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The Macroprudential Toolkit: Effectiveness and Interactions

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Abstract

We use a DSGE model with financial frictions and with macroprudential limits on both banks and mortgage borrowers, in the form of capital requirements and maximum debt-service ratios. We then examine: (i) the impact of different combinations of macroprudential limits on key macroeconomic aggregates; (ii) their interaction with each other and with monetary policy; and (iii) their effects on the volatility of key macroeconomic variables and on welfare. We find that capital requirements on banks are the optimal tool when faced with a financial shock, as they nullify the effects of financial frictions and reduce the effects of the shock on the real economy. Instead, limits on mortgage debt-service ratios are optimal following a housing demand shock, as they disconnect the housing market from the real economy, reducing the volatility of inflation. Hence, no policy on its own is sufficient to deal with a wide range of shocks.

I. Introduction and motivation

Since the 2008 global financial crisis, policymakers have designed macroprudential policies that help stabilize debt and prevent or lessen the impact of future financial shocks. However, with many of these policies still untested, policymakers are facing the challenge of understanding their interactions with monetary policy or with the rest of the macroprudential toolkit. The task is even harder when, unlike for monetary policy, the objectives of macroprudential policy are much broader in nature and cannot be defined

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numerically. For example, the Bank of England's Financial Stability Objective is 'to protect and enhance the stability of the financial system of the United Kingdom'. It does this via its Financial Policy Committee (FPC) whose responsibility 'in relation to the achievement by the Bank of its Financial Stability Objective relates primarily to the identification of, monitoring of, and taking of action to remove or reduce systemic risks with a view to protecting and enhancing the resilience of the UK financial system.[1]

The topic is relevant and interesting, considering the special attention paid to macroprudential policy after the financial crisis and in recent years. Even though macroprudential policies have been studied in the literature, there are still gaps to fill in the analysis of macroprudential shocks and the optimal use of policies. Policy practitioners are still unsure on how to implement many of these policies, since macroprudential measures, by their nature, pursue several objectives simultaneously, and there is an ample range of instruments to be used. The main innovation of this work clearly lies in the elaboration of the theoretical transmission channels of a macroprudential shock and is thus perfectly suited to fill parts of this gap in the literature.

The ample range of potential risks to be monitored and addressed as well as the availability of multiple macroprudential tools adds complexity to the task of choosing optimal policy by central bankers. For example, macroprudential household tools designed to mitigate risks on household balance sheets have to be set in conjunction with tools that address risks for the financial sector, as these can also have implications for housing markets and household debt. Additionally, household behaviour can also affect the wider economy via aggregate demand effects, hence the composition of household balance sheets may also be of interest to the monetary policymaker. This raises the importance of policy interactions, not only between different macroprudential tools, but also between macroprudential and price stability tools.

In particular, this paper contributes to the existing literature on the optimal use of monetary and macroprudential policy by considering a macroprudential toolkit that includes both capital requirements for banks and affordability constraints on mortgage borrowers. Most previous papers on the topic have looked at the interactions of one tool at a time with monetary policy but not at a broader macroprudential toolkit with macroprudential limits on both lenders and borrowers. We show that having macroprudential credit contraints on either lenders or borrowers, but not on both, is not sufficient to maintain financial resilience when faced with different types of negative shocks. However, we find that having a more comprehensive set of macroprudential limits can amplify monetary policy shocks, highlighting the need to understand broader policy interactions. Our setup allows us to explore a rich set of interactions between policies acting on bank balance sheets, household balance sheets and firms' production decisions. To the standard Dynamic Stochastic General Equilibrium Model (DSGE) model of Smets and Wouters (2007), we follow Iacoviello (2015) and add household borrowing and an endogenous leverage constraint on banks, resulting from the possibility of bankers absconding with their assets a la (Gertler and Karadi, 2011). The financial and real frictions in the model give rise to meaningful roles for macroprudential and monetary policy. However, unlike the existing academic literature, which focuses on a very limited

[1] See the Remit and Recommendations for the Financial Policy Committee in the UK.

set of tools, we model the actual policy toolkit used by central banks at the moment. We do this by augmenting the model in two important ways.

First, we add capital requirements on banks. We do this via a maximum leverage ratio set by the policymaker. Further, we assume that banks see leverage limits as an absolute maximum and they will expend effort (i.e. incur costs) to avoid reaching it. This approach ties in with the data, as in practice banks keep excess capital buffers over and above their capital requirements.

Second, we examine the role of affordability constraints (i.e. maximum mortgage debt-service relative to income) on mortgage lending and their interaction with monetary policy. Most of the existing literature on household and bank leverage has considered the policy design of either LTV limits or capital requirements. But affordability constraints can be used to stress test households' debt levels. This is a crucial innovation in this paper, since DSR limits have not been studied much in the literature and they are to be used by policymakers. We follow the macroprudential framework introduced in the UK in 2014 and model affordability constraints as stressed debt-service ratios (DSR)[2] on households' balance sheets. We augment the standard DSR measure which captures debt repayments as a proportion of labour income, by adding a fixed buffer on top of the mortgage interest rate. This tests whether borrowers can still afford their mortgage payments should credit conditions tighten. Additionally, a change in the monetary policy rate will have a direct effect on DSR ratios by increasing interest repayments. As such, adding this tool in the model introduces an additional channel of monetary and macroprudential policy interaction, which is missing in the literature with just collateral constraints. We note however that, given mortgage loans in our model are all assumed to be for one period only, this affordability constraint is equivalent to a loan-to-income (LTI) constraint, where the LTI ratio depends on the current interest rate. The analysis of the channel of transmission of affordability constraints is very useful for countries that are actively using this tool, that is, the UK. Our results shed some light to policy makers on how to make the best use of the DSR on the pursuit of financial stability, and in conjunction with the rest of the available macroprudential toolkit.

Affordability constraints were introduced in the UK in June 2014 (Bank of England, 2017). The FPC argued that this tool allows them to guard against an increase in the number of highly indebted households. A high proportion of highly leveraged households can lead to demand externalities if they are forced to deleverage following a negative aggregate shock, cutting back on spending and amplifying the economic bust. The FPC did not expect their recommendation to restrain housing market activity unless lending standards declined. We interpret this as implying that the affordability constraint would 'kick in' if lending rose too strongly relative to income.

There are two key issues we examine in this paper. First, we investigate the interaction of macroprudential tools with each other and with monetary policy. Second, we examine the gains from adding each policy to the macroprudential toolkit in terms of reducing the volatility of key macroeconomic variables. In order to assess the impact of the different macroprudential policy tools and their interaction with each other, we adopt the following approach. We first develop a baseline model in which we have frictions in the banking

[2] We use affordability constraints and DSR limits interchangeably throughout the paper.

sector and where we calibrate credit conditions in steady state to match UK data. We then consider the impact of adding a maximum leverage ratio on banks imposed by the macroprudential policymaker. Next, we examine the impact of introducing DSR limits on household borrowing as a sole macroprudential policy. We finally introduce a model with both capital requirements and affordability constraints. In each case, we examine the volatilities of household borrowing, house prices, output and inflation as well as welfare.[3] To understand the interaction between different tools, we examine the responses of macroeconomic variables to productivity, housing demand, financial and monetary policy shocks.

We find that capital requirements on banks are the optimal tool when faced with a financial shock, as they nullify the effects of financial frictions by diminishing the impact of the shock on the spread between lending and deposit rates. Instead, limits on mortgage DSR are optimal following a housing demand shock, as they disconnect the housing market from the real economy, reducing the volatility of inflation. Hence, no policy on its own is sufficient to deal with both financial and housing shocks, although they both have hit the economy simultaneously in previous crisis episodes.

Our results disentangle in a simple way all the mechanisms behind the interactions between different macroprudential tools, adding the DSR to the existing literature. These findings give a good roadmap to policy makers on how to make an optimal use of the available macroprudential toolkit, which was not clear yet, given that the range of tools studied so far was not complete. This ample spectrum gives an overall picture for the implementation of macroprudential policies, certainly very useful for practitioners from policymaking institutions.

The remainder of the paper is structured as follows. In the next section we briefly review the literature that is most relevant to our paper before going on to describe the model in section III and section IV and its calibration in section V. Section VI describes our quantitative experiments, examining the effects of the various macroprudential tools and their interactions with each other and with monetary policy. Section VII derives a welfare-based loss function against which we assess our macroprudential policy tools. Section VIII concludes.

II. Literature review

In this section, we review some of the existing literature on macroprudential policy tools that is most relevant to this paper.

A substantial corpus of evidence establishes the existence of quantitatively relevant channels through which macroprudential tools might influence aggregate demand and through which monetary policy might have an effect on bank profitability and risk-taking (e.g. Korinek and Simsek, 2016; Woodford, 2010; Cúrdia and Woodford, 2009; Farhi and Werning, 2016; (Aguilar *et al.*, 2019). In particular some authors (Angelini, Neri, and Panetta, 2014; Rubio and Carrasco-Gallego, 2015; Rubio and Yao, 2020; De Paoli and Paustian, 2017; Carrillo *et al.*, 2017) have explicitly turned to the question of how monetary and macroprudential policies should be coordinated in a world featuring

[3] Our use of the welfare function is not original; rather our contribution is studying the effects of capital requirements and DSR limits on welfare.

both nominal rigidities and financial frictions. These papers evaluate the optimal policy response of monetary policy and macroprudential actions either on LTV limits or on capital requirements when the economy is faced with aggregate shocks, such as to productivity or monetary policy. In most of these papers, the objective of macroprudential policy is to avoid excessive lending, that is, to minimize the variances of total lending or the ratio of loans to output. The extent to which policies are complementary or substitutes for each other, depends on the nature of the shock. For example, shocks to net worth or productivity create no tension between policies targeting output and inflation on the one side and bank lending on the other. However, there are welfare losses when the committees are non-cooperative in the case of cost-push shocks. In this case, monetary and macroprudential policies become strategic complements with both policies tightened more than in the case of coordination.

Our model contributes to this literature in two important ways. First, we introduce DSR limits on household balance sheets to cap mortgage borrowing. This tool acts to reduce the overall indebtedness of the household sector relative to nominal income. It is different from collateral constraints because it imposes constraints relative to borrowers' income rather than to the value of their house. By modelling this tool relative to a regulatory stress rate buffer on existing mortgage rates rather than a standard LTI limit, we introduce additional interactions between macroprudential and monetary policy. Second, we consider the interaction of monetary policy with this richer macroprudential toolkit that includes macroprudential limits for both banks and households. This allows us to examine not only the coordination between macroprudential and monetary policy tools, but also the optimal interaction of policies within the macroprudential toolkit.

To our knowledge, affordability constraints have not been addressed in the literature so far, although some authors have examined tools acting on limiting household debt relative to income. Marcus Mølbak Ingholt (2018) compares LTV limits on mortgage lending with LTI limits in terms of smoothing responses to shocks. Greenwald (2018) examines a mortgage-payments-to-income limit in a DSGE model, and finds that it amplifies the transmission mechanism from policy rates to debt, house prices and economic activity. The paper also finds that a relaxation of payments-to-income standards is essential to match the recent boom. Fazio, Gimber, and Miles (2019) study the impact of debt limits on housing markets and find that they might have distributional effects. However, unlike our model, none of these papers have a banking sector. The introduction of a banking sector in our model opens up a new transmission channel that the above-mentioned papers are not able to capture.

In terms of model setup, there are two papers that use a similar model to ours in the literature on policy coordination. First, Ferrero, Harrison, and Nelson (2018) introduce a DSGE model with housing, heterogeneous households, loan-to-value (LTV) limits on mortgage lending and capital requirements on financial intermediaries, to study how monetary and macroprudential policies should optimally respond to shocks. The authors derive a welfare-based loss function containing four (quadratic) terms. Two of them stem from the standard NK model where the policymaker seeks to stabilize the output gap and inflation. The remaining terms come from the desire of the policymaker to stabilize the distribution of non-durable consumption and housing consumption between borrowers and savers. Monetary policy is constrained by the zero bound. In a similar fashion, Rubio and Yao (2020) also study optimal macroprudential and monetary policy in a low interest-rate environment.

Second, Gelain and Ilbas (2017) study the implications of macroprudential policy in the context of an estimated (Smets and Wouters, 2007) type DSGE model for the USA, featuring a financial intermediation sector, subject to (Gertler and Karadi, 2011) financial frictions. Macroprudential policy aims at stabilizing nominal credit growth and the output gap by setting a lump-sum levy on bank capital. Monetary policy pursues a standard inflation targeting mandate using the short term interest rate. The paper focuses on testing how the variations in the macroprudential objectives affect the coordination between macro and monetary policies. In addition, the paper derives optimal policy rules and optimal weights under the assumption that the two policymakers cannot coordinate. In both papers macroprudential policy is always binding and the interaction between various macroprudential policy tools is not considered.

Finally, Hinterschweiger *et al.* (2021) use a DSGE model with default to assess various macroprudential tools. However, unlike us, the paper neither does consider affordability constraints within the macroprudential toolkit nor does it consider the interaction of macroprudential policy with monetary policy. We also concentrate on the ability of macroprudential policy to reduce the volatility of economic variables, rather than on default. And, by assuming an efficient steady state, we are able to derive a utility-based welfare measure that does not arbitrarily weight steady-state utility against its volatility.

III. Baseline model

Our novelty comes from analysing a set of comprehensive macroprudential tools, whose interaction with each other, and with monetary policy has not been studied before. Since macroprudential measures can affect several objectives simultaneously (e.g. credit, house prices, monetary policy interaction, etc), the existing literature still lacks a clear understanding of how different macroprudential tools are transmitted to the real economy and what is the value added having multiple tools active at the same time. To study the theoretical transmission channels of different macroprudential tools, we start from a standard baseline model, to which will we add a rich set of macroprudential tools. This section describes the key features of our baseline model, with additional details provided in Appendix A.[4]

The household and housing sectors follow Iacoviello (2015). We have two types of households: patient ones, who save via bank deposits, and impatient ones, who borrow from banks against housing collateral. Unlike Iacoviello (2015), we do not impose the collateral limit exogenously, to mimic regulatory intervention, but we calibrate it to hit the average mortgage borrowing to GDP in the UK. That is because, even in the absence of a

[4]Our model is a closed economy one and does not capture cross-country macroprudential spillovers. In a globally interconnected banking system, there can be spillovers from domestic macroprudential policies to foreign banks and vice versa, for example, through the presence of foreign branches in the domestic economy. The lack of reciprocity of some macroprudential instruments may result in 'leakages', which may in turn decrease the effectiveness of macroprudential policies. These effects are out of the scope of this paper. For a DSGE model, which takes into account these effects, see for instance Rubio 2020, who considers a two-country DSGE model with housing and credit constraints. Results in that paper show that, when there are some sort of reciprocity agreements on macroprudential policies across countries, financial stability and welfare gains are larger than in a situation of non-reciprocity.

regulatory LTV limit, as in the UK, lenders' themselves will lend only up to a proportion of housing collateral, according to their internal risk management policies. Evidence from the Bank of England [5] suggests that after the crisis, the vast majority of loans had LTV ratios between 75% and 90%, even in the absence of any regulatory intervention. This suggests that, even without policy, borrowing in the UK is constrained by the value of housing collateral imposed by lenders themselves, which we model in our baseline.

Patient households have a higher discount factor than impatient households. Hence, they value future consumption relative to current consumption by more than the impatient households. Both types of households obtain utility from consumption, housing and leisure. In line with typical new Keynesian models (e.g. Smets and Wouters, 2007), we have a perfectly competitive final-goods sector whose firms combine intermediate goods to produce the final good. Intermediate-goods-producing firms combine the labour of patient and impatient households to produce intermediate goods. They face price adjustment costs and have to borrow from banks to finance their working capital (i.e. wage payment) needs. Finally, we have a banking sector that accepts deposits from the patient households and lends money to impatient households and firms. Following Gertler and Karadi (2011), banks face a costly enforcement problem. Specifically, we assume that banks are able to divert a fraction of their assets to their owners, albeit at the expense of not being able to continue as a bank. To stop this from happening, it must always be more profitable for the banks to continue operating than to divert funds. This incentive constraint acts as a friction in the banking sector that limits leverage and creates a spread between loan and deposit rates. The central bank operates a Taylor Rule and in equilibrium, goods market, housing market and credit markets clear.

IV. Baseline model with macroprudential tools

Relative to the baseline model described above, we add two macroprudential tools[6].

Macroprudential limits on banks – that is, capital requirements

First we consider the effects of adding a regulatory limit on how much credit lenders' themselves can extend, in the form of a maximum leverage ratio constraint on banks. In essence, this captures banks' capital requirements. In particular, we suppose that the macroprudential policymaker sets a maximum leverage ratio Lev. Banks regard Lev as an absolute maximum, exerting effort and incurring costs in order to avoid reaching it. These costs get larger, the closer the bank gets to the maximum leverage limit. In other words, there are sanctions applied to banks that overstep the line. Our penalty function

[5]See page 5 of June 2017 Financial Stability Report.

[6]Our model could be useful to consider an even wider range of tools, than just DSR limits and capital requirements. For instance, LTV limits could be easily added by strengthening the aggregate collateral constraints imposed in the baseline case. In addition, the model could be adapted to include an LTI tool, instead of a DSR limit, by setting the stress buffer in equation (A5) to 0. However, adapting the model to include sectoral capital requirements (SCR), rather than aggregate capital requirements may be more challenging. To apply SCRs policymakers must be able to identify the sector of financial activity where systemic threats are emerging. As such, a more sophisticated baking sector, with more granular assets and risk levels is needed to be able to assess the performance of SCRs on financial risks.

is designed to reflect how regulatory requirements operate in reality. Failure to meet regulatory requirements triggers supervisory attention and remedial plans, which are more complex (and thus more costly) the larger the gap.[7] Specifically, we suppose that banks face the following penalty cost function:

$$\left(\frac{\phi_b}{(Lev-\boldsymbol{\varphi}_t)}-\frac{\phi_b}{(Lev-\boldsymbol{\varphi})}\right)n_t, \tag{1}$$

where φ_t is their leverage in period t and φ is steady-state leverage.

The banking sector net worth will evolve according to:

$$n_t=\zeta\left(R_{L,t-1}L_{t-1}(1+\tau_b)-R_{t-1}D_{t-1}-\left(\frac{\phi_b}{(Lev-\boldsymbol{\varphi}_{t-1})}-\frac{\phi_b}{(Lev-\boldsymbol{\varphi})}\right)n_{t-1}-nA_{n,t}\right)$$
$$+(1-\zeta)\nu. \tag{2}$$

And the Bellman equation for the banking sector will now be given by:

$$\psi_t=\beta_P E_t\left(\frac{P_t}{P_{t+1}}\right)E_t\left(\frac{c_{P,t}}{c_{P,t+1}}\right)E_t\left(1-\boldsymbol{\zeta}+\boldsymbol{\zeta\phi}_{t+1}\right)$$
$$E_t\left((R_{L,t}(1+\boldsymbol{\tau}_b)-R_t)\boldsymbol{\varphi}_t+R_t-\frac{\phi_b}{(Lev-\boldsymbol{\varphi}_t)}+\frac{\phi_b}{(Lev-\boldsymbol{\varphi})}-\frac{n}{n_t}A_{n,t+1}\right). \tag{3}$$

Subject to equation (A8).

The first-order conditions for this problem imply:

$$\varphi_t=\text{Lev}-\sqrt{\frac{\phi_b}{R_{L,t}(1+\tau_b)-R_t}}\ \text{ and }\ \theta\varphi_t<\psi_t, \tag{4}$$

where we have assumed that the maximum leverage ratio (with associated penalty cost function) has been calibrated such that imposing it results in the diversion risk constraint always being slack. We discuss this in more detail in section VI, below.

Macroprudential limits on households – that is, debt-service ratio constraints

Second, we add a regulatory limit on household borrowing, which in essence is a debt-service-ratio (DSR) limit on impatient households' balance sheets. Specifically, we assume that the representative impatient household i faces the following constraint:

$$L_{i,t}\le\rho_L L_{i,t-1}+(1-\rho_L)\frac{\text{DSR }h_{i,t}w_{I,t}}{R_{L,t}-1+\text{stress}}, \tag{5}$$

where DSR is the maximum debt service ratio – that is, the proportion of impatient households' wage income being used to pay the interest on a loan – at which the loan

[7] See Bank of England, Supervisory Statement, 2021.

would still be considered 'affordable' at the stressed interest rate set by the macroprudential policy maker. stress denotes by how much the interest rate is stressed when considering affordability. Intuitively, the constraint checks whether a borrower would still be able to afford the interest payments on their loan if the interest rate they had to pay were to rise by the amount implied by the stress parameter. Given that mortgage loans in our model are all assumed to be for one period only, this affordability constraint is equivalent to a LTI constraint, where the LTI ratio depends on the current interest rate. In addition, following Iacoviello (2015), we assume that impatient households only adjust slowly to their borrowing limits. That is because, mortgage borrowing limits are typically imposed when mortgages are taken out; thus they will not effectively apply to all mortgage lending. Since we have a one period loan in this model, the absence of slow adjustment to the new borrowing limits would in essence imply that borrowing constraints apply on all mortgages in every period. Given this intuition, we can then interpret ρ_L as the proportion of existing mortgages and $1 - \rho_L$ as the proportion of new mortgages, on which credit limits apply.[8]

As the DSR limit is imposed by regulators for macroprudential policy reasons, we assume that this is more binding than banks' internal risk limits that govern mortgage borrowing in the baseline version of the model. Otherwise, if unregulated lending would be prudent and consistent with financial stability, there would be no further need for additional macroprudential tools. As such, the macroprudential DSR limit is the only binding constraint in the version of the model where the policy is imposed. And in turn, this renders the collateral limit in the baseline model slack in all periods where the macroprudential DSR limit is switched on. We discuss the impact of introducing a binding macroprudential credit limit on household borrowing in section VI.

The addition of an affordability constraint changes the following first-order conditions for the impatient households, relative to the baseline model:

$$\frac{1}{c_{I,t}}(1-\mu_t) = \beta_I E_t \frac{R_{L,t} - \rho_L \mu_{t+1}}{(1+\pi_{t+1})c_{I,t+1}}, \tag{6}$$

$$\frac{jA_{j,t}}{H_{I,t}} = \frac{q_t}{c_{I,t}} - \beta_I E_t \frac{q_{t+1}}{c_{I,t+1}}, \tag{7}$$

$$w_{I,t}\left(1 + \frac{\mu_t(1-\rho_L)\text{DSR}}{R_{L,t} - 1 + \text{stress}}\right) = h^{\xi}_{I,t}, \tag{8}$$

where μ is now the Lagrange multiplier on the affordability constraint. The housing demand equation is now simplified as impatient borrowers no longer benefit from having more housing to relax their collateral constraint. Against that, impatient households are now prepared to supply more labour for a given wage, since doing so will relax their affordability constraint[9].

[8]We introduce the borrowing constraint with an inequality. However, given the differences in discount factors among agents, this constraint will always be binding. See Iacoviello (2005) for further discussion.

[9]Typically, in models with borrowing constraints, income effects on the labour-supply decision are important. With the type of preferences used in standard real business cycle models, labour effort is determined together with the

TABLE 1
Parameter values

Parameter	Description	Value
β_P	Discount rate for patient households	0.9925
β_I	Discount rate for impatient households	0.985
j	Weight on housing in utility function	0.1377
ξ	Inverse Frisch elasticity of labour supply	1.83
σ	Proportion of total wage bill going to impatient households	0.33
ε	Elasticity of demand for differentiated intermediate goods	6
χ	Size of price adjustment costs	70
ϕ_π	Coefficient on inflation in Taylor rule	1.5
ϕ_y	Coefficient on output in Taylor rule	0.125
ρ_R	Interest rate smoothing in Taylor rule	0.81
ρ_L	Inertia in credit constraint	0.7
θ	Proportion of assets that can be diverted	0.1
ζ	Bank survival rate	0.975
ν	Capital of newly formed banks as a fraction of bank assets	0.05
ϕ_b	Scale parameter of penalty cost function	0.0526
$\varphi_{\max}$	Maximum leverage ratio	20
LTV	Average collateral constraint in the baseline case	0.4833
DSR	Debt-service ratio	0.1323
stress	Stress rate (annualized)	3pp

V. Calibration

Before displaying our quantitative experiments, we first discuss our calibration and what this means for the implied steady-state relationships in our model. We calibrate the parameters of the model either to match the previous literature or to hit steady-state targets. Our parameter choices for the baseline model are shown in Table 1.

The discount rate for patient households is 0.9925, implying a risk-free rate of 3% per annum. The discount rate for impatient households is set to 0.985, following Ferrero *et al.* (2018). Notice that the discount rate for patient households is higher than the one for the impatient ones. The steady-state version of equation (6), implies the following steady-state value for the Lagrange multiplier on the impatient households' borrowing constraint:

$$\mu = \frac{1 - \beta_I R_L}{1 - \beta_I \rho_L}. \tag{9}$$

Given the calibration of the two discount factors, the impatient households will be constrained in their ability to borrow (i.e. either by lenders' own collateral constraints in the baseline case, or more tightly, by the macroprudential affordability limit in the case where the policy is switched on). However, we set the banking subsidy, τ_b, to ensure a zero spread in steady state.

intertemporal consumption choice. When consumption is reduced, individuals tend to work more to compensate and smooth consumption. Other types of preferences, namely GHH preferences have the property of shutting down the income effect on the labour-supply decision. In these preferences, labour and consumption are non-separable. This makes labour effort to be determined independently from the intertemporal consumption-savings choice (See Rubio, 2011 for more details).

Based on the estimation results reported in Smets and Wouters (2007), we set the inverse Frisch elasticity to 1.83. Following Iacoviello (2015), we set the inertia in the borrowing constraint equal to 0.7 and the share of the total wage bill going to impatient households equal to 0.33. We set the elasticity of substitution, ϵ, equal to 6. Absent the production subsidy, this would imply a mark-up of 1.2 in the intermediate goods sector, in line with the results in Macallan, Millard, and Parker (2008). We then set the size of the price adjustment costs, χ, such that the coefficient on (log) real marginal cost in the new Keynesian Phillips curve, $\frac{\epsilon-1}{\chi}$, was equal to 0.0852. This is the value that would be obtained in a Calvo (1983) model of price-setting with prices assumed to be adjusted once a year, on average. We set the survival rate for banks equal to 0.975, implying an average expected life for a retail bank of 10 years, the proportion of assets that can be diverted to 10% and the amount of capital that new banks start off with equal to 1/20 of the steady-state assets of the banking sector. Finally, we used standard values for the Taylor rule.

We set the maximum leverage ratio to 20 (i.e. minimum capital requirement of 5%). Then, by setting the scale parameter on the penalty cost function to 0.0526, we ensure a steady-state leverage ratio of 10, roughly in line with the average leverage in the UK banking sector. Note that this is lower than the steady-state leverage ratio in the baseline model, which equals 11.5. This implies that capital requirements bind in the steady state.

We turn to the data to choose a target for the steady-state housing wealth to output ratio. Figure 1a shows that this ratio has risen over time from about 6 ½ in the 1980s and 1990s to around 12 in 2019. Hence, we set the weight of housing in the utility function, j equal to 0.1377, which ensures a steady-state value for the housing wealth to output ratio of 12 in the model.

Figure 1b shows that in the UK, the ratio of mortgage borrowing to GDP is currently around 2.9. Given that, we set of lenders' own collateral constraint to 0.4833, ensuring that the steady-state ratio of mortgage borrowing to GDP in our model is also equal to 2.9.[10] For the macroprudential affordability constraint, we set the stress buffer to 0.0075.

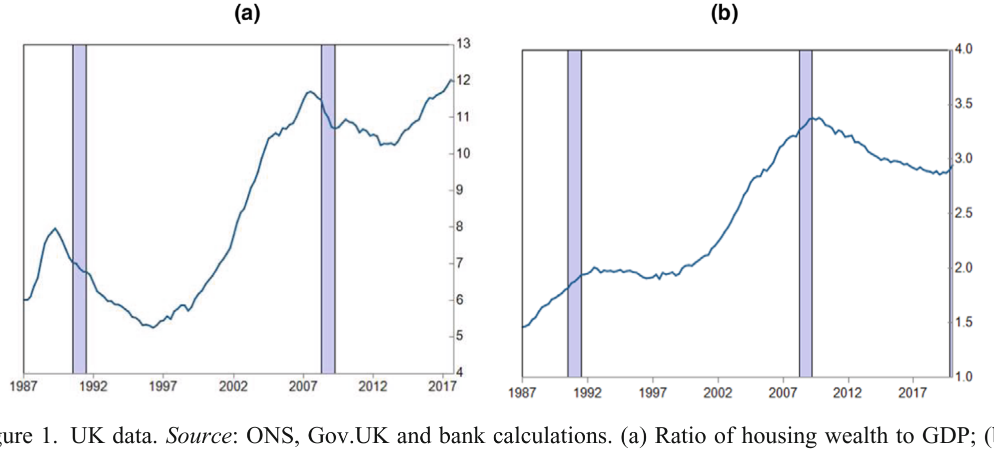

Figure 1. UK data. *Source*: ONS, Gov.UK and bank calculations. (a) Ratio of housing wealth to GDP; (b) Ratio of mortgage borrowing to GDP [Colour figure can be viewed at wileyonlinelibrary.com]

[10]For further data on UK capital requirements and housing indicators, see Appendix E.

This implies a 3 percentage point buffer per annum on top of the current interest rate when assessing principal and interest repayments for mortgage borrowing relative to labour income. Given that we set the subsidy to firms so as to ensure that real marginal cost is unity in steady state, and the subsidy to banks to ensure that the interest rate spread is zero in steady state, the steady-state versions of equations (A1) and (A17) imply:

$$\frac{L_M}{y} = \frac{\sigma \mathrm{DSR}}{\frac{1}{\beta_P} - 1 + \mathrm{stress}}. \tag{10}$$

Given our other parameters, we also set the DSR limit to ensure that the steady-state ratio of mortgage borrowing to GDP, $\frac{L_M}{y}$, is equal to 2.9, as in the baseline model. Hence, we calibrate the macroprudential tool to yield in equilibrium, the same mortgage borrowing to GDP ratio as the baseline model without macroprudential tools in place. This is done to reflect how macroprudential credit limits on borrowers are intended to work in reality. For instance, the macroprudential policymaker in the UK argues that credit limits on borrowers are not expected to restrain housing market activity in equilibrium, but only when lenders' underwriting standards ease during the cycle.[11] As a result, macroprudential tools act as an insurance mechanism, becoming more binding only following shocks that loosen lenders' credit conditions outside of the equilibrium path. Hence, even with the same steady-state calibration, we expect that following a shock, lending in the baseline model will be less constrained than lending in the model with the macroprudential DSR limits switched on. That is because the macroprudential tool, unlike lenders' own internal risk management (in the baseline case), depends directly on economic fundamentals. As we show in equation 34, the DSR tool links lending to income and interest rates.

This calibration implies a value for the DSR of 0.1323. This value for the DSR is low relative to the value of 0.4 that is applied in the UK in practice. However, this is a result of having only one-period loans in our model. For a long-term mortgage, the DSRs fall over the lifetime of the mortgage as income rises.

VI. Macroprudential tools: effects and interactions

The novelty of our work comes from analysing a more comprehensive set of macroprudential tools, which includes limits on both lenders and borrowers, in the form of macroprudential DSR constraints and capital requirements. This is important given the increased use in recent years in many countries, of macroprudential policies targeted to the household sector. The addition of a housing tool to capital requirements, reinforces however questions around the conduct of macroprudential policy and its interplay with monetary policy.

This section examines these interactions in more detail to assess how different macroprudential tools can be complements or substitutes to each other and how they can smooth cycles and deal with economic shocks. We start by examining the effects of capital requirements, showing that they can nullify the effects of the financial frictions in the model and reduce the effects of shocks on the interest rate spread. We then examine

[11] See Financial Stability Report, June 2017.

the interactions of all our macroprudential and monetary policy tools by simulating four versions of the model with four different configurations of macroprudential policies in place: i) the baseline model with no active macroprudential tool in place; ii) a model with only macroprudential capital requirements; iii) a model with capital requirements and affordability constraints; and iv) a model with affordability constraints only. In each case, we use Dynare to calculate the volatilities of the key macroeconomic variables and their impulse responses to aggregate shocks.

The role of capital requirements

Before analysing policy interactions, we first use our model to examine the implications of capital requirements. The purpose of capital requirements is to ensure the resilience of banks in the face of shocks. Minimum capital requirements are normally set by microprudential regulators. However, macroprudential regulators typically have the ability to raise capital requirements above the regulatory minimum, in response to cyclical movements in either aggregate or sector-specific financial risks. The purpose of this additional capital is to ensure the resilience of the banking sector as a whole if risks crystallize. In practice, macroprudential capital requirements help ensure that frictions within the banking sector do not amplify the effects of shocks passing through the banking sector onto the real economy.

In the context of our model, the key friction is the ability of bankers to divert a proportion of their assets to consumption. This friction gives rise to a spread between lending and deposit rates. By raising the level of capital in the banking system above the level in the baseline economy, capital requirements can prevent this 'diversion constraint' from binding, thus eliminating the key friction in the banking sector. Similarly, the key financial shock affecting the banking sector in our model is an increase in non-performing loans. Capital requirements can make the system more resilient to such a shock by requiring banks to set aside more buffers. In what follows, we compare the baseline model with the model with capital requirements in order to examine the extent to which capital requirements are able to neutralize the financial friction and increase the resilience of the banking sector to negative shocks to non-performing loans.

Figure 2 plots the ratio of banks' stock-market value to assets, that is, $\frac{\psi}{\varphi}$, following shocks to productivity, housing demand and non-performing loans when capital requirements are switched on in the model. The shocks are deliberately large: amounting to three SDs. The reason for doing this was to illustrate the ability of capital requirements to neutralize the effects of financial frictions, even in extremely rare circumstances.[12] The calibration implies that the shocks give a 3.24% fall in productivity, a 7.36% rise in house

[12]Note: The model is solved using a log-linear approximation around the non-stochastic steady state. As with any such approximation, the approximation is fine provided the model does not get too far away from this steady state. The approximation of the penalty function related to capital requirements is fine while the leverage ratio is close to its steady-state value of 10. For a leverage ratio between 4 and 14, the approximation error is less than 0.3% of bank net worth. For large shocks, such as those applied in this exercise, the log-linear approximation may not be so good. That is because, the 'true' penalty becomes much larger than the approximate penalty as the leverage ratio approaches the imposed limit. Nonetheless, this suggests that under a large shock, banks will have even more incentive to ensure sufficient capital than implied by the approximation. And this likely means that the effects of the shocks on credit spreads will be even smaller than implied by the approximate (log-linear) model.

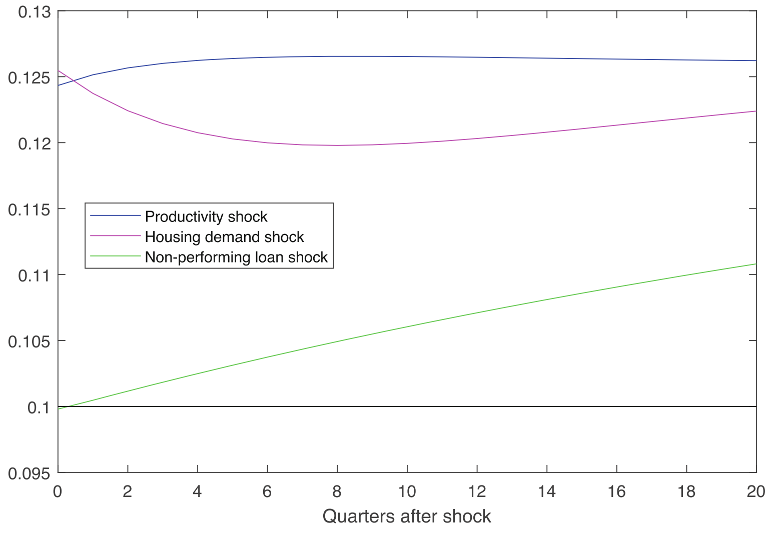

Figure 2. Ratio of banks stock market value to their divertable assets (i.e. $\frac{\psi}{\varphi}$) [Colour figure can be viewed at wileyonlinelibrary.com]

prices and a 1,844 basis point rise in the lending spread in the baseline model. For the persistence of the shocks we set the autocorrelation coefficients to 0.95, 0.98 and 0.02 for the productivity, housing demand and non-performing loans shocks, respectively. [13]

Without capital requirements, the ratio of banks' stock market value to the value of their divertable assets will be constant and equal to 0.1 (the black line in Figure 2); this is the 'banking sector friction'. With capital requirements in place, the stock-market value of the banking sector is higher than the value of divertable assets, and hence the friction depicted in the black line, does not bind. Figure 2 shows that this generally holds. To push the $\frac{\psi}{\varphi}$ ratio below the black line, the model needs a very extreme shock to non-performing loans (i.e. resulting in at least a 1,844 basis point rise in the lending spread, as depicted by the green line). As a result, Figure 2 shows that the introduction of capital requirements has effectively neutralized the effect of the banking sector friction.

Figure 3 plots the behaviour of the spread to each of our three extreme shocks when capital requirements are switched off (in the top plot) vs. when they are included in the model (in the bottom plot). This spread is the nearest equivalent in our model to the 'excess bond premium', which Gilchrist and Zakrajšek (2012) found to be a good leading indicator for the risk of a recession in the near term, which we can think of as 'GDP-at-Risk'. Adrian, Boyarchenko, and Giannone (2019) and Aikman *et al.* (2019) suggest that GDP-at-risk can serve as a useful measure of financial instability.) In our model, movements in the spread can be thought of as proxies for the resilience of the banking sector. That is, if the shocks translate into large movements in the spread, then the financial sector is not shielding the real economy from the negative effects of the shocks. By passing through the impact of negative financial volatility to the real

[13]These values, and those for the SDs of the shocks are estimated from UK data. We discuss the estimation of the shock processes in section VII.

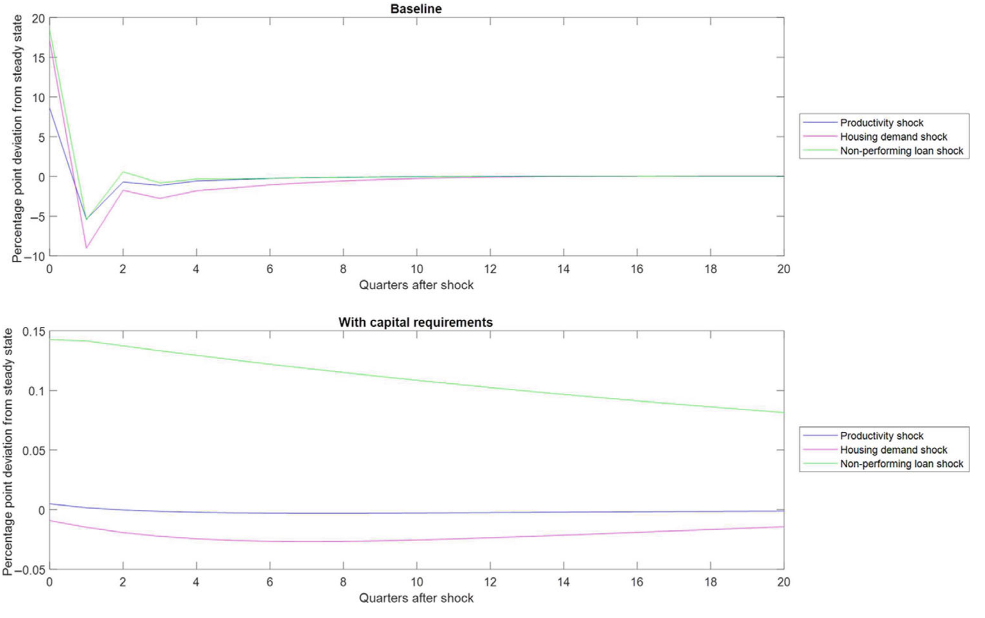

Figure 3. Behaviour of the spread [Colour figure can be viewed at wileyonlinelibrary.com]

economy via the change in lending spreads, the financial sector can be thought of as being less resilient.

Figure 3 shows that the introduction of capital requirements within the model can greatly dampen the effects of all of our shocks on the lending spread. The lending spread barely moves (by less than two basis points annualized) in response to either a severe productivity or housing demand shock. A 3-SD shock to non-performing loans raises the lending spread by 1,844 basis points in the baseline model, but only by 14 basis points once capital requirements are imposed in the bottom plot. This implies that capital requirements are able to insulate the real economy from the effects of a financial shock, since the lending spread is the channel through which such a shock leads to real economic effects.

In this subsection, we have shown that capital requirements act to increase the resilience of the banking sector by neutralizing the effects of the diversion friction and by substantially reducing the response of the lending spread to shocks. In particular, the introduction of capital requirements enables banks to absorb shocks to their balance sheets without the effects being passed through to the real economy via higher lending spreads.

The interaction of macroprudential and monetary policy tools following aggregate shocks

Next, we examine the interaction of our macroprudential tools with each other and with monetary policy. To achieve this, we gradually switch on different policies in our model, and examine their impact on output, lending, inflation, house prices, labour supply variables, financial variables and the interest rate following aggregate economic shocks. In this section, we consider 1 SD shocks to productivity, housing demand and non-performing loans.

Housing demand shock

Figure 4 plots the responses of macroeconomic variables to a housing demand shock that leads to an approximately 3% rise in house prices. There are two important results coming out of this experiment. First, when lending to households is constrained by DSR limits (i.e. blue and magenta lines), the economy does not respond to the housing demand shock, except for an increase in house prices. Affordability constraints limit the impact of housing market shocks on household borrowing and the real economy since, when borrowing is not linked to housing wealth, a shock to house prices does

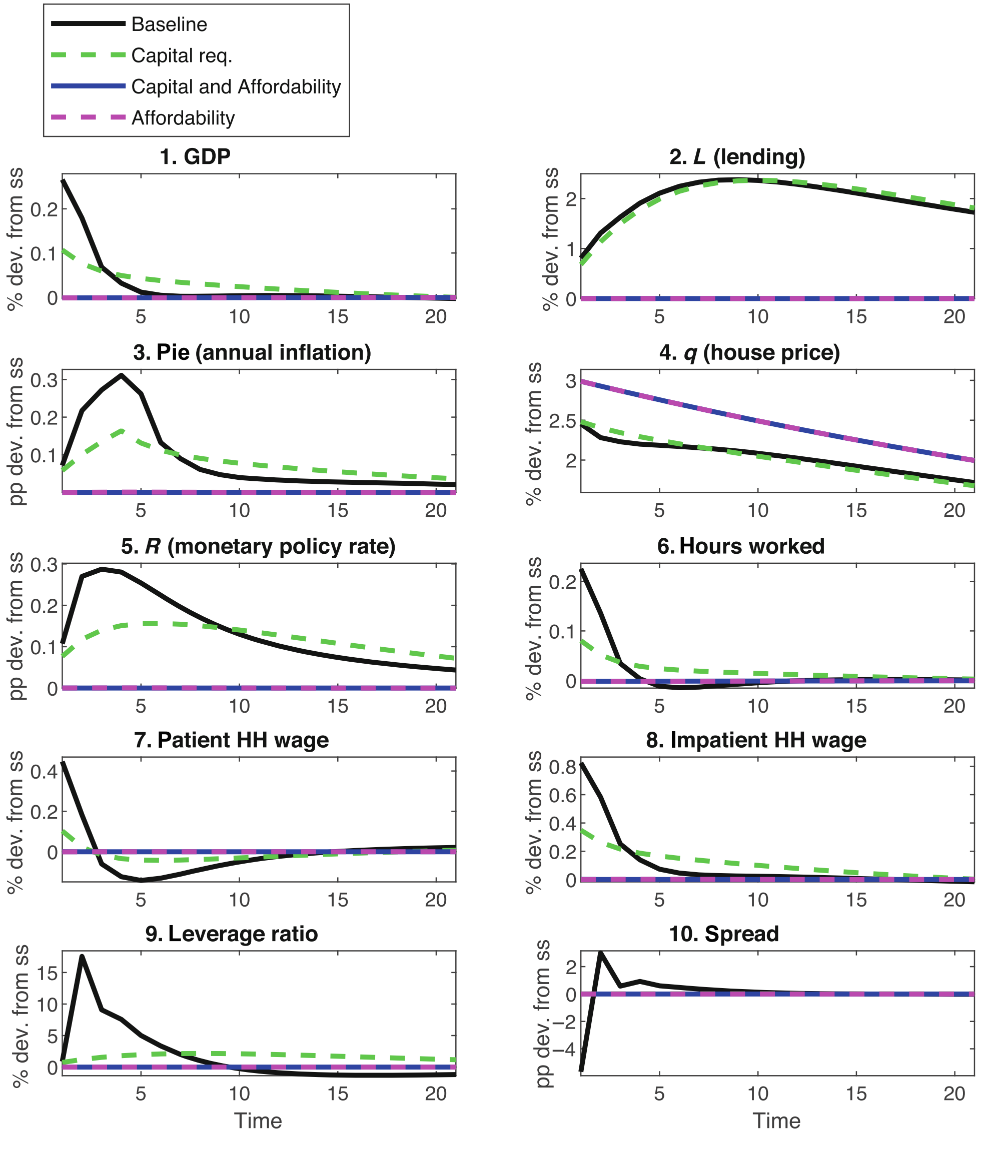

Figure 4. Responses to a housing demand shock (≈3% rise in prices) [Colour figure can be viewed at wileyonlinelibrary.com]

not influence credit constraints or how much households can borrow. In booms this may impose a cost in terms of lost GDP growth, which does not increase as much as in experiments where DSR limits are switched off, as shown in Panel 1. However, this mechanism also prevents GDP growth from falling due to a negative shock in house prices, limiting the effects of a crisis. As a result, the key benefit of DSR limits arises from limiting the volatility in economic variables following the housing demand shock.

Second, monetary policy responds less to the housing demand shock when capital requirements are added to the baseline case, as shown in Plot 5 of Figure 4. In the baseline case, bankers' lending standards are eased when house price rise, as each unit of collateral becomes more valuable and fuels a rise in lending. But introducing capital requirements requires lenders to increase debt prudently, effectively dampening how much lending can react following of the house price shock. This decreases the effect of the shock on GDP and inflation. Hence, macroprudential capital requirements contribute to price stability in the face of a housing demand shock, helping monetary policy achieve its primary objective.

Technology shock

We also investigate the responses of variables to a positive technology shock, shown in Figure 5. In all models, the productivity shock leads to positive changes in output. This incentivizes borrowers to purchase more housing, driving up house prices and lending. However, when affordability constraints are switched on (i.e. the blue and magenta lines), the impact on lending and subsequently on output is higher.

While a DSR limit removes the link between house price movements and borrowing, it introduces other links to household incomes and to the monetary policy rate. A decrease in the official rate following the technology shock, as shown in plot 5 of Figure 5, directly loosens the borrowing constraint on households. In addition, hours worked decrease by less in models with DSR limits, and impatient household wages are higher. These also alter the household's borrowing constraints, affecting their credit access and as a consequence, house prices and economic activity. As a result, inflation decreases less when DSR limits are switched on, requiring a less aggressive response from the monetary policy maker. This suggests that, when faced with a technology shock, housing policies implemented via DSR ratios may support the objectives of the monetary policymaker.

Financial shock

We also examine the responses of variables to the financial (non-performing loans) shock, which lowers the net worth of the banks. Figure 6 shows that in simulations without capital requirements in place, the financial shock increases bank spread substantially upon impact, which lowers bank lending. Absent a monetary policy response, a rise in spreads would imply higher costs for firms and, hence, lower output and higher inflation. However, the resulting monetary policy response makes the impact on most real economy variables modest in all but the simulation with just the affordability constraints switched on (i.e. the magenta line).

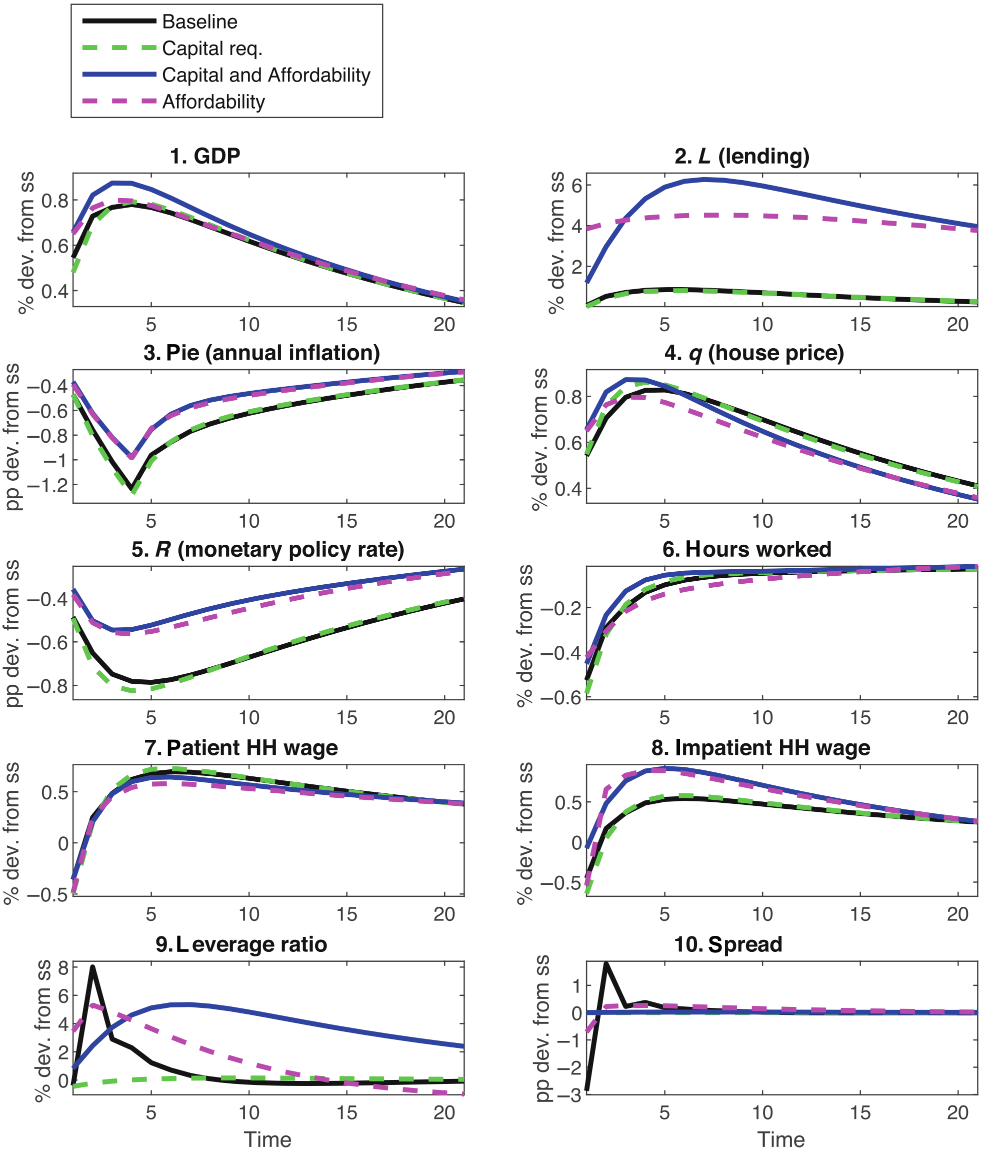

Figure 5. Technology shock [Colour figure can be viewed at wileyonlinelibrary.com]

When DSR limits are the only macroprudential policy used, GDP, house prices and employment decrease significantly relative to the other simulations. These effects are mostly driven by the interaction between labour income and the DSR constraints on households. The initial rise in spreads makes lending more expensive, increasing both the real cost of production for firms and the cost of borrowing for households. Employment decreases, and workers' incomes are reduced. This tightens budget constraints and results in lower output and lower household lending. In turn, this leads to a drop in inflation and the subsequent decrease in the policy rate. The eventual lower base rate acts to loosen DSR constraints, supporting a recovery in household borrowing and in their demand

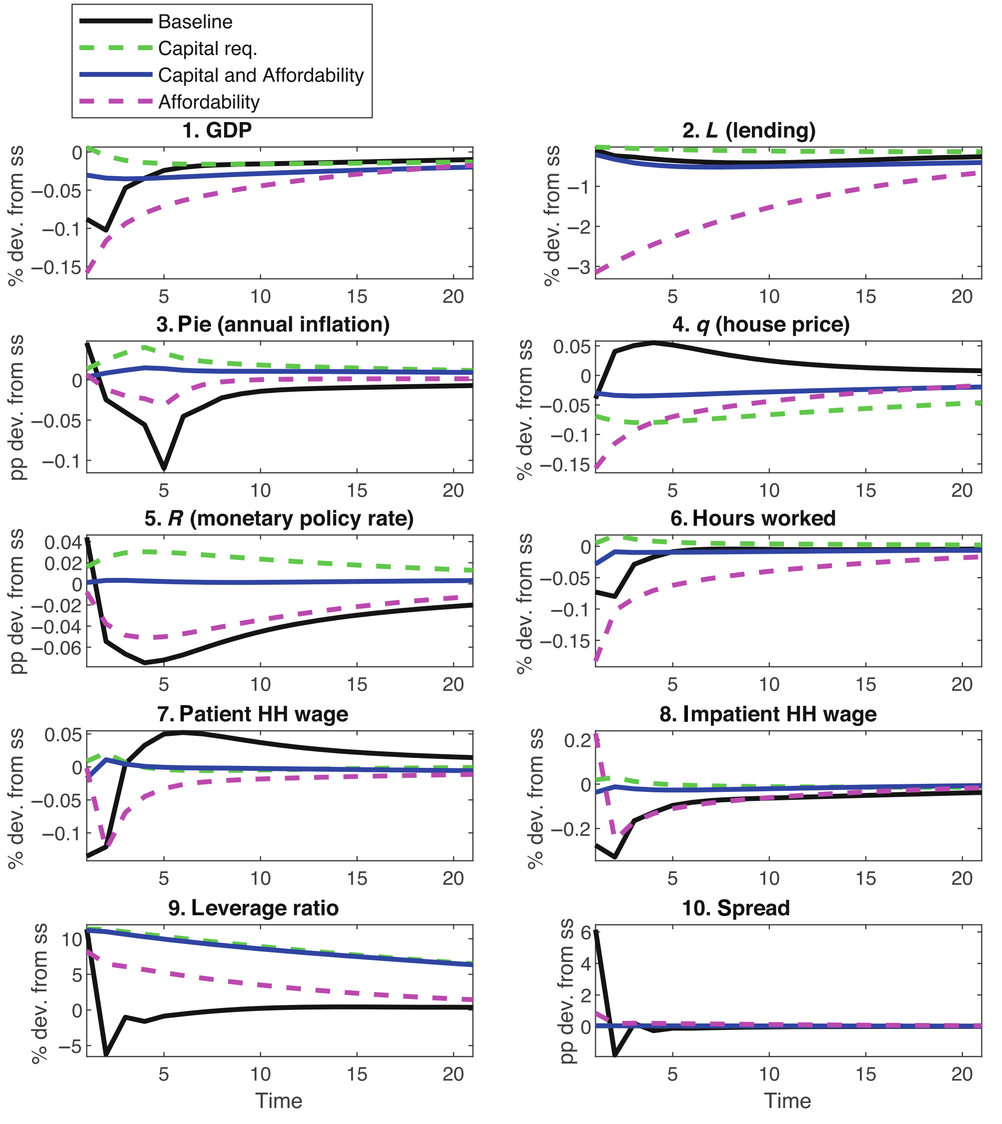

Figure 6. Financial shock [Colour figure can be viewed at wileyonlinelibrary.com]

for housing. These effects are substantially more muted when capital requirements are added to DSR limits. That is because leverage limits on commercial banks make financial variables less sensitive to financial shocks, as previously discussed in section VI.

These results suggest that macroprudential policy implemented only through affordability constraints may aggravate the effect of financial shocks on the real economy, due to the feedback loops between real economic variables and DSR limits. In this case, macroprudential DSR limits and monetary policy have conflicting objectives and become strategic substitutes (i.e. the tighter are the macroprudential affordability constraints, the more monetary policy has to be loosened to compensate for the negative impact on economic activity).

Monetary policy
To further examine the interaction between macroprudential and monetary policy tools, we investigate the responses of macro variables to a monetary policy shock, which leads to a 1% rise in annualized rates. Figure 7 shows that the real economy behaves almost identically in all four models, with output, inflation and labour supply variables all falling by roughly similar magnitudes regardless of which macroprudential policy is switched on. However, the impact of the monetary policy shock on the financial sector depends significantly on macroprudential tools. For instance, the monetary policy shock leads to a larger contraction in lending when affordability constraints are switched on and this is further aggravated if capital requirements are also added on top. This effect occurs for two reasons. First, the monetary policy contraction leads to a drop in GDP which results in lower household income. As borrowing is backed by household earnings, a loss of income leads to an immediate tightening of credit constraints and of overall lending. Second, the rise in risk-free rates leads to a subsequent rise in the mortgage lending rate. This further tightens households' credit constraints by increasing the proportion of interest payments that households have to pay back for any given loan size – that is, increases the denominator in equation (A17). These results suggest that capital requirements, DSR limits and monetary policy can have important spill-overs on each other, highlighting the importance of coordination between policymakers. However, although the macroprudential policies and monetary policy can have important spill-overs on each other, the consequence for the real economy of these spill-overs is limited.

The interaction of macroprudential tools with each other

This section highlights the interaction between capital requirements and DSR limits in further detail. While many jurisdictions in advance economies have macroprudential capital requirements in place, very few have complemented these with macroprudential DSR or LTI limits. As such, it is important to examine the value added of introducing credit limits on borrowers, if lenders' already face capital requirements on their lending.

To understand how DSR ratios evolve following economic shocks, we conduct the following experiment. For the versions of the model where DSR limits are switched off – that is, the baseline with and without capital requirements – we calculate the prevailing DSR rates in the economy. For each shock, we then examine the prevailing ratio relative to our calibration in section V – that is, a 0.1323 limit for the DSR ratio.

This exercise allows us to investigate if macroprudential DSR limits and capital requirements are complements – that is, they are both more binding or tighter at the same time – or substitutes to each other – that is, when one is looser the other one is tighter. This is an important exercise for policymaking. For instance, if we find that the two macroprudential tools are complements, then capital requirements will interact with and have positive spill-overs for borrowers' debt-service ratios. In this case the macroprudential policymaker can address risks coming from the housing market using capital requirements. However, if the two tools are substitutes, then they will respond

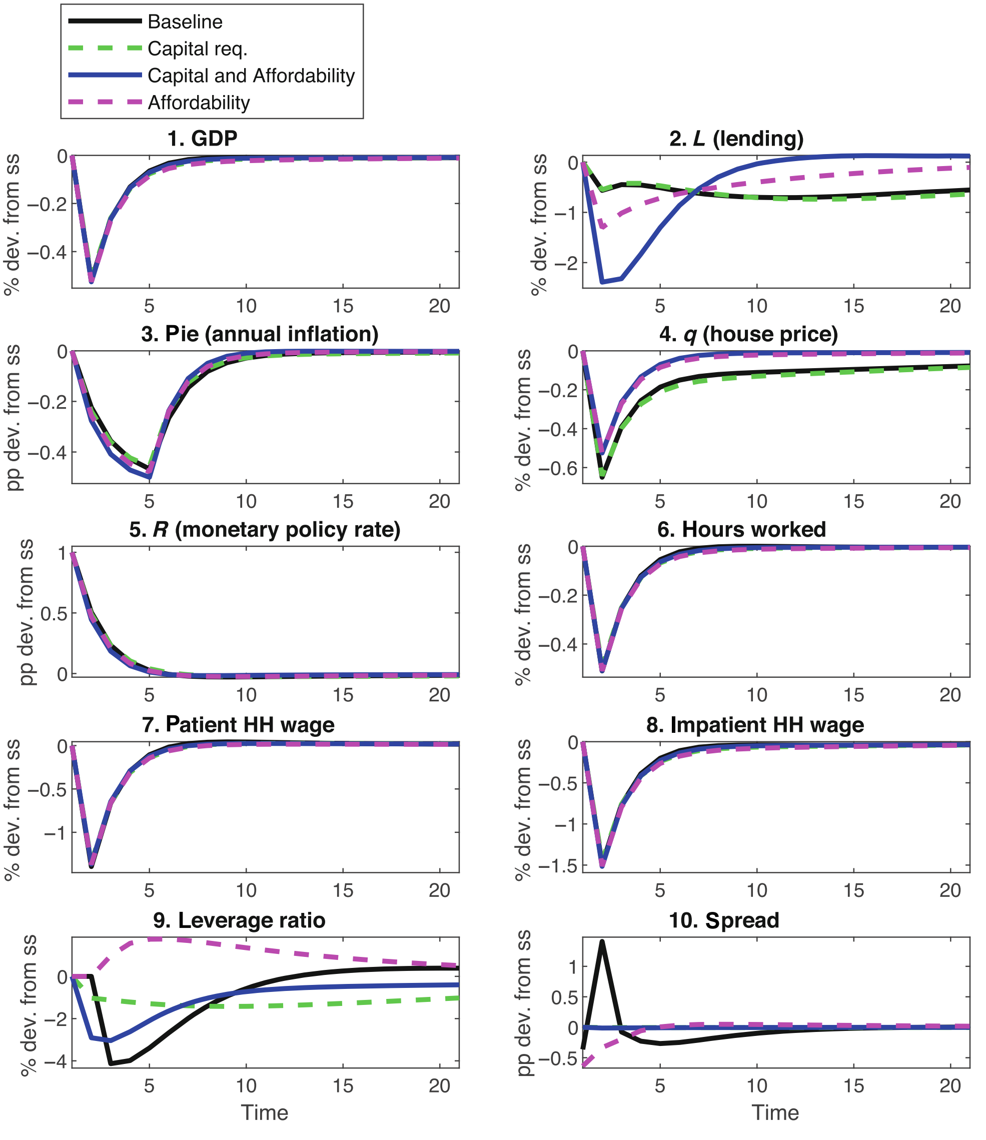

Figure 7. Monetary policy shock (1pp rise in rates) [Colour figure can be viewed at wileyonlinelibrary.com]

to boom-bust cycles differently and hence the policymaker may need to assess the effectiveness of each tool separately.

The implied DSR ratio for the version of the model where it is switched off, is calculated as:

$$\text{DSR} = \frac{L_{M,t}(R_{L,t} - 1 + \text{stress})}{h_{I,t} w_{I,t}} \tag{11}$$

Figure 8 shows the implied responses of the DSR ratio in the baseline case and in the version with capital requirements as the only macroprudential tool, following

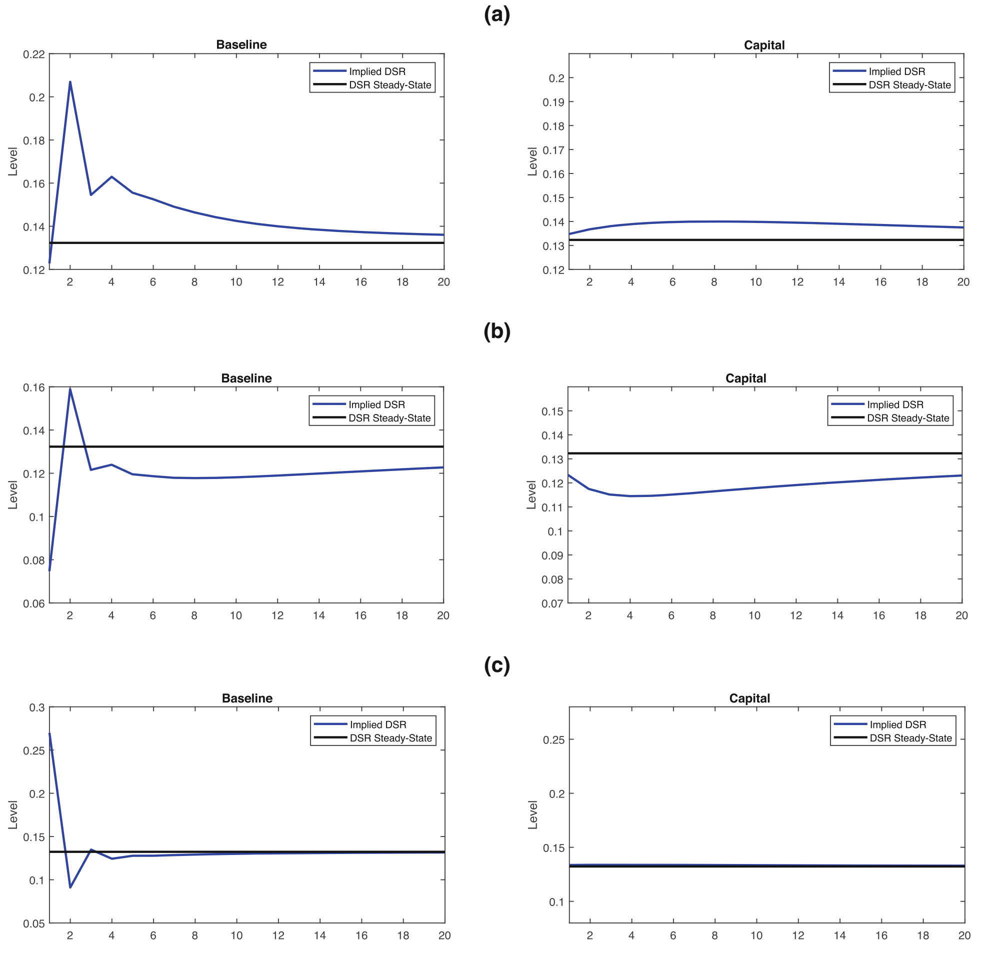

Figure 8. Impact of capital requirements on debt-service ratios following aggregate shocks. (a) Housing demand shock; (b) Technology shock; (c) Financial shock [Colour figure can be viewed at wileyonlinelibrary.com]

a housing demand shock, a technology shock and a financial shock. For each shock, the blue lines show the implied DSR rate and the black lines show the steady-state calibration.

Figure 8a shows the results for the housing demand shock. In the baseline simulation (i.e. left panel), the implied DSR rises sharply to 0.2 following the housing shock. This is a 56% increase in the average DSR ratio from the steady-state value that we impose in the versions of the model where the affordability constraint is switched on. This increase in DSR is due to a large increase in borrowing and in the monetary policy rate, in the baseline simulation. The house price appreciation of nearly 3%, relaxes collateral constraints and allows households to access more debt. The LTV ratio in the economy remains constant over time due to movements in house prices, but the additional debt in the economy raises debt service ratios.

DSRs are however less responsive to the housing demand shock when the model is augmented by capital requirements in the right panel. This occurs because lending and interest rates respond less to the shock when capital requirements are imposed on banks, as shown in Figure 4. This limits the fluctuations in DSRs, but not perfectly. DSRs are still above the equilibrium level by nearly 6% at the peak. And the larger the housing shock, which in our case implies only a 3% rise in house prices, the more average DSRs will deviate from sustainable levels even when capital requirements are in place. This can be problematic particularly if average rises in DSRs are unevenly distributed across households. For instance, if average DSRs rise because more vulnerable households access more debt, then this can increase economic risks. Evidence from the 2007 crisis has shown that borrowers with higher mortgage DSR are substantially more likely to default in stress (FSR, 2017) and were also more likely to pull away from consumption to meet mortgage payments, thus amplifying the recession (Bunn and Rostom, 2015). As a result, capital requirements may not be sufficient to maintain economic stability following a large housing demand shock.

Figure 8b shows the results for the technology shock. In the baseline model, the DSR rises and the drops. The increase in early periods occurs because, the technology shock increases household borrowing and decreases hours worked by the impatient household. Higher debt is thus serviced by lower labour income leading to a rise in DSRs. In subsequent periods, employment recovers more quickly than the monetary policy rate, which loosens DSRs. In contrast, adding capital requirements leads to DSR levels that are consistently below the steady-state level. As shown in Figure 5, adding capital requirements leads to a larger loosening in monetary policy. This effect outweighs the initial increase in aggregate borrowing and weighs down on DSRs. As such, following a productivity shock, the two macroprudential tools are complements. That is, having just capital requirements in place, also keeps household borrowing under control.

Finally, Figure 8c shows the implied DSR ratio for the financial shock. It nearly doubles initially, in the baseline simulation. As shown in Figure 6, this result is caused by the negative implications of the shock on borrowers' wages and labour supply decisions, which tighten DSRs. However, similar to a technology shock, adding capital requirements to the baseline model, stabilizes DSRs.

Putting these results together suggests that DSR and capital requirements are complement tools following all shocks, except the ones to housing demand. DSR tools are specifically designed at addressing housing-related risks, while capital requirements are a blunt tool meant to boost lenders resilience following shocks. And while in theory capital requirements could be set tight enough to ensure the economy is perfectly resilient to housing shocks as well, this may prove to be very costly. Macroprudential capital requirements affect all types of lending, not just mortgage lending, thus affecting sectors of the economy that may not be exposed to housing-related risks in the first place.

Trade-offs between different calibrations and designs of housing tools

In this section we look at different calibrations for the housing tool, to highlight trade-offs that policymakers may face when deciding which tool design to choose or how tight tools should be set. Housing tools benefit the economy by enhancing borrowers' resilience and preventing the build-up of unsustainable levels of debt. However, these tools will also incur some costs in terms of forgone GDP growth, as they also prevent a temporary increase in housing market activity.[14]. The tighter the tools are set, the higher the benefits in terms of enhancing borrowers' resilience, but also the higher their costs. And, LTI and stressed DSR tools also operate through slightly different channels, impacting costs and benefits further.

To illustrate these trade-offs, we run an experiment where we compare the performance of: i) a stressed DSR tool where the stress buffer is set at 3pp as in the experiments above; ii) a stressed DSR where the stress buffer is decreased to 1pp; iii) a LTI tool[15]; iv) an economy where average collateral requirements are 20% tighter compared to the Baseline model presented before and no affordability limits are in place. The final scenario allows us to examine what the effect of LTV limits would be, as they would impose stricted collateral requirements compared to the Baseline model.

The benefits of tighter calibration of housing tools is most visible in a downturn, where the effects of a negative aggregate shock are smaller, the tighter are the housing tools. That is because tighter tools guard against the build-up of aggregate household debt, making borrowers less likely to have to deleverage significantly in stress. Figure 9 shows the baseline model, against different calibrations for DSR tools, an LTI limit and tighter collateral requirements. A negative shock to non-performing loans lowers the net worth of the banks and leads to fall in lending, real-economic activity and incomes. However, the shock has the lowest impact in the model with tighter collateral constraints (dotted black line), as lending and house prices respond the least of all models, which reduces the pressure on output. Among income-based housing tools, the model with the tighter stressed DSR (pink line) has the least severe impact on output. Unlike LTV limits, DSR and LTI type tools tighten when household incomes are squeezed, reducing the ability of households to borrow in stress. While looser monetary policy acts to directly offset these effects when DSR tools are in place (via lowering interest repayments), it does not directly impact LTI limits. As a result, the model with the LTI limits in place, is the most sensitive to the negative financial shock.

Nonetheless, income based-measures are better than collateral requirements to ensuring household resilience against housing demand shocks, due to their ability to remain countercyclical in booms. Figure 10 compares the same models as in Figure 9, following an approximately 3% rise in house prices. The house price appreciation directly loosens collateral constraints, fuelling leverage and prompting a rise in the monetary policy stance. Similar to Figure 4, housing demand shocks do not significantly affect households' ability to borrow when income-based measures are in place. With DSR or LTI limits in place, borrowing is linked to incomes and interest rates, as opposed to house prices. Figure 11

[14]See Bank of England, Financial Stability Report December 2019.
[15]In our model with short term mortgage contracts, an LTI tool is in essence equivalent to a stressed DSR tool where the stress buffer is set to 0 in Equation (5).

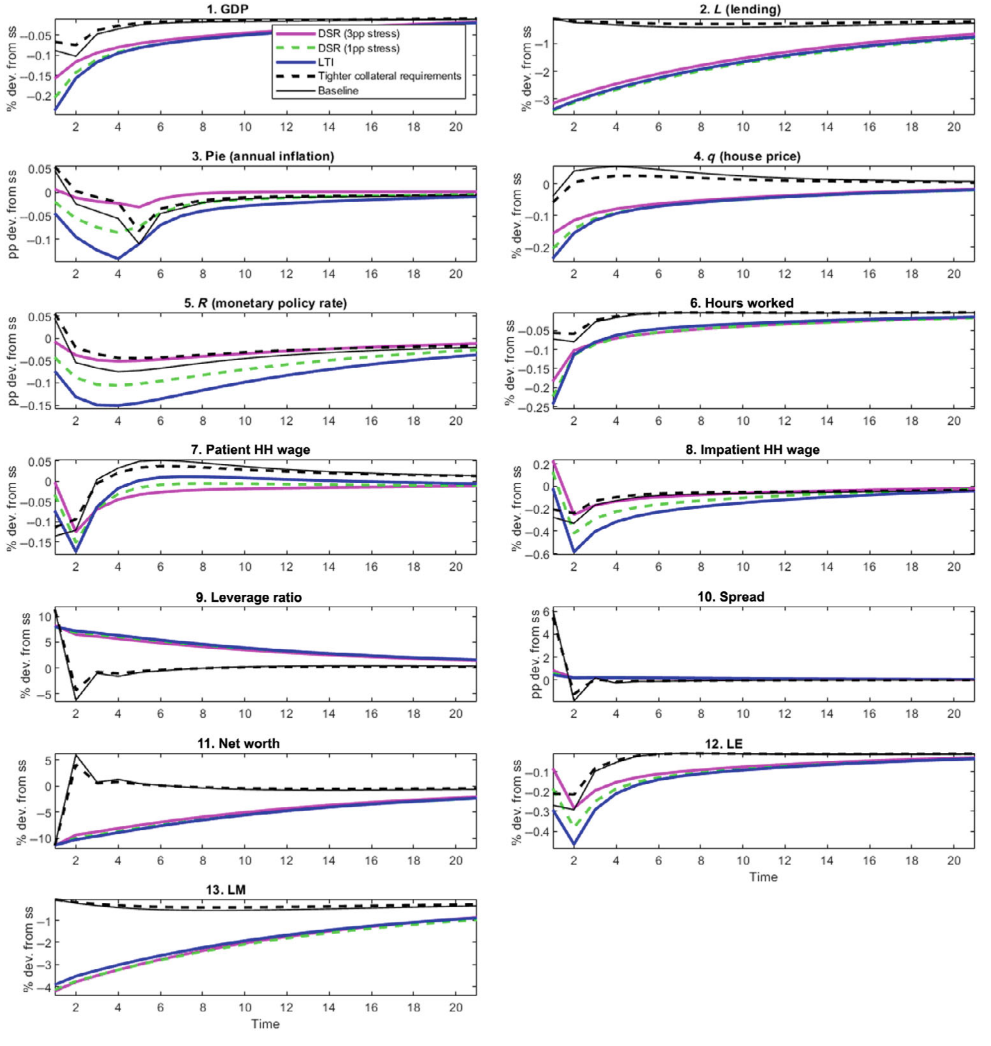

Figure 9. Financial shock: LTV, LTI, and different calibration for DSR [Colour figure can be viewed at wileyonlinelibrary.com]

replicates Figure10 without the models with collateral limits in place, to examine in more detail the performance of income-based housing tools. While the housing demand shock does not directly affect borrowing in models with DSR or LTI limits, it still affects households via general equilibrium effects, albeit these are small. Figure 11 shows that imposing an LTI tool allows households to borrow more, out of the all three income-based measures plotted in Figure 11. An LTI tool does not stress test individual borrowers' ability to repay their mortgages at the prevailing income and potential future higher rates. As a result, it imposes less burden on individual households, and the least cost on GDP. In addition, as borrowing only depends on total incomes when LTI tools are in place, households are more incentivized to increase hours worked to be able to afford housing when house prices rise, further supporting output performance. Nonetheless, Figure 11

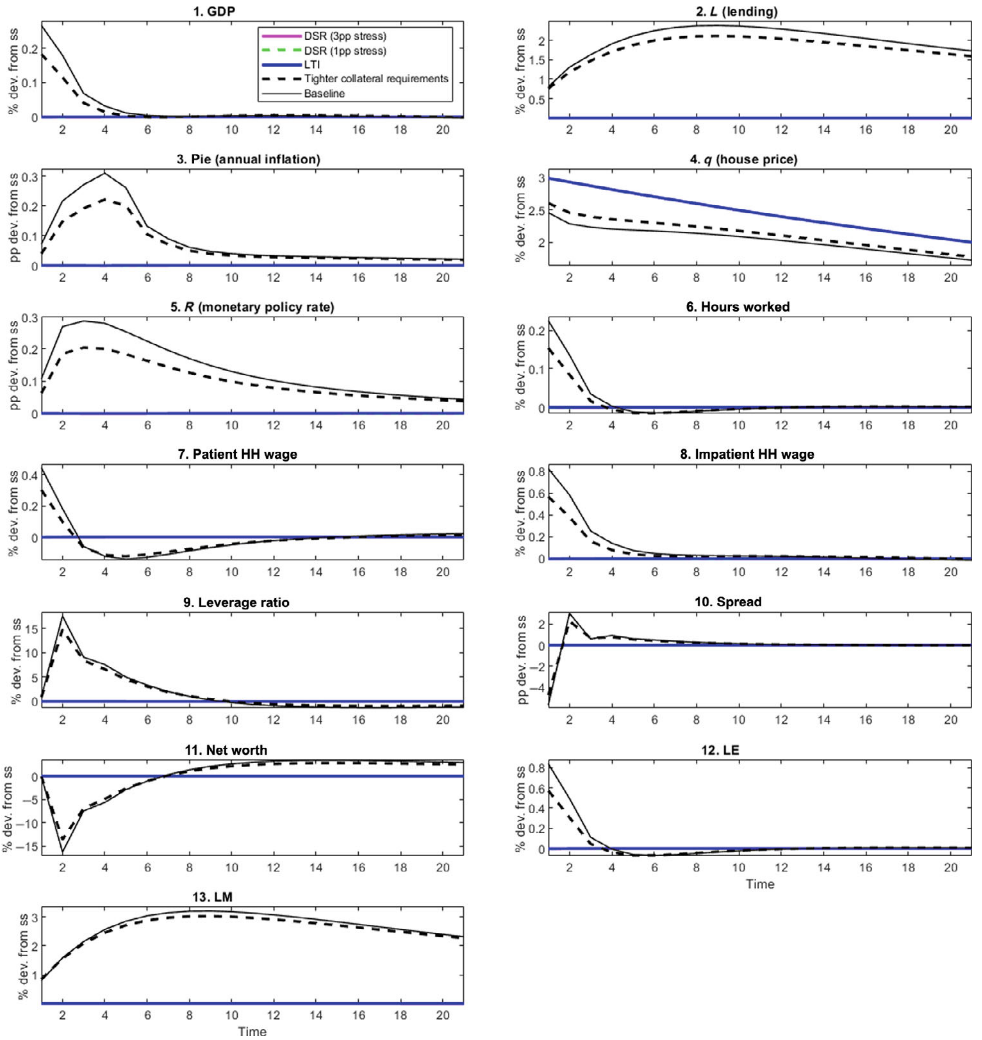

Figure 10. Housing demand shock: LTV, LTI, and different calibration for DSR [Colour figure can be viewed at wileyonlinelibrary.com]

shows that if taming credit is an objective when house prices rise, DSR-type limits are more efficient in constraining leverage growth.

VII. The impact of macroprudential tools on the volatility of key macroeconomic variables and welfare

In this section, we examine the extent to which the adoption of macroprudential policy tools can improve welfare by stabilizing output, inflation, lending and house prices. First, we derive the welfare-based loss function for our model against which we evaluate the performance of the various macroprudential tools. Our discussion of the loss function follows Ferrero *et al.* (2018) and Rubio and Yao (2020). Following these authors, we

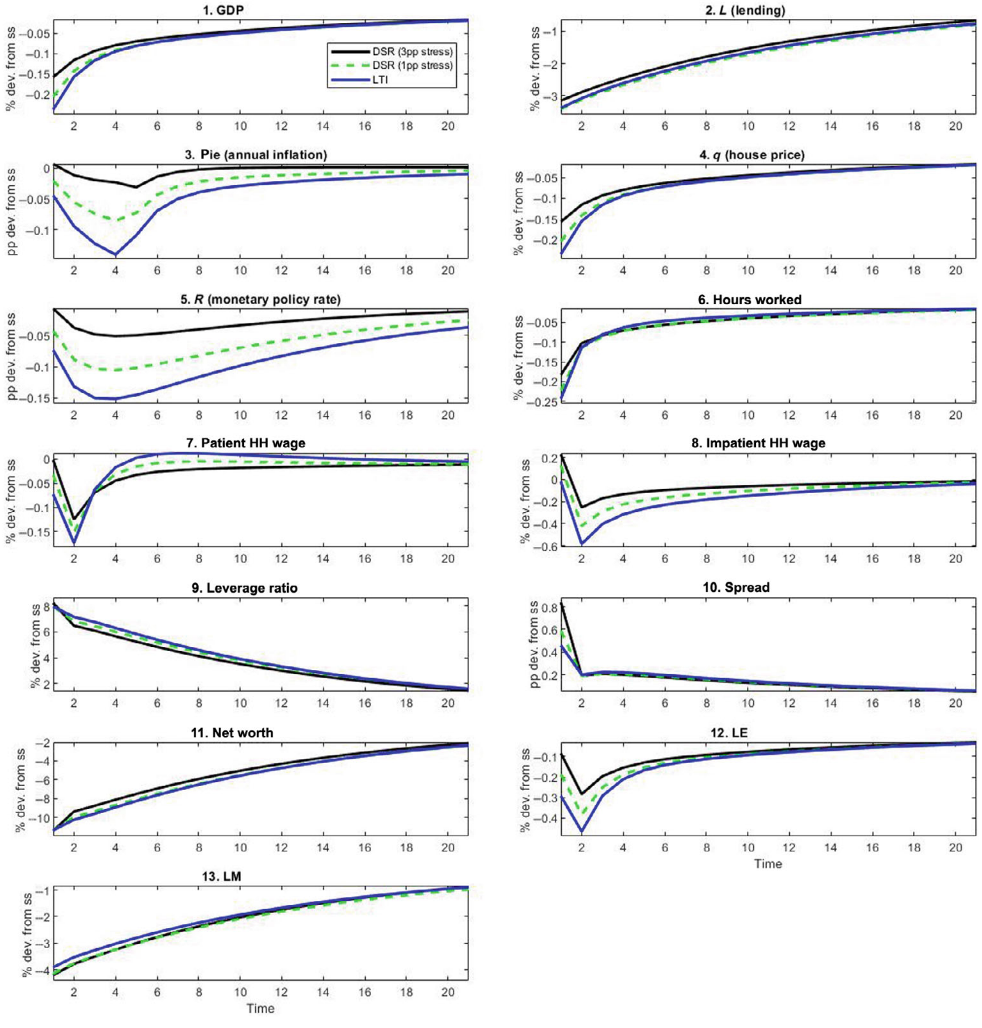

Figure 11. Housing demand shock: LTI vs. different calibration for DSR [Colour figure can be viewed at wileyonlinelibrary.com]

derive the loss function by taking a weighted-average of the per-period utility functions of patient and impatient households where the savers are given an arbitrary weight of ω. We assume that the planner discounts the future at the discount rate of the savers, β_P. A second-order approximation of the resulting objective function around a zero-inflation steady state in which the LTV constraint is assumed to bind gives:

$$L \approx \frac{1}{2}E_0\sum_{t=0}^{\infty} \beta_P^t \left(\hat{y}_t^2 + \lambda_\pi \pi_t^2 + \lambda_c \tilde{c}_t^2 + \lambda_H \tilde{H}_t^2\right), \quad (12)$$

where $\hat{y}$ denotes the log deviation of output from its efficient steady-state level, $\tilde{c}$ represents the consumption gap, defined as the log difference in consumption between patient and impatient households relative to the log difference between their consumption levels in the efficient steady state, and $\tilde{H}$ corresponds to the housing gap, defined as the log difference in housing held by patient and impatient households relative to the log difference between their housing levels in the efficient steady state. The efficient steady state is defined and derived in Appendix B of this paper.

The weights on inflation, the consumption gap and the housing gap are derived in Appendix C of this paper and are given by:

$$\lambda_\pi = \frac{\chi}{1+\xi}, \quad \lambda_c = \frac{1+\xi-4\sigma(1-\sigma)}{4(1+\xi)^2} \ \ \textit{and} \ \ \lambda_H = \frac{j}{4(1+\xi)}.$$

As in Ferrero *et al.* (2018), the loss function adds terms in the consumption and housing gaps to the standard output gap and inflation terms found in standard New Keynesian macroeconomic models. These terms are generated by incomplete financial markets where households are unable to completely share consumption and housing risk between them. Risk-sharing is further limited by the collateral constraint faced by impatient households. The goal of macroprudential policy in this setup is to limit the welfare losses that arise out of incomplete risk-sharing.

Since the performance of different tools in smoothing financial and real economic variables is likely to depend on the relative importance of each of the shocks in driving the economy, it is important that we have a good estimate of the relative volatilities of the three shocks, as well as their persistence. As such, we estimate our shock processes: productivity, A_z, housing demand, A_H, and non-performing loans, A_n. In each case, we assume that the (log of the) shock follows an AR(1) process.[16] We estimate the SDs and first-order autocorrelation coefficients of the shocks using Bayesian techniques and quarterly UK data for GDP growth, real house prices and the spread of effective mortgage interest rates over the Bank of England base rate for the period 1999–2018. Table 2 shows the priors and the full results from the estimation. We then set our parameter values in line with the mean estimated values. As such, the SD of the productivity shock is set to 1.08% and its autocorrelation to 0.95, which is in line with existing literature (e.g. Smets and Wouters, 2007). We set the SD of the housing demand shock to 6.93% and its autocorrelation to 0.98. Finally, we set the SD of the financial shock to 11.69% and its autocorrelation to 0.02.

Table 3 shows the results of stochastically simulating the model. For each of the four versions of the model considered earlier, we show the SDs of total bank lending, L, output, y, inflation, π and real house prices, q following all three shocks we considered earlier. In addition, we show the implied welfare loss based on our loss function. We obtain the implied welfare loss (shown in the last column of Table 3) by computing equation (10) for each of our four versions of the model. Each variable in equation (10), is computed

[16] We acknowledge that estimation of our shocks is subject to limitations. First, estimating the shocks using AR(1) processes may be too simplistic. Second, there are limitations in identifying the three shocks based on the information in the time series of real GDP growth, real house prices and the mortgage spreads. For example, these economic variables may be driven by shocks other than just technology, housing or financial shocks which are unaccounted for by our model and may be driving the results. However, although not perfect, we argue that a Bayesian estimation approach still provides a more accurate magnitude of the shocks compared to a simple calibration. As these shock magnitudes feature in welfare calculations, it is desirable to produce estimates that are closer to reality rather than making simple guesses.

TABLE 2

Estimation of shock processes

	Prior			Estimated Max Posterior		Posterior
Parameter	*Type*	*Mean*	*SE*	*Mode*	*SE*	*Mean*
σ productivity shock	Inv gamma	0.01	∞	0.0105	0.0010	0.0108
σ housing demand shock	Inv gamma	0.035	∞	0.0503	0.0235	0.0693
σ financial shock	Inv gamma	0	∞	0.1174	0.0106	0.1169
ρ productivity shock	Beta	0.5	0.2	0.9551	0.0229	0.9472
ρ housing demand shock	Beta	0.5	0.2	0.9857	0.0114	0.9766
ρ financial shock	Beta	0.5	0.2	0.0089	0.0101	0.0155

TABLE 3

Volatility of macro variables when a housing demand, productivity and financial shock hit all at once

	$\sigma_{House\ Prices}$ (%)	$\sigma_{Lending}$ (%)	σ_{π} (%)	σ_{y} (%)	Welfare loss
Baseline	13.08	11.92	0.91	2.92	0.0032
Cap. Req	13.04	11.85	0.9	2.90	0.0031
DSR	15.31	26.32	0.71	2.98	0.0028
Cap. Req and DSR	15.34	27.66	0.7	3.13	0.0029

using its SD after the simulation is run. In essence, the welfare loss column shows the loss incurred by the households when output, inflation, consumption and housing levels differ from their efficient steady-state levels. The deviation of these variables from their efficient levels will be different in each of our four models, as they differ in terms of the macroprudential tools imposed. Hence, the lower the welfare loss, the smaller the total weighted deviations of economic variables from their potential. Relative to the baseline model, imposing capital requirements leads to reductions in the volatilities of macro variables, when all three shocks hit at once. However, these are marginal and hence have little significant effect on the welfare loss. Switching on affordability constraints leads to an increase in the volatility of real house prices, lending, and output, mainly due to the more intense response of macro variables to the technology shock when DSR limits are imposed, as shown in Figure 5. However, switching on affordability constraints leads to a large decrease in the volatility of inflation. And this results in an improvement in welfare when DSR limits are used as macroprudential tools, compared to the other versions of the model. Adding capital requirements to the model with affordability constraints leads to a slight worsening of welfare as the volatilities of output, lending and house prices are increased.

To investigate these results further, we decompose the variance in lending, real house prices, output and inflation into the proportions driven by each of our shocks. The results are shown in Table 4. The introduction of affordability constraints wipes out any effect of the housing demand shock on all variables other than house prices. This is because affordability constraints ensure that borrowing is no longer linked to house prices. The introduction of capital requirements reduces the contribution of the financial shock to lending, output and inflation volatility. That is, capital requirements can help protect the real economy from financial shocks. These results suggest that capital requirements are a

TABLE 4
Variance decomposition

	Baseline			Cap. Req			DSR			Cap. Req and DSR		
	ε_{Az}	ε_{Aj}	ε_{An}	ε_{Az}	ε_{Aj}	ε_{An}	ε_{Az}	ε_{Aj}	ε_{An}	ε_{Az}	ε_{Aj}	ε_{An}
Lending	5.51	92.37	2.11	5.2	94.25	0.55	90.33	0	9.67	98.94	0	1.06
Output	98.30	1.39	0.31	99.45	0.54	0.01	99.06	0	0.94	99.97	0.02	0.03
Inflation	95.07	3.98	0.95	98.43	1.46	0.1	99.91	0	0.09	99.94	0	0.06
House prices	6.11	93.88	0.01	6.27	93.66	0.07	3.75	96.21	0.04	4.05	95.94	0.01

TABLE 5
Volatility of macro variables following a housing demand shock only

	$\sigma_{House\ Prices}$ (%)	$\sigma_{Lending}$ (%)	σ_{π} (%)	σ_{y} (%)	Welfare loss
Baseline	12.67	11.46	0.34	0.18	0.0002
Cap. Req	12.62	11.51	0.11	0.21	0.0002
DSR	15.02	0.03	0.00	0.00	0.00
Cap. Req and DSR	15.02	0.04	0.00	0.00	0.00

TABLE 6
Volatility of macro variables following a financial shock only

	$\sigma_{House\ Prices}$ (%)	$\sigma_{Lending}$ (%)	σ_{π} (%)	σ_{y} (%)	Welfare loss
Baseline	0.14	1.73	0.09	0.16	0.0000
Cap. Req	0.35	0.88	0.03	0.03	0.0000
DSR	0.29	8.18	0.02	0.29	0.0002
Cap. Req and DSR	0.15	2.85	0.02	0.06	0.0000

good addition, from a macroprudential standpoint, to DSR tools in the face of financial shocks.

Table 4 provides evidence that capital requirements and DSR tools are most potent in reducing the volatility of macroeconomic variables when they address the specific risks they were designed to mitigate: that is, a financial shock and a housing demand shock, respectively. These effects are hidden in Table 3, when all three shocks are switched at once. Instead, Table 5 and Table 6 show the performance of tools following individual shocks. In both tables, we also show the welfare loss, computed in the same way as discussed in Table 3.

In a housing-driven boom in Table 5, DSR tools are substantially more able to reduce economic volatility, and thus increase welfare, when compared to both the baseline and with the version with capital requirements. Lending, inflation and output are all more stable in response to the shock when DSR limits are in place. As the objective of these tools is to make household debt less reactive to house price volatility, they act to reduce the procyclical feedback loop between lending and house price. While in good times DSR limits may impose a constraint on economic growth by not allowing the economy to move in response to the shock, they also introduce economic benefits by not allowing GDP to decrease in a housing demand bust. Adding capital requirements to DSR limits leads to nearly no changes, suggesting a redundancy between these two macroprudential tools following housing demand shocks.

Similarly, Table 6 shows that following a financial shock, capital requirements are more able to reduce economic volatility compared to the baseline or with DSR limits alone. The model with capital requirements reduces lending volatility in half and brings output and inflation volatility close to 0. Similarly, adding capital requirements to a model with DSR limits in place, reduces the SD of lending and output by more than three times. This leads to a reduction in welfare loss, when the economy is hit by a financial shock.

As a result, the performance of macroprudential DSR tools and capital requirements is shock-dependent. They affect economic variables most, when faced with the aggregate shocks they were designed to mitigate in the first place.

VIII. Conclusion

In this paper, we examine two macroprudential policies: capital requirements on banks and affordability constraints on mortgage borrowing. We consider the interaction of macroprudential policies with each other as well as with monetary policy. Additionally, we assess the effects of each policy on: macroeconomic stability, as measured by the SDs of output and inflation; on financial stability, as measured by the SDs of bank lending and house prices; and on welfare.

We first showed that capital requirements reduce the effects of various shocks on the spread between lending and deposit rates, and in turn, on the real-economy. And in particular, capital requirements are especially able to reduce economic volatility following financial shocks. We also found that introducing DSR limits in a housing demand driven boom, can lead to a significant decrease in the response of lending, consumption and inflation, since they disconnect the housing market from the real economy. Capital requirements alone cannot prevent DSR from rising following a shock to house prices, unless they are tightened substantially. This suggests that having both macroprudential tools in place is more efficient when dealing with a range of financial stability risks, as tools can be targeted at the shocks that they are best equipped to address.

In terms of interactions with monetary policy, we found that interest rate movements had stronger effects on lending with DSR limits in place due to the direct impact of base rates on debt-servicing. These results are further amplified if capital requirements are also in place, highlighting the importance of coordination between policymakers.

Future research on the interaction between policies should consider allowing policy tools to vary over the cycle and work out the welfare implications of optimal simple macroprudential policy rules. For instance, it is important to examine the optimal degree of countercyclicality in capital requirements or in the DSR stress buffer. This would better inform macroprudential policymakers on the effectiveness of different tools in smoothing aggregate shocks over the business cycle.

Appendix A: Baseline model

The household and housing sectors follow Iacoviello (2015). We have two types of households: patient ones, who save via bank deposits, and impatient ones, who borrow from banks against housing collateral. Unlike Iacoviello (2015), we do not impose the collateral limit exogenously, to mimic regulatory intervention, but we calibrate it to hit the average mortgage borrowing to GDP in the UK. That is because, even in the absence of a regulatory LTV limit, as in the UK, lenders' themselves will lend only up to a proportion of housing collateral, according to their internal risk management policies. Evidence from the Bank of England[17] suggests that after the crisis, the vast majority of loans had LTV ratios between 75% and 90%, even in the absence of any regulatory intervention. This suggests that, even without policy, borrowing in the UK is constrained by the value of housing collateral imposed by lenders themselves, which we model in our baseline.

Patient households have a higher discount factor than impatient households. Hence, they value future consumption relative to current consumption by more than the impatient households. Both types of households obtain utility from consumption, housing and leisure. In line with typical new Keynesian models (e.g. Smets and Wouters, 2007), we have a perfectly competitive final-goods sector whose firms combine intermediate goods to produce the final good. Intermediate-goods-producing firms combine the labour of patient and impatient households to produce intermediate goods. They face price adjustment costs and have to borrow from banks to finance their working capital (i.e. wage payment) needs. Finally, we have a banking sector that accepts deposits from the patient households and lends money to impatient households and firms. Following Gertler and Karadi, (2011), banks face a costly enforcement problem. Specifically, we assume that banks are able to divert a fraction of their assets to their owners, albeit at the expense of not being able to continue as a bank. To stop this from happening, it must always be more profitable for the banks to continue operating than to divert funds. This incentive constraint acts as a friction in the banking sector that limits leverage and creates a spread between loan and deposit rates.

Patient households

We start by describing the problem faced by patient households. We assume that there is a unit continuum of these households and that they maximize the present discounted value of their current and future streams of utility, subject to a budget constraint. They obtain utility from consumption, housing and leisure – that is, obtain disutility from working. We can write the problem facing patient household i mathematically as:

$$\text{Maximize} \quad E_0 \sum_{t=0}^{\infty} \beta_P^t \left[ln(c_{i,t}) + jA_{H,t} ln(H_{i,t}) - \frac{1}{1+\xi} h_{i,t}^{1+\xi} \right],$$

$$\begin{aligned} \text{Subject to}: \quad D_{i,t} + Q_t H_{i,t} &= Q_t H_{i,t-1} + R_{t-1} D_{i,t-1} + W_{P,t} h_{i,t} \\ &+ \Pi_t - P_t c_{i,t} - P_t T_P - \tau_H Q_t H_{i,t}, \end{aligned}$$

where c_i denotes consumption of household i, H_i indicates housing held by household i, h_i corresponds to hours worked by household i, D_i denotes bank deposits held by household i, Q represents the price of a unit of housing, R corresponds to the interest rate paid on

[17] See page 5 of June 2017 Financial Stability Report.

bank deposits (which will be equal to the central bank's policy rate), W_P denotes the wage paid to patient households, P represents the aggregate price level, Π denotes profits of the firms and banks returned to the patient households, who we assume own them, net of money used by patient households to provide initial capital to new banks, and T_P corresponds to lump-sum taxes. In order to deliver an efficient steady state in the housing market, we introduce a constant tax on patient households' housing denoted by τ_H. To generate volatility in house prices, we add a 'housing demand' shock common to all (i.e. both patient and impatient) households, denoted by A_H.

Assuming all patient households are identical, the first-order conditions for this problem imply:

$$\frac{1}{c_{P,t}} = \beta_P R_t E_t \frac{1}{(1+\pi_{t+1})c_{P,t+1}}, \tag{A1}$$

$$\frac{(1+\tau_H)q_t}{c_{P,t}} - \frac{jA_{j,t}}{H_{P,t}} = \beta_P E_t \frac{q_{t+1}}{c_{P,t+1}}, \tag{A2}$$

$$w_{P,t} = h_{P,t}^{\xi} c_{P,t}, \tag{A3}$$

where c_P denotes aggregate consumption by patient households, H_P represents the aggregate housing stock owned by patient households, π denotes the rate of inflation, q denotes real house prices and w_P corresponds to the real wage paid to patient households. Equation (1) is the familiar patient household's intertemporal Euler equation, relating consumption today to the real interest rate and expected consumption tomorrow. Equation (2) is the housing demand equation for patient households, which shows that the higher is the real cost of housing, the less housing will be demanded. Finally, equation (3) is the labour supply equation for patient households, which shows that the higher the real wage paid to patient households is, the more hours of labour they will supply.

Impatient households

We assume that there is a unit continuum of impatient households, who also maximize the present discounted value of their current and future streams of utility. Again, they obtain utility from consumption, housing and leisure (i.e. obtain disutility from working). In addition to a budget constraint, however, they also face a collateral (LTV) constraint on their borrowing. We assume that this constraint is imposed on them by the banks themselves (rather than by regulators) for internal risk management purposes. Following (Iacoviello, 2015), we assume that impatient households discount the future at a greater rate than the patient households, that is $\beta_I < \beta_P$. We can write the problem facing impatient household i mathematically as:

$$\text{Maximize} \;\; E_0 \sum_{t=0}^{\infty} \beta_I^t \left[ln(c_{i,t}) + jA_{H,t} ln(H_{i,t}) - \frac{1}{1+\xi} h_{i,t}^{1+\xi} \right]$$

$$\text{Subject to}: \;\; L_{i,t} = Q_t(H_{i,t} - H_{i,t-1}) + R_{L,t-1}L_{i,t-1} - W_{I,t}h_{i,t} + P_t c_{i,t} + P_t T_I, \tag{A4}$$

$$L_{i,t} \leq \rho_L L_{i,t-1} + (1 - \rho_L)\text{LTV } H_{i,t} E_t Q_{t+1}, \tag{A5}$$

where c_i denotes consumption of impatient household i, H_i represents housing held by household i, h_i corresponds to hours worked by household i, L_i denotes bank lending to household i, R_L denotes the interest rate charged on bank loans, W_I denotes the wage paid to impatient households, LTV is the average loan-to-value limit targeted by the banks on their lending, and T_I denotes lump-sum taxes, including those used to achieve an efficient allocation of consumption in steady state.[18] Note that, following Iacoviello (2015), we assume that impatient households only adjust slowly to their borrowing limits. The intuitive justification for allowing impatient consumers to adjust slowly to the mortgage borrowing limits is that these limits are typically imposed when mortgages are taken out; thus they will not effectively apply to all mortgage lending. Given this intuition, we can interpret ρ_L as the proportion of existing mortgages and $1 - \rho_L$ as the proportion of new mortgages.[19]

The first-order conditions for this problem imply:

$$\frac{1}{c_{I,t}}(1 - \mu_t) = \beta_I E_t \frac{R_{L,t} - \rho_L \mu_{t+1}}{(1 + \pi_{t+1}) c_{I,t+1}}, \tag{A6}$$

$$\frac{jA_{j,t}}{H_{I,t}} = \frac{q_t}{c_{I,t}} - \frac{\mu_t (1 - \rho_L)\text{LTV}, E_t[q_{t+1}(1 + \pi_{t+1})]}{c_{I,t}} - \beta_I E_t \frac{q_{t+1}}{c_{I,t+1}}, \tag{A7}$$

$$w_{I,t} = h_{I,t}^{\xi} c_{I,t}, \tag{A8}$$

where c_I denotes aggregate consumption by impatient households, H_I represents the aggregate housing stock owned by impatient households and w_I corresponds to the real wage paid to impatient households. Equation (6) is the intertemporal Euler equation for impatient households. Note that in addition to the real interest rate they pay on their borrowing and their expected future consumption, the consumption of impatient households will also depend on the tightness of the LTV constraint on their borrowing, as picked up by the Lagrange multiplier, μ. Equation (7) is the housing demand equation for impatient households. This equation shows that in addition to its utility value, a marginal unit of housing yields extra value to impatient households by loosening their collateral constraint, enabling them to borrow and consume more. This effect is picked up by the term: $\frac{\mu_t(1-\rho_L)LTV \; E_t[q_{t+1}(1+\pi_{t+1})]}{c_{I,t}}$. Equation (8) is the labour supply equation for impatient households showing that the higher the real wage is, the more hours of labour they will supply.

[18]In the UK, the Financial Policy Committee has the power to direct banks to set LTV limits at levels of their choosing for owner–occupier and/or buy-to-let mortgages. But, as the Committee has not used these powers yet, we assume that banks set the LTV ratio. We calibrate it to match the ratio of mortgage borrowing to GDP, as described in section IV.

[19]We introduce the collateral constraint with an inequality. However, given the differences in discount factors among agents, this constraint will always be binding. See Iacoviello (2005) for further discussion.

Firms

As is standard in the new Keynesian literature, we assume that there is a unit continuum of monopolistically competitive intermediate-goods-producing firms and a representative perfectly competitive firm that combines intermediate goods to produce a final good. We assume that the intermediate-goods-producing firms face costs of adjusting prices a la Rotemberg (1982). They also have to borrow to finance their working capital needs, creating a direct link between the financial sector and output and inflation. In what follows we present the optimization problem for the two types of firms.

Final-goods-producing firms. The representative final goods firm operates in a perfectly competitive market and produces a final good by combining inputs of intermediate goods. These final goods are then consumed or invested. We can write the problem for this firm mathematically as follows:

$$\text{Maximize } P_t y_t - \int_{l=0}^{1} P_{l,t} y_{l,t} dl,$$

$$\text{Subject } to: \; y_t = \left(\int_{l=0}^{1} y_{l,t}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}},$$

where y denotes final goods output, y_l represents output of intermediate firm l and P_l corresponds to the price of output for the intermediate firm l.

The first-order condition for this firm gives the demand function for the output of individual firms:

$$y_{l,t} = \left(\frac{P_t}{P_{j,t}} \right)^{\epsilon} y_t. \tag{A9}$$

Intermediate-goods-producing firms. We assume a unit continuum of firms producing differentiated intermediate goods in a monopolistically competitive market. These firms face costs of adjusting prices. In addition, they have to borrow to finance their wage bill (what we think of as working capital). As a result of this constraint any shocks that have an effect on the rate of interest on bank lending will have a direct effect on firms' costs and, hence, output and inflation. This is an important channel of transmission for macroprudential policy since, by reducing the effects of shocks on bank lending rates, macroprudential policy can have beneficial effects on the real economy by reducing output and inflation volatility. Since the firms are owned by the patient households, they discount their profits using the patient households' stochastic discount rate. We can write the problem facing the intermediate firm l mathematically as:

$$\text{Maximize } \sum_{t=0}^{\infty} \frac{\beta_P^t}{P_t c_{P,t}} \Bigg[(1+\tau_P) P_{l,t} y_{l,t} + n A_{n,t} - W_{P,t} h_{P,l,t} - W_{I,t} h_{I,l,t}$$
$$+ L_{l,t} - R_{L,t-1} L_{l,t-1} - \frac{\chi}{2} \left(\frac{P_{l,t}}{P_{l,t-1}} - 1 \right)^2 P_t y_t \Bigg],$$

Subject to:

$$L_{l,t} = W_{P,t}h_{P,l,t} + W_{I,t}h_{I,l,t} \tag{A10}$$

$$y_{l,t} = A_{z,t}h_{P,l,t}^{(1-\sigma)}h_{I,l,t}^{\sigma}, \tag{A11}$$

$$y_{l,t} = \left(\frac{P_t}{P_{l,t}}\right)^{\epsilon} y_t,$$

Where τ_P is a subsidy to make steady-state production efficient[20], $h_{P,l}$ is the labour input of patient households within firm l, $h_{I,l}$ is the labour input of impatient households within firm l and L_l is borrowing by firm l. All intermediate firms are subject to an aggregate technology shock, A_Z. Following Iacoviello (2015), we assume that firms default on an exogenous amount nA_n of their loans from banks, where n denotes the steady-state net worth of the banking sector and A_n follows an exogenous process. This will act as an exogenous shock to bank balance sheets.

If we assume a symmetric equilibrium, the first-order conditions for this problem imply:

$$\frac{(1-\sigma)y_t}{h_{P,t}}rmc_t = \frac{R_{L,t}}{R_t}w_{P,t} \tag{A12}$$

$$\frac{\sigma y_t}{h_{I,t}}rmc_t = \frac{R_{L,t}}{R_t}w_{I,t} \tag{A13}$$

$$\pi_t(1+\pi_t) = \frac{(1-\epsilon)(1+\tau_p)}{\chi} + \frac{\epsilon}{\chi}rmc_t + \frac{1}{R_t}E_t\pi_{t+1}(1+\pi_{t+1})^2\frac{y_{t+1}}{y_t} \tag{A14}$$

Equations (12) and (A1) represent the demand for each type of labour; in each case, the lower the wage, the more labour is demanded. Note that the wage is multiplied by the interest rate spread, reflecting the fact that firms have to borrow to pay their wage bill. Again, it is this channel that provides a direct link from the financial sector to firms' costs and, hence, output and inflation. Equation (14) is the new Keynesian Phillips curve, which relates inflation today to expected future inflation, expected future output growth and real marginal cost.

Banks

Our modelling of the banking sector follows Gertler and Karadi (2011) with an endogenously-generated interest rate spread and leverage ratio. We assume that banks issue loans to impatient households and firms and finance these out of patient household deposits and their own net worth, n. To ensure that banks cannot accumulate retained earnings to achieve full equity finance, we follow Gertler and Karadi (2011) and assume that each period, banks have an *iid* probability $1-\zeta$ of exiting. Hence, the expected

[20]The taxes/subsidies we use throughout our model ensure that the steady state in this economy is efficient. This enables us to derive analytically the welfare loss function in terms of variables expressed as gaps relative to their efficient steady-state values. The dynamics of the model should not be affected by this choice but it allows us to obtain a more rigorous analysis of the welfare implications of the measures.

lifetime of a bank is $1/(1-\zeta)$. When banks exit, their accumulated net worth is distributed as dividends to the patient households. Each period, exiting banks are replaced with an equal number of new banks which initially start with a net worth of $L\nu$, where L is the steady state value of the banking sector's assets, provided by the patient households. A bank that survived from the previous period – bank b, say – will have net worth, n_b, given by:

$$n_{b,t} = R_{L,t-1}L_{b,t-1}(1+\tau_b) - R_{t-1}D_{b,t-1} - nA_{n,t} \tag{A15}$$

where τ_b is a subsidy which ensures a steady-state spread of zero (the efficient level), L_b is the total lending of bank b to impatient households and firms and D_b are deposits from patient households held at bank b. As we explained earlier, $nA_{n,t}$ denotes non-performing loans, acting as an exogenous shock to bank balance sheets.

Total net worth at time t, n_t of the banking sector will be given by:

$$n_t = \zeta(R_{L,t-1}L_{t-1}(1+\tau_b) - R_{t-1}D_{t-1} - nA_{n,t}) + (1-\zeta)L\nu. \tag{A16}$$

Each period, banks (whether new or existing) finance their loan book with newly issued deposits and net worth:

$$L_{b,t} = D_{b,t} + n_{b,t}. \tag{A17}$$

Following Gertler and Karadi (2011), we introduce the following friction into the banks' ability to issue deposits. After accepting deposits and issuing loans, banks have the ability to divert some of their assets for the personal use of their owners. Although the patient households are both the owners of the banks and the depositors in the model, we assume that each household is 'large' enough that we could imagine the banks owners and depositors being separate individuals, with the owners prepared to divert assets towards their own personal use. Specifically, they can sell up to a fraction θ of their loans in period t and spend the proceeds during period t. But, if they do, their depositors will force them into bankruptcy at the beginning of period $t+1$. When deciding whether or not to divert funds, bank b, will compare the franchise value of the bank, V_b, against the gain from diverting funds, θL_b. Hence, depositors will ensure that banks satisfy the following incentive constraint:

$$\theta L_{b,t} \le V_{b,t}. \tag{A18}$$

The problem for bank b is to choose L_b and D_b each period to maximize its franchise value subject to its incentive constraint, equation (A6), its balance sheet constraint (A5) and the evolution of its net worth (A3).

$$\text{Maximize } V_{b,t} = P_t E_t \sum_{j=1}^{\infty} \beta_P^j \zeta^{j-1}(1-\zeta)\frac{1}{c_{P,t+j}P_{t+j}}$$
$$(R_{L,t+j-1}L_{b,t+j-1}(1+\tau_b) - R_{b,t+j-1}D_{t+j-1} - nA_{n,t+j}).$$

We can note that both the objective and constraints of the bank are constant returns to scale. As a result, we can rewrite the optimization problem for bank b in terms of choosing its leverage ratio, $\varphi_b = \frac{L_b}{n_b}$, to maximize the ratio of its franchise value to net worth, $\psi_b = \frac{V_b}{n_b}$. Given constant returns to scale, we can aggregate up across all banks. Doing so, we obtain the aggregate Bellman equation for the franchise value of the banking sector as a whole:

$$\psi_t = \beta_H E_t\left(\frac{P_t}{P_{t+1}}\right) E_t\left(\frac{c_{P,t}}{c_{P,t+1}}\right) E_t(1-\zeta+\zeta\psi_{t+1})$$
$$E_t\left((R_{L,t}(1+\tau_b)-R_t)\varphi_t + R_t - \frac{n}{n_t}A_{n,t+1}\right). \tag{A19}$$

$$\text{Subject to: } \theta\varphi_t \leq \psi_t, \tag{A20}$$

where we note that constant returns to scale implies that all banks will choose the same leverage ratio, φ.

Monetary policy

The central bank operates a Taylor Rule of the form:

$$lnR_t = (1-\rho_R)ln(R) + \rho_R lnR_{t-1} + (1-\rho_R)\left[\phi_\pi \pi_t + \phi_y ln\left(\frac{y_t}{y}\right)\right] + \epsilon_{R,t}, \tag{A21}$$

where y denotes the steady-state level of output and ϵ_R is a white-noise shock.

Market clearing

Aggregating the budget constraints for each sector implies the goods market clearing condition:

$$y_t = \frac{c_t}{1-\frac{\chi}{2}\pi_t^2}. \tag{A22}$$

We assume a fixed stock of housing equal to unity:

$$H_{P,t} + H_{I,t} = 1. \tag{A23}$$

And:

$$L_{M,t} + L_{E,t} = L_t, \tag{A24}$$

where L_M and L_E denote total lending to households and firms, respectively.

Appendix B: The efficient steady state

In this annex, we define the conditions under which a zero-inflation steady state is efficient and show that we can obtain an efficient steady state in our decentralized economy by setting taxes and subsidies.

Consider a social planner who maximizes a weighted average of patient and impatient households' period utility function, subject to the aggregate resource constraint and market clearing in the housing and labour markets. Price adjustment costs are zero in a zero inflation steady state.

Maximize:

$$U = \omega U(c_P, H_P, h_P) + (1-\omega)U(c_I, H_I, h_I).$$

Subject to

$$h_P^{(1-\sigma)} h_I^{\sigma} = c_P + c_I.$$

And

$$H_P + H_I = 1.$$

Let μ_1 and μ_2 be the Lagrange multipliers on the resource and housing constraints, respectively. Then the first-order conditions will imply:

$$\omega U_{c,P} = \mu_1, \tag{B1}$$

$$(1-\omega)U_{c,I} = \mu_1, \tag{B2}$$

$$\omega U_{H,P} = \mu_2, \tag{B3}$$

$$(1-\omega)U_{H,I} = \mu_2, \tag{B4}$$

$$\omega U_{h,P} = -\mu_1(1-\sigma)\frac{y}{h_P}, \tag{B5}$$

$$(1-\omega)U_{h,I} = -\mu_1 \frac{\sigma y}{h_I}, \tag{B6}$$

where Uc, U_H and U_h are the marginal utilities of consumption, housing and hours worked, respectively, for household type j. Combining equations (B3), (B4), (B5) and (B6) gives:

$$\frac{U_{c,P}}{U_{H,P}} = \frac{U_{c,I}}{U_{H,I}} = \frac{\mu_1}{\mu_2}. \tag{B7}$$

In addition, equations (B7) and (B8) imply that the marginal rate of substitution between consumption and each type of labour is equal to the marginal rate of transformation between each type of labour and output.

$$\frac{U_{h,P}}{U_{c,P}} = (1-\sigma)\frac{y}{h_P}. \tag{B8}$$

$$\frac{U_{h,I}}{U_{c,I}} = \sigma\frac{y}{h_I}. \tag{B9}$$

Furthermore, if Pareto weights are set to match the population weights, that is, $\omega = \frac{1}{2}$, then in the efficient steady state:

$$c_P = c_I = \frac{y}{2}. \tag{B10}$$

$$H_P = H_I = \frac{1}{2}. \tag{B11}$$

Next, we show that by choosing taxes and subsidies we can achieve the efficient steady state in the decentralized economy. We set the subsidy to firms, τ_P, equal to $\frac{1}{(\varepsilon-1)}$. The zero-inflation steady-state version of the New Keynesian Phillips curve now implies:

$$\text{rmc} = \frac{(\varepsilon - 1)(1 + \tau_p)}{\varepsilon} = 1. \tag{B12}$$

This implies:

$$R_L = \frac{1}{\beta_P} \frac{(\beta_P + \zeta(\varphi - 1) - (1 - \zeta)\varphi\nu\beta_P)}{\zeta\varphi(1 + \tau_b)}. \tag{B13}$$

If we set the subsidy to banks, τ_b, equal to $\frac{\beta_P}{\zeta\varphi^*}(1 - \frac{\zeta}{\beta_P} - (1 - \zeta)\varphi^*\nu)$ where φ^* is the degree of leverage in the efficient steady state, then:

$$R = R_L = \frac{1}{\beta_P}. \tag{B14}$$

And:

$$\theta\varphi^* = (1 - \zeta + \zeta\theta\varphi^*)\left(\frac{\beta_P}{\zeta}\left(1 - \frac{\zeta}{\beta_P} - (1 - \zeta)\varphi^*\nu\right) + 1\right), \tag{B15}$$

which can be used to solve for φ^*.

The steady-state versions of equations (3), (8), (12) and (A1) imply:

$$\frac{U_{h,P}}{U_{c,P}} = w_P = (1 - \sigma)\frac{y}{h_P}. \tag{B16}$$

$$\frac{U_{h,I}}{U_{c,I}} = w_I = \sigma\frac{y}{h_I}. \tag{B17}$$

Evaluating the Euler equation for impatient households at the efficient steady state gives:

$$\mu = \frac{1 - \frac{\beta_I}{\beta_P}}{1 - \beta_I\rho_L}. \tag{B18}$$

The Lagrange multiplier will be positive in the efficient steady state so long as $\beta_P > \beta_I$. Hence, the housing demand equation for impatient households in steady state implies:

$$\frac{c_I}{H_I} = \frac{(1-\beta_I - \mu(1-\rho_L)\text{LTV})\, q}{j} = \frac{\left(1-\beta_I - \frac{1-\frac{\beta_I}{\beta_P}}{1-\beta_I\rho_L}(1-\rho_L)\text{LTV}\right) q}{j}. \tag{B19}$$

Similarly, for patient households we obtain:

$$\frac{c_P}{H_P} = \frac{(1+\tau_H - \beta_P)q}{j}. \tag{B20}$$

Equation (B9) then implies that to obtain an efficient steady state, we need to set the housing tax equal to:

$$\tau_H = \beta_P - \beta_I - \frac{\left(1-\frac{\beta_I}{\beta_P}\right)}{(1-\beta_I\rho_L)}(1-\rho_L)\text{LTV}. \tag{B21}$$

The LTV constraint then implies the efficient household debt to GDP ratio:

$$\frac{L_M}{y} = \text{LTV}\frac{q}{2y}. \tag{B22}$$

From the steady-state budget constraint for the impatient households we have:

$$\sigma = \frac{1-\beta_P}{\beta_P}\frac{L_M}{y} + \frac{c_I}{y} + \frac{T_I}{y} \Rightarrow \frac{T_I}{y} = -\left(\frac{1-\beta_P}{\beta_P}\text{LTV}\frac{q}{2y} + \frac{1}{2} - \sigma\right). \tag{B23}$$

The impatient households need to receive a subsidy (net of taxes) proportional to GDP given by the term in brackets on the right-hand side of equation (C1). Given such a subsidy, they will enjoy the same consumption and housing as the patient households, in line with our efficiency conditions (B11) and (B12).

Appendix C: Derivation of the loss function

This annex describes the derivation of the loss function shown in section VII of the paper. Following Ferrero *et al.*, (2018), the welfare objective of the policymaker is defined as the present discounted value of the utility of the two types of household, weighted by arbitrary weights, ω and $1-\omega$, and discounted at the patient households' discount rate, β_P:

$$W_0 = E_0 \sum_{t=0}^{\infty} \beta_P^t (\omega U_{P,t} + (1-\omega)U_{I,t}).$$

Given the functional forms:

$$U_{P,t} = lnc_{P,t} + jlnH_{P,t} - \frac{1}{(1+\xi)} h_{P,t}^{1+\xi}.$$

$$U_{I,t} = lnc_{I,t} + jlnH_{I,t} - \frac{1}{(1+\xi)} h_{I,t}^{1+\xi}.$$

A second-order approximation of U around the efficient steady state gives:

$$\begin{aligned} U_t - U \approx\ & \omega U_c \left(c_{P,t} - \frac{y}{2} + \frac{1}{2}\frac{U_{cc}}{U_c}\left(c_{P,t} - \frac{y}{2}\right)^2 \right) \\ & + (1-\omega) U_c \left(c_{I,t} - \frac{y}{2} + \frac{1}{2}\frac{U_{cc}}{U_c}\left(c_{I,t} - \frac{y}{2}\right)^2 \right) \\ & + \omega U_H \left(H_{P,t} - \frac{1}{2} + \frac{1}{2}\frac{U_{HH}}{U_H}(H_{P,t} - \frac{1}{2})^2 \right) \\ & + (1-\omega) U_H \left(H_{I,t} - \frac{1}{2} + \frac{1}{2}\frac{U_{HH}}{U_H}\left(H_{I,t} - \frac{1}{2}\right)^2 \right) \\ & + \omega U_h \left(h_{P,t} - h_P + \frac{1}{2}\frac{U_{hh}}{U_h}(h_{P,t} - h_P)^2 \right) \\ & + (1-\omega) U_h \left(h_{I,t} - h_I + \frac{1}{2}\frac{U_{hh}}{U_h}(h_{I,t} - h_I)^2 \right). \end{aligned}$$

Using the first-order conditions for the efficient steady state derived in Annex 1 we obtain:

$$\begin{aligned} U_t - U \approx\ & \mu_1 \left(c_{P,t} - \frac{y}{2} + \frac{1}{2}\frac{U_{cc}}{U_c}(c_{P,t} - \frac{y}{2})^2 \right) + \mu_1 \left(c_{I,t} - \frac{y}{2} + \frac{1}{2}\frac{U_{cc}}{U_c}\left(c_{I,t} - \frac{y}{2}\right)^2 \right) \\ & + \mu_2 \left(H_{P,t} - \frac{1}{2} + \frac{1}{2}\frac{U_{HH}}{U_H}(H_{P,t} - \frac{1}{2})^2 \right) \\ & + \mu_2 \left(H_{I,t} - \frac{1}{2} + \frac{1}{2}\frac{U_{HH}}{U_H}\left(H_{I,t} - \frac{1}{2}\right)^2 \right) \\ & - \mu_1 (1-\sigma) \frac{y}{h_P} \left(h_{P,t} - h_P + \frac{1}{2}\frac{U_{hh}}{U_h}(h_{P,t} - h_P)^2 \right) \\ & - \mu_1 \sigma \frac{y}{h_I} \left(h_{I,t} - h_I + \frac{1}{2}\frac{U_{hh}}{U_h}(h_{I,t} - h_I)^2 \right). \end{aligned}$$

Given the functional form for preferences, we note that:

$$\frac{U_{cc}}{U_c} = -\frac{2}{y}$$

$$\frac{U_{HH}}{U_H} = -2$$

$$\frac{U_{hh}}{U_h} = \frac{\xi}{h}.$$

Substituting in gives:

$$U_t - U \approx \mu_1 \left(c_{P,t} - \frac{y}{2} - \frac{1}{y}\left(c_{P,t} - \frac{y}{2}\right)^2 \right) + \mu_1 \left(c_{I,t} - \frac{y}{2} - \frac{1}{y}\left(c_{I,t} - \frac{y}{2}\right)^2 \right)$$
$$+ \mu_2 \left(H_{P,t} - \frac{1}{2} - \left(H_{P,t} - \frac{1}{2}\right)^2 \right) + \mu_2 \left(H_{I,t} - \frac{1}{2} - \left(H_{I,t} - \frac{1}{2}\right)^2 \right)$$
$$- \mu_1(1-\sigma)\frac{y}{h_P}\left(h_{P,t} - h_P + \frac{1}{2}\frac{\xi}{h}(h_{P,t} - h_P)^2 \right)$$
$$- \mu_1 \sigma \frac{y}{h_I}\left(h_{I,t} - h_I + \frac{1}{2}\frac{\xi}{h}(h_{I,t} - h_I)^2 \right). \tag{C1}$$

Now the aggregate resource constraint is given by:

$$c_{P,t} + c_{I,t} = y_t\left(1 - \frac{\chi}{2}\pi_t^2\right). \tag{C2}$$

We can approximate any variable x using $x_t = x\left(1 + \hat{x}_t + \frac{1}{2}\hat{x}_t{}^2\right)$. Taking a second-order approximation of equation (C4) and ignoring terms independent of policy gives:

$$c_{P,t} + c_{I,t} - y = y\left(\hat{y}_t + \frac{1}{2}\hat{y}_t{}^2 - \frac{1}{2}\chi\pi_t^2\right). \tag{C3}$$

We can also note that:

$$H_{P,t} - \frac{1}{2} + H_{I,t} - \frac{1}{2} = 0. \tag{C4}$$

Substituting equations (C5) and (C6) into equation (C3) gives:

$$U_t - U \approx \mu_1 y\left(\hat{y}_t + \frac{1}{2}\hat{y}_t{}^2 - \frac{1}{2}\chi\pi_t^2\right) - \frac{\mu_1}{y}\left(\left(c_{P,t} - \frac{y}{2}\right)^2 + \left(c_{I,t} - \frac{y}{2}\right)^2\right)$$
$$- \mu_2\left(\left(H_{P,t} - \frac{1}{2}\right)^2 + \left(H_{I,t} - \frac{1}{2}\right)^2\right)$$
$$- \mu_1(1-\sigma)y\left(\frac{h_{P,t} - h_P}{h_P} + \frac{\xi}{2}\left(\frac{h_{P,t} - h_P}{h}\right)^2\right)$$
$$- \mu_1\sigma y\left(\frac{h_{I,t} - h_I}{h_I} + \frac{\xi}{2}\left(\frac{h_{I,t} - h_I}{h_I}\right)^2\right). \tag{C5}$$

To eliminate the remaining first-order terms from equation (C7), we express variables in terms of log-deviations from the efficient steady-state values and drop terms of order 3 and higher:

$$U_t - U \approx \mu_1 y\left(\hat{y}_t + \frac{1}{2}\hat{y}_t{}^2 - \frac{1}{2}\chi\pi_t^2\right) - \frac{\mu_1 y}{4}\left(\hat{c}_{P,t}^2 - \hat{c}_{I,t}^2\right)$$
$$- \mu_1 y\left((1-\sigma)\hat{h}_{P,t} + \sigma\hat{h}_{I,t}\right) - \mu_1 y\left(\frac{(1-\sigma)}{2}\hat{h}_{P,t}^2 + \frac{\sigma}{2}\hat{h}_{I,t}^2\right)$$
$$- \frac{\mu_1 \xi y}{2}\left((1-\sigma)\hat{h}_{P,t}^2 + \sigma\hat{h}_{I,t}^2\right) - \frac{\mu_2}{4}\left(\hat{H}_{P,t}^2 + \hat{H}_{I,t}^2\right). \quad \text{(C6)}$$

Log-linearizing the production function around the efficient steady state implies:

$$\hat{y}_t = \hat{A}_{z,t} + (1-\sigma)\hat{h}_{P,t} + \sigma\hat{h}_{I,t}.$$

Substituting into equation (C8) and dropping the term in $\hat{A}_{z,t}$, as it is independent of policy, implies:

$$U_t - U \approx \frac{\mu_1 y}{2}(\hat{y}_t^2 - \chi\pi_t^2) - \frac{\mu_1 y}{4}\left(\hat{c}_{P,t}^2 + \hat{c}_{I,t}^2\right)$$
$$- \frac{\mu_1(1+\xi)y}{2}\left((1-\sigma)\hat{h}_{P,t}^2 + \sigma\hat{h}_{I,t}^2\right) - \frac{\mu_2}{4}\left(\hat{H}_{P,t}^2 + \hat{H}_{I,t}^2\right). \quad \text{(C7)}$$

The log-linearized version of the housing market equilibrium condition around the efficient steady state implies:

$$\hat{H}_{P,t} = -\hat{H}_{I,t} \Rightarrow \hat{H}_{P,t}^2 + \hat{H}_{I,t}^2 = \frac{1}{2}\left(\hat{H}_{P,t} - \hat{H}_{I,t}\right)^2.$$

Substituting back into equation (C9) and collecting the output, consumption and labour terms implies:

$$U_t - U \approx \frac{-\mu_1 y}{2}\left(\frac{1}{2}\left(\hat{c}_{P,t}^2 + \hat{c}_{I,t}^2\right) - \hat{y}_t^2 + (1+\xi)\left((1-\sigma)\hat{h}_{P,t}^2 + \sigma\hat{h}_{I,t}^2\right)\right)$$
$$- \frac{\mu_2}{8}\left(\hat{H}_{P,t} - \hat{H}_{I,t}\right)^2 - \frac{(\mu_1 y\chi)}{2}\pi_t^2. \quad \text{(C8)}$$

Next, use:

$$\frac{1}{2}\left(\hat{c}_{P,t}^2 + \hat{c}_{I,t}^2\right) - \hat{y}_t^2 = \frac{1}{2}\left(\hat{c}_{P,t}^2 - \hat{y}_t^2\right) + \frac{1}{2}\left(\hat{c}_{I,t}^2 - \hat{y}_t^2\right)$$
$$= \frac{1}{2}\left(\left(\hat{c}_{P,t} + \hat{y}_t\right)\left(\hat{c}_{P,t} - \hat{y}_t\right) + \left(\hat{c}_{I,t} + \hat{y}_t\right)\left(\hat{c}_{I,t} - \hat{y}_t\right)\right)$$
$$= \frac{1}{2}\left(\left(\frac{3}{2}\hat{c}_{P,t} + \frac{1}{2}\hat{c}_{I,t}\right)\left(\frac{1}{2}\hat{c}_{P,t} - \frac{1}{2}\hat{c}_{I,t}\right) - \left(\frac{3}{2}\hat{c}_{I,t} + \frac{1}{2}\hat{c}_{P,t}\right)\left(\frac{1}{2}\hat{c}_{P,t} - \frac{1}{2}\hat{c}_{I,t}\right)\right)$$
$$= \frac{1}{4}\left(\hat{c}_{P,t} - \hat{c}_{I,t}\right)^2.$$

Substituting back into equation (C10) implies:

$$U_t - U \approx \frac{-\mu_1 y}{2}\left(\frac{1}{4}(\hat{c}_{P,t} - \hat{c}_{I,t})^2 + (1+\xi)\left((1-\sigma)\hat{h}_{P,t}^2 + \sigma \hat{h}_{I,t}^2\right)\right)$$
$$- \frac{\mu_2}{8}(\hat{H}_{P,t} - \hat{H}_{I,t})^2 - \frac{\mu_1 y \chi}{2}\pi_t^2. \quad \text{(C9)}$$

Next, the labour supply equations imply:

$$\frac{w_{P,t} h_{P,t}}{w_{I,t} h_{I,t}} = \frac{1-\sigma}{\sigma}.$$

Combining implies:

$$h_{I,t} = \left(\frac{\sigma}{1-\sigma}\right)^{\frac{1}{1+\xi}} h_{P,t} \left(\frac{c_{P,t}}{c_{I,t}}\right)^{\frac{1}{1+\xi}}.$$

Combining with the production function implies:

$$y_t = A_{z,t} h_{P,t} \left(\frac{\sigma}{1-\sigma}\frac{c_{P,t}}{c_{I,t}}\right)^{\frac{\sigma}{1+\xi}}$$
$$\Rightarrow \hat{h}_{P,t} = \hat{y}_t - \hat{A}_{z,t} - \frac{\sigma}{1+\xi}(\hat{c}_{P,t} - \hat{c}_{I,t})$$
$$\Rightarrow \hat{h}_{I,t} = \hat{y}_t - \hat{A}_{z,t} - \frac{1-\sigma}{1+\xi}(\hat{c}_{P,t} - \hat{c}_{I,t}).$$

Hence:

$$(1-\sigma)\hat{h}_{P,t}^2 + \sigma \hat{h}_{I,t}^2$$
$$= (1-\sigma)\left(\hat{y}_t - \hat{A}_{z,t} - \frac{\sigma}{(1+\xi)}(\hat{c}_{P,t} - \hat{c}_{I,t})\right)^2 + \sigma\left(\hat{y}_t - \hat{A}_{z,t} - \frac{1-\sigma}{(1+\xi)}(\hat{c}_{P,t} - \hat{c}_{I,t})\right)^2$$
$$= (\hat{y}_t - \hat{A}_{z,t})^2 + \frac{\sigma(1-\sigma)}{(1+\xi)^2}(\hat{c}_{P,t} - \hat{c}_{I,t})^2.$$

Substituting back into equation 70 and ignoring terms independent of policy gives:

$$U_t - U \approx \frac{-\mu_1 y}{2}\left(\frac{1+\xi+4\sigma(1-\sigma)}{4(1+\xi)}(\hat{c}_{P,t} - \hat{c}_{I,t})^2 + (1+\xi)\hat{y}_t^2\right)$$
$$- \frac{\mu_2}{8}(\hat{H}_{P,t} - \hat{H}_{I,t})^2 - \frac{\mu_1 y \chi}{2}\pi_t^2. \quad \text{(C10)}$$

Using the first-order conditions for the efficient steady state to express μ_2 in terms of $\mu_1 y$:

$$\mu_2 = \frac{\mu_1 U_{H,I}}{U_{C,I}} = \mu_1 y j.$$

Substituting into equation (71) gives:

$$U_t - U \approx \frac{-\mu_1 y}{2}\left(\frac{1+\xi+4\sigma(1-\sigma)}{4(1+\xi)}(\hat{c}_{P,t}-\hat{c}_{I,t})^2 + (1+\xi)\hat{y}_t^2\right) - \frac{j}{4}(\hat{H}_{P,t}-\hat{H}_{I,t})^2 - \chi\pi_t^2.$$

The welfare-based loss function can be expressed in terms of quadratic and gap variables as:

$$W_0 = \frac{-\mu_1 y}{2}(1+\xi)E_0\sum_{t=0}^{\infty}\beta_P^t\left(\hat{y}_t^2 + \lambda_1\pi_t^2 + \lambda_2(\hat{c}_{P,t}-\hat{c}_{I,t})^2 + \lambda_3(\hat{H}_{P,t}-\hat{H}_{I,t})^2\right),$$

where $\lambda_1 = \frac{\chi}{(1+\xi)}$, $\lambda_2 = \frac{(1+\xi+4\sigma(1-\sigma))}{4(1+\xi)^2}$ and $\lambda_3 = \frac{j}{4(1+\xi)}$.

Appendix D: Log-linear equations of the model

This annex presents the log-linearized version of the model based on a Taylor series expansion of the equations of the model around the efficient non-stochastic steady state derived in B.

$$\hat{c}_{P,t} = E_t\hat{c}_{P,t+1} - (\hat{R}_t - E_t\pi_{t+1}),$$

$$\hat{H}_{P,t} = \frac{\beta_P}{(1+\tau_H-\beta_P)}E_t(\hat{q}_{t+1}-\hat{c}_{P,t+1}) - \frac{(1+\tau_H)}{(1+\tau_H-\beta_P)}(\hat{q}_t-\hat{c}_{P,t}) + \hat{A}_{j,t},$$

$$\hat{w}_{P,t} = \xi\hat{h}_{P,t} + \hat{c}_{P,t},$$

$$\sigma(\hat{w}_{I,t}+\hat{h}_{I,t}) + \frac{L_M}{y}\left(\hat{L}_{M,t} - \frac{1}{\beta_P}(\hat{L}_{M,t-1}+\hat{R}_{L,t-1}-\pi_t)\right) - \frac{q}{2y}(\hat{H}_{I,t}-\hat{H}_{I,t-1}) = \frac{1}{2}\hat{c}_{I,t}$$

$$\hat{L}_{M,t} = \rho_L(\hat{L}_{M,t-1}-\pi_t) + (1-\rho_L)E_t(\hat{q}_{t+1}+\pi_{t+1}+\hat{H}_{I,t}),$$

$$\hat{c}_{I,t} = E_t\hat{c}_{I,t+1} - \left(\frac{1}{1-\beta_P\rho_L\mu}\hat{R}_{L,t} - \frac{\beta_P\rho_L\mu}{(1-\beta_P\rho_L\mu)}\hat{\mu}_{t+1} - E_t\pi_{t+1} + \frac{\mu}{(1-\mu)\hat{\mu}_t}\right),$$

$$\hat{H}_{I,t} = \frac{\beta_I}{(1-\mu(1-\rho_L)LTV-\beta_I)}E_t(\hat{q}_{t+1}-\hat{c}_{I,t+1}) + \frac{\mu(1-\rho_L)LTV}{(1-\mu(1-\rho_L)LTV-\beta_I)}E_t(\hat{\mu}_t+\hat{q}_{t+1}+\pi_{t+1}-\hat{c}_{I,t}) - \frac{1}{(1-\mu(1-\rho_L)LTV-\beta_I)}(\hat{q}_t-\hat{c}_{I,t}) + \hat{A}_{j,t},$$

$$\hat{w}_{I,t} = \xi \hat{h}_{I,t} + \hat{c}_{I,t},$$

$$\hat{L}_{E,t} = (1-\sigma)(\hat{w}_{P,t} + \hat{h}_{P,t}) + \sigma(\hat{w}_{I,t} + \hat{h}_{I,t}),$$

$$\hat{y}_t = \hat{A}_t + (1-\sigma)\hat{h}_{P,t} + \sigma \hat{h}_{I,t},$$

$$\hat{w}_{P,t} = r\hat{m}c_t + \hat{y}_t - \hat{h}_{P,t} + \hat{R}_t - \hat{R}_{L,t}$$

$$\hat{w}_{I,t} = r\hat{m}c_t + \hat{y}_t - \hat{h}_{I,t} + \hat{R}_t - \hat{R}_{L,t},$$

$$\pi_t = \beta_P E_t \pi_{t+1} + \frac{\varepsilon}{\chi}\mathrm{r\hat{m}c}_t,$$

$$\hat{n}_t = \frac{\zeta\varphi(1+\tau_b)}{\beta_P}(\hat{R}_{L,t-1} + \hat{L}_{t-1}) - \zeta\frac{(\varphi-1)}{\beta_P}(\hat{R}_{t-1} + \hat{D}_{t-1}) - \frac{\zeta}{\beta_P}(1+\varphi\tau_b)\pi_t,$$

$$\hat{n}_t = \varphi \hat{L}_t - (\varphi - 1)\hat{D}_t,$$

$$\hat{\varphi}_t = \hat{L}_t - \hat{n}_t,$$

$$\hat{\psi}_t = \hat{\varphi}_t,$$

$$\hat{\psi}_t = \frac{\varphi\tau_b}{(\varphi\tau_b+1)}\hat{\varphi}_t + \frac{\varphi(1+\tau_b)}{(\varphi\tau_b+1)}(\hat{R}_{L,t} - \hat{R}_t) + \frac{\zeta\psi}{(1-\zeta+\zeta\psi)}\widehat{\psi_{t+1}}$$

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)(\nu_\pi \pi_t + \nu_y \hat{y}_t) + \varepsilon_{R,t},$$

$$\hat{y}_t = \frac{1}{2}(\hat{c}_{P,t} + \hat{c}_{I,t}),$$

$$\hat{H}_{P,t} + \hat{H}_{I,t} = 0,$$

$$\frac{L}{y}\hat{L}_t = \frac{L_M}{y}\hat{L}_{M,t} + \hat{L}_{E,t}.$$

Appendix E: Data on UK housing indicators and capital requirements

In this Appendix, we show additional data on the quantities affected by macroprudential tools, such as lenders' capital requirements and the share of mortgages extended at high LTV and high LTI ratios in the UK (Figures E1-E3).

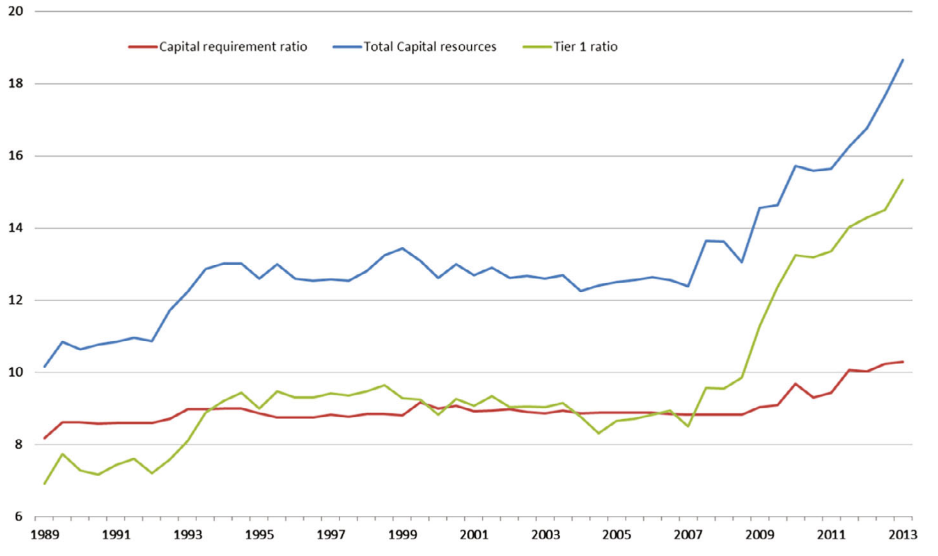

Figure E1. Capital requriements in the UK over time. *Source*: de-Ramon, Francis and Milonas (2016) [Colour figure can be viewed at wileyonlinelibrary.com]

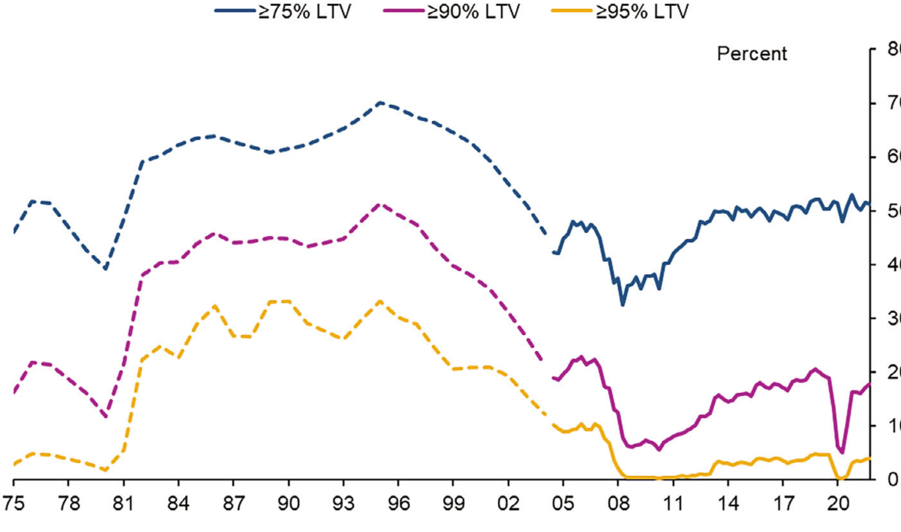

Figure E2. Share of new mortgage lending at or above different LTV ratios$^{(a)(b)}$. *Source*: 5% Sample Survey of Building Society Mortgage Completions (BSM), Product Sales Database (PSD), Survey of Mortgage Lenders (SML) and Bank calculations. (a) The dashed lines show data based on BSM (up to and including 1991) and SML (1992–2004). The solid lines show PSD (from 2005 onwards). (b) BSM and SML surveys contain samples of lenders' mortgage lending, so may not be fully reflective of the total mortgage market [Colour figure can be viewed at wileyonlinelibrary.com]

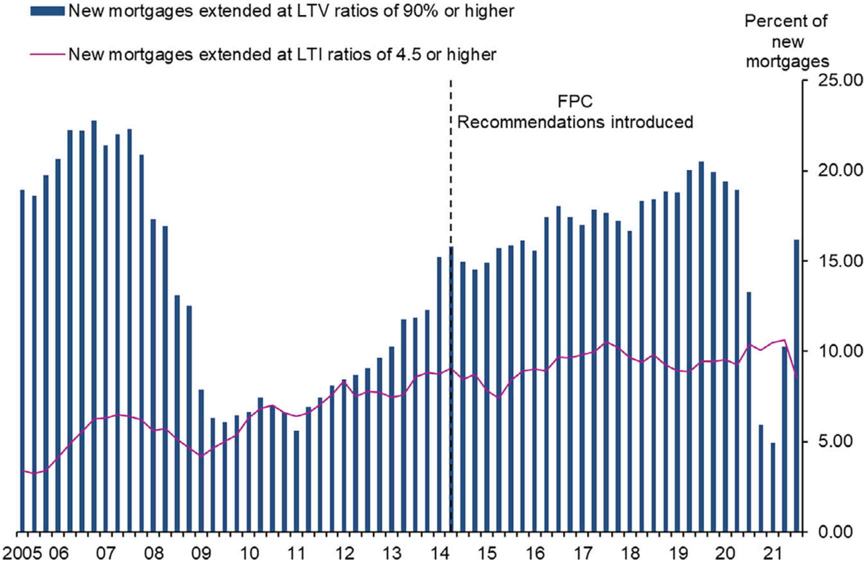

Figure E3. Share of mortgages with an loan to income ratio of 4.5 or higher and an loan to value ratio of 90% or higher[(a)] *Source*: FCA Product Sales Database (PSD) and Bank calculations. (a) In 2014, the FPC (i.e. the macroprudential regulator in the UK) introduced two measures: i) a stressed DSR test, which assessed whether borrowers could still afford their mortgage payments if mortgages rates were 3 percentage points higher than their contractual reversion rate; ii) a loan to income (LTI) flow limit, which limits the number of mortgages extended at LTI ratios of 4.5 or higher to 15% of a lender's new mortgage lending. The chart shows that since 2014 (dotted line), the share of mortgages with high LTI ratios has not increased significantly and the share of lending at high LTV ratios is below its pre-global financial crisis level [Colour figure can be viewed at wileyonlinelibrary.com]

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