# Dressed dense atomic gases

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The interaction between atomic transition dipoles and photons leads to the formation of many-body states with collective dissipation and long-ranged interactions. Here, we put forward and explore a scenario in which a dense atomic gas — where the separation of the atoms is comparable to the transition wavelength — is weakly excited by an off-resonant laser field. We develop the theory for describing such dressed many-body ensemble and show that collective excitations are responsible for the emergence of many-body interactions, i.e. effective potentials that cannot be represented as a sum of binary terms. We illustrate how collective effects may be probed experimentally through microwave spectroscopy. We analyze time-dependent line-shifts, which are sensitive to the phase pattern of the dressing laser and show that the strong interactions lead to a dramatic slow down of the relaxation dynamics. Our study offers a new perspective on dense atomic ensembles interacting with light and promotes this platform as a setting for the exploration of rich non-equilibrium many-body physics.

*Introduction* — There is currently substantial interest in probing and understanding the physics of dense atomic ensembles, as they represent a paradigmatic many-body system whose properties are governed solely by the fundamental interaction of photons with matter. These systems feature strong dipole-dipole interactions, which have been investigated in the context of cold molecular physics [1], to identify novel states based on two-particle entanglement [2], or for their impact for high precision measurement with cold neutral atoms [3, 4]. A number of experimental results, such as the observation of the collective Lamb and Lorentz-Lorenz shifts [5–9] as well as the emergence of sub-[10, 11] and superradiance [12–15] have confirmed theoretical predictions made as early as in the 1950's [16].

In this work we develop a theory for dressed dense atomic gases, a scenario in which excited atomic states are weakly coupled to the atomic ground state via an off-resonant laser. Laser-dressing has become popular in recent years in the context of Rydberg atoms [19, 20], where it is used to tailor interaction potentials for the purpose of simulating exotic types of matter [21–23] or the generation of entangled many-body states for quantum enhanced measurements [24–26]. We show here that dressing dense atomic gases with low-lying electronic states may produce strong effective interactions already at the level of second order in the strength of the dressing laser (in contrast to Rydberg gases where this is a fourth-order effect). The resulting interatomic potential is in general of many-body type, i.e. it cannot be decomposed as the sum of binary interaction terms. We discuss collective properties that can be probed via standard microwave (MW) spectroscopy [19, 20, 27–32], revealing laser-phase sensitive line shifts and an interaction-induced slowdown of the relaxation dynamics. Beyond shedding light on fundamental aspects of matter-photon systems, our results are of relevance for the understanding of interactions and decoherence mechanisms in technological applications based on dense atomic ensembles held in off-resonant optical fields. These are, e.g., lattice clocks, where dipole-dipole induced shifts are either to be avoided or used to create many-body entanglement, allowing to go beyond the Heisenberg limit [3, 18, 33–35], as well as quantum simulation and computing platforms [36–39]. Importantly, MW spectroscopy allows to explore many-

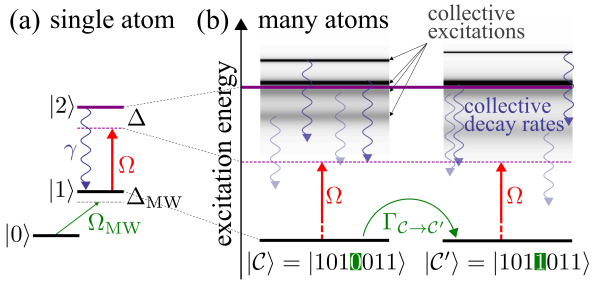


FIG. 1. **Dressed atomic gas.** (a) Sketch of the single-atom level structure. The off-resonant dressing laser (Rabi frequency $\Omega$, detuning $\Delta$) weakly drives the transition $|1\rangle \to |2\rangle$ that has a decay rate $\gamma$. The resulting level shifts can be probed via microwave (MW) spectroscopy on the $|0\rangle \to |1\rangle$ transition (Rabi frequency $\Omega_{\rm MW}$ detuning $\Delta_{\rm MW}$). (b) Atoms exchange (virtual) photons on the transition $|1\rangle \to |2\rangle$, which lead to the formation of delocalised many-body states (collective excitations) with collective decay rates. Sketched are the collective energy levels for two different atomic configurations, $|\mathcal{C}\rangle$ and $|\mathcal{C}'\rangle$. The many-body level structure depends on the number and spatial arrangement of atoms in the state $|1\rangle$, and the energies of the collective excitation states are shifted and broadened with respect to the single atom state. The MW field effectuates transitions between the states $|\mathcal{C}\rangle$ and $|\mathcal{C}'\rangle$ at a rate $\Gamma_{\mathcal{C}\to\mathcal{C}'}$ permitting a spectroscopic analysis of the dressed state manifold.

body effects which cannot be probed using standard optical spectroscopy, similar to the MW spectroscopy observation of the Lamb shift [40]. *Many-body model* — We employ a model consisting of $N$ three-level atoms located at positions $\mathbf{r}_\alpha$. As shown in Fig. 1a, the levels $|1\rangle$ and $|2\rangle$ are weakly coupled with an off-resonant dressing laser with detuning $\Delta$ and single atom Rabi frequencies $\Omega_\alpha$ ($|\Omega_\alpha| \ll |\Delta|$). The resulting (light-)shift and decay rate of atoms from state $|2\rangle$ into state $|1\rangle$ can be probed by a MW field on the $|0\rangle \to |1\rangle$ transition.

For the time being we take the MW field to be absent and focus on the dynamics that takes place between the levels $|1\rangle$ and $|2\rangle$. This is governed by the dressing laser and its interplay with the inter-particle interactions, which are obtained by integrating out the free photon modes [41–43]. The dynamics is governed by a Markovian master equation of the form $\partial_t \rho = \mathcal{L}\rho$. Here $\rho$ is the density matrix of the atomic system and $\mathcal{L}$ is the master operator which we decompose as $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$. The two terms correspond to a "fast" and a "slow" dynamics **(care must be taken that this scale separation remains valid also for strong interactions)**. The slow dynamics is given by the coupling to the dressing laser,

$$\mathcal{L}_1 \bullet = -i \sum_\alpha^N \left[\Omega_\alpha^* b_\alpha^+ + \Omega_\alpha b_\alpha, \bullet\right], \tag{1}$$

with $b_\alpha^+ = |2\rangle_\alpha\langle 1|$ [44], while the fast dynamics is given by

$$\mathcal{L}_0 \bullet = -i \sum_k^N \Delta \left[b_\alpha^+ b_\alpha, \bullet\right] - i \sum_{\alpha\neq\beta}^N V_{\alpha\beta} \left[b_\alpha^+ b_\beta, \bullet\right] + \sum_{\alpha\beta}^N G_{\alpha\beta} \left(b_\alpha \bullet b_\beta^+ - \frac{1}{2}\left\{b_\beta^+ b_\alpha, \bullet\right\}\right), \tag{2}$$

which depends on the interaction and dissipation matrices

$$V_{\alpha\beta} = \frac{3\gamma}{4}\left\{-\left[1-(\hat{d}\cdot\hat{r}_{\alpha\beta})^2\right]\frac{\cos\kappa_{\alpha\beta}}{\kappa_{\alpha\beta}} + \left[1-3(\hat{d}\cdot\hat{r}_{\alpha\beta})^2\right]\left(\frac{\sin\kappa_{\alpha\beta}}{\kappa_{\alpha\beta}^2}+\frac{\cos\kappa_{\alpha\beta}}{\kappa_{\alpha\beta}^3}\right)\right\} \tag{3}$$

$$G_{\alpha\beta} = \frac{3\gamma}{2}\left\{\left[1-(\hat{d}\cdot\hat{r}_{\alpha\beta})^2\right]\frac{\sin\kappa_{\alpha\beta}}{\kappa_{\alpha\beta}} + \left[1-3(\hat{d}\cdot\hat{r}_{\alpha\beta})^2\right]\left(\frac{\cos\kappa_{\alpha\beta}}{\kappa_{\alpha\beta}^2}-\frac{\sin\kappa_{\alpha\beta}}{\kappa_{\alpha\beta}^3}\right)\right\}. \tag{4}$$

Both matrices are functions of the reduced length $\kappa_{\alpha\beta} = kr_{\alpha\beta}$, product of the separation between the atoms $r_{\alpha\beta} = |\mathbf{r}_\alpha - \mathbf{r}_\beta|$ and the wave number $k$ of the atomic transition $|1\rangle \to |2\rangle$. They furthermore depend on the decay rate $\gamma$ of this transition, the relative direction of its dipole moment $\hat{d}$ and the interatomic separations $\hat{r}_{\alpha\beta} = (\mathbf{r}_\alpha - \mathbf{r}_\beta)/r_{\alpha\beta}$.

*Dressed many-body states and effective equation of motion* — Each (classical) atomic configuration of the form

$$|\mathcal{C}\rangle = \otimes_\alpha |\xi_\alpha\rangle \tag{5}$$

with $\xi_\alpha = 0, 1$ evolves independently and is weakly coupled (via $\mathcal{L}_1$) to a set of collective many-body states whose coherent and dissipative dynamics is governed by the master operator $\mathcal{L}_0$. Since $\mathcal{L}_0$ couples only the states $|1\rangle$ and $|2\rangle$, the form of the excited collective states is dictated by the number as well as the arrangement of atoms in state $|1\rangle$ of a given configuration $|\mathcal{C}\rangle$, sketched for two configurations in Fig. 1b. The dressing lets each configuration acquire a collective energy shift and decay rate but does not couple different configurations.

To understand the dynamics of the dressed atomic ensemble we seek an effective equation of motion for the density matrix $\mu = \sum_{\mathcal{C}\mathcal{C}'} \mu_{\mathcal{C}\mathcal{C}'} |\mathcal{C}\rangle\langle\mathcal{C}'|$. We outline the main steps of the calculation, leading to the main result [Eq. (8)]. We begin with the general formula for second order perturbation theory [45]:

$$\partial_t \mu = \mathcal{L}_{\text{eff}}\mu = \mathcal{P}\int_0^\infty dt\, \mathcal{L}_1 e^{\mathcal{L}_0 t}\mathcal{L}_1 \mu \tag{6}$$
$$= -\mathcal{P}\int_0^\infty dt \sum_{\gamma\alpha}\left[\Omega_\gamma^* b_\gamma^+ + \Omega_\gamma b_\gamma, e^{\mathcal{L}_0 t}\left(\left[\Omega_\alpha^* b_\alpha^+ + \Omega_\alpha b_\alpha, \mu\right]\right)\right].$$

Here $\mathcal{P} = \lim_{t\to\infty} e^{\mathcal{L}_0 t}$ is the projector on the stationary subspace of $\mathcal{L}_0$, which is spanned by the configurations (5). To carry out the calculation explicitly, we focus on the evolution of the basis vectors $|\mathcal{C}\rangle\langle\mathcal{C}'|$. We exploit that $b_\alpha |\mathcal{C}\rangle\langle\mathcal{C}'| = |\mathcal{C}\rangle\langle\mathcal{C}'|\, b_\alpha^+ = 0$ and furthermore

$$\partial_t\left(b_\alpha^+ |\mathcal{C}\rangle\langle\mathcal{C}'|\right) = \mathcal{L}_0\left(b_\alpha^+ |\mathcal{C}\rangle\langle\mathcal{C}'|\right) = \sum_\beta M_{\alpha\beta}^{\mathcal{C}}\left(b_\beta^+ |\mathcal{C}\rangle\langle\mathcal{C}'|\right),$$

with the symmetric matrix $\mathbf{M}^{\mathcal{C}}$, whose components are

$$M_{\alpha\beta}^{\mathcal{C}} = -i\Delta\delta_{\alpha\beta} - iV_{\alpha\beta} - \frac{1}{2}G_{\alpha\beta} \tag{7}$$

and involve the coefficients (3) and (4). Here the superscript $\mathcal{C}$ emphasizes that $\mathbf{M}^{\mathcal{C}}$ depends on the structure of configuration $|\mathcal{C}\rangle$, i.e. on the number and arrangement of atoms in state $|1\rangle$. Integrating the equation of motion for the operators $b_\alpha^+ |\mathcal{C}\rangle\langle\mathcal{C}'|$ yields

$$\mathcal{P}\int_0^\infty dt\, b_\beta e^{\mathcal{L}_0 t}\left(b_\alpha^+ |\mathcal{C}\rangle\langle\mathcal{C}'|\right) = -\Lambda_{\alpha\beta}^{\mathcal{C}} |\mathcal{C}\rangle\langle\mathcal{C}'|,$$

with the matrix $\mathbf{\Lambda}^{\mathcal{C}} = -\int_0^\infty dt\, e^{t\mathbf{M}^{\mathcal{C}}} = \left(\mathbf{M}^{\mathcal{C}}\right)^{-1}$. Here we have exploited that $\mathcal{P} b_\alpha b_\beta^+ |\mathcal{C}\rangle\langle\mathcal{C}'| = \delta_{\alpha\beta} |\mathcal{C}\rangle\langle\mathcal{C}'|$. The computation of the other terms is analogous and involves the expression $\mathcal{P} b_\alpha^+ |\mathcal{C}\rangle\langle\mathcal{C}'|\, b_\beta = \Theta_{\alpha\beta}^{\mathcal{C}\mathcal{C}'} |\mathcal{C}\rangle\langle\mathcal{C}'|$ where

$$\Theta_{\alpha\beta}^{\mathcal{C}\mathcal{C}'} = \left[\int_0^\infty ds\, e^{s\mathbf{M}^{\mathcal{C}}}\mathbf{G}^{\mathcal{C}\mathcal{C}'} e^{s\mathbf{M}^{\mathcal{C}'*}}\right]_{\alpha\beta}.$$

The coefficients of the matrix $G_{\alpha\beta}^{\mathcal{C}\mathcal{C}'}$ are given by Eq. (4), and the indices $\alpha$ and $\beta$ run through the atoms in state

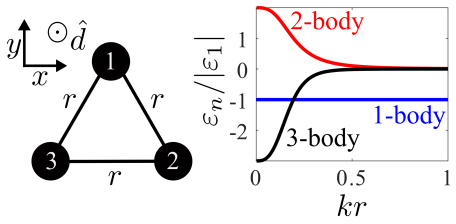


FIG. 2. **Few-body interactions.** Three atoms on an equilateral triangle adressed by a laser with uniform Rabi frequency, $\Omega = 5\gamma$, and detuning $\Delta = 100\gamma$. The atomic dipole moment is pointing in the $z$-direction, perpendicular to the atomic plane. The right panel shows the single, two- and three-body contribution, $\varepsilon_n$, to the (interaction) energy.

$|1\rangle$ of the configurations $|\mathcal{C}\rangle$ and $|\mathcal{C}'\rangle$, respectively. **The explicit calculation of $\Theta^{\mathcal{C}\mathcal{C}'}_{\alpha\beta}$ requires the inversion of a matrix with a dimension that is at most $2N \times 2N$. In general this cannot be done analytically, but very efficient numerical algorithms exist.**

Putting all terms together yields the effective equation of motion for the density matrix basis states:

$$\partial_t |\mathcal{C}\rangle\langle\mathcal{C}'| = \mathcal{L}_{\text{eff}} |\mathcal{C}\rangle\langle\mathcal{C}'| = -i\Delta_{\mathcal{C}\mathcal{C}'} |\mathcal{C}\rangle\langle\mathcal{C}'| . \tag{8}$$

This equation shows that each element $|\mathcal{C}\rangle\langle\mathcal{C}'|$ evolves according to the (complex) energy difference

$$\begin{aligned} -i\Delta_{\mathcal{C}\mathcal{C}'} = {} & \vec{\Omega}^*_{\mathcal{C}} \cdot \mathbf{\Lambda}^{\mathcal{C}} \cdot \vec{\Omega}_{\mathcal{C}} + \vec{\Omega}_{\mathcal{C}'} \cdot \mathbf{\Lambda}^{\mathcal{C}'*} \cdot \vec{\Omega}^*_{\mathcal{C}'} \\ & - \vec{\Omega}^*_{\mathcal{C}} \cdot \mathbf{\Lambda}^{\mathcal{C}} \cdot \mathbf{\Theta}^{\mathcal{C}\mathcal{C}'} \cdot \vec{\Omega}_{\mathcal{C}'} - \vec{\Omega}^*_{\mathcal{C}} \cdot \mathbf{\Theta}^{\mathcal{C}\mathcal{C}'} \cdot \mathbf{\Lambda}^{\mathcal{C}'*} \cdot \vec{\Omega}_{\mathcal{C}'}, \end{aligned} \tag{9}$$

where the vectors $\vec{\Omega}_{\mathcal{C}}$ contain the dressing laser Rabi frequencies of all atoms in the state $|1\rangle$ contained in $|\mathcal{C}\rangle$. The real and imaginary part of $\Delta_{\mathcal{C}\mathcal{C}'}$ are the energy difference and dephasing rate of the configuration $|\mathcal{C}\rangle$ relative to $|\mathcal{C}'\rangle$ (see further below). Note that $\Delta_{\mathcal{C}\mathcal{C}} = 0$, i.e. the diagonal elements of the density matrix $\mu$ do not evolve in time.

*Few atom system* — First, we consider a system where three atoms are positioned on an equilateral triangle, as shown in Fig. 2. The atoms lie in the $xy$-plane and the dipole moment of the $|1\rangle \to |2\rangle$ transition points into the $z$-direction. For the sake of simplicity we assume that the dressing laser Rabi frequency is uniform and real, i.e. we neglect the spatial dependence of the laser phase. We are interested in the question whether the effective interaction energy is the sum of binary interactions or whether there are three-body interactions involved. The single-body energy is given by

$$\varepsilon_1 = -\text{Re}\left[\Delta_{|000\rangle|100\rangle}\right] = -\frac{4\Delta\Omega^2}{\gamma^2 + 4\Delta^2}, \tag{10}$$

which is the familiar light shift. Analogously, we obtain the two-atom interaction potential

$$\begin{aligned} \varepsilon_2 &= -\text{Re}\left[\Delta_{|100\rangle|110\rangle}\right] - \varepsilon_1 \\ &= \frac{8\Omega^2}{\gamma^2 + 4\Delta^2} \frac{(V_{12} + \Delta)(\gamma G_{12} + 4\Delta V_{12})}{(\gamma + G_{12})^2 + 4(\Delta + V_{12})^2}. \end{aligned} \tag{11}$$

As one can observe in Fig. 2, for very small interparticle distances, $kr \ll 1$, $\varepsilon_2$ shows a characteristic flat-top shape, while for large $kr$ one finds $\varepsilon_2 \to \frac{2\Omega^2}{\Delta^2} V_{12}$.

The phenomenology is similar to the dressing of Rydberg states. The difference is that the strength of the dressed state potential is proportional to $\Omega^2/\Delta^2$ rather than $\Omega^4/\Delta^4$ [24]. This is a consequence of the fact that (typically) the dominant interaction between Rydberg states is not exchange but rather a density-density interaction, so that two atoms have to be virtually excited to interact. A further difference is that the interaction energy of a dressed many-body state $|\mathcal{C}\rangle$ cannot be constructed as the sum of binary interactions [46]. This is a direct consequence of the fact that the dressing laser (virtually) excites collective states whose structure strongly depends on the particular arrangement of atoms in state $|1\rangle$. In Fig. 2 we display the three-body potential $\varepsilon_3 = -\text{Re}\left[\Delta_{|110\rangle|111\rangle}\right] - 2\varepsilon_2 - \varepsilon_1$. It saturates when $kr \to 0$ while for large separations it behaves approximately as $\varepsilon_3 \to \frac{\Omega^2}{2\Delta^3}(G_{12}^2 - 12V_{12}^2)$. **Similar considerations hold for the imaginary part of Eq. 9, i.e. also the dissipation rates display in general a many-body character.**

*Many-body MW spectroscopy* — The energy spectrum of the dressed state manifold, which is inaccessible through optical spectroscopy of the $|1\rangle \to |2\rangle$ transition, can be probed by coupling the transition $|0\rangle \to |1\rangle$ with a MW field of detuning $\Delta_{\text{MW}}$ and Rabi frequency $\Omega_{\text{MW}}$. Examples for the experimental realization of such a protocol are given in Refs. [19, 32, 47]. We assume that the MW Rabi frequency is **weak compared to the (collective) dephasing rate, $\Omega_{\text{MW}} \ll (\Omega/\Delta)^2\gamma$,** which allows us to adopt a description of the dynamics in term of rate equations [48–50]. The transition rate between two configurations $|\mathcal{C}\rangle$ and $|\mathcal{C}'\rangle$ is only non-zero when they differ by one atom in the $|1\rangle$ state (e.g. the ones displayed in Fig. 1b) and is given by

$$\Gamma_{\mathcal{C}\to\mathcal{C}'} = \frac{2\,\Omega^2_{\text{MW}} \text{Im}\left(\Delta_{\mathcal{C}\mathcal{C}'}\right)}{\left[\text{Im}\left(\Delta_{\mathcal{C}\mathcal{C}'}\right)\right]^2 + \left[\Delta_{\text{MW}}\left(n_{\mathcal{C}'} - n_{\mathcal{C}}\right) - \text{Re}\left(\Delta_{\mathcal{C}\mathcal{C}'}\right)\right]^2},$$

where $n_{\mathcal{C}}$ is the number of atoms in state $|1\rangle$ contained in configuration $|\mathcal{C}\rangle$. The resulting rate equations permit the calculation of the density of atoms in state $|1\rangle$, $n_1(t)$, as well as the MW absorption rate, $\partial_t n_1(t)$. In Fig. 3 **we consider for the sake of simplicity a lattice system** and show data for a chain of 40 atoms. Here we consider two situations: one where the dressing laser is irradiated from the top, i.e. the laser phases are identical for each atom (Fig. 3b), and from the left, i.e. the laser phase changes from atom to atom (Fig. 3c). In both cases we vary the lattice spacing $a$, interpolating between the strongly interacting and the non-interacting limit ($ka \to \infty$).

For short excitation times, both $n_1(t)$ and the absorption signal exhibit a peak near $\Delta^0_{\text{MW}} = \Omega^2/\Delta = 0.5 \times \gamma$

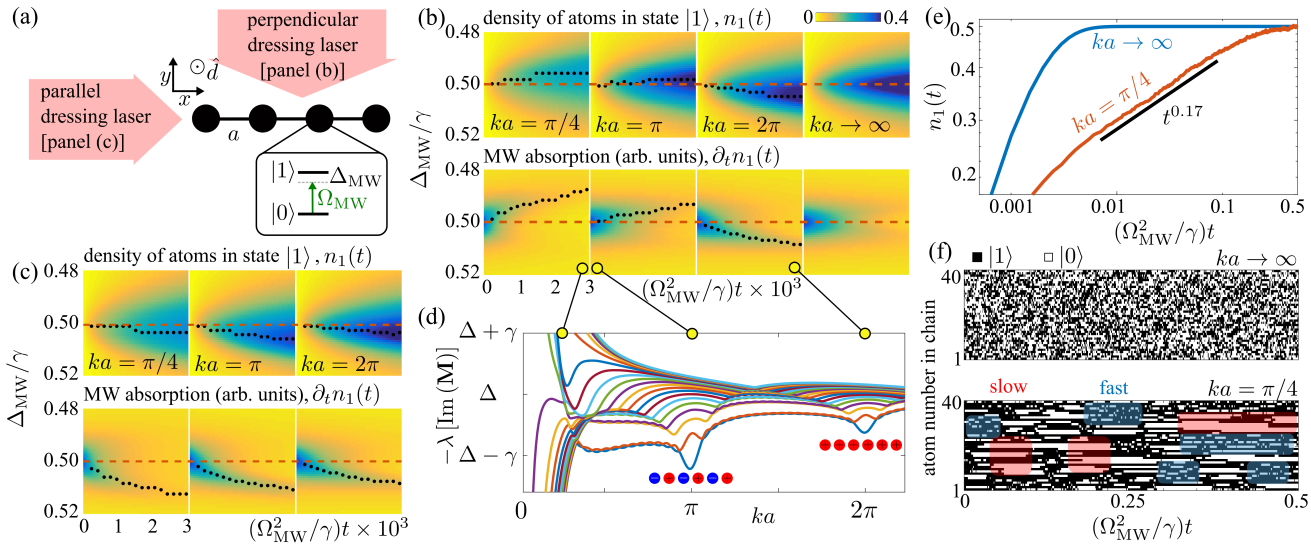


FIG. 3. **MW spectroscopy.** (a) One-dimensional chain of atoms with spacing $a$. The atomic transition dipoles are pointing into the $z$-direction, and the laser is either irradiated perpendicular or parallel to the chain. (b) Time-evolution of the excitation density (upper row) and the MW absorption rate (lower row) for various values of the lattice spacing $ka$ and the MW detuning $\Delta_{\rm MW}$. The black dotted line indicates the position of the maximum (peak) value at a given time. The laser parameters are $\Delta = 50\gamma$ and $\Omega = 5\gamma$ with the laser being irradiated perpendicular to the chain of $N = 40$ atoms. (c) Same as in (b), but with the laser irradiated parallel to the chain. (d) Eigenvalues of the imaginary part of the matrix $\mathbf{M}$ [Eq. (7)] for $N = 20$. The colored dots indicate the phase pattern of the atoms in the lowest energy eigenstate at $ka = \pi$ and $ka = 2\pi$, i.e. alternating vs. uniform. (e) Long time-evolution of the excitation density of a resonantly excited chain ($\Delta_{\rm MW} = \Delta^0_{\rm MW}$) of $N = 40$ atoms. In the non-interacting limit ($ka \to \infty$) the approach to the steady-state follows an exponential with time constant $\tau = \frac{1}{4}(\Omega/\Delta)^2(\gamma/\Omega^2_{\rm MW})$. For a dense gas ($ka = \pi/4$) relaxation slows down dramatically and the time-dependence of the density follows a power-law. (f) Typical trajectories (white/black: atom in state $|0\rangle/|1\rangle$) are distinctively different in the two regimes. The non-interacting one is without structure, while in the interacting case collective effects manifest in heterogeneous dynamics. Here relaxation timescales are non-uniform and may vary drastically in different space-time regions. Examples for regions featuring fast/slow relaxation are marked in blue/red.

(the light-shift in the non-interacting limit). As time passes the position of the peak (marked by a black dotted line in the individual sub-panels) departs from $\Delta^0_{\rm MW}$, indicating the presence of interactions. In the density plots of the excitation density one observes only a small shift as time progresses. Interaction effects are far more pronounced in the MW absorption spectrum, where observe a clear shift of the absorption line when interactions are present.

The direction of the shift depends on the angle between the dressing laser and the atomic chain. This effect can be qualitatively understood by inspecting the eigenvalues of the imaginary part of the matrix $\mathbf{M}$, Eq. (7), which can be interpreted as the energy of collective states that are off-resonantly excited by the dressing laser. In Fig. 3d we show for the purpose of illustration these eigenvalues for a chain of 20 atoms as a function of $ka$, assuming that all of the 20 atoms are in the state $|1\rangle$ and thus participate in the dressing [51].

A special situation is encountered when the lattice spacing is a multiple of $\pi/k$. Here, the excitation spectrum displayed in Fig. 3d possesses pronounced dips with lower energy. They decrease effectively the detuning and thus enhance the coupling of the dressing laser to the collective states. These dips occur because at these points the interaction matrix $V_{\alpha\beta}$ [Eq. (3)] has either only positive entries or the sign of the entries alternates as $(-1)^{\alpha-\beta}$. The eigenstates corresponding to the collective excitation with lowest energy thus have an alternating or uniform phase pattern, respectively, as depicted in Fig. 3d through the red/blue circles. This phase pattern influences the coupling strength of the dressing laser to the collective states and thereby the collective light shift. This is particularly visible in Fig. 3b,c for $ka = \pi$. When the dressing laser is irradiated from the side, its phase pattern is alternating and therefore the time-dependent shift of the MW absorption is drastically different to the case in which each atom experiences a uniform laser phase. This is a clear signature for the presence of collective excitations at the heart of the dressing of dense atomic gases. In contrast, such sensitivity to the laser phase does not occur in conventional Rydberg dressing, as excited atoms interact here via a density-density interaction, which is phase insensitive.

Finally, let us briefly discuss a significant difference between the non-interacting and strongly interacting re-

gime. In the latter, the stationary state value of the excitation density — which in all cases is 0.5 — is reached extremely slowly, as shown in Fig. 3e. Moreover, the approach to stationarity proceeds via a power-law. At the level of individual trajectories, i.e. the time and spatially resolved excitation density, strong interactions manifest in heterogeneous dynamics, where different space-time regions relax at vastly different speeds (see Fig. 3f). This so-called dynamic heterogeneity is often found in glass-forming substances [52].

*Conclusions and outlook* — We have developed a theory of dressed dense atomic gases and showed that these systems feature many-body interactions as well as delocalized excitations and non-trivial relaxation behavior that can be probed via MW spectroscopy. In our simulations we so far focused on small ensembles in one dimension. In the future it would be interesting to shed further light on the observed power-law relaxation and also to study large (inhomogeneous and disordered) three-dimensional systems for which the theory is applicable as well. Moreover, it would be desirable to study coherent collective quantum phenomena, moving away from the limit described via rate equations.

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