatrix} N(t)-\tilde{N} \\ q(t)-\tilde{q} \\ B(t)-\tilde{B} \end{bmatrix} \tag{39}$$

$$\text{where} \quad \tilde{\Omega} \equiv \frac{d\tilde{Y}}{dN} = \mu\frac{\nu\phi}{\mu-\nu}\left(\varepsilon_{HN} - 1 + \frac{1}{\mu}\right) \tag{40}$$

In steady state the effect of entry on aggregate output is ambiguous $\tilde{\Omega} \gtreqless 0$. The labor returns parameter ν is an important determinant of this as it dictates ε_{HN} the labor elasticity to entry (Proposition 2). For $\nu \geq 1$ entry always decreases aggregate output, but for $\nu < 1$ all outcomes are possible.[20]

Proposition 3 (Entry and Aggregate Output)**.** *The effect of entry on aggregate output $\tilde{\Omega}$ in steady-state can be classified as follows:*

$$\tilde{\Omega} \gtreqless 0 \iff 1-\nu \gtreqless \eta(\mu-1)$$

3.3.1 Industry Dynamics Solution

The determinant and trace of the industry dynamics $\{N, q\}$ sub-system $\mathbf{B} \in \mathbb{R}^2$ in (39) are

$$\det(\mathbf{B}) = \Delta = \frac{\frac{d\tilde{\pi}}{dN}}{\gamma} = -\frac{\nu\eta\phi}{\gamma\,(1-\nu+\eta)\,\tilde{N}(\bar{\lambda})} < 0$$
$$tr(\mathbf{B}) = r$$

$\det(\mathbf{B})$ is negative as $1-\nu+\eta > 0$ and is increasing in $\bar{\lambda}$.[21] The root to the characteristic polynomial corresponding to the subsystem is

$$\Gamma(\bar{\lambda}) = \frac{r}{2}\left(1 \pm \frac{1}{r}\left[r^2 - 4\Delta(\tilde{N}(\bar{\lambda}))\right]^{\frac{1}{2}}\right)$$

The discriminant (square root term) is positive since the determinant is negative ($\Delta < 0$). This implies two distinct real roots. And since the discriminant exceeds 1, then so does its square root so there will be one positive and one negative root. Hence the system is saddle-path stable, with a negative real root Γ and a positive real root Γ^U. Furthermore the trace is positive so the sum of the eigenvalues is positive implying the positive eigenvalue is larger than the absolute value of the negative eigenvalue. Our focus is the stable root

[20] This result can be expanded to study the optimal golden rule number of firms. See supplementary Appendix B.

[21] See Appendix A.1 for proof.

which is negative

$$\Gamma = \frac{1}{2}\left(r - \left[r^2 - 4\Delta)\right]^{\frac{1}{2}}\right)$$

Lemma 3. *The stable eigenvalue is increasing in* $\bar{\lambda}$

Proof. See Appendix A.1. □

The solution to the linearized subsystem is

$$N(t) = \tilde{N} + \exp[\Gamma(\bar{\lambda})t](N_0 - \tilde{N}) \tag{41}$$

take derivative to get the net entry rate $E = \dot{N} = \Gamma \exp[\Gamma t](N_0 - \tilde{N})$ and substitute $q = \gamma E$ for the sunk cost solution

$$q(t) = \gamma\Gamma \exp[\Gamma t](N_0 - \tilde{N}) \tag{42}$$

The derivative of the solution is $\dot{q} = \Gamma^2 \gamma \exp(\Gamma t)(N_0 - \tilde{N})$, so the growth (shrinkage) in the cost of entry (firm NPV) is given in absolute terms by the stable eigenvalue

$$\left|\frac{\dot{q}}{q}\right| = \Gamma$$

with the sign being determined by whether profits are positive (firms accumulation) or negative (decumulation).

3.3.2 Bonds Solution

Combining (29c) and (8) provides a condition that the solution for bonds must satisfy in the long run.[22]

$$0 = B_0 + \int_0^\infty e^{-rt}\left[Y - \frac{q^2}{2\gamma} - C\right] dt \tag{43}$$

The two terms must cancel out, which has an intuitive interpretation. The first term is the initial position of bond holdings. $B_0 > 0$ implies the country begins as a borrower, $B_0 < 0$ implies it begins as a creditor. The second term represents trade surplus if positive and deficit if negative. Therefore (43) states that if a country begins as a borrower, at some point over the time horizon it must run a trade deficit.

Linearizing the differential equation in bonds gives

$$\dot{B}(t) = \tilde{\Omega}\left[N(t) - \tilde{N}\right] - \frac{\tilde{q}}{\gamma}\left[q(t) - \tilde{q}\right] + r\left[B(t) - \tilde{B}\right]$$

[22]We show this in Appendix B.

where $\tilde{q} = 0$. Then substitute in the $N(\bar{\lambda}, t)$ solution (41) restricts the differential equation to be a linear first-order nonhomogeneous differential equation in $B(t)$

$$\dot{B}(t) = \tilde{\Omega}\left[\exp[\Gamma t](N_0 - \tilde{N})\right] + r\left[B(t) - \tilde{B}\right] \tag{44}$$

If the economy starts with bonds $B(0) = B_0$ the solution to (44) is

$$B(t) = \tilde{B} + \frac{\tilde{\Omega}}{\Gamma(\bar{\lambda}) - r}\exp[\Gamma(\bar{\lambda})t](N_0 - \tilde{N}) \tag{45}$$

where $\frac{d\dot{B}}{dN}|^{\tilde{}} = \tilde{\Omega}$ implies the effect of entry on aggregate output equals the effect of entry on the flow of bonds evaluated at steady state. $\tilde{\Omega}$ affects how accumulation of firms $N_0 \to \tilde{N}$ so $N_0 - \tilde{N} < 0$ changes stock of bonds $B(t)$. $\tilde{\Omega} > 0$ then entry strengthens home production and increases bond investment, whereas $\tilde{\Omega} < 0$ then entry weakens home production and decreases bond investment. In the Walrasian case $(\mu = 1, \nu < 1)$, $\tilde{\Omega} > 0$ and the accumulation of firms leads to a reduction in bonds. The main mechanism here is that there is a positive effect of N on labor supply and output $(Y_{HN} > 0)$, so that having too few firms means that wages, labor income and home production are below their steady state level. To maintain consumption, this low level of income is compensated by higher than steady state imports, financed by running down bonds. An *increase* in firms per se makes wages higher. However, the number of firms is increasing because it is below the steady-state. The stock of bonds decreases because entry implies that the initial level of N was low in the first place, not because the accumulation of firms lowers income.

However, given $\mu > 1, \nu < 1$, if μ is large enough then bonds will increase as firms are accumulated. This is because the level of profits along the path to equilibrium is large: whilst the number of firms is below equilibrium, the extra profits generated are enough to exceed the adjustment costs and lower wage. In addition, there is a capacity effect, so that productivity is higher whilst the number of firms is below equilibrium (for $\mu > 1$, free-entry leads to excessive number of firms in steady-state). In the case of $\nu \geq 1$, the flow of entry leads to an increase in the stock of bonds: this is because N has a negative effect on wages and profits, so that N below its steady state implies income above the steady state.

3.4 Steady-state Bonds

The linearized dynamics give an explicit solution for steady state bonds as a function of $\bar{\lambda}$ and the initial conditions N_0, B_0. Evaluate (45) at $t = 0$ implies

$$\tilde{B}(\bar{\lambda}) = B_0 - \frac{\tilde{\Omega}}{\Gamma(\tilde{N}(\bar{\lambda})) - r}(N_0 - \tilde{N}(\bar{\lambda})) \tag{46}$$

therefore the steady-state bond condition (46) and steady-state arbitrage condition (36) give the excess demand condition (38) in terms of $\bar{\lambda}$ only

$$\tilde{w}\tilde{H}(\bar{\lambda}) + r\tilde{B}(\bar{\lambda}) - \bar{C}(\bar{\lambda}) = 0 \tag{47}$$

We can solve this for the steady-state consumption index $\bar{\lambda}$, which then provides $\tilde{C}(\bar{\lambda})$, $\tilde{H}(\bar{\lambda})$, $\tilde{N}(\bar{\lambda})$, and $\tilde{B}(\bar{\lambda})$. We cannot solve (47) analytically since it is highly nonlinear in $\bar{\lambda}$. However we can show analytically that a unique solution exists, and then solve for this numerically. A useful lemma to show uniqueness (and other results) is that the steady-state excess demand function is strictly increasing in inverse consumption, so is decreasing in consumption given N_0 begins within a neighbourhood of $\tilde{N}$.

Lemma 4 (Excess Demand Monotonically Increasing)**.** *The steady-state market-clearing condition is monotonically increasing in* $\bar{\lambda}$

$$\tilde{w}\frac{d\tilde{H}}{d\lambda} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{d\tilde{C}}{d\bar{\lambda}} > 0 \tag{48}$$

if the following sufficient condition holds

$$\left(\varepsilon_{HN} - 1 + \frac{1}{\mu}\right)\left(\frac{N_0}{\tilde{N}(\bar{\lambda})} - 1\right) \geq -\left(\frac{\varepsilon_{HN} - 1}{\Gamma(\bar{\lambda})} + \frac{1}{r\mu}\right)(r - 2\Gamma(\bar{\lambda})) \tag{49}$$

Proof. See appendix A.2. □

The right-hand side of (49) is strictly negative and the left-hand side is ambiguous. This condition is weaker than the simpler sufficient condition $N_0 - \tilde{N}(\bar{\lambda}) \to 0$ which is commonly assumed and ensures the left-hand side is zero.[23] The condition always holds if there is entry $N_0 < \tilde{N}$ and $\varepsilon_{HN} - 1 + \frac{1}{\mu} < 0$ (i.e. $\tilde{\Omega} < 0$) implying the left-hand side is positive.

Corollary 1 ($\bar{\lambda}$ Uniqueness)**.** *If* (49) *holds then there is a unique* $\bar{\lambda}$ *that solves* (47)*.*

Proof. Lemma 4 shows that given (49) the steady state market clearing condition is strictly monotonic in $\bar{\lambda}$. Hence, if a steady-state exists it is a *unique* steady state solution for $\bar{\lambda}$. □

[23] See Turnovsky 1997, p.68 (footnote 8) for a justification of this.

4 Technology Shock

4.1 Comparative Statics

An improvement in technology A reduces employment per firm but output per firm (firm scale) (12) is unaffected. Consequently an improvement in technology increases wages.[24]

$$\frac{d\tilde{h}}{dA} = -\frac{\tilde{h}}{\nu A} < 0, \qquad \frac{d\tilde{w}}{dA} = \frac{\tilde{w}}{\nu A} > 0$$

Therefore in the long run technological progress crowds-out labor at the product-level but output is unaffected (aggregate output will expand as there are more products each requiring less labor). These comparative statics are simple as they only depend on exogenous variables. However, the aggregate endogenous variables $\{\bar{C}, \tilde{N}, \tilde{B}\}$ ((6), (36), (46)), excluding $\tilde{q}$ which is zero, are a function of A directly but also depend on $\bar{\lambda}(A)$. Therefore technology change has a direct (partial) and an indirect (consumption) effect.[25]

Proposition 4 (Long-run Effect of Technology)**.** *A permanent increase in technology has the following long-run effects on aggregate variables:*

$$\begin{aligned}
\frac{d\bar{C}}{dA} &> 0 \\
\frac{d\tilde{N}}{dA} &> 0 \\
\frac{d\tilde{B}}{dA} &\gtreqless 0 \iff \tilde{\Omega} \lesseqgtr 0 \\
\frac{d\tilde{H}}{dA} &\gtreqless 0 \iff B_0 \gtreqless \frac{\tilde{\Omega}}{\Gamma - r} N_0 \\
\frac{d\tilde{Y}}{dA} &= \tilde{y}\frac{d\tilde{N}}{dA} > 0
\end{aligned}$$

From the steady-state market clearing condition, the implicit function theorem implies that technology unambiguously increases consumption. This rise in consumption (indirect effect) decreases aggregate labor and number of firms, whereas the direct partial effects of increased technology increase labor and number of firms. Overall, the partial effect dominates in the number of firms case, whereas it is ambiguous in the labor case. The increase in the stock of firms implies an increase in aggregate output, and a bond response that depends on the how entry affects aggregate output $\tilde{\Omega}$.[26] The effect on the labor supply is ambiguous because there is a conflict of income and substitution effects: the higher wage causes a substitution effect for less leisure and more consumption, which

[24] An increase in steady-state wages is equivalent to an increase in labor productivity since $\tilde{w} = \frac{\tilde{y}}{\tilde{h}}$.

[25] We call the indirect effect a consumption effect as $\bar{\lambda}(A)$ is inverse consumption by (6).

[26] $\tilde{\Omega}$ is the general derivative of aggregate output with respect to number of firms *evaluated at* steady-state. It is not the steady-state derivative.

increases labor. Whereas the income effect increases leisure and decreases labor. Which effect dominates depends on the level of initial wealth. From (46) $B_0 - \frac{\tilde{\Omega}}{\Gamma - r} N_0$ is the initial value of wealth in terms of bonds.[27] If $\tilde{\Omega} > 0$, that is $\nu < 1$ and μ small enough, then a sufficient condition for employment to increase $\frac{d\tilde{H}}{dA} > 0$ is that bond holdings are non-negative $B_0 \geq 0$. Likewise, if $\tilde{\Omega} < 0$, (for which $\nu \geq 1$ is sufficient) then a sufficient condition for employment to decrease $\frac{d\tilde{H}}{dA} < 0$ is that bond holdings are non-positive $B_0 \leq 0$.

Bonds respond in the opposite direction to the entry effect on output. If technology-induced entry increases GDP, then bonds decrease (less borrowing is necessary). If technology-induced entry decreases GDP, then bonds increase (more borrowing is necessary). Since steady-state bonds only depend on technology through $\tilde{N}$, the bond response follows the number of firms increase: $\frac{d\tilde{B}}{dA} = \frac{d\tilde{B}}{dN} \frac{d\tilde{N}}{dA}$, and to a first-order approximation $\operatorname{sgn} \frac{d\tilde{B}}{dN} \approx \operatorname{sgn} -\tilde{\Omega}$.[28] Similarly the increase in number of firms determines that aggregate output increases as long-run output per firm (firm scale) is constant.

4.2 Comparative Dynamics

From the dynamic solution for number of firms (41), we can see that on impact $t = 0$ of a shock the number of firms is fixed $N(0) = N_0$, whereas entry adjusts $E(0) = \Gamma(N_0 - \tilde{N})$, which affects the stock of firms an instance later. In other words number of firms is a stock (state) variable, and entry is a flow (jump) variable. Thus entry jumps the economy onto its stable manifold instantaneously as the shock hits, subsequently the number of firms responds as the economy evolves along this manifold. Therefore the difference between the impact and long-run effects depend on the effect of entry.

Proposition 5. *On impact of a technology shock, the response of hours and wages relative to their long-run level depending depends on labor returns to scale:*

$$\frac{dH(0)}{dA} - \frac{dH(\infty)}{dA} \gtreqless 0 \iff \nu \gtreqless 1$$
$$\frac{dw(0)}{dA} - \frac{dw(\infty)}{dA} \gtreqless 0 \iff \nu \gtreqless 1$$

On impact, relative to the initial position under the old technology, the labor effect is ambiguous. The reason is the same as the ambiguity in the long run (competing income and substitution effects). However, if we look at the difference between the impact and long-run effect, this depends on whether there is an increasing or decreasing MPL at

[27] From (46), $-\frac{\tilde{\Omega}}{\Gamma - r} N_0 = \tilde{B} - B_0 - \frac{\tilde{\Omega}}{\Gamma - r} \tilde{N}$ thus the term $-\frac{\tilde{\Omega}}{\Gamma - r} N_0$ is the present value of the bonds that would have been decumulated/accumulated if $\tilde{N} = 0$.

[28] The approximation arises from assuming we begin close to steady-state $N_0 - \tilde{N} \to 0$. From (46) removes the effect of the eigenvalue responding to $\tilde{N}$.

the firm level. We can thus get undershooting of employment ($\nu < 1$) or overshooting ($\nu > 1$) on impact relative to the new long-run level depending on whether entry increases or decreases the marginal product. The intuition for the result is that timing differences between firms adjusting and aggregate labor adjusting cause variation in labor per firm which affects its efficiency at the firm-level due to non-constant returns. When there are increasing returns to labor, subsequent firm entry always decreases the efficiency at which labor is employed which means aggregate hours converge downwards towards their long-run level. The opposite holds when there are decreasing returns to labor at the firm level: an additional firm employs labor more productively so as entry takes places wages and hours increase to their new long-run level.

Table 1 summarizes the combination of static (Proposition 4) and dynamic effects (Proposition 5) on labor. Rows capture the static effect that labor might in the long-run increase, decrease or remain constant depending on initial wealth. Columns capture the dynamic effect that labor might initially overshoot, undershoot or equate to its long-run level.

	$\nu > 1$	$\nu < 1$	$\nu = 1$
$B_0 > \frac{\tilde{\Omega}}{\Gamma - r} N_0$	Increase, Overshoot	Increase, Undershoot	Increase, Constant
$B_0 < \frac{\tilde{\Omega}}{\Gamma - r} N_0$	Decrease, Overshoot	Decrease, Undershoot	Decrease, Constant
$B_0 = \frac{\tilde{\Omega}}{\Gamma - r} N_0$	Constant, Overshoot	Constant, Undershoot	Constant, Constant

Table 1: Conditions for Taxonomy of Labor Dynamics

4.3 Empirical Evidence

In the theoretical model we derived the result that the short-run response of labor depends on whether the marginal product of labor is increasing or decreasing. In most models of entry, such as Bilbiie, Ghironi, and Melitz 2012, there is a constant marginal product of labor, so that there is no short-run impact on labor. Chang and Hong 2006 conduct an SVAR analysis of labor responses to technology shocks across US manufacturing industries. They show that of their 2-digit industry estimates, 14 industries show a positive response (4 significant) while 6 industries show a negative response (1 significant).[29] Additionally they provide estimates of returns to scale using the methodology of Basu, Fernald, and Kimball 2006 (BFK). The BFK methodology is to run a log-linear regression of output on inputs with a common coefficient γ^{BFK} on capital and employment for each industry, with an additional coefficient β on hours per worker.[30] We add the BFK superscript to distinguish their gamma parameter from our usage of γ as the

[29] *Instruments* and *Non-electornic* are zero at 3 decimal places but positive with greater precision. Statistical significance is at the 10% level. *Misc* are significant with greater precision than reported in Table 2: $\frac{SRR}{SD} = 0.01626/0.0098 = 1.6492 > t^{crit.} = 1.6449$.

[30] See Basu, Fernald, and Kimball 2006 equation 18, p. 1424.

congestion parameter. The coefficient γ^{BFK} is interpreted as returns to scale which is reported by Chang and Hong (Table 5) for their dataset. In terms of our model, in which there is only labor, we can interpret the increasing or decreasing marginal product of labor $\nu \gtreqless 1$ either as the coefficient γ^{BFK} (i.e. interpreting labor input as employment) or as the sum of the coefficients γ^{BFK} and β (i.e. the coefficient on total hours, the product of employment and hours-per-worker). Chang and Hong (Table 5) provide estimates of γ^{BFK} for 20 two-digit industries (ten durables and ten non-durables) plus an estimate of β for durables $\beta^D = 0.17$ and non-durables $\beta^{ND} = 0.76$ (β is assumed constant across industries within each sector). Our theory predicts a positive relationship between labor returns to scale (ν) and short run responses (SRR) of labor to technology shocks that is supported by their evidence. In Table 2 the SRR of labor for 2-digit industries, and standard deviations, are taken directly from Chang and Hong replication files, while the labor returns to scale are proxied by the returns to scale reported in their table 5. Our main result is the levels prediction that short-run responses are positive with increasing returns to labor $\nu > 1$ and negative with decreasing returns to labor $\nu < 1$. The results show that 14 of 20 industries respond the way we would expect,[31] and of the 5 significant (asterisk) responses reported by Chang and Hong all but textile conform to our theory.[32]

Chang and Hong find that there are increasing returns in the majority of industries (14 out of 20) in terms of γ^{BFK}. Estimates of β are both positive: if we combine β with γ^{BFK}, all of the industries have increasing returns so that all of the sectors with a negative or zero short-run impact are inconsistent with our theory: this is 7 industries, meaning 13 are theory consistent. Hence, Chang and Hong's results are broadly supportive of our theoretical result: 13 or 14 of the industries are consistent with our results whether we use γ^{BFK} or $\gamma^{\text{BFK}} + \beta$ as our measure of ν.

[31] This includes *Instruments* which has no short-run response and is the closest estimate to constant returns.

[32] In the appendix we report the results as a scatter plot.

SIC	Industry	RTS	SRR	SD
23	Apparel	1.24	0.012	0.009
28	Chemicals	1.52	-0.004	0.004
36	Electronic	1.53	-0.009	0.012
34	Fab. Metal	1.29	0.024	0.090
20	Food	0.38	0.001	0.003
25	Furniture	1.18	0.021	0.009*
38	Instruments	0.97	0.000	0.011
31	Leather	0.39	-0.002	0.012
24	Lumber	0.92	-0.028	0.011*
33	Metal	1.29	0.012	0.017
39	Misc	1.41	0.016	0.010*
35	Non-electronic	1.67	0.000	0.013
26	Paper	1.48	0.001	0.008
29	Petrol	0.53	-0.004	0.007
27	Printing	1.49	-0.001	0.008
30	Rubber	1.15	0.022	0.010*
32	Stone	1.36	0.009	0.008
22	Textile	0.86	0.017	0.006*
21	Tobacco	1.08	0.005	0.006
37	Transport	1.12	0.018	0.013

Table 2: Chang and Hong 2006 Results Comparison

5 Entry Regulation Shock

We interpret γ in the cost of entry equation (15) as red tape. When red tape increases firm entry costs become more sensitive to the flow of entry. For example, if a resource needed to setup a firm is in inelastic supply, like a government office that provides certificates to enter an industry, then a rise in red tape amplifies congestion. This makes entry more costly, and a firm may wait until a less congested period to attain certification. A 'deregulatory' policy decreases γ.[33] Data reported in Figure 1 indicate that red tape, proxied by procedures to start a business, is positively related to the length of time it takes to start a firm which proxies pace of business formation.[34]

[33] We adopt the term deregulatory shock following Bilbiie, Ghironi, and Melitz 2007 and authors who interpret entry costs as influenced by regulation (Blanchard and Giavazzi 2003; Poschke 2010; Barseghyan and DiCecio 2011). Whereas these focus on differences in fixed exogenous sunk costs and changes in the steady-state stock of operating firms, our interest is endogenous sunk costs and changes in speed of adjustment of firms.

[34] Figure 1 represents 2016 World Bank Doing Business data for 211 countries. Venezuela is the 20 procedures 230 days outlier. New Zealand is the 0.5 days 1 procedure point. Ebell and Haefke 2009

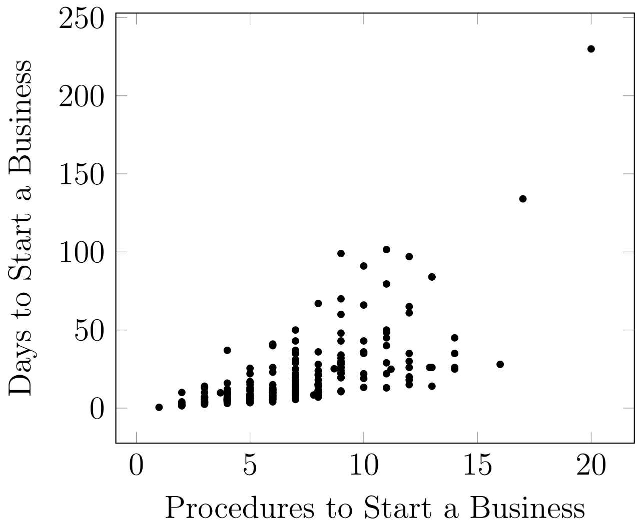

Figure 1: Red Tape and Business Churn

Proposition 6. *The economy's speed of adjustment is monotonically decreasing in regulation of business creation.*

The magnitude of the stable root captures the economy's speed of adjustment, as it dictates the speed of adjustment of the sole state variable (number of firms) through the exponential term of (41). Taking the derivative of the stable root, which is negative, with respect to the regulatory parameter gives[35]

$$\Gamma_\gamma = \Gamma_\Delta \Delta_\gamma = \frac{\Delta_\gamma}{(r^2 - 4\Delta)^{\frac{1}{2}}} = \frac{-\Delta}{\gamma(r^2 - 4\Delta)^{\frac{1}{2}}} > 0 \tag{50}$$

The stable root is increasing in the discriminant and the discriminant $\Delta_\gamma = -\frac{\Delta}{\gamma}$ is increasing in the regulatory parameter. Therefore an increase in regulation, increases the the value of the negative root moving it closer to zero and implying slower adjustment. The result implies that economies with less red tape recover faster following a shock. In the context of labor responses to technology shocks, it implies that labor achieves its new steady state faster. The implication that less red tape, helps business churn and aids the dissipation of shocks supports recent policy work and academic literature.[36]

6 Quantitative Exercise

The assumption of an SOE facilitated an explicit analytical solution, at the cost of leaving out capital and assuming an exogenous world interest rate. Focusing on only labor input

report similar trends in number of procedures and days to start-up for OECD data.

[35] This result is for a given steady-state $\tilde{N}(\bar{\lambda}))$ as γ will also affect $\tilde{N}$ through $\bar{\lambda}$.

[36] See The Case for Fiscal Policy to Support Structural Reforms (IMF, 2017) Cacciatore, Duval, et al. 2016a; Cacciatore, Duval, et al. 2016b.

kept our model close to the original framework of Gali 1999, and subsequent work that has adhered to this restriction (Mandelman and Zanetti 2014). In this section, we show that with capital and an endogenous interest rate the results will still stand. We use a discrete-time, closed-economy RBC framework which shares our key assumption of sluggish firm entry costs due to congestion and allows for an increasing or decreasing MPL.[37]

The production function of the firm with capital is $y = Ak^{\alpha}h^{\beta} - \phi$, which implies the slope of the firm-level marginal cost curve is $\nu = \alpha + \beta$. For $\nu = \alpha + \beta > 1$ it is downward sloping, and, as in our theoretical SOE model, the markup and Frisch elasticity provide a limit to the extent of increasing return consistent with existence. Our experiments analyze the effect of changing β on short-run hours responses, holding other variables constant at their calibrated levels. Figure 2 shows labor hours transition over t for different values of β in response to a permanent 1% technology shock. Changes in β for a given value of α represent changes in the slope of the marginal cost curve ν due to a change in the slope of the MPL. We vary β from 0.2 to 1.1, which shows that for low values of β (0.2 to 0.4) the initial ($t = 0$) short-run response is negative, but the SRR is positive for larger values of β. Additionally, the increase of SRR with β (or ν) is monotonic as our theory predicts (Figure 3 stresses this point). When there is a negative SRR it is followed by overshooting of hours and a return to the long run from above – this dynamic is noted in the empirical evidence of Basu, Fernald, and Kimball 2006.

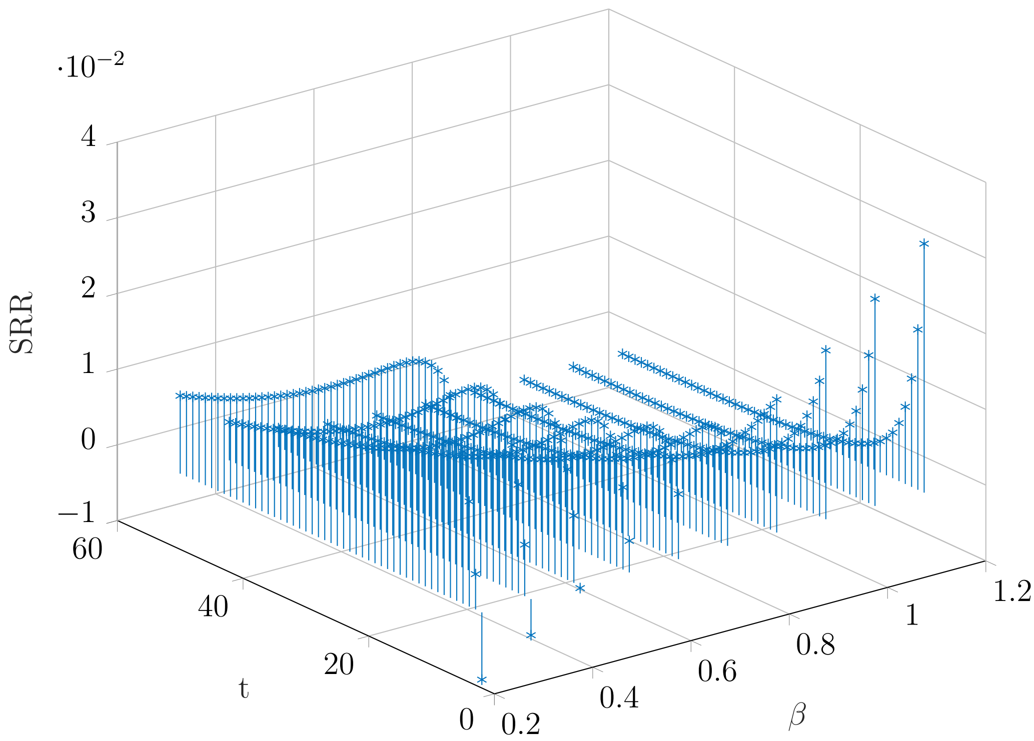

Figure 2: Hours Transition Paths as β Changes

[37]The full model and calibration are given in supplementary Appendix C.

Figure 3 shows the $t = 0$ short-run hours response on the y-axis for a range of β values on the x-axis. Crucially it shows how these responses differ according to degree of entry adjustment costs (γ is the entry adjustment parameter). This illustrates the importance of dynamic firm entry for the result. When there is instantaneous adjustment of firms $\gamma = 0$, SRR are always positive. They only become negative when entry sluggishness increases.

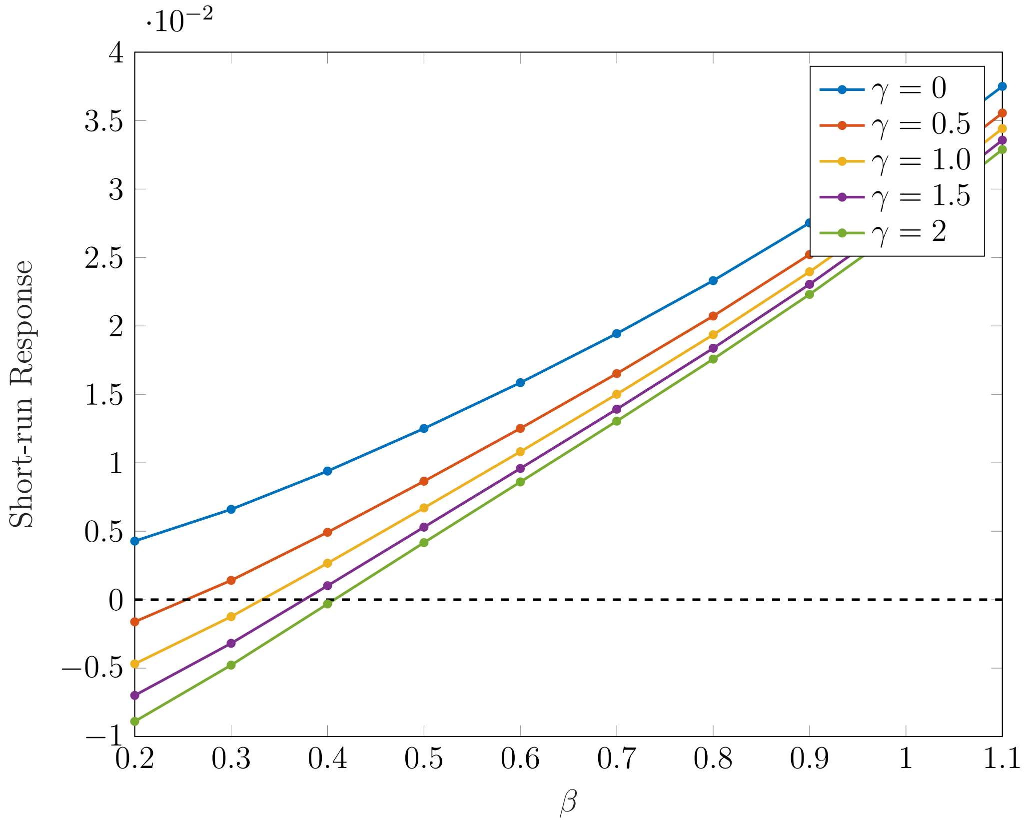

Figure 3: Short-run Response at $t = 0$ of Hours as β Changes

7 Conclusion

This paper studies the effect of dynamic entry on short-run labor responses to technology shocks. The main insight is that if firm entry is slow to react, then the response of labor to technology shocks will depend on whether labor is employed with decreasing, increasing or constant returns to scale at the firm level. Furthermore the persistence of these deviations will depend on the level of regulation and consequently on the pace of firms' adjustment.

Our core analysis provides an analytically tractable small open economy model without capital. This provides a clear, micro-founded mechanism which is novel relative to the predominantly reduced-form debate. However, we also extend our model to a more complex quantitative setting and the results remain significant. Hence we conclude that the assumption of a constant marginal product of labor – adopted by much of the literature – may be excessively restrictive. Several empirical studies verify heterogeneity in labor returns to scale across industries, and we match these to short-run responses.

The intuition for our result relies on variations in labor at the firm level which affects the efficiency at which it is employed. Therefore other mechanisms, aside from firm entry, that cause variation in employment at the firm level may lead to similar dynamics when teamed with non constant returns to labor. Further research may investigate this channel by looking at sluggish labor adjustment from search frictions as in related papers by Mandelman and Zanetti 2014; Cacciatore and Fiori 2016.

References

Acemoglu, D. (2009). *Introduction to Modern Economic Growth*. Introduction to Modern Economic Growth v. 1. Princeton University Press.

Barseghyan, Levon and Riccardo DiCecio (2011). "Entry costs, industry structure, and cross-country income and TFP differences". In: *Journal of Economic Theory* 146.5, pp. 1828–1851.

Basu, Susanto, John G. Fernald, and Miles S. Kimball (2006). "Are technology improvements contractionary?" In: *The American Economic Review* 96.5, pp. 1418–1448.

Bergin, Paul R. and Ching-Yi Lin (2012). "The dynamic effects of a currency union on trade". In: *Journal of International Economics* 87.2, pp. 191–204.

Bilbiie, Florin O., Fabio Ghironi, and Marc J. Melitz (2012). "Endogenous Entry, Product Variety, and Business Cycles". In: *Journal of Political Economy* 120.2, pp. 304–345.

Bilbiie, Florin O, Fabio Ghironi, and Marc J Melitz (2019). "Monopoly Power and Endogenous Product Variety: Distortions and Remedies". In: *American Economic Journal: Macroeconomics*.

Bilbiie, Florin, Fabio Ghironi, and Marc J. Melitz (2007). *Endogenous Entry, Product Variety, and Business Cycles*. NBER Working Papers 13646. National Bureau of Economic Research, Inc.

Blanchard, Olivier and Jordi Gali (2010). "Labor markets and monetary policy: A New Keynesian model with unemployment". In: *American economic journal: macroeconomics* 2.2, pp. 1–30.

Blanchard, Olivier and Francesco Giavazzi (2003). "Macroeconomic effects of regulation and deregulation in goods and labor markets". In: *The Quarterly Journal of Economics* 118.3, pp. 879–907.

Cacciatore, Matteo, Romain Duval, Giuseppe Fiori, and Fabio Ghironi (2016a). "Market reforms in the time of imbalance". In: *Journal of Economic Dynamics and Control* 72.C, pp. 69–93.

— (2016b). "Short-term pain for long-term gain: Market deregulation and monetary policy in small open economies". In: *Journal of International Money and Finance* 68.C, pp. 358–385.

Cacciatore, Matteo and Giuseppe Fiori (2016). "The Macroeconomic Effects of Goods and Labor Marlet Deregulation". In: *Review of Economic Dynamics* 20, pp. 1–24.

Cantore, Cristiano, Filippo Ferroni, and Miguel A. Leon-Ledesma (2017). "The dynamics of hours worked and technology". In: *Journal of Economic Dynamics and Control* 82.C, pp. 67–82.

Caputo, M.R. (2005). *Foundations of Dynamic Economic Analysis: Optimal Control Theory and Applications*. Cambridge University Press.

Chang, Yongsung and Jay H Hong (2006). “Do technological improvements in the manufacturing sector raise or lower employment?” In: *American Economic Review* 96.1, pp. 352–368.

Christiano, Lawrence J., Martin Eichenbaum, and Robert Vigfusson (2003). *What Happens After a Technology Shock?* NBER Working Papers 9819. National Bureau of Economic Research, Inc.

Datta, Bipasa and Huw Dixon (2002). “Technological Change, Entry, and Stock-Market Dynamics: An Analysis of Transition in a Monopolistic Industry”. In: *American Economic Review* 92.2, pp. 231–235.

Devereux, Michael B, Allen C Head, and Beverly J Lapham (1996). “Aggregate fluctuations with increasing returns to specialization and scale”. In: *Journal of economic dynamics and control* 20.4, pp. 627–656.

Dixit, Avinash K. and Joseph E. Stiglitz (1977). “Monopolistic Competition and Optimum Product Diversity”. In: *The American Economic Review* 67.3, pp. 297–308.

Ebell, Monique and Christian Haefke (2009). “Product Market Deregulation and the U.S. Employment Miracle”. In: *Review of Economic Dynamics* 12.3, pp. 479–504.

Etro, Federico (2009). *Endogenous market structures and the macroeconomy.* Springer Science & Business Media.

Etro, Federico and Andrea Colciago (2010). “Endogenous Market Structures and the Business Cycle*”. In: *The Economic Journal* 120.549, pp. 1201–1233.

Francis, Neville and Valerie A. Ramey (2005). “Is the technology-driven real business cycle hypothesis dead? Shocks and aggregate fluctuations revisited”. In: *Journal of Monetary Economics* 52.8, pp. 1379–1399.

Gali, Jordi (1999). “Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?” In: *American Economic Review* 89.1, pp. 249–271.

Hornstein, Andreas (1993). “Monopolistic competition, increasing returns to scale, and the importance of productivity shocks”. In: *Journal of Monetary Economics* 31.3, pp. 299–316.

Kim, Jinill (2004). “What determines aggregate returns to scale?” In: *Journal of Economic Dynamics and Control* 28.8, pp. 1577–1594.

King, Robert G, Charles I Plosser, and Sergio T Rebelo (1988). “Production, growth and business cycles: I. The basic neoclassical model”. In: *Journal of Monetary Economics* 21.2-3, pp. 195–232.

Ko, Jun-Hyung and Hyeog Ug Kwon (2015). “Do technology shocks lower hours worked?–Evidence from Japanese industry level data”. In: *Journal of Macroeconomics* 44, pp. 138–157.

Lewis, Vivien (2009). “Business Cycle Evidence On Firm Entry”. In: *Macroeconomic Dynamics* 13 (05), pp. 605–624.

Mandelman, Federico S and Francesco Zanetti (2014). “Flexible prices, labor market frictions and the response of employment to technology shocks”. In: *Labour Economics* 26, pp. 94–102.

Mankiw, N Gregory and Michael D Whinston (1986). “Free entry and social inefficiency”. In: *The RAND Journal of Economics*, pp. 48–58.

Oxborrow, David and Stephen J Turnovsky (2017). “Closing the small open economy model: A demographic approach”. In: *Review of International Economics* 25.1, pp. 44–75.

Poschke, Markus (2010). “The Regulation of Entry and Aggregate Productivity”. In: *Economic Journal* 120.549, pp. 1175–1200.

Rebei, Nooman (2014). “What (really) accounts for the fall in hours after a technology shock?” In: *Journal of Economic Dynamics and Control* 45, pp. 330–352.

Stokey, Nancy L (2008). *The Economics of Inaction: Stochastic Control models with fixed costs*. Princeton University Press.

Turnovsky, S.J. (1997). *International Macroeconomic Dynamics*. MIT Press.

Uribe, M. and S. Schmitt-Grohé (2017). *Open Economy Macroeconomics*. Princeton University Press.

A Main Appendix

A.1 Jacobian

The Jacobian matrix of the 3-dimensional system is as follows where elements are evaluated at steady state:

$$\mathbf{J} = \begin{bmatrix} 0 & \frac{d\dot{N}}{dq} & 0 \\ \frac{d\dot{q}}{dN} & \frac{d\dot{q}}{dq} & 0 \\ \frac{d\dot{B}}{dN} & \frac{d\dot{B}}{dq} & \frac{d\dot{B}}{dB} \end{bmatrix} \Bigg|^{\tilde{\cdot}} = \begin{bmatrix} 0 & \frac{1}{\gamma} & 0 \\ -\frac{\tilde{d\pi}}{dN} & r & 0 \\ \frac{\tilde{dY}}{dN} & -\frac{\tilde{d\mathcal{C}}}{dq} & r \end{bmatrix} \tag{51}$$

where

$$\frac{\tilde{d\mathcal{C}}}{dq} = \frac{\tilde{q}}{\gamma} \tag{52}$$

$$\frac{\tilde{d\pi}}{dN} = \frac{\tilde{\pi} + \phi}{\tilde{N}(\bar{\lambda})} \left(\frac{-\eta\nu}{1 - \nu + \eta} \right) \tag{53}$$

$$\frac{\tilde{dY}}{dN} = A\tilde{h}^{\nu} \left(1 + \nu \left(\frac{1 - \tilde{h}}{\tilde{h}} \right) \right) - \phi \tag{54}$$

where $\tilde{q} = \tilde{\pi} = 0$ (from (31) and (32)) and (33) gives $\tilde{h}$ as a function of exogenous parameters, but $\tilde{N}(\bar{\lambda})$ depends on endogenously determined steady-state consumption index given in (36). In the results that follow, the trace, determinant, eigenvalue relationships are useful

$$\Delta = \Gamma\Gamma^{U} \tag{55}$$

$$r = \Gamma + \Gamma^{U} \tag{56}$$

$$\Delta = \Gamma(r - \Gamma) \tag{57}$$

$$(r^2 - 4\Delta)^{\frac{1}{2}} = r - 2\Gamma \tag{58}$$

Proof of Lemma 3. The determinant of the entry subsystem $\det(\mathbf{B}) = \Delta(\tilde{N}(\bar{\lambda}))$ is increasing in $\bar{\lambda}$.

$$\Delta_{\lambda} = \Delta_{N}\tilde{N}_{\lambda} = -\frac{\Delta}{\tilde{N}} \cdot \frac{\tilde{N}}{\eta\bar{\lambda}} = -\frac{\Delta}{\eta\bar{\lambda}} > 0 \tag{59}$$

The stable root is increasing in the determinant

$$\Gamma_{\Delta} = -\frac{r}{2} \left(\frac{1}{2} \left(1 - \frac{4\Delta}{r^2} \right)^{\frac{-1}{2}} \cdot \frac{-4}{r^2} \right) \tag{60}$$

$$= \frac{1}{(r^2 - 4\Delta)^{\frac{1}{2}}} = \frac{1}{r - 2\Gamma} > 0 \tag{61}$$

and therefore increasing in the number of firms

$$\frac{d\Gamma}{d\tilde{N}} = \Gamma_\Delta \Delta_N = \frac{\Gamma(\Gamma - r)}{r - 2\Gamma}\frac{1}{\tilde{N}} > 0 \tag{62}$$

Therefore the stable root is increasing in $\bar{\lambda}$

$$\Gamma_{\bar{\lambda}} = \Gamma_\Delta \Delta_\lambda = \Gamma_\Delta \Delta_{\tilde{N}} \tilde{N}_\lambda > 0 \tag{63}$$

This can be written

$$\Gamma_{\bar{\lambda}} = -\frac{\Delta}{\eta\bar{\lambda}(r^2 - 4\Delta)^{\frac{1}{2}}} = \frac{1}{\eta\bar{\lambda}}\frac{\Gamma(\Gamma - r)}{r - 2\Gamma} > 0$$

□

A.2 Steady-state Proofs

Proof of Proposition 3.

$$\tilde{\Omega} = \left(\varepsilon_{HN} - 1 + \frac{1}{\mu}\right) Y_H \tilde{h}$$

$$\tilde{\Omega} = \nu\frac{\phi}{1 - \frac{\nu}{\mu}}\left(\varepsilon_{HN} - 1 + \frac{1}{\mu}\right) = \frac{\nu\phi\mu}{\mu - \nu}\left(\frac{1}{\mu} - \frac{\eta}{1 - \nu + \eta}\right)$$

$$\operatorname{sgn}\tilde{\Omega} = \operatorname{sgn}\left[\varepsilon_{HN} - \left(\frac{\mu - 1}{\mu}\right)\right]$$

where $\operatorname{sgn}\varepsilon_{HN} = \operatorname{sgn}(1 - \nu)$ since $\varepsilon_{HN} = \frac{1-\nu}{1-\nu+\eta}$ from (28). □

Repeating the steady-state bond condition here

$$\tilde{B}(\bar{\lambda}, A) = B_0 - \frac{\tilde{\Omega}}{\Gamma(\tilde{N}(\bar{\lambda})) - r}(N_0 - \tilde{N}(\bar{\lambda})) \tag{46}$$

The total derivative of steady-state bonds with respect to inverse consumption is

$$\frac{d\tilde{B}}{d\bar{\lambda}} = -\tilde{\Omega}\left(\frac{d\left(\frac{N_0 - \tilde{N}(\bar{\lambda})}{\Gamma(N(\bar{\lambda})) - r}\right)}{d\bar{\lambda}}\right) = \tilde{\Omega}\left[\frac{(\Gamma(\bar{\lambda}) - r)\frac{d\tilde{N}}{d\bar{\lambda}} + [N_0 - \tilde{N}(\bar{\lambda})]\frac{d\Gamma(\tilde{N})}{d\bar{\lambda}}}{(\Gamma(\bar{\lambda}) - r)^2}\right] \tag{64}$$

The response of steady-state bonds to inverse consumption $\bar{\lambda}$ is ambiguous because both $\tilde{\Omega}$ and $[N_0 - \tilde{N}(\bar{\lambda})]$ are ambiguously signed. Since this model is path-dependent (steady-state depends on initial conditions $\tilde{N}(\bar{\lambda}, N_0)$ due to (46)), we cannot evaluate at $N_0 = \tilde{N}$, which removes the changing eigenvalue effect (see Caputo 2005, p. 475-477 for this common approach).[38] Instead we follow Turnovsky 1997, p.68 (footnote 8) and assume this component $[N_0 - \tilde{N}]$ is small, which – to a linear approximation – removes the changing eigenvalue effect.

Lemma 5. *The effect of a change in the consumption index on bonds is*

$$\frac{d\tilde{B}}{d\bar{\lambda}} = \frac{\tilde{\Omega}}{\Gamma(\tilde{N}(\bar{\lambda})) - r}\left[\frac{\Gamma}{r-2\Gamma}\left(\frac{r}{\Gamma} - 3 + \frac{N_0}{\tilde{N}}\right)\right]\frac{\tilde{N}}{\bar{\lambda}\eta} \tag{65}$$

Proof. From (46) a change in consumption index only affects steady-state bonds indirectly through its effect on steady-state stock of firms

$$\frac{d\tilde{B}}{d\bar{\lambda}} = \frac{d\tilde{B}}{d\tilde{N}}\frac{d\tilde{N}}{d\bar{\lambda}} \tag{66}$$

Then steady-state stock of firms affects bonds directly $\frac{\partial \tilde{B}}{\partial \tilde{N}}$ through $\tilde{N}$ and indirectly $\frac{d\tilde{B}}{d\Gamma}\frac{d\Gamma}{d\tilde{N}}$ through the eigenvalue $\Gamma(\tilde{N}(\bar{\lambda}))$:

$$\frac{d\tilde{B}}{d\tilde{N}} = \frac{\partial \tilde{B}}{\partial \tilde{N}} + \frac{d\tilde{B}}{d\Gamma}\frac{d\Gamma}{d\tilde{N}} = \frac{\tilde{\Omega}}{\Gamma(\tilde{N}(\bar{\lambda})) - r}\left[1 + \left(\frac{N_0 - \tilde{N}(\bar{\lambda})}{\Gamma(\tilde{N}(\bar{\lambda})) - r}\right)\frac{d\Gamma}{d\tilde{N}}\right] \tag{67}$$

Therefore the effect of a change in consumption index on bonds through eigenvalues is an indirect-indirect effect.

$$\frac{d\tilde{B}}{d\bar{\lambda}} = \frac{d\tilde{B}}{d\tilde{N}}\frac{d\tilde{N}}{d\bar{\lambda}} = \left(\frac{\partial \tilde{B}}{\partial \tilde{N}} + \frac{d\tilde{B}}{d\Gamma}\frac{d\Gamma}{d\tilde{N}}\right)\frac{d\tilde{N}}{d\bar{\lambda}} \tag{68}$$

$$= \frac{\tilde{\Omega}}{\Gamma(\tilde{N}(\bar{\lambda})) - r}\left[1 + \left(\frac{N_0 - \tilde{N}(\bar{\lambda})}{\Gamma(\tilde{N}(\bar{\lambda})) - r}\right)\frac{d\Gamma}{d\tilde{N}}\right]\frac{d\tilde{N}}{d\bar{\lambda}} \tag{69}$$

Using (62) the term in square brackets simplifies

$$\frac{d\tilde{B}}{d\bar{\lambda}} = \frac{\tilde{\Omega}}{\Gamma(\tilde{N}(\bar{\lambda})) - r}\left[\frac{\Gamma}{r-2\Gamma}\left(\frac{r}{\Gamma} - 3 + \frac{N_0}{\tilde{N}}\right)\right]\frac{d\tilde{N}}{d\bar{\lambda}} \tag{70}$$

Therefore substituting in (91) gives (65). □

Corollary 2. *If* $\frac{N_0}{\tilde{N}(\bar{\lambda})} < 3 - \frac{r}{\Gamma}$ *then*

$$\operatorname{sgn}\frac{d\tilde{B}}{d\bar{\lambda}} = -\operatorname{sgn}\tilde{\Omega} \tag{71}$$

Proof. From (65) this result ensures the term in curled parenthesis is negative. □

[38] Attempting this approach here introduces another fixed point problem since changing N_0 to equal $\tilde{N}$ will in turn change $\tilde{N}$ due to path-dependency.

Hence a sufficient condition is $\frac{N_0}{\tilde{N}} < 3$, which allows for both entry and exit $-\tilde{N} < N_0 - \tilde{N} < 2\tilde{N}$. The economic interpretation is that the initial stock of firms (market size) is greater than zero and less than three times the steady-state stock of firms. This is more general than the (commonly assumed) stronger condition that the initial condition is arbitrarily close to steady state $\frac{N_0}{\tilde{N}} \to 1$. This condition simply ensures we ignore the changing eigenvalue effect.

Corollary 3. *If* $[N_0 - \tilde{N}(\bar{\lambda})] \to 0$ *then*

$$\operatorname{sgn} \frac{d\tilde{B}}{d\bar{\lambda}} = -\operatorname{sgn} \tilde{\Omega} \tag{72}$$

Proof. From (67) as $N_0 - \tilde{N}(\bar{\lambda}) \to 0$

$$\frac{d\tilde{B}}{d\tilde{N}} \approx \frac{\partial \tilde{B}}{\partial \tilde{N}} = \frac{\tilde{\Omega}}{\Gamma(\tilde{N}(\bar{\lambda})) - r} \tag{73}$$

$$\frac{d\tilde{B}}{d\bar{\lambda}} \approx \frac{\partial \tilde{B}}{\partial \tilde{N}} \frac{d\tilde{N}}{d\bar{\lambda}} = \frac{\tilde{\Omega}}{\Gamma(\tilde{N}(\bar{\lambda})) - r} \frac{\tilde{N}}{\bar{\lambda}\eta} \tag{74}$$

□

Lemma 6 (Steady-state Existence)**.** *There exists at least one* $\bar{\lambda}$ *that solves the steady-state market clearing condition*

$$\tilde{w}\tilde{H}(\bar{\lambda}) + r\tilde{B}(\bar{\lambda}) - \bar{C}(\bar{\lambda}) = 0 \tag{47}$$

Proof of Lemma 6. We use the intermediate-value theorem. Split the steady-state excess demand function into two functions: an income function $f(\bar{\lambda}) = \tilde{w}\tilde{H}(\bar{\lambda}) + rB(\bar{\lambda})$ and an expenditure function $g(\bar{\lambda}) = C(\bar{\lambda})$, so we have $f(\bar{\lambda}) - g(\bar{\lambda}) = 0$. Analyze the functions for the limits of $\bar{\lambda}$. Existence follows from the functional forms for $H(\bar{\lambda}, A) = (\bar{\lambda}w)^{\frac{1}{\eta}}$ and $C(\bar{\lambda}) = \frac{1}{\bar{\lambda}}$. Also that $\tilde{B}$ is bounded in (46) since $\tilde{N}$ is bounded as it is proportional to $\tilde{H}$, which lies in $[0, 1]$. $\lim_{\lambda \to 0} H = 0$ and $\lim_{\lambda \to 0} C = \infty$ so expenditure exceeds income. $\lim_{\lambda \to \infty} H = 1$ and $\lim_{\lambda \to \infty} C = 0$, so income exceeds expenditure. Hence for at least one intermediate value of λ (47) is satisfied. □

Proof of Lemma 4. We aim to show that the steady-state market clearing condition is increasing in $\bar{\lambda}$

$$\tilde{w}\frac{d\tilde{H}}{d\lambda} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{d\tilde{C}}{d\bar{\lambda}} > 0 \tag{48}$$

Since $\frac{d\tilde{C}}{d\bar{\lambda}} < 0$, a sufficient condition is to show that $\tilde{w}\frac{d\tilde{H}}{d\lambda} + r\frac{d\tilde{B}}{d\bar{\lambda}} > 0$. That is, we show that the positive labor effect always dominates the (potentially) negative bond effect.

$$\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} = \frac{\tilde{Y_H}}{\mu}\frac{d\tilde{H}}{d\bar{\lambda}} + r\tilde{\Omega}\left[\frac{(\Gamma - r)\frac{d\tilde{N}}{d\bar{\lambda}} + [N_0 - \tilde{N}]\frac{d\Gamma}{d\bar{\lambda}}}{(\Gamma - r)^2}\right] \tag{75}$$

Substitute $\tilde{\Omega} = \left(\varepsilon_{HN} - 1 + \frac{1}{\mu}\right)\tilde{Y_H}\tilde{h}$ and $\frac{d\tilde{N}}{d\bar{\lambda}} = \frac{d\tilde{H}}{d\bar{\lambda}}\frac{1}{\tilde{h}}$

$$\begin{aligned} &= \left[\frac{Y_H}{\mu}\frac{d\tilde{H}}{d\bar{\lambda}}(\Gamma - r) + r\left(\varepsilon_{HN} - 1 + \frac{1}{\mu}\right)Y_H\frac{d\tilde{H}}{d\bar{\lambda}}\right. \\ &\quad \left. + \frac{r\left(\varepsilon_{HN} - 1 + \frac{1}{\mu}\right)Y_H\tilde{h}(N_0 - \tilde{N})}{\Gamma - r}\frac{d\Gamma}{d\bar{\lambda}}\right]\frac{1}{\Gamma - r} \end{aligned} \tag{76}$$

$$\begin{aligned} &= \left[\frac{1}{\mu}(\Gamma - r) + r\left(\varepsilon_{HN} - 1 + \frac{1}{\mu}\right)\right. \\ &\quad \left. + \frac{r\left(\varepsilon_{HN} - 1 + \frac{1}{\mu}\right)\tilde{h}(N_0 - \tilde{N})}{(\Gamma - r)\frac{d\tilde{H}}{d\bar{\lambda}}}\frac{d\Gamma}{d\bar{\lambda}}\right]\frac{Y_H\frac{d\tilde{H}}{d\bar{\lambda}}}{\Gamma - r} \end{aligned} \tag{77}$$

Cancel $\frac{r}{\mu}$ and use that $\frac{d\tilde{H}}{d\bar{\lambda}} = \frac{d\tilde{N}}{d\bar{\lambda}}\tilde{h}$

$$= \left[\frac{1}{\mu}\Gamma + r\left(\varepsilon_{HN} - 1\right) + \frac{r\left(\varepsilon_{HN} - 1 + \frac{1}{\mu}\right)(N_0 - \tilde{N})}{\Gamma - r}\frac{\frac{d\Gamma}{d\bar{\lambda}}}{\frac{d\tilde{N}}{d\bar{\lambda}}}\right]\frac{Y_H\frac{d\tilde{H}}{d\bar{\lambda}}}{\Gamma - r} \tag{78}$$

Remembering $\varepsilon_{HN} - 1 < 0$, the first two terms are negative and the third term (the changing eigenvalue term $\frac{d\Gamma}{d\bar{\lambda}}$) is ambiguous. As with signing $\tilde{B}_{\bar{\lambda}}$, a sufficient condition to remove the problematic changing eigenvalue term is $N_0 - \tilde{N} \to 0$. Although a weaker, but messier, sufficient condition is:

$$\left(\varepsilon_{HN} - 1 + \frac{1}{\mu}\right)\left(\frac{N_0}{\tilde{N}} - 1\right)\frac{\Gamma}{r - 2\Gamma} \leq -\left(\frac{\Gamma}{r\mu} + \varepsilon_{HN} - 1\right) \tag{79}$$

$$\left(\varepsilon_{HN} - 1 + \frac{1}{\mu}\right)\left(\frac{N_0}{\tilde{N}} - 1\right) \geq -\left(\frac{\varepsilon_{HN} - 1}{\Gamma} + \frac{1}{r\mu}\right)(r - 2\Gamma) \tag{80}$$

The right-hand side is negative so this condition always holds if there is entry $N_0 < \tilde{N}$ and $\varepsilon_{HN} - 1 + \frac{1}{\mu} < 0$ implying $\tilde{\Omega} < 0$. Or if there is exit $N_0 > \tilde{N}$ and and $\varepsilon_{HN} - 1 + \frac{1}{\mu} > 0$ implying $\tilde{\Omega} > 0$.

□

A.3 Dynamics

Rather than defining steady-state as a function of $\tilde{h}(A), \tilde{w}(A)$ as in (36) and (37), since both depend on A and we are investigating changes in A it is useful substitute out. Repeating $\tilde{B}$, expressing dependence on A, is also useful. A only affects $\tilde{B}$ through $\tilde{N}$, which it affects directly and indirectly: $\tilde{N}(A, \bar{\lambda}(A))$ via (81).

$$\tilde{N}(\bar{\lambda}, A) = \left(\bar{\lambda}\frac{\nu}{\mu}\right)^{\frac{1}{\eta}} A^{\frac{1+\eta}{\nu\eta}} \left(\frac{\mu-\nu}{\mu\phi}\right)^{\frac{1-\nu+\eta}{\nu\eta}} \tag{81}$$

$$\tilde{H}(\bar{\lambda}, A) = \tilde{h}(A)\tilde{N}(\bar{\lambda}, A) = \left(\bar{\lambda}\frac{\nu}{\mu}\right)^{\frac{1}{\eta}} A^{\frac{1}{\nu\eta}} \left(\frac{\mu-\nu}{\mu\phi}\right)^{\frac{1-\nu}{\nu\eta}} \tag{82}$$

$$\tilde{B}(\tilde{N}(A, \bar{\lambda}(A))) = B_0 - \frac{\tilde{\Omega}}{\Gamma(\tilde{N}(A, \bar{\lambda}(A))) - r}(N_0 - \tilde{N}(\tilde{N}(A, \bar{\lambda}(A))) \tag{46}$$

Technology change has a direct (partial) and an indirect (consumption) effect on the core endogenous model variables

$$\frac{dX}{dA} = \frac{\partial X}{\partial A} + \frac{dX}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA}, \quad X \in \{\bar{C}, \tilde{N}, \tilde{B}\} \tag{83}$$

The direct (partial) effects of A holding $\bar{\lambda}$ constant are simple to calculate. There is no partial effect on consumption, only an indirect effect.

$$\frac{\partial \bar{C}}{\partial A} = 0 \tag{84}$$

$$\frac{\partial \tilde{N}}{\partial A} = \frac{(1+\eta)\tilde{N}}{\nu\eta A} > 0 \tag{85}$$

$$\frac{\partial \tilde{B}}{\partial A} \approx \frac{\tilde{\Omega}}{\Gamma - r}\frac{\partial \tilde{N}}{\partial A} \gtreqless 0 \implies \operatorname{sgn}\frac{\partial \tilde{B}}{\partial A} = \operatorname{sgn} -\tilde{\Omega} \tag{86}$$

$$\frac{\partial \tilde{H}}{\partial A} = \frac{\tilde{H}}{\nu A\eta} > 0 \tag{87}$$

From the steady state market clearing condition (47), we can use the implicit function theorem to infer that technology decreases the marginal utility of consumption and therfore increase consumption (since through (6) consumption and marginal utility are inversely related).

Proposition 7 (Technology Effect on Steady-state Consumption)**.**

$$\frac{d\bar{\lambda}}{dA} < 0 \tag{88}$$

$$\frac{d\bar{C}}{dA} = \frac{d\bar{C}}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA} > 0 \tag{89}$$

$$\frac{d\bar{C}}{d\bar{\lambda}} = -\frac{1}{\bar{\lambda}^2} < 0 \tag{90}$$

Therefore an increase in technology increases consumption (decreases marginal utility), which, from (36) and (37), will have an indirect effect of decreasing numbers of firms and labor. This is because consumption crowds out investment in firms.

$$\frac{d\tilde{N}}{d\bar{\lambda}} = \frac{\tilde{N}}{\eta\bar{\lambda}} > 0 \tag{91}$$
$$\frac{d\tilde{B}}{d\bar{\lambda}} = \frac{d\tilde{B}}{d\tilde{N}}\frac{d\tilde{N}}{d\bar{\lambda}} \approx \frac{\tilde{\Omega}}{\Gamma - r}\frac{d\tilde{N}}{d\bar{\lambda}} \implies \operatorname{sgn}\frac{d\tilde{B}}{d\bar{\lambda}} = -\operatorname{sgn}\tilde{\Omega} \tag{92}$$
$$\frac{d\tilde{H}}{d\bar{\lambda}} = \tilde{h}\frac{d\tilde{N}}{d\bar{\lambda}} = \frac{\tilde{H}}{\eta\bar{\lambda}} > 0 \tag{93}$$

Proof of Proposition 7. The total derivative of (47) with respect to technology is

$$\frac{d\tilde{w}}{dA}\tilde{H} + \tilde{w}\left(\frac{\partial\tilde{H}}{\partial A} + \frac{d\tilde{H}}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA}\right) + r\left(\frac{\partial\tilde{B}}{\partial A} + \frac{d\tilde{B}}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA}\right) - \frac{dC}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA} = 0 \tag{94}$$

Therefore

$$\frac{d\bar{\lambda}}{dA} = -\frac{\frac{d\tilde{w}}{dA}\tilde{H} + \tilde{w}\frac{\partial\tilde{H}}{\partial A} + r\frac{\partial\tilde{B}}{\partial A}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{dC}{d\bar{\lambda}}} < 0 \tag{95}$$

The denominator is positive under sufficient condition (49) or stronger sufficient condition $N_0 - \tilde{N} \to 0$. Let's focus on the numerator

$$\frac{d\tilde{w}}{dA}\tilde{H} + \tilde{w}\frac{\partial\tilde{H}}{\partial A} + r\frac{\partial\tilde{B}}{\partial A} \tag{96}$$

which appears to be ambiguous. We shall show it is positive implying (95) is negative.

$$\frac{d\tilde{w}}{dA}\tilde{H} + \tilde{w}\frac{\partial\tilde{H}}{\partial A} + r\frac{\partial\tilde{B}}{\partial A} \tag{97}$$
$$= \frac{\tilde{w}}{\nu A}\tilde{H} + \tilde{w}\frac{\tilde{H}}{\nu A\eta} + r\frac{\tilde{\Omega}}{\Gamma - r}\frac{(1+\eta)\tilde{N}}{\nu\eta A} = \frac{1+\eta}{\nu A}\left[\frac{\tilde{w}\tilde{H}}{(1+\eta)} + \frac{\tilde{w}\tilde{H}}{(1+\eta)\eta} + r\frac{\tilde{\Omega}}{\Gamma - r}\frac{\tilde{N}}{\eta}\right] \tag{98}$$
$$= \frac{1+\eta}{\nu A}\left[\frac{\tilde{w}\tilde{H}}{\eta} + r\frac{\tilde{\Omega}}{\Gamma - r}\frac{\tilde{N}}{\eta}\right] = \frac{1+\eta}{\nu A}\left[\frac{\frac{\tilde{Y}_H}{\mu}\tilde{H}}{\eta} + r\frac{\tilde{\Omega}}{\Gamma - r}\frac{\tilde{N}}{\eta}\right] \tag{99}$$

Substitute $\tilde{\Omega} = (\varepsilon_{HN} - 1 + \frac{1}{\mu})\tilde{Y_H}\frac{\tilde{H}}{\tilde{N}}$

$$= \frac{1+\eta}{\nu A}\left[\frac{\frac{\tilde{Y_H}}{\mu}\tilde{H}}{\eta} + r\frac{(\varepsilon_{HN} - 1 + \frac{1}{\mu})\tilde{Y_H}\frac{\tilde{H}}{\tilde{N}}}{\Gamma - r}\frac{\tilde{N}}{\eta}\right] = \frac{(1+\eta)\tilde{Y_H}\tilde{H}}{\nu A\eta}\left[\frac{1}{\mu} + r\frac{(\varepsilon_{HN} - 1 + \frac{1}{\mu})}{\Gamma - r}\right] \tag{100}$$

$$= \frac{(1+\eta)\tilde{Y_H}\tilde{H}}{\nu A\eta}\frac{1}{(\Gamma - r)}\left[\frac{\Gamma}{\mu} + r(\varepsilon_{HN} - 1)\right] = \frac{(1+\eta)\tilde{N}(\tilde{y} + \phi)}{A\eta}\frac{1}{(\Gamma - r)}\left[\frac{\Gamma}{\mu} + r(\varepsilon_{HN} - 1)\right] > 0 \tag{101}$$

Using $\frac{\tilde{H}}{\eta\bar{\lambda}} = \frac{d\tilde{H}}{d\bar{\lambda}}$ we can show

$$= \frac{(1+\eta)\bar{\lambda}}{\nu A}\frac{\tilde{Y_H}\frac{d\tilde{H}}{d\bar{\lambda}}}{(\Gamma - r)}\left[\frac{\Gamma}{\mu} + r(\varepsilon_{HN} - 1)\right] \tag{102}$$

Substitute (78) (ignore changing eigenvalue effect)

$$= \frac{(1+\eta)\bar{\lambda}}{\nu A}\left(\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}}\right) > 0 \tag{103}$$

Therefore

$$\frac{d\bar{\lambda}}{dA} = -\frac{\frac{d\tilde{w}}{dA}\tilde{H} + \tilde{w}\frac{\partial\tilde{H}}{\partial A} + r\frac{\partial\tilde{B}}{\partial A}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{dC}{d\bar{\lambda}}} = -\frac{(1+\eta)\bar{\lambda}}{\nu A}\left(\frac{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{dC}{d\bar{\lambda}}}\right) < 0 \tag{104}$$

□

Proof of Proposition 4.

Firms

$$\frac{d\tilde{N}}{dA} = \frac{\partial\tilde{N}}{\partial A} + \frac{d\tilde{N}}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA} \tag{105}$$

$$= \frac{(1+\eta)}{\nu\eta A}\tilde{N} - \frac{\tilde{N}}{\bar{\lambda}\eta}\left[\frac{(1+\eta)\bar{\lambda}}{\nu A}\left(\frac{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{dC}{d\bar{\lambda}}}\right)\right] \tag{106}$$

$$= \frac{\partial\tilde{N}}{\partial A}\left[1 - \frac{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{dC}{d\bar{\lambda}}}\right] = \frac{\partial\tilde{N}}{\partial A}\left[\frac{-\frac{d\bar{C}}{d\bar{\lambda}}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{dC}{d\bar{\lambda}}}\right] > 0 \tag{107}$$

Bonds

$$\frac{d\tilde{B}}{dA} = \frac{\partial\tilde{B}}{\partial A} + \frac{d\tilde{B}}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA} = \frac{d\tilde{B}}{d\tilde{N}}\frac{\partial\tilde{N}}{\partial A} + \frac{d\tilde{B}}{d\tilde{N}}\frac{d\tilde{N}}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA} \tag{108}$$

$$= \frac{d\tilde{B}}{d\tilde{N}}\left[\frac{\partial\tilde{N}}{\partial A} + \frac{d\tilde{N}}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA}\right] = \frac{d\tilde{B}}{d\tilde{N}}\frac{d\tilde{N}}{dA} \tag{109}$$

From (67) if $N_0 - \tilde{N} \to 0$ then $\frac{d\tilde{B}}{d\tilde{N}} = \frac{\partial \tilde{B}}{\partial \tilde{N}} + \frac{d\tilde{B}}{d\Gamma}\frac{d\Gamma}{d\tilde{N}} = \frac{\tilde{\Omega}}{\Gamma - r}\left(1 + \frac{N_0 - \tilde{N}}{\Gamma - r}\frac{d\Gamma}{d\tilde{N}}\right) \approx \frac{\tilde{\Omega}}{\Gamma - r}$ thus

$$\frac{d\tilde{B}}{dA} \approx \frac{\tilde{\Omega}}{\Gamma - r}\frac{d\tilde{N}}{dA} \gtreqless 0 \implies \operatorname{sgn}\frac{d\tilde{B}}{dA} = \operatorname{sgn} -\tilde{\Omega} \tag{110}$$

Labor

$$\frac{d\tilde{H}}{dA} = \frac{\partial \tilde{H}}{\partial A} + \frac{d\tilde{H}}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA} = \frac{\tilde{H}}{\nu A \eta} + \frac{\tilde{H}}{\nu \bar{\lambda}}\frac{d\bar{\lambda}}{dA} = \frac{\partial \tilde{H}}{\partial A}\left[1 + \frac{\nu A}{\bar{\lambda}}\frac{d\bar{\lambda}}{dA}\right] \tag{111}$$

Substitute out (104)

$$= \frac{\partial \tilde{H}}{\partial A}\left(1 - \frac{(1+\eta)\left(\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}}\right)}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{d\bar{C}}{d\bar{\lambda}}}\right) \tag{112}$$

$$= \frac{\frac{\partial \tilde{H}}{\partial A}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{d\bar{C}}{d\bar{\lambda}}}\left(-\eta\left(\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}}\right) - \frac{d\bar{C}}{d\bar{\lambda}}\right) \tag{113}$$

Substitute out $\frac{d\tilde{H}}{d\bar{\lambda}} = \frac{\tilde{H}}{\bar{\lambda}\eta}$, $\frac{d\tilde{B}}{d\bar{\lambda}} \approx \frac{\tilde{\Omega}}{\Gamma - r}\frac{d\tilde{N}}{d\bar{\lambda}}$ and $\frac{d\bar{C}}{d\bar{\lambda}} = -\frac{1}{\bar{\lambda}^2} = -\frac{\bar{C}}{\bar{\lambda}}$

$$= \frac{\frac{\partial \tilde{H}}{\partial A}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{d\bar{C}}{d\bar{\lambda}}}\frac{1}{\bar{\lambda}}\left(\bar{C} - \tilde{w}\tilde{H} - r\frac{\tilde{\Omega}}{\Gamma - r}\tilde{N}\right) \tag{114}$$

In steady state $\tilde{C} - \tilde{w}\tilde{H} = r\tilde{B}$

$$\frac{d\tilde{H}}{dA} = \frac{\frac{\partial \tilde{H}}{\partial A}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{d\bar{C}}{d\bar{\lambda}}}\frac{1}{\bar{\lambda}}\left(r\tilde{B} - r\frac{\tilde{\Omega}}{\Gamma - r}\tilde{N}\right)$$

From (46) $\tilde{B} - \frac{\tilde{\Omega}}{\Gamma - r}\tilde{N} = B_0 - \frac{\tilde{\Omega}}{\Gamma - r}N_0$

$$\frac{d\tilde{H}}{dA} = \frac{\frac{\partial \tilde{H}}{\partial A}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{d\bar{C}}{d\bar{\lambda}}}\frac{r}{\bar{\lambda}}\left(B_0 - \frac{\tilde{\Omega}}{\Gamma - r}N_0\right)$$

□

Proof of Proposition 5.
Labor Totally differentiating $H = H(\bar{\lambda}, N, A)$ keeping N fixed yields.

$$\frac{dH(0)}{dA} = \frac{dH}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA} + \frac{\partial H}{\partial A} \tag{115}$$

$$= -\frac{\partial H}{\partial A}\left[\frac{(1 - \nu + \eta)(w\frac{dH}{d\bar{\lambda}} + r\frac{dB}{d\bar{\lambda}}) - \nu\frac{dC}{d\bar{\lambda}})}{\nu\left(w\frac{dH}{d\bar{\lambda}} + r\frac{dB}{d\bar{\lambda}} - \frac{dC}{d\bar{\lambda}}\right)}\right] \tag{116}$$

As in the long-run case, the income and substitution effects of a technological improvement work in opposite directions. The difference between the long-run and impact multiplier is accounted for by the effect of entry, so that

$$\frac{dH(0)}{dA} - \frac{dH(\infty)}{dA} = \frac{dH}{dN}\frac{dN}{dA} = \frac{dH}{dN}\left[\frac{\partial N}{\partial A} + \frac{dN}{d\bar{\lambda}}\frac{d\bar{\lambda}}{dA}\right] \tag{117}$$

$$= \frac{dH}{dN}\frac{\partial \tilde{N}}{\partial A}\left[\frac{-\frac{d\bar{C}}{d\bar{\lambda}}}{\tilde{w}\frac{d\tilde{H}}{d\bar{\lambda}} + r\frac{d\tilde{B}}{d\bar{\lambda}} - \frac{dC}{d\bar{\lambda}}}\right] \tag{118}$$

$$\text{sgn}\left[\frac{dH(\infty)}{dA} - \frac{dH(0)}{dA}\right] = \text{sgn}\ \ H_N = \text{sgn}\left[1-\nu\right]$$

Wages

$$\frac{dw(0)}{dA} = \frac{1}{\mu}Y_{HH}\frac{dH(0)}{dA} + \frac{w}{A\nu} \tag{119}$$

Hence

$$\frac{dw(0)}{dA} - \frac{dw(\infty)}{dA} = \frac{1}{\mu}Y_{HH}\frac{dH(0)}{dA} \tag{120}$$

$$\text{sgn}\left[\frac{dw(0)}{dA} - \frac{dw(\infty)}{dA}\right] = \text{sgn}\left[\nu - 1\right] \tag{121}$$

The difference between the long-run and short run wage effect depends on whether an increase in employment increases the MPL ($\nu > 1, Y_{HH} > 0$), or decreases it ($\nu < 1$, $Y_{HH} < 0$). □

B Supplementary Appendix I: Extra Results

B.1 Household Optimization

The Hamiltionian and optimality conditions are

$$\hat{\mathcal{H}}(t) = U(C, H) + \lambda(t)[rB + wH + \Pi - C] \tag{122}$$

$$\hat{\mathcal{H}}_C = 0: \quad \Longrightarrow \quad U_C(C) - \lambda = 0 \tag{123}$$

$$\hat{\mathcal{H}}_H = 0: \quad \Longrightarrow \quad U_H(H) + \lambda w = 0 \tag{124}$$

$$\hat{\mathcal{H}}_B = \rho\lambda - \dot{\lambda}: \quad \Longrightarrow \quad \lambda r = \rho\lambda - \dot{\lambda} \tag{125}$$

$$\hat{\mathcal{H}}_\lambda = \dot{B}: \quad \Longrightarrow \quad \dot{B} = rB + wH + \Pi - C \tag{126}$$

The presence of a small open economy and international capital markets $\rho = r$ means that the household can completely smooth its consumption so (125) implies $\dot{\lambda} = 0$. Therefore marginal utility of wealth is unchanging over time. $\lambda = \bar{\lambda}$ combined with additively separable preferences $u_{CH} = 0$ this implies from (123) that consumption is constant and in a one-one relationship with marginal utility of wealth.[39]

$$\bar{C} = C(\bar{\lambda}) \tag{127}$$

This relationship from (123) then implies labor only varies with real wage from (124)

$$H = H(\bar{\lambda}, w) = H(\bar{C}, w) \tag{128}$$

This represents the households labor supply.

B.2 General Equilibrium Effect of Entry on Output

There are two ways to think of the effect of an entrant on aggregate output $\frac{dY}{dN}$, and they offer different intuitions. The first begins with $Y = Ny$ and the second begin with $Y = AN^{1-\nu}H^{\nu} - N\phi$.

B.2.1

Entry has an ambiguous effect on aggregate output if there are decreasing returns $\nu < 1$ so that $\varepsilon_{HN} > 0$. This is because entry strengthens labor supply which can increase output. Whereas with constant or increasing returns $\nu \geq 1$ an entrant always decreases

[39] We could not make the final step from (123) if $u_{CH} \neq 0$. Imposing additive separability and therefore constant consumption, we simplify analysis of dynamics as C can be treated as fixed.

aggregate output.

$$\frac{dY}{dN} = y + N\frac{dy}{dN} = \varepsilon_{HN}(1+\eta)Ah^{\nu} - \phi \tag{129}$$

The first equality states that an entrant contributes its own output y but has a *business stealing* (Mankiw and Whinston 1986) effect on the output of all other incumbents. In the appendix we show this business stealing effect is strictly negative $N\frac{dy}{dN} = \nu(y+\phi)(\varepsilon_{HN} - 1) < 0$. The second equality of states that an entrant has a negative effect by bringing in an extra fixed cost, but it has another positive, negative or zero effect depending on the labor elasticity to entry ε_{HN}.

$$\frac{dY(N, y(N, H))}{dN} = \frac{d[Ny]}{dN} = y + N\frac{dy}{dN}$$

An entrant always causes 'business stealing' from other firms: a fall in output at the firm level or analogously, by (24), a fall in an each incumbents' profits.

$$\frac{dy}{dN} < 0 \tag{130}$$

$$\frac{dy}{dN} = \frac{d\left(AN^{-\nu}H^{\nu} - \phi\right)}{dN} \tag{131}$$

$$= -\nu\frac{(y+\phi)}{N} + \nu\frac{(y+\phi)}{H}\frac{dH}{dN} \tag{132}$$

$$= \nu\frac{(y+\phi)}{N}\left[\varepsilon_{HN} - 1\right] < 0 \tag{133}$$

$$= Y_H\frac{h}{N}\left[\varepsilon_{HN} - 1\right] \tag{134}$$

Therefore the aggregate business stealing effect is

$$N\frac{dy}{dN} = \nu(y+\phi)(\varepsilon_{HN} - 1) \tag{135}$$

This also implies the effect on operating profits is negative and less than proportional

$$\frac{d\pi}{dN} = \left(1 - \frac{\nu}{\mu}\right)\frac{dy}{dN} < 0 \tag{136}$$

At the aggregate level it is not clear whether the negative business stealing effect of an entrant aggregated across all incumbents offsets the positive effect of the new firms' extra output.

$$\frac{dY}{dN} = \frac{d(Ny)}{dN} \tag{137}$$
$$= y + N\frac{dy}{dN} \tag{138}$$
$$= y + \nu A h^{\nu}(\varepsilon_{HN} - 1) \tag{139}$$
$$= A h^{\nu}(1 - (1 - \varepsilon_{HN})\nu) - \phi \tag{140}$$
$$= \frac{(1-\nu)(1+\eta)}{1-\nu+\eta} A h^{\nu} - \phi \tag{141}$$
$$= \varepsilon_{HN}(1+\eta) A h^{\nu} - \phi \tag{142}$$

The final representation makes clear the crucial effect of returns to scale. It reads that an entrant has a negative effect by bringing in an extra fixed cost, but it has another positive negative or zero effect depending on ε_{HN}.

B.2.2

The partial derivatives of aggregate output (20) with respect to inputs are:[40]

$$\frac{\partial Y}{\partial N} = (1-\nu)Ah^{\nu} - \phi = y - \nu A h^{\nu} = \frac{Y}{N}\left(1 - \nu\left(1 + s_{\phi}\right)\right) \gtreqless 0 \tag{143}$$
$$Y_H \equiv \frac{dY}{dH} = A\nu (H/N)^{\nu-1} = A\nu h^{\nu-1} = \frac{Y}{H}\nu\left(1 + s_{\phi}\right) > 0 \tag{144}$$

where $s_{\phi} \equiv \frac{N\phi}{Y}$ is the share of fixed costs in output.[41] Since aggregate output is homogeneous of degree 1 in inputs

$$Y = \frac{\partial Y}{\partial N} N + Y_H H \tag{145}$$

Substituting this and $w = \frac{1}{\mu} Y_H$ into $N\pi = Y - wH$ gives

$$\pi = Y_N + \left(1 - \frac{1}{\mu}\right) Y_H \frac{H}{N} \tag{146}$$

Alternatively use (146), where the first term is the partial derivative effect of an

[40] Aggregate and firm-level derivatives are equivalent $Y_H = y_h$.

[41] The N derivative is partial, as in general equilibrium the total derivative would recognize that a variation in N affects H, so $\frac{dY}{dN} = \frac{\partial Y}{\partial N} + \frac{dY}{dH}\frac{dH}{dN}$ or $\varepsilon_{YN} = \frac{\partial Y}{\partial N}\frac{N}{Y} + \varepsilon_{YH}\varepsilon_{HN}$. When there are constant or increasing returns to labor $\nu \geq 1$, an entrant always decreases aggregate output due to the fixed cost it brings in addition to the division of labor across more units where it has a lower productivity. When $\nu < 1/(1+s_{\phi}) < 1$ aggregate output will increase in response to entry because the decreasing returns to labor ($\nu < 1$) mean that when an entrant divides aggregate labor into smaller units it employs labor more productively than the incumbents did prior to its entry. This positive effect of output is stronger than the negative fixed cost effect. Since N is independent of H then the partial and total derivative are equivalent for $\frac{dY}{dH} = \frac{\partial Y}{\partial H}$.

entrant which we have explained is ambiguous based on ν, and the second term is the labor response which is also ambiguous based on ν.

$$\frac{dY(N,H)}{dN} = \frac{d[AN^{1-\nu}H^{\nu} - N\phi]}{dN} = \frac{\partial Y}{\partial N} + Y_H \frac{dH}{dN} \tag{147}$$

$$= \pi - \left(1 - \frac{1}{\mu}\right) Y_H \frac{H}{N} + Y_H \frac{dH}{dN} \tag{148}$$

$$= \pi - \left(1 - \frac{1}{\mu} - \varepsilon_{HN}\right) Y_H h \tag{149}$$

$$\varepsilon_{YN} = s_\pi - \left(1 - \frac{1}{\mu} - \varepsilon_{HN}\right) \varepsilon_{YH}, \quad \text{where } s_\pi = \frac{N\pi}{Y} \tag{150}$$

In terms of profits this can be written $\frac{dY}{dN} = \frac{\partial Y}{\partial N} + Y_H \frac{dH}{dN} = \pi - \left(1 - \frac{1}{\mu} - \varepsilon_{HN}\right) Y_H h$ which is useful when we analyze zero-profit steady state. The first term is the partial derivative effect of an entrant (143) which we have explained is ambiguous based on ν, and the second term is the labor response which is also ambiguous based on ν.

Since y and π are in a one-one relationship, the business stealing effect can also be interpreted as entrants diminishing profits, from (23) $\frac{d\pi}{dN} = \frac{dy}{dN}\left(1 - \frac{\nu}{\mu}\right) < 0$.

B.2.3 Steady-state Effect of Entry on Aggregate Output

The ambiguity of aggregate output response to entry has a long tradition in welfare analysis of firm-entry. These discussions are traditionally focused on *'business stealing'* (Mankiw and Whinston 1986) and variety effects.[42] In our framework, ν creates the possibility that entry increases, decreases or has no effect on aggregate output. This implies there can be an insufficient, excess or optimal number of firms in steady-state. Optimal implies the number of firms that maximizes steady-state aggregate output, conditional on a markup existing. There is no maximum with perfect competition $\mu = 1$, always a lack of entry due to a positive labor effect and no negative markup (business stealing) effect. Etro 2009; Etro and Colciago 2010 provide a discussion of 'golden rule' number of firms when there is endogenous imperfect competition, constant returns and love-of-variety. The golden rule number of firms is that which maximizes consumption and therefore output in steady-state. They show that imperfect competition causes excessive entry in steady-state, which our result corroborates ($\mu > 1$ and $\nu = 1$ implies $1 - \nu < \eta(\mu - 1)$, so excess entry). Bilbiie, Ghironi, and Melitz 2019 provide a discussion of the welfare effects of entry taking transition into account, rather than focusing on steady-state.

The derivation of $\tilde{\Omega}$ shows that the outcome depends on whether the negative business stealing effect $-1 < -\left(\frac{\mu-1}{\mu}\right) \leq 0, \ \mu \in [1, \infty)$ dominates the labor elasticity to entry effect $-1 < \frac{-\eta}{1-\nu+\eta} < \varepsilon_{HN} < 1$ (the left-hand bound holds with $\nu \to 0$ and $\eta \to \infty$), which may be positive, negative or zero. For a standard calibration, $\eta = 1$ and $\mu = 1.5$, the

[42] A textbook reference is Acemoglu 2009 (Ch. 12).

condition

$$\tilde{\Omega} \gtreqless 0 \iff 1 - \nu \gtreqless \eta(\mu - 1)$$

becomes $\tilde{\Omega} \gtreqless 0 \iff 0.5 \gtreqless \nu$.

Excess Entry $\tilde{\Omega} < 0$: If there are constant $\nu = 1$ or increasing $\nu > 1$ returns to labor, $\varepsilon_{HN} \leq 0$, then the fall in labor reinforces the negative business stealing effect, so there is unambiguously a negative effect of entrants on aggregate output in steady state. This is a sufficient condition but is not necessary, providing the business stealing effect is large enough it can override even a positive labor elasticity effect that arises with decreasing returns $\nu < 1$.

1. Example: Positive labor elasticity effect, dominated by negative business stealing effect $\nu = 0.9,\ \eta = 1$ therefore $\varepsilon_{HN} = 0.\overline{09}$ with $\mu = 1.15$ business stealing is -0.13.

2. Constant Returns Special Case $\nu = 1$: The labor effect is zero, so only the negative business stealing effect is present. The smaller the markup $\mu \to 1$ the smaller the negative business stealing effect. But it cannot equal 1 due to the existence condition $\nu < \mu$.

With large markups this outcome is likely. With less divisible labor $\eta \to 0$ this outcome is more likely.

Lack of Entry $\tilde{\Omega} > 0$: If there are decreasing returns $\nu < 1$ then $0 < \varepsilon_{HN} < 1$ and the boost in labor from entry works against the negative business stealing effect, so there can be too little entry if this positive effect dominates the negative business stealing effect. $\varepsilon_{HN} > 0$, hence $\nu < 1$, is necessary but not sufficient, sufficiency requires it is positive *and* larger than the negative business stealing effect.

1. Example: Positive labor elasticity effect dominates negative business stealing effect $\nu = 0.9,\ \eta = 1$ therefore $\varepsilon_{HN} = 0.\overline{09}$ with $\mu = 1.05$ business stealing is -0.05.

2. Perfect Competition Special Case $\mu = 1, \nu < 1, \tilde{\Omega} > 0$: There is no negative business stealing effect, and the the existence condition $\nu < \mu$ enforces decreasing returns. Therefore entry always has a positive effect, implying lack of entry in steady state in the Walrasian (perfect competition) economy.

Optimal Entry $\tilde{\Omega} = 0$: A necessary condition is that the ambiguous labor elasticity effect is positive $\varepsilon_{HN} > 0$, so it can counterbalance the negative business stealing effect. Therefore a necessary condition is decreasing returns $\nu < 1$.

1. Example: $\nu = 0.9,\ \eta = 1,\ \mu = 1.1$

B.3 Bonds

The dynamic equation (29c) is a first-order, linear, nonhomogeneous ordinary differential equation in B. Rewrite in standard form

$$\dot{B} - rB = Y - \frac{q^2}{2\gamma} - C \tag{151}$$

Multiply by the integrating factor e^{-rt}

$$e^{-rt}\dot{B} - re^{-rt}B = e^{-rt}\left[Y - \frac{q^2}{2\gamma} - C\right] \tag{152}$$

The left-hand side is the result of a product rule differentiation, hence integrating yields

$$e^{-rt}B = \kappa + \int_0^\infty e^{-rt}\left[Y - \frac{q^2}{2\gamma} - C\right] dt \tag{153}$$

To find the constant of integration κ, evaluate at $t = 0$ and use the initial condition $B(0) = B_0$

$$B(0) = \kappa = B_0 \tag{154}$$

Substitute this back in (153), then evaluate at $t \to \infty$. Use the transversality condition (8) which makes the left-hand side zero as $\lambda = \bar{\lambda}$. Therefore

$$0 = B_0 + \int_0^\infty e^{-rt}\left[Y - \frac{q^2}{2\gamma} - C-\right] dt \tag{43}$$

B.4 Firm Problem

B.4.1 Final Good Profit Maximization

The final goods producer maximizes profits taking all prices as given. That is, it operates as a price-taker because of perfect competition:

$$\max_{y_i} \quad PY - \int_0^N p_i y_i \, di \tag{155}$$

subject to

$$Y = N^{\varsigma - \frac{\theta}{\theta-1}}\left[\int_0^N y_i^{(\theta-1)/\theta} \, di\right]^{\theta/(\theta-1)} \tag{156}$$

where p_i is the price of firm i's product and P is the aggregate price level. The N^ς multiplier captures any variety effect. We assume $\varsigma = 1$ so no variety effect which implies an increase in the range of intermediates does not affect the unit cost function.

A common case (following Dixit and Stiglitz 1977) is $\varsigma = \frac{\theta}{\theta-1}$ which leads to a variety effect. We remove the variety effect because it will create an additional mechanism adding to the main result that we want to distill. Without removing love of variety, N will enter the labor market equilibrium condition, even with constant returns to scale. The corresponding aggregate price index is

$$P = N^{-\left(\varsigma - \frac{\theta}{\theta-1}\right)} \left(\int_0^N p_i^{1-\theta} \, di \right)^{\frac{1}{1-\theta}}$$

At symmetry $P = N^{1-\varsigma} p$, variety effects (e.g. the Dixit-Stiglitz case $\varsigma = \frac{\sigma}{\sigma-1}$) cause an increase in the number of firms to decrease the aggregate price index as there are efficiency gains. Substituting (156) into (155) and taking the first-order condition yields the constant elasticity demand for each product

$$y_i = \left(\frac{p_i}{P} \right)^{-\theta} \frac{Y}{N^{\varsigma(1-\theta)+\theta}} \tag{157}$$

Rearranged for inverse demand, this gives:

$$\frac{p_i}{P} = \left(\frac{Y}{N^{\varsigma(1-\theta)+\theta} y_i} \right)^{\frac{1}{\theta}} \tag{158}$$

B.4.2 Firm-level Production Function

When $\nu < 1$, $\phi > 0$ there is a U-shaped average cost curve with increasing marginal cost. This is compatible with both perfect and imperfect competition. When $\nu = 1, \phi = 0$, there are constant returns to scale: AC = MC. When $\nu = 1, \phi > 0$, there is a constant MC and decreasing AC. When $\nu > 1$ there is decreasing AC and MC. The extent of increasing returns to labor $\nu > 1$ is limited by the degree of imperfect competition. In the two cases with globally increasing returns to scale, equilibrium can only exist with imperfect competition. Expressed as elasticities the MPL and its slope are:

$$\varepsilon_{yh} \equiv y_h \frac{h}{y} = \nu \left(1 + s_\phi\right) \tag{159}$$

$$\varepsilon_{y_h h} \equiv y_{hh} \frac{h}{y_h} = \nu - 1 \tag{160}$$

The fixed cost implies that labor returns to scale ν are not equivalent to overall returns to scale measured as average cost over marginal cost[43]

$$\frac{AC}{MC} = \nu(1 + s_\phi) \tag{162}$$

where $s_\phi \equiv \frac{\phi}{y}$ is the fixed cost share in output.

B.4.3 Firm-level Profit Maximization

An individual firm operates under imperfect competition. It is a price-setter because it can have some influence on its own price through the inverse demand function

$$\max_h \pi_i = p_i y_i - Pwh_i \tag{163}$$

$$\text{s.t.} \quad \frac{p_i}{P} = \left(\frac{Y}{N^{\varsigma(1-\theta)+\theta} y_i}\right)^{\frac{1}{\theta}} \tag{158}$$

$$y_i = Ah_i^\nu - \phi \tag{12}$$

where $w = W/P$ is the real wage and W is the nominal wage. Substituting in the constraints and treating firms as symmetric gives profit as a function of h

$$\pi = P\left(\frac{Y}{N^{\varsigma(1-\theta)+\theta}}\right)^{\frac{1}{\theta}} (Ah^\nu - \phi)^{1-\frac{1}{\theta}} - Pwh \tag{164}$$

The first order condition with respect to h is

$$\pi_h = P\left(\frac{Y}{N^{\varsigma(1-\theta)+\theta}}\right)^{\frac{1}{\theta}} \left(1 - \frac{1}{\theta}\right) (Ah^\nu - \phi)^{-\frac{1}{\theta}} \cdot A\nu h^{\nu-1} - Pw \tag{165}$$

To find the profit maximizing outcome set $\pi_h = 0$

$$\pi_h = P\left(\frac{Y}{N^{\varsigma(1-\theta)+\theta} y}\right)^{\frac{1}{\theta}} \left(1 - \frac{1}{\theta}\right) \cdot A\nu h^{\nu-1} - Pw = 0 \tag{166}$$

[43] The cost function dual of our production function is $\text{TC} = \text{MC}\,\nu(y+\phi)$. This follows because factor prices equal their marginal revenue product, in the case for labor $w = \text{MR} \times \text{MPL}$. An optimizing firm produces where $MR = MC$, hence as labor is the sole input $\text{TC} = wh = \text{MC} \times \text{MPL} \times h = \text{MC}\,\nu(y+\phi)$. Multiply by $\frac{1}{y\,\text{MC}}$ to get AC / MC which captures overall returns to scale. Furthermore, where w is nominal wage, as labor is the only input, total costs are $\text{TC} = wh = w\left(\frac{y+\phi}{A}\right)^{\frac{1}{\nu}}$ so that marginal cost is

$$\text{MC} = \frac{w}{\nu A}\left(\frac{y+\phi}{A}\right)^{\frac{1-\nu}{\nu}} = \frac{\text{TC}}{\nu(y+\phi)} \tag{161}$$

and average cost is $\text{AC} = \frac{\text{TC}}{y}$ which in the U-shaped AC case ($\nu < 1$ and $\phi > 0$) will achieve minimum at firm scale $y^{\text{MES}} = \frac{\nu\phi}{1-\nu}$, the firm's *minimum efficient scale* (MES).

Use the inverse demand definition (158) gives

$$\pi_h = p_i \left(1 - \frac{1}{\theta}\right) \cdot A\nu h^{\nu-1} - Pw = 0 \tag{167}$$

This gives the optimizing condition that

$$\frac{Pw}{p_i} = \frac{\nu}{\mu} A h_i^{\nu-1} \tag{168}$$

where $\mu = \frac{\theta}{\theta-1}$.

Symmetric Equilibrium: We have $\frac{P}{p} = N^{1-\varsigma}$ therefore

$$w = N^{\varsigma-1} \frac{\nu}{\mu} \frac{y+\phi}{h} \tag{169}$$

which can be rearranged as an expression for profit-maximizing labor demand h_i conditional on the wage w. $\varsigma = 1$ removes variety effects.

Second-order condition for profit maximization

In the increasing returns case $\nu > 1$, the second-order condition for profit maximization is not always satisfied, so we give a necessary condition for this. However, our later condition $\nu < \mu$ is sufficient for this second-order necessary condition to hold. The second-order condition for a maximum requires $\pi_{hh} < 0$

$$\pi_{hh} = -\frac{1}{\theta} \frac{(\pi_h + w)}{y} \frac{(y+\phi)\nu}{h} + \frac{(\pi_h + w)(\nu - 1)}{h} = \frac{\pi_h + w}{h} \left[\nu \left(1 - \frac{1}{\theta} - \frac{\phi}{\theta y}\right) - 1 \right] \tag{170}$$

The term in square brackets provides a necessary and sufficient condition for a maximum

$$\pi_{hh} < 0 \qquad \Longleftrightarrow \qquad 1 - \frac{1 + s_\phi}{\theta} < \frac{1}{\nu}, \tag{171}$$

where $s_\phi \equiv \frac{\phi}{y}$ is the fixed cost share in output. The second-order condition for maximization $\pi_{hh} < 0$ is always satisfied when $\nu \leq 1$. Throughout the paper we impose a more restrictive assumption that is important for another reason – it is necessary for a well-defined steady-state. The assumption is that the markup $\mu \equiv \frac{\theta}{\theta-1}$ exceeds the MPL slope $\nu < \mu$. A by-product of the assumption is that it is sufficient (but not necessary) for the second-order profit-maximization condition to hold. Rearranging the condition and using the markup definition show this is the case

$$\frac{1}{\mu - 1} \left(1 - \frac{\mu}{\nu}\right) < s_\phi \tag{172}$$

If we impose $\nu < \mu$ then the left-hand side is always negative whereas the right-hand side is always positive. Hence the second-order condition for profit maximization is satisfied.

The $\nu \leq \mu$ restriction implies that, for profit maximizing output, MR must intersect MC from above (the second order condition for profit maximization). A higher degree of monopoly μ (more differentiated products) implies steeper MR which allows steeper downward sloping MC (higher ν). Horizontal MC only exists if MR is downward sloping, so some monopoly power exists. Increasing marginal costs $\nu < 1$ is compatible with any level of imperfect competition $\mu \in [1, \infty)$ including perfect competition.

B.5 General Equilibrium Labor Behavior

The elasticity $\varepsilon_{HN} = \frac{1-\nu}{1-\nu+\eta}$ is less than 1, it approaches 1 in the indivisible labor limit.

$$\lim_{\eta \to 0} \varepsilon_{HN} = 1 \tag{173}$$

$$\lim_{\eta \to \infty} \varepsilon_{HN} = \begin{cases} 0^+ & \nu < 1 \\ 0^- & \nu > 1 \end{cases} \tag{174}$$

The elasticity of hours to number of firms is constant and bounded. It is bounded by $\frac{-\eta}{1-\nu+\eta} < \varepsilon_{HN} < 1$. The upper bound occurs with indivisible labor $\eta \to 0$. The lower bound follows from $\nu < 1 + \eta$ so that (working right to left) $\frac{-\eta}{1-\nu+\eta} < \frac{1-(1+\eta)}{1-\nu+\eta} < \frac{1-\nu}{1-\nu+\eta} = \varepsilon_{HN}$. If $\nu = 1$ then $\varepsilon_{HN} = 0$. If $\nu < 1$ then $0 < \varepsilon_{HN} < 1$. And if $\nu > 1$ then $-\infty < \varepsilon_{HN} < 0$.

B.6 Extra Figures

Figure 4 plots a scatter of the Chang and Hong results from Table 2. Red triangles represent the 14 observations that are consistent with our theory.

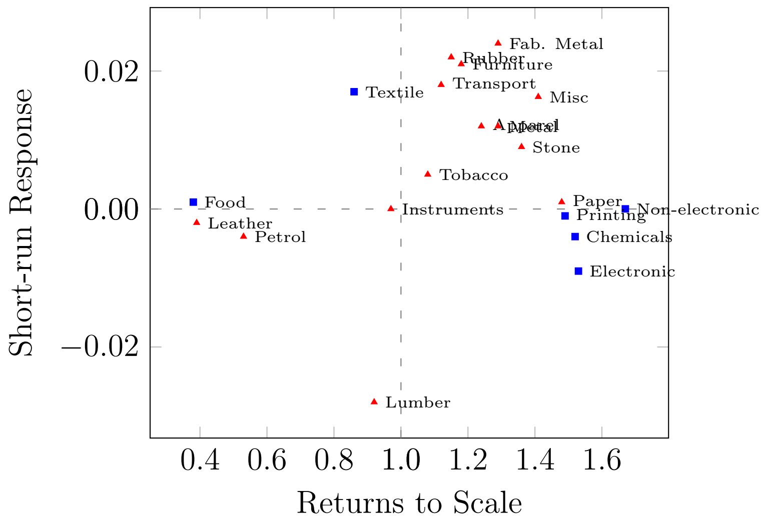

Figure 4: Empirical Evidence

C Supplementary Appendix II: Quantitative Model

This section develops a discrete-time, closed-economy, RBC model with entry and capital adjustment costs. Entry costs are denominated in output terms.

$$\max_{C_t,H_t,I_t,E_t,K_{t+1},N_{t+1},B_{t+1}} E_0 \sum_{t=0}^{\infty} \varrho^t u(C_t, H_t) \tag{175}$$

$$\text{subject to} \tag{176}$$

$$C_t + I_t + \Theta\left(\frac{I_t}{K_t}\right) K_t + E_t + \Psi\left(\frac{E_t}{N_t}\right) N_t + B_{t+1}$$

$$\leq \pi_t N_t + w_t H_t + R_t K_t + (1 + r_t) B_t \tag{177}$$

$$K_{t+1} = I_t + (1 - \delta_K) K_t \tag{178}$$

$$N_{t+1} = E_t + (1 - \delta_N) N_t \tag{179}$$

The model represents a standard closed-economy RBC model with the addition of adjustment costs in firm entry.

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \varrho^t \Bigg\{ u(C_t, H_t) \tag{180}$$

$$+ \lambda_t \Bigg[\pi_t N_t + w_t H_t + R_t K_t + (1 + r_t) B_t$$

$$- C_t - I_t - \Theta\left(\frac{I_t}{K_t}\right) K_t - E_t - \Psi\left(\frac{E_t}{N_t}\right) N_t - B_{t+1} \Bigg] \tag{181}$$

$$+ \varkappa_t [I_t + (1 - \delta_K) K_t - K_{t+1}] \tag{182}$$

$$+ \varpi_t [E_t + (1 - \delta_N) N_t - N_{t+1}] \Bigg\} \tag{183}$$

We get the following first-order conditions and transversality

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \implies u_C = \lambda_t \tag{184}$$

$$\frac{\partial \mathcal{L}}{\partial H_t} = 0 \implies u_H = -\lambda_t w_t \tag{185}$$

$$\frac{\partial \mathcal{L}}{\partial I_t} = 0 \implies \varkappa_t = \lambda_t \left(1 + \Theta'\left(\frac{I_t}{K_t}\right)\right) \tag{186}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = 0 \implies$$

$$\varkappa_t = \varrho \, \mathbb{E}_t \left(\lambda_{t+1}\left[R_{t+1} - \Theta\left(\frac{I_{t+1}}{K_{t+1}}\right) + \Theta'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}}\right] + (1-\delta_K)\varkappa_{t+1}\right) \tag{187}$$

$$\frac{\partial \mathcal{L}}{\partial E_t} = 0 \implies \varpi_t = \lambda_t \left(1 + \Psi'\left(\frac{E_t}{N_t}\right)\right) \tag{188}$$

$$\frac{\partial \mathcal{L}}{\partial N_{t+1}} = 0 \implies$$

$$\varpi_t = \varrho \, \mathbb{E}_t \left(\lambda_{t+1}\left[\pi_{t+1} - \Psi\left(\frac{E_{t+1}}{N_{t+1}}\right) + \Psi'\left(\frac{E_{t+1}}{N_{t+1}}\right)\frac{E_{t+1}}{N_{t+1}}\right] + (1-\delta_N)\varpi_{t+1}\right) \tag{189}$$

$$\frac{\partial \mathcal{L}}{\partial B_{t+1}} = 0 \implies \lambda_t = \varrho \, \mathbb{E}_t \, \lambda_{t+1}(1 + r_{t+1}) \tag{190}$$

The shadow values λ_t, $\varkappa_t$ and ϖ_t represent the value of an additional unit of consumption; the value of an additional unit of capital; and the value of an additional firm. Divide through by λ_t to get in terms of the consumption good (since λ_t is the marginal utility of consumption by (184)) and define $q_t \equiv \frac{\varkappa_t}{\lambda_t}$ and $s_t \equiv \frac{\varpi_t}{\lambda_t}$. The interpretation of these terms is the marginal value of capital and a firm in consumption unit terms. [44]

$$q_t = \varrho \, \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\left[R_{t+1} - \Theta\left(\frac{I_{t+1}}{K_{t+1}}\right) + \Theta'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}} + (1-\delta_K)q_{t+1}\right]\right) \tag{192}$$

$$s_t = \varrho \, \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\left[\pi_{t+1} - \Psi\left(\frac{E_{t+1}}{N_{t+1}}\right) + \Psi'\left(\frac{E_{t+1}}{N_{t+1}}\right)\frac{E_{t+1}}{N_{t+1}} + (1-\delta_N)s_{t+1}\right]\right) \tag{193}$$

[44] With no adjustment costs and zero death, (193) reduces to the firm asset pricing equation in Bilbiie, Ghironi, and Melitz 2012 (191) (assuming zero death in their model which is implausible on technical grounds). They solve the problem of maximizing utility by choosing consumption C_t, labor L_t, and mutual fund holdings X_{t+1}, rather than choosing N_t and E_t, and capital is ignored in the basic setup. The constraint is $w_t L_t + X_t N_t(\pi_t + s_t) = C_t + X_{t+1} s_t (N_t + E_t)$ where they use notation $d_t = \pi_t$, $v_t = s_t$ and $H_t = E_t$. Bilbiie, Ghironi, and Melitz 2012 *assume* time-to-build law of motion, where some new entrants die before production, $(1-\delta_N)(N_t + E_t) = N_{t+1}$ this ensures a contraction in the firm asset pricing equation by substitution $\frac{N_{t+1}}{N_t + E_t} = 1 - \delta_N$

$$s_t = \mathbb{E}_t \, \varrho \frac{\lambda_{t+1}}{\lambda_t}\frac{N_{t+1}}{(N_t + E_t)}(\pi_{t+1} + s_{t+1}) = (1-\delta_N)\varrho \, \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t}(\pi_{t+1} + s_{t+1})\right] \tag{191}$$

Dividing (185) by (184) gives

$$-\frac{u_H}{u_C} = w \tag{194}$$

Then from (186) and (188) we have

$$q_t = 1 + \Theta'\left(\frac{I_t}{K_t}\right) \tag{195}$$

$$s_t = 1 + \Psi'\left(\frac{E_t}{N_t}\right) \tag{196}$$

And from (190) we get the asset pricing relationship, where $1 + r_{t+1}$ can be taken outside the E_t operator as it is known at the start of the period[45]

$$u'(C_t) = \varrho\,\mathbb{E}_t[u'(C_{t+1})(1 + r_{t+1})] \tag{197}$$

Capital adjustment costs are a function of *net investment* $I_t - \delta_K K_t = K_{t+1} - K_t$ which is investment in capital less capital depreciation. Similarly firm creation adjustment costs are a function of *net entry* $E_t - \delta_N N_t = N_{t+1} - N_t$ which is investment in firms less firm death. Net investment and net entry equal the change in stocks over a time period, and are zero in steady-state as stocks are unchanging. Hence adjustment costs are zero in steady-state.

The production function now includes capital

$$y_t = A k_t^{\alpha} h_t^{\beta} - \phi \tag{198}$$

$$Y_t = N_t y_t \tag{199}$$

$$K_t = N_t k_t \tag{200}$$

$$H_t = N_t h_t \tag{201}$$

We continue to work with imperfect competition in the product market so that

$$R_t = \frac{1}{\mu} A\alpha K_t^{\alpha-1} H_t^{\beta} \tag{202}$$

$$w_t = \frac{1}{\mu} A\beta K_t^{\alpha} H_t^{\beta-1} \tag{203}$$

The aggregate operating profit expression

$$\pi_t N_t = Y_t - w_t H_t - R_t K_t \tag{204}$$

[45] r_t is the consumption-based interest rate on holdings of bonds between $t-1$ and t which is known with certainty as of $t-1$. Therefore $1+r_{t+1}$ can be taken outside the time t expectations operator (E_t).

Allows us to update the aggregate resource constraint

$$C_t + I_t + \Theta\left(\frac{I_t}{K_t}\right)K_t + E_t + \Psi\left(\frac{E_t}{N_t}\right)N_t + B_{t+1}$$
$$\leq \pi_t N_t + w_t H_t + R_t K_t + (1+r_t)B_t \tag{205}$$
$$Y_t = C_t + I_t + \Theta\left(\frac{I_t}{K_t}\right)K_t + E_t + \Psi\left(\frac{E_t}{N_t}\right)N_t \tag{206}$$

where we drop B_t.

If we specify adjustment costs as

$$\Theta\left(\frac{I_t}{K_t}\right) = \frac{\vartheta}{2}\left(\frac{I_t}{K_t} - \delta_K\right)^2 \tag{207}$$
$$\Psi\left(\frac{E_t}{N_t}\right) = \frac{\gamma}{2}\left(\frac{E_t}{N_t} - \delta_N\right)^2 \tag{208}$$

and assume isoelastic preferences with separable subutilities (as in the baseline RBC model King, Plosser, and Rebelo 1988)

$$U(C_t, L_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \chi\frac{H_t^{1+\eta}}{1+\eta} \tag{209}$$

where $\eta \equiv \frac{1}{\varphi} \in (0, \infty)$ and $\varphi \geq 0$ is the Frisch elasticity of labor supply to wages and the intertemporal elasticity of substitution in labor supply. Isoelastic utility implies there is constant elasticity of marginal utility with respect to consumption $u_{CC}\frac{u_C}{C} = -\sigma$ (this is the inverse elasticity of intertemporal subsitution) and labor $u_{HH}\frac{u_H}{H} = \eta$. At $\eta = 0$ workers have indivisible labor, and as $\eta \to \infty$ labor responds more strongly. $\sigma \to 1$ is the logarithmic utility case in which income and substitution effects cancel out.

Table summarizes the equilibrium conditions

Static	
Labor Supply	$\chi H_t^{\eta} C_t^{\sigma} = w$
Labor Demand	$w_t = \frac{1}{\mu} A N^{1-\alpha-\beta} \beta K_t^{\alpha} H_t^{\beta-1} = \frac{\beta}{\mu} \frac{Y_t + N_t \phi}{H_t}$
Capital Cost	$R_t = \frac{1}{\mu} A N^{1-\alpha-\beta} \alpha K_t^{\alpha-1} H_t^{\beta} = \frac{\alpha}{\mu} \frac{Y_t + N_t \phi}{K_t}$
q	$q_t = 1 + \vartheta \left(\frac{I_t}{K_t} - \delta_K \right)$
s	$s_t = 1 + \gamma \left(\frac{E_t}{N_t} - \delta_N \right)$
Agg. Accnt.	$Y_t = C_t + I_t + \frac{\vartheta}{2} \left(\frac{I_t}{K_t} - \delta_K \right)^2 K_t + E_t + \frac{\gamma}{2} \left(\frac{E_t}{N_t} - \delta_N \right)^2 N_t$ equivalently: $\pi_t N_t + w_t H_t + R_t K_t = C_t + I_t + \frac{\vartheta}{2} \left(\frac{I_t}{K_t} - \delta_K \right)^2 K_t + E_t + \frac{\gamma}{2} \left(\frac{E_t}{N_t} - \delta_N \right)^2 N_t$
Agg. Profit	$\pi_t N_t = Y_t - w_t H_t - R_t K_t$ equivalently: $\pi_t N_t = \left(1 - \frac{\nu}{\mu} \right) (Y_t + N_t \phi) - N_t \phi$
Firm Prod.	$y_t = A k_t^{\alpha} h_t^{\beta} - \phi$
Agg. Output	$Y_t = N_t y_t$
Agg. Capital	$K_t = N_t k_t$
Agg. Hours	$H_t = N_t h_t$
Dynamic	
Consp. Euler	$1 = \varrho \, \mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} (1 + r_{t+1}) \right]$, where $r_{t+1} = R_{t+1} - \delta_K$
Entry Arb.	$s_t = \varrho \, \mathbb{E}_t \left(\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\pi_{t+1} - \frac{\gamma}{2} \left[\frac{E_{t+1}}{N_{t+1}} - \delta_N \right)^2 + \gamma \left(\frac{E_{t+1}}{N_{t+1}} - \delta_N \right) \frac{E_{t+1}}{N_{t+1}} + (1 - \delta_N) s_{t+1} \right) \right]$
Capital Arb.	$q_t = \varrho \, \mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(R_{t+1} - \frac{\vartheta}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta_K \right)^2 + \vartheta \left(\frac{I_{t+1}}{K_{t+1}} - \delta_K \right) \frac{I_{t+1}}{K_{t+1}} + (1 - \delta_K) q_{t+1} \right) \right]$
Firms LOM	$N_{t+1} = E_t + (1 - \delta_N) N_t$
Capital LOM	$K_{t+1} = I_t + (1 - \delta_K) K_t$

C.1 Steady State

$$\tilde{q} = 1 \tag{210}$$

$$\tilde{s} = 1 \tag{211}$$

$$\tilde{R} = \frac{1}{\varrho} - (1 - \delta_K) \tag{212}$$

$$\tilde{\pi} = \frac{1}{\varrho} - (1 - \delta_N) \tag{213}$$

$$\tilde{r} = \frac{1}{\varrho} - 1 \tag{214}$$

$$\tilde{y} = \left(1 - \frac{\nu}{\mu}\right)^{-1} (\tilde{\pi} + \phi) - \phi \tag{215}$$

$$\tilde{k} = \frac{\alpha}{\mu} \frac{\tilde{y} + \phi}{\tilde{R}} \tag{216}$$

$$\tilde{c} = \tilde{y} - \delta_K \tilde{k} - \delta_N \tag{217}$$

$$\tilde{h} = \left(\frac{\mu \tilde{R}}{\alpha A \tilde{k}^{\alpha - 1}}\right)^{\frac{1}{\beta}} \tag{218}$$

$$\tilde{H} = \left[\left(\frac{\beta(\tilde{y} + \phi)}{\mu \tilde{c} \chi}\right) \left(\frac{\tilde{c}}{\tilde{h}}\right)^{1-\sigma}\right]^{\frac{1}{\sigma + \eta}} \tag{219}$$

$$\tilde{N} = \frac{\tilde{H}}{\tilde{h}} \tag{220}$$

$$\tilde{C} = \tilde{N} \tilde{c} \tag{221}$$

$$\tilde{Y} = \tilde{N} \tilde{y} \tag{222}$$

$$\tilde{K} = \tilde{N} \tilde{k} \tag{223}$$

$$\tilde{I} = \tilde{K} \delta_K \tag{224}$$

$$\tilde{E} = \tilde{N} \delta_N \tag{225}$$

The *per firm* variables $\tilde{\pi}, \tilde{y}, \tilde{k}, \tilde{c}$ are independent of technology in the long-run steady state. Hours per firm $\tilde{h}$ on the other hand decreases as technology increases. So at the firm-level production function: technology increases, capital is unchanged and hours decrease to offset the technology increase, so that output per firm is unchanged. The aggregate variables are all affected by A through the change in the number of firms (the extensive margin). The intensive margin plays no role, except for hours $\tilde{h}$. $\tilde{H}$ will not be affected by technology with logarithmic utility $\sigma = 1$ as income and substitution effects cancel out.

C.2 Calibration

Table 3 gives the parameter values we use to simulate the RBC model with firm entry.

ϱ	0.980	Discount factor
χ	1.000	Labor disutility weight
δ_K	0.024	Depreciation Rate
α	0.200	MPK slope
κ	0.100	Capital adjustment cost
γ	$\in (0, 2)$	Firm adjustment cost
δ_N	0.200	Death rate
β	$\in (0.1, 1.1)$	MPL slope
μ	1.500	Markup
ϕ	0.300	Fixed cost
η	0.976	Inv. Frisch elasticity
σ	0.970	Inv. EIS

Table 3: RBC with Entry Parameter Values

C.3 Intratemporal Condition

In both the RBC and SOE setups labor supply is determined by the intratemporal condition and

$$-\frac{u_H}{u_C} = w$$

Labor demand, with imperfect competition, is determined by wage market equilibrium that states wage is equal to the marginal revenue product of labor

$$w = \frac{1}{\mu}\frac{dY}{dH}$$

Given isoelastic utility and Cobb-Douglas production, equating labor demand and supply gives hours as a function of technology, capital, consumption and number of firms

$$H(A, K, C, N) = \left(\frac{N^{1-\nu} A\beta K^{\alpha}}{\mu\chi C^{\sigma}}\right)^{\frac{1}{1+\eta-\beta}} \tag{226}$$

This reduces to the SOE presentation (27), by defining $\nu \equiv \alpha + \beta$, switching-off capital $\alpha = 0$, imposing logarithmic consumption utility $\sigma = 1$ and setting labor disutility weight $\chi = 1$. In either model, the general response of hours to a technology shock, where elasticities are defined as $\varepsilon_{ab} \equiv \frac{da}{db}\frac{b}{a}$, is

$$\varepsilon_{HA} = \left(\frac{1}{1+\eta-\beta}\right)(1 - \sigma\varepsilon_{CA} + \alpha\varepsilon_{KA} + (1-\nu)\varepsilon_{NA}) \tag{227}$$

In the SOE model with no capital, a permanent positive technology shock increases A and causes an immediate permanent increase in C to its new long-run level. The technology increase raises hours and the consumption increase lowers hours. After impact the only variable left to adjust is the number of firms N which is a state variable, so predetermined on impact. After impact, the number of firms will gradually increase to its new long-run level and whether this increases or decreases hours from the initial jump depends on the sign of the power $1-\nu$. Hence whether the long-run steady state level of hours is approached from above (positive short-run response) or approached from below (negative short-run response) is entirely determined by firm adjustment. Table 1 summarized the long-run hours response and whether it is approached from above of below.

In the RBC model consumption will not remain constant after immediately jumping as it does in the SOE model. If there is logarithmic consumption utility long-run hours will revert to their initial position, hence C must rise to offset the increase in A. If we assume $\alpha = 0$ and $1-\nu = 0$, it is easy to see that consumption is the only variable that can offset the rise in A such that hours will be unchanged in the long run. It will jump and then increase until it reaches its steady-state level. If $1-\nu < 0$ then the adjustment in number of firms will 'help' C to offset the A increase, hence C will not need to increase as much as N will be doing some of the job.

The long-run response is the same regardless of entry. It is a function of σ, β, η and will be $\tilde{\varepsilon}_{HA} = 0$ with $\sigma = 1$ and will be $\tilde{\varepsilon}_{HA} = -\frac{1}{\beta}$ with $\eta = 1$. In steady state capital per firm and consumption per firm are independent of A which implies the long-run elasticities of aggregate capital, aggregate consumption and total number of firms are all they same – the rise in capital and consumption is perfectly offset by a rise in number of firms such that per firm quantities are fixed $\tilde{\varepsilon}_{CA} = \tilde{\varepsilon}_{KA} = \tilde{\varepsilon}_{NA} = \frac{1+\eta}{\beta(\sigma+\eta)}$.

$$\tilde{\varepsilon}_{HA} = \frac{1-\sigma}{\beta(\sigma+\eta)} \gtreqless 0 \iff \sigma \lesseqgtr 1 \tag{228}$$

With log consumption utility $\sigma \to 1$, consumption will increase such that it exactly offsets the increase in technology leaving long-run hours unchanged.