

# A Simulation-based Optimisation Approach for Inventory Management of Highly Perishable Food

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**Abstract:** The taste and freshness of perishable foods decrease dramatically with time. Effective inventory management requires understanding of market demand as well as balancing customers needs and preferences with products' shelf life. The objective is to avoid food overproduction as this leads to waste and value loss. In addition, product depletion has to be minimised, as it can result in customers renegeing. This study tackles the production planning of highly perishable foods (such as freshly prepared dishes, sandwiches and desserts with shelf life varying from 6 to 12 hours), in an environment with highly variable customers demand. In the scenario considered here, the planning horizon is longer than the products' shelf life. Therefore, food needs to be replenished several times at different intervals. Furthermore, customers demand varies significantly during the planning period. We tackle the problem by combining discrete-event simulation and particle swarm optimisation (PSO). The simulation model focuses on the behaviour of the system as parameters (i.e. replenishment time and quantity) change. PSO is employed to determine the best combination of parameter values for the simulations. The effectiveness of the proposed approach is applied to some real-world scenario corresponding to a local food shop. Experimental results show that the proposed methodology combining discrete event simulation and particle swarm optimisation is effective for inventory management of highly perishable foods with variable customers demand.

## 1 INTRODUCTION

Effective planning is important in production systems that aim at effective management and coordination of related activities and resources for an organisation (Makui et al., 2016). A common aim in such systems is to achieve optimal production planning and inventory management to meet (often variable) products demand over the planning horizon (Ramezani et al., 2012). Within the food production sector, producers aim at delivering optimal planning to manage supply/demand effectively. Perishability is a key attribute that cannot be ignored in food supply chain management. Critical decisions must be made regarding the replenishment of food items in the right time and quantity with the goal of maximising profit while minimising complete stock depletion and waste.

This paper tackles inventory management of highly perishable food with a very limited shelf life and with variable customers demand. We focus on a scenario in the hospitality sector, that of shops selling foods such as freshly prepared dishes, sandwiches and

desserts. The production planning period is longer than the shelf life of the products. This means that products need to be replenished frequently during the planning period. In addition, customers demand for the foods varies greatly over time. A particular strain is put into production, when rare and/or extreme external events occur. For instance, when demand changes dramatically due to weather conditions, special events (e.g. holiday season, football matches) and promotions. Production planning and inventory management under such conditions is difficult and subject to high uncertainty.

Planning problems involving such complexity often require the use of simulation models to understand the behaviour of the system as parameters change. The abstracted simulation model is capable of encompassing the stochasticity and the variability found in the real-world, therefore assisting in addressing the problem's complexity. Also, detailed reports regarding the dynamic of the system and the simulation outputs can be extracted for further analysis and to be employed as inputs to future optimisation exercises

(Duong and Wood, 2018). We apply discrete event simulation (DES) to understand the behaviour of the system as parameters (i.e. replenishment time and quantity) change. Discrete events at particular times such as sales and food expiring are simulated as the changes of state in the system. A Particle Swarm Optimisation (PSO) algorithm (Shi and Eberhart, 1998) is implemented to determine the best combination of parameter values that leads to an optimal solution. We investigate the optimisation of replenishment time and quantities in the tackled scenario. Experimental results using real-world data from a food shop demonstrate the effectiveness of the proposed method for tackling inventory management of highly perishable foods with variable customers demand.

Problems related to the planning of perishable inventories and ordering policies arise in different sectors including food, chemicals, drugs and blood banks (Vila-Parrish et al., 2008). Regarding modelling techniques and mathematical methods, linear programming, integer or mixed integer programming are the most dominant methods, please refer to (Soto-Silva et al., 2016) for model details. Literature reviews on perishable inventory systems can be found in (Ferguson et al., 2007), (Bakker et al., 2012), (Yu et al., 2012), (Bushuev et al., 2015). Perishable inventory research to date has made a lot of progress on food sectors of grocery retailers context, where the shelf life usually ranges from several days to months. To the best of our knowledge, this is the first attempt to address the problem of inventory planning of perishable food with shelf life of just a few hours.

The remainder of this paper is organised as follows: the problem is described in detail in Section 2; the solution method is presented in Section 3; the computational experiments follow in Section 4; finally conclusions are drawn in the last section.

## 2 PROBLEM DESCRIPTION

We model the food production of a food shop that produces and sells fresh prepared dishes, sandwiches and desserts. The shop usually opens at 7:00AM selling items produced early that morning and leftover items produced the night before. It is expected that the number of leftover items from the night before is enough to cover demand until around 10:00AM. After that, food items produced after the shop opens are served to customers. Depending on the demand, product shelves will be replenished several times in a day. To aim for freshness, food items are marked as expired after 6 to 12 hours (exact time depends on the product type) on the shelf. The expired product is

subsequently thrown away. If food items are prepared and placed in the shelves too early and not sold out before they have perished then waste may occur. However, items not prepared on time to supply demand may result in customers renegeing (i.e. customers leaving because the preferred item is out of stock). Due to this dynamic complexity, the store manager faces a challenging problem of both preparing enough items and reducing waste. The goal is to determine the replenishment time and replenishment quantity for each item that needs to be prepared in advance, in order to minimise food wastage and running out of inventory (i.e., ‘a stock out’).

The items demand may change dramatically in a day or week as shown in Figures 1 and 2, respectively. In this paper we only predict the next day hourly demand. Prediction beyond that is out of the scope of this paper because of the sensitivity to the effect of store promotions, weather, traffic conditions and special events (e.g. holiday, football match) which is not yet considered here.

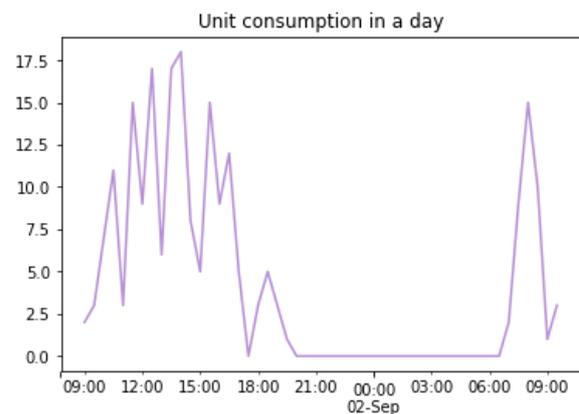


Figure 1: Demand fluctuates considerably in a day.

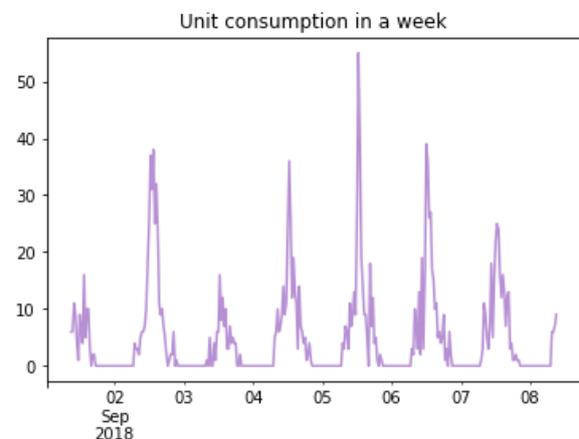


Figure 2: Demand fluctuates considerably in a week.

### 2.1 Discrete Event Simulation Model

The following notation is used in the DES model:

- $e$ : the number of items expiring.
- $r$ : the number of customers renegeing.
- $w_e$ : the weight of  $e$  in the objective function .
- $w_r$ : the weight of  $r$  in the objective function .
- $t_i$ : the  $i$ th replenishment time,  $t_i < t_{i+1}$  and  $t_i \in T$ .
- $q_i$ : the replenishment quantity at  $t_i$ ,  $q_i \in Q$ .
- $(t_i^l, t_i^u)$ : the lower bound and the upper bound of  $t_i$  respectively, in minutes.
- $(q_i^l, q_i^u)$ : the lower bound and the upper bound of  $q_i$  respectively, in units.
- $l$ : the shelf life of an item, in minutes.
- $(r_l, r_u)$ : the lower bound and the upper bound for calculating  $(t_i^l, t_i^u)$ , in minutes.

A solution to the inventory management problem is encoded by two n-tuples  $s = \{t_0, t_1, t_2, \dots, t_n\}, \{q_0, q_1, q_2, \dots, q_n\}$ , representing replenishment times and replenishment quantities, respectively.

The problem can be formally defined as follows:

$$\min w_e e + w_r r \tag{1}$$

subject to

$$t_i^l \leq t_i \leq t_i^u \quad \forall i \in I \tag{2}$$

$$q_i^l \leq q_i \leq q_i^u \quad \forall i \in I \tag{3}$$

## 3 SOLUTION METHOD

We use the solution framework illustrated in Figure 3 to explore optimal replenishment time and quantity for each item. The framework has 2 phases: 1) generate data for the simulation and 2) execute simulation-based optimisation. The first phase for generating data, denoted DG, generates a set of traces for the simulation according to predicted demand. For this, 5 replications are performed in this work, hence generating 5 traces. The aim of performing multiple replications is to produce multiple samples in order to obtain a better estimate of mean performance. The second phase, applies discrete event simulation to simulate each generated trace and the best combination of parameters values (i.e. replenishment time and quantity) for the simulation is determined by the PSO.

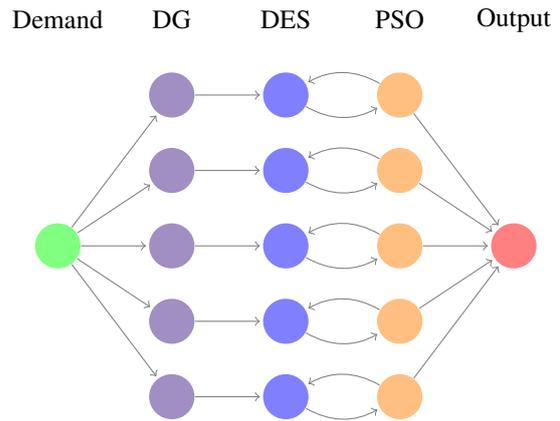


Figure 3: Solution framework. The first layer (green circle) represents the demand; the second layer is the data generation, followed by the discrete event simulation phase (DES) coupled with the particle swarm optimisation (PSO).

### 3.1 Data Generation

We generate trace data based on predicted hourly sales. Each trace contains transaction time and the number of units sold at that time. We first analyse historical customer arrival intervals for an item within a recent month, as an example shown in Figure 4.

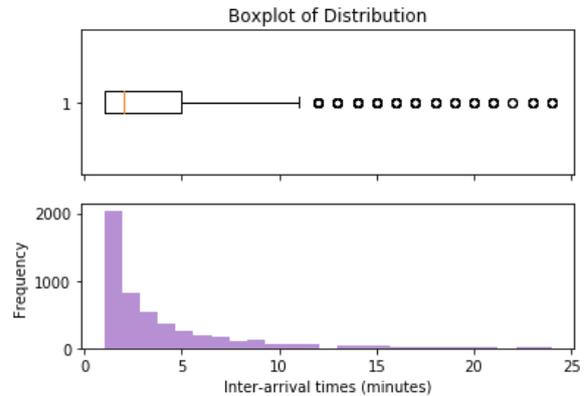


Figure 4: Frequency of customer arrival intervals for a given item in a recent month.

The upper part of the graph shows the distribution box plot. In the lower part, the horizontal axis shows the interval in minutes of customer arrivals while the vertical axis shows the frequency. The figure shows that the inter-arrival times distribution is right skewed with fat tails (with mean=3.67, median=2). In fact, the shape of the distribution is similar to an exponential distribution, namely Erlang distribution when  $K = 1$ . Further exploration suggested that Erlang distribution (mean=3.67, and  $K=0.85$ ) fits well with the historical data. We use a QQ (quantile quantile) plot of inter-arrival distribution with the Erlang distribu-

tion to do a more thorough check for the distribution. It can be seen that most dots fit the line, suggesting the shape of the data matches the shape of the probability density function of Erlang.

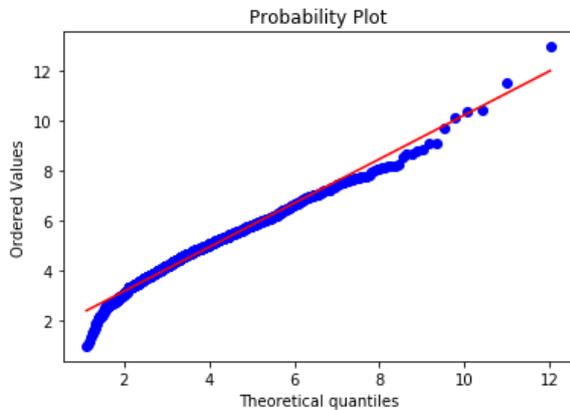


Figure 5: QQ (quantile quantile) plot of historical data and simulated Erlang data.

Next, we generate the number of units that would be sold in each transaction. This number is randomly drawn with some probability from historical data. Note that not all numbers are randomly generated as we have to ensure the total units in each hour matches predicted values. Therefore, it is necessary to fix some values instead of generating them randomly.

## 3.2 Simulation-based Optimisation

At this stage, DES is applied to simulate each generated trace (Section 3.1). For each simulation with certain parameters values (i.e. replenishment time and quantity), we calculate the objective function value (Section 2.1), which is the solution fitness in the PSO, and update parameters values. Finding the best combination of parameters values (i.e. replenishment time and quantity) for the simulation is guided by the PSO. The reason to use simulation to obtain the solution fitness is twofold: firstly, simulation can generate random samples; secondly, due to the stochastic and dynamic complexity nature, the objective function is not easy to be mathematically expressed and computed directly.

### 3.2.1 Discrete Event Simulation

DES has been widely applied in production planning due to its capability for modelling uncertainties that lie in a planning process. A DES models the operation of a system as a discrete sequence of events in time. Each event, such as a customer consuming a product, occurs at a particular time and changes the state of the

system. In the context of inventory management, the optimisation procedure can be used to find the most suitable parameters for the simulation in order to find the best possible solutions to a problem. Optimisation techniques that have been applied to inventory management can be widely classified into mathematical programming (e.g. linear/non-linear programming) and direct search methods (e.g. gradient based, statistical models, (meta)-heuristics). Please refer to the review papers by (Arisha and Abo-Hamad, 2010) (De Meyer et al., 2014) for more details.

### 3.2.2 Particle Swarm Optimisation

We implemented PSO to determine the best combination of parameters values for the simulation model (Section 2.1). The reasons to adopt PSO as optimiser for this problem are threefold: PSO is simple in implementation; PSO uses simple fast to execute mathematical operators and is efficient in memory requirement, both essential for simulation based optimisation; and PSO is a general parameter value optimisation method which requires no assumptions about the problem being optimised.

PSO is a population-based meta-heuristic introduced by Eberhart and Kennedy (Kennedy, 1995) that has been successfully applied to several inventory management applications (Tsai and Yeh, 2008) (Xu et al., 2013) (Park and Kyung, 2014). PSO is inspired on the social behaviour patterns of a biological group such as a flock of birds or a school of fish. In the PSO method, a decision variable is regarded as a particle, and the population of decision variables is considered as a swarm. PSO was initially proposed for continuous function optimisation. In this work we truncate continuous values to integers when implementing PSO as the number of units and time are both expressed by integer values. Our experiments here indicate that this truncation does not affect significantly the performance of the method, as it was also found in other works (Laskari et al., 2002).

PSO is initialised with a group of random particles (solutions) and then the algorithm searches for optima by updating generations of particles. In each iteration of PSO, each particle is updated within its given bounds by following two “best” values. The first one is the best solution that the particle and its neighbours have achieved so far (local best). The other one is the best solution found so far by any particle in the whole population (global best). Based on these two best values, a particle updates its velocity and positions. More details of how PSO works can be found in (Kennedy, 1995) and (Shi and Eberhart, 1998). There are many variants (Rini et al., 2011) of PSO, but in this paper we implemented the standard

one (Kennedy, 1995). The fitness of a particle is computed by means of the simulation as mentioned previously, hence the approach being a simulation-based optimisation technique.

## 4 EXPERIMENTS

The proposed solution method was implemented in Python and run on a PC with Intel i7 2.40GHZ processor and 4GB RAM, similar computing equipment available in a typical food shop in the scenario considered here.

### 4.1 Problem Instance Data

A set of problem instances reflecting characteristics of the real-world scenarios considered was generated by a prediction model. Each instance contains predicted hourly sales of an item in a day. Five instances of each item are generated. The instances groups are called *I6* and *I12*, where the number corresponds to shelf life. For example, *I6* are the instances where the shelf life of the item is 6 hours.

### 4.2 Parameters Tuning

Considering the two objectives of minimising waste and minimising the number of customers renegeing, we can think of trade-off solutions exhibiting a compromise between these two objectives. This is because arguably minimising waste (producing less) could be in conflict with minimising renegeing (producing more). However, the desirable overall optimal solution for our case will obviously have zero waste and zero customers renegeing.

Since the desired optimal solution will have 0 as the value for waste and customers renegeing, the goal is to find the parameter values that quickest converge to 0 in the objective function. The *Irace* package (López-Ibáñez et al., 2011) is applied for parameter tuning using 4 selected instances (*I6\_1, I6\_2, I12\_1, I12\_2*).

A standard implementation of the PSO algorithm requires 4 parameters:

- Swarm size ( $n$ ): Number of particles in the population.
- Inertia ( $\omega$ ): Inertia weight in a particle's movement.
- Personal best attraction ( $\phi_i$ ): Weight for particle's pull towards its own best solution achieved so far.

- Neighbour best attraction ( $\phi_g$ ): Weight for pull towards the global best solution.

The parameters, their range considered in the tuning (based on some preliminary runs) and the best values found are given in Table 1. The maximum number of experiments is set to 1000 and other parameters for *Irace* are set to default values.

Table 1: Parameter Tuning for the PSO Using *Irace*.

Parameters	Type	Range	Best
$n$	C	(5, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 120, 140, 160, 180, 200, 240, 280, 320)	50
$\omega$	R	(-2, 2)	0.637
$\phi_i$	C	(-4, -3.5, -3, -2.5, -2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4)	0.5
$\phi_g$	C	(-4, -3.5, -3, -2.5, -2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4)	1.5

C: Categorical  
R: Real

### 4.3 Algorithm Performance

In order to analyse the performance of the proposed solution method, we test the algorithm on 10 problem instances using the best-suited parameter values given in Table 1. In order to obtain statistically sound results, all experiments are conducted with 10 independent runs (mean values were recorded) over all 10 problem instances. To obtain a better estimate of mean performance, each experiment run is based on the mean values of 5 simulations. Each simulation is terminated after reaching an optimal solution (i.e. with objective function value equal to 0 representing the best fitness). For each instance, all 10 runs start with the same initial solution created at random.

The computation time for each problem instance is given in Table 2. It can be seen that all instances were solved well under 2 minutes. Solving the group

Table 2: Computation Times.

Instance	Time(s)
I6_1	92.4
I6_2	81.6
I6_3	70.8
I6_4	60
I6_5	65
I12_1	31.2
I12_2	42
I12_3	34.8
I12_4	49.2
I12_5	34

of instances *I6* requires more decision variables (because of more frequent replenishment), hence taking longer computation time compared to instances *I12*.

We now describe some examples of replenishing plans. Figure 6), is an example of such plan for an instance with 6 hours shelf life. The horizontal axis shows the length of the planning period (i.e. 1 day), while the vertical axis shows the number of units of an item. The *Unit consumption* line corresponds to the total number of units sold during the day. The *Unit left* line corresponds to the units available on the shelf. The *Reneging* line is for the number of customers renegeing at different times during the day. The *Garbage size* line is the cumulative sum of items that have been thrown away due to expiration.

Figure 6 shows a non-optimal solution for instance *I6\_1* found during the algorithm execution and before reaching an optimal solution. This figure captures the simulation scenario in which a large number of items were replenished (unit left line) at around 10 am. This large replenishment was sufficient to cover the high demand (unit consumption line) from 11 am to around 3 pm. Due to the faltering demand from 3pm onward, the number of units left is higher than the consumption by 4pm. However, because of the 6 hours shelf life of the item, those leftovers have to be thrown away at 4pm. Since there was no fresh items produced and placed timely on the shelf at 4pm, some customers left because the item they wanted was out of stock. A similar situation occurred later at night. It can be seen that this is a non-optimal solution or inventory management plan because the replenishment time and quantity have led to both item wastage and customer renegeing.

Figure 7 shows an optimal solution for the same instance *I6\_1* discussed above, in which there is no item wastage and no customers renegeing. The replenishment times are represented by the vertical bars and the corresponding replenishment time and quantity are given in the legend. In this optimal solution, it can be seen that although the unit left line is well

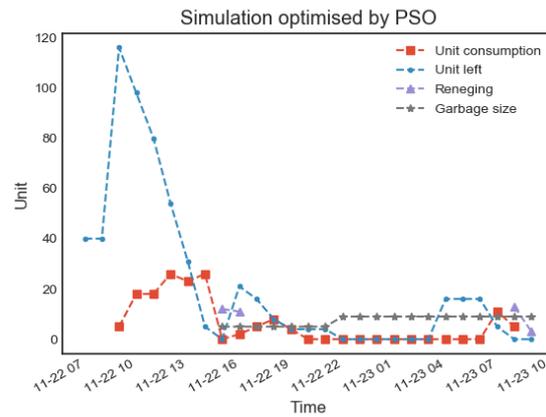


Figure 6: Non-optimal solution for instance *I6\_1*.

above the unit consumption line, there is no wastage or renegeing because replenishment has been done effectively to supply demand. A typical convergence rate of one simulation (i.e. simulation based on 1 trace) of this solution is given in Figure 8. The horizontal axis corresponds to number of iterations while the vertical axis corresponds to fitness values. It can be seen in the figure that convergence is very fast in the first 250 iterations and it slows down after that. Finally, an optimal solution was found just before 600 iterations.

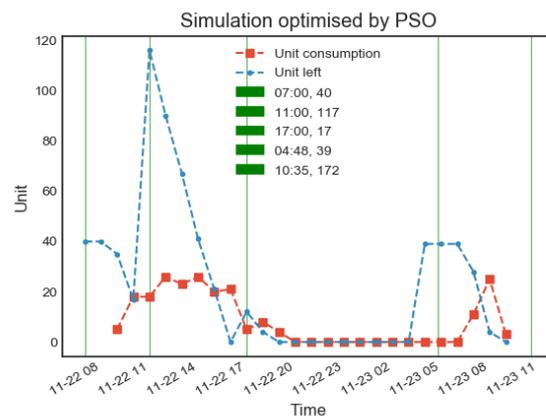


Figure 7: Optimal solution for instance *I6\_1*.

Another example of a replenishment plan, this time for an instance for an item with 12 hours shelf life, is shown in Figure 9. This plan is a non-optimal solution obtained before an optimal solution was found for problem instance *I12\_1*. Similar to the previous example, this figure captures the simulation of a scenario in which a large number of items are replenished at around 10am. Items have not been sold out by 10pm and hence they are thrown away after the 12 hours shelf life passes. A number of items were produced around 11pm. Demand increased from 7am in the next day and customers consumed all items by

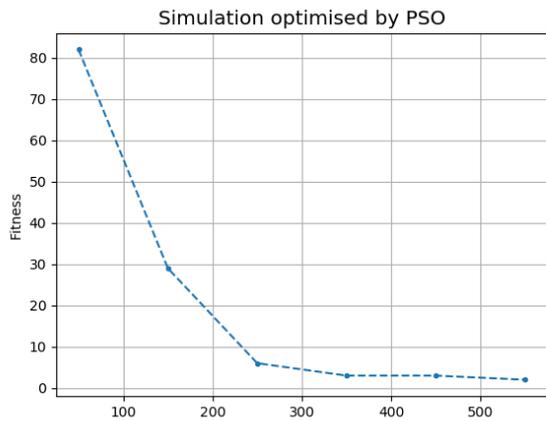


Figure 8: Convergence rate of solving instance I6\_1.

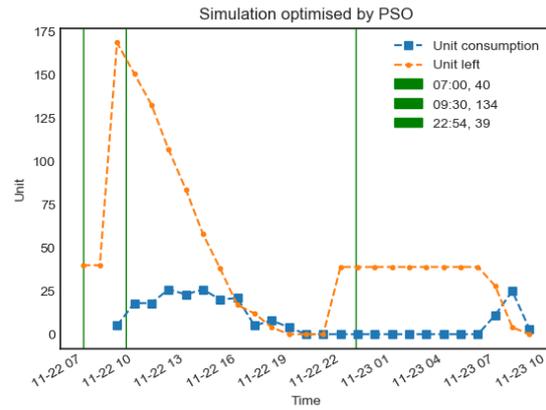


Figure 10: Optimal solution for instance I12\_1.

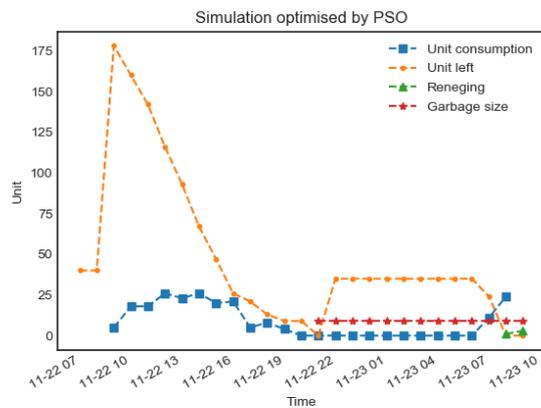


Figure 9: Non-optimal solution for instance I12\_1.

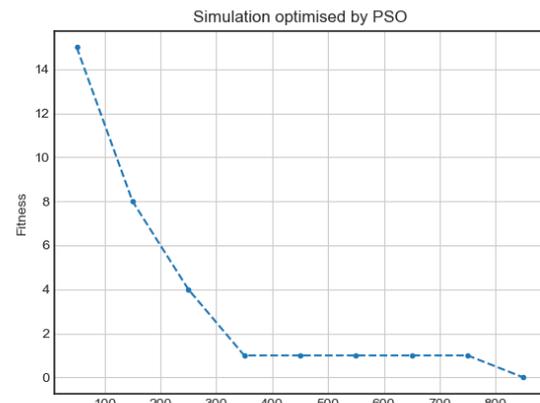


Figure 11: Convergence rate of solving instance I12\_1.

around 9am. As no replenishing takes place since then, customers that arrived after 9am left after not being able to supply their demand.

Figure 10 shows an optimal replenishment plan with no item wastage or customers renegeing. The convergence rate of one simulation for this solution is shown in Figure 11. This figure shows that convergence is very fast within the first 350 iterations and an optimal solution is found before 900 iterations.

## 5 CONCLUSIONS

In this paper, we addressed an inventory management problem considering a scenario in which food items have very short shelf life and they need to be replenished several times during the day in order to satisfy demand, avoid wastage and avoid customers renegeing. This problem arises in many restaurants and other places in the hospitality sector. In the scenario tackled here, the planning horizon is longer than the shelf life of the food items. In addition, demand varies considerably within the planning horizon. A replen-

ishment plan consists of determining the times during the planning horizon in which food items should be replenished and the corresponding quantities in order to minimise waste and the number of customers renegeing.

We model this inventory problem using Discrete Event Simulation (DES) to understand the dynamics of the system as parameters change. Then, Particle Swarm Optimisation (PSO) is employed to determine the best combination of parameter values for the simulations. Experimental results on 10 problem instances have demonstrated the effectiveness of the proposed simulation-based optimisation approach for solving different problem instances. This solution framework could be easily adapted to other similar inventory replenishment scenarios. For instance, to minimise perishable wastage in restaurants, supermarkets and hospitals. Particular benefit could also be obtained when venues need to comply with more strict regulations due to the nature of the clientele, such as pregnant women, immunocompromised individuals, and transplant patients.

Although the solution quality and computation

time in this work are acceptable by the typical store manager, the abstract model created for simulation still does not yet reflect the entire complexity of the shop production dynamics. Further work will be undertaken to improve the current model to enable even better decision support. One aspect to be considered is the inclusion of the food preparation process and defrosting times, rather than just focusing on determining replenishment times and quantity. Also, an in-depth study comparing the PSO to other optimisation methods could also be an interesting future research direction.

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## REFERENCES

- Arisha, A. and Abo-Hamad, W. (2010). Simulation optimisation methods in supply chain applications: a review.
- Bakker, M., Riezebos, J., and Teunter, R. H. (2012). Review of inventory systems with deterioration since 2001. *European Journal of Operational Research*, 221(2):275–284.
- Bushuev, M. A., Guiffrida, A., Jaber, M., and Khan, M. (2015). A review of inventory lot sizing review papers. *Management Research Review*, 38(3):283–298.
- De Meyer, A., Cattrysse, D., Rasinmäki, J., and Van Orshoven, J. (2014). Methods to optimise the design and management of biomass-for-bioenergy supply chains: A review. *Renewable and sustainable energy reviews*, 31:657–670.
- Duong, L. N. K. and Wood, L. C. (2018). Discrete event simulation in inventory management. In *Encyclopedia of Information Science and Technology, Fourth Edition*, pages 5335–5344. IGI Global.
- Ferguson, M., Jayaraman, V., and Souza, G. C. (2007). Note: An application of the eoq model with nonlinear holding cost to inventory management of perishables. *European Journal of Operational Research*, 180(1):485–490.
- Kennedy, R. (1995). J. and eberhart, particle swarm optimization. In *Proceedings of IEEE International Conference on Neural Networks IV*, pages, volume 1000.
- Laskari, E. C., Parsopoulos, K. E., and Vrahatis, M. N. (2002). Particle swarm optimization for integer programming. In *Evolutionary Computation, 2002. CEC'02. Proceedings of the 2002 Congress on*, volume 2, pages 1582–1587. IEEE.
- López-Ibáñez, M., Dubois-Lacoste, J., Stützle, T., and Birattari, M. (2011). The irace package, iterated race for automatic algorithm configuration. Technical report, Technical Report TR/IRIDIA/2011-004, IRIDIA, Université Libre de Bruxelles, Belgium.
- Makui, A., Heydari, M., Aazami, A., and Dehghani, E. (2016). Accelerating benders decomposition approach for robust aggregate production planning of products with a very limited expiration date. *Computers & Industrial Engineering*, 100:34–51.
- Park, K. and Kyung, G. (2014). Optimization of total inventory cost and order fill rate in a supply chain using pso. *The International Journal of Advanced Manufacturing Technology*, 70(9-12):1533–1541.
- Ramezani, R., Rahmani, D., and Barzinpour, F. (2012). An aggregate production planning model for two phase production systems: Solving with genetic algorithm and tabu search. *Expert Systems with Applications*, 39(1):1256–1263.
- Rini, D. P., Shamsuddin, S. M., and Yuhaniz, S. S. (2011). Particle swarm optimization: technique, system and challenges. *International journal of computer applications*, 14(1):19–26.
- Shi, Y. and Eberhart, R. (1998). A modified particle swarm optimizer. In *Evolutionary Computation Proceedings, 1998. IEEE World Congress on Computational Intelligence.. The 1998 IEEE International Conference on*, pages 69–73. IEEE.
- Soto-Silva, W. E., Nadal-Roig, E., González-Araya, M. C., and Pla-Aragones, L. M. (2016). Operational research models applied to the fresh fruit supply chain. *European Journal of Operational Research*, 251(2):345–355.
- Tsai, C.-Y. and Yeh, S.-W. (2008). A multiple objective particle swarm optimization approach for inventory classification. *International journal of production Economics*, 114(2):656–666.
- Vila-Parrish, A. R., Ivy, J. S., and King, R. E. (2008). A simulation-based approach for inventory modeling of perishable pharmaceuticals. In *Simulation Conference, 2008. WSC 2008. Winter*, pages 1532–1538. IEEE.
- Xu, J., Zeng, Z., Han, B., and Lei, X. (2013). A dynamic programming-based particle swarm optimization algorithm for an inventory management problem under uncertainty. *Engineering Optimization*, 45(7):851–880.
- Yu, Y., Wang, Z., and Liang, L. (2012). A vendor managed inventory supply chain with deteriorating raw materials and products. *International Journal of Production Economics*, 136(2):266–274.