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Cooperative R&D for a New Product under Convex Production Costs[*,†]

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Abstract: The role of knowledge spillover for cooperative research and development (R&D), where firms commit before R&D about sharing R&D outcomes and choosing joint profit maximising R&D investments, is well known. In a duopoly model of product innovation with a stochastic non-tournament R&D process, we show that the firms may prefer cooperative R&D compared to non-cooperative R&D in the presence of convex production costs, even if there is no knowledge spillover. Thus, we provide a new reason for cooperative R&D. Consumer surplus and the expected welfare can be higher under cooperative R&D compared to non-cooperative R&D. We further show that the convex production costs create the incentive for technology licensing ex-post R&D. In the presence of licensing ex-post R&D, the firms prefer joint profit maximising R&D investments with the option for licensing ex-post R&D, while the consumers and the society may prefer cooperative R&D. Hence, a proper distribution scheme, such as a tax/subsidy policy, might be required to encourage firms to undertake cooperative R&D. Our analysis may provide some implications for vaccine research.

Key Words: Cooperative R&D; Non-cooperative R&D; Technology licensing; Welfare
JEL Classifications: D43; L13; L24; O31

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1 Introduction

The seminal papers by Katz (1986) and d'Aspremont and Jacquemin (1988) inspired the literature on cooperative R&D (reviewed below). With deterministic R&D, where success in R&D is certain and the marginal costs of production reduce with higher R&D investments, these papers show that cooperative R&D helps firms to internalise the effects of knowledge spillover and to avoid duplicative research by internalising the effects of a firm's R&D on the other firm.

Choi (1993) extends this literature by considering uncertain R&D, where the success in R&D is uncertain and the probability of success in R&D increases with higher R&D investment. He shows that cooperative R&D helps to internalise the effects of knowledge spillover but may increase the intensity of competition in the product market by sharing valuable information about the new technology. Hence, if knowledge spillover is not high, the firms may not have the incentive for cooperative R&D (Choi, 1993).

Considering uncertain R&D like Choi (1993), this paper examines the preference for cooperative R&D to find a new product *in the absence of knowledge spillover*. We use a duopoly model of product innovation with no knowledge spillover, stochastic non-tournament R&D process and convex production costs, creating soft capacity constraints (Cabon-Dhersin and Drouhin, 2020) and the possibility of technology licensing ex-post R&D,[1] to examine the firms', consumers' and the society's preference for cooperative R&D to find a new product. Our model

[1] Our focus on product innovation helps to convey our message in the simplest way. It is worth highlighting that firms invest significant amount in product innovations, yet the literature on cooperative R&D did not pay much attention to this aspect. Imai (1992) mentioned Japanese firms spend 40% of their research budget towards product innovation. 23.9 % of European Union enterprises did product innovations during 2012–2014. Although some papers on cooperative R&D considering stochastic R&D process, such as Choi (1993), can be considered for process and product innovations, the focus of the literature on cooperative R&D is mostly on process innovation.

of product innovation is similar to Federico, Langus and Valletti (2017, 2018), Denicolò and Polo (2018) and Choi (1993).

We consider the following four arrangements for our analysis:

Non-cooperation (NC): Here firms behave completely non-cooperatively. They choose their R&D investments to maximise their own profits and they neither commit before R&D about sharing R&D outcomes nor engage in technology licensing ex-post R&D.

Cooperative R&D (CO): Here firms commit before R&D about sharing R&D outcomes and choosing R&D investments to maximise joint profits. This is like research joint venture in Choi (1993), research joint venture cartel in Kamien, Muller and Zang (1992) and Erkal and Piccinin (2010) and research joint venture in Miyagiwa and Ohno (2002). Hence, this arrangement allows firms to perfectly coordinate their R&D efforts by choosing the joint profit maximising R&D investments and sharing the R&D outcomes. Like the previous literature, such as Choi (1993), we have ignored synergic benefits under this arrangement. It is intuitive that the presence of synergic benefits will increase its incentive.

Non-cooperative R&D with technology licensing (NCL): Here, like NC, firms choose their R&D investments to maximise their own profits and do not commit before R&D about sharing R&D outcomes. However, they may engage in technology licensing after R&D if both firms are not successful in R&D. This is like non-cooperative R&D with licensing in Marjit and Mukherjee (2004), Mukherjee (2005) and Miyagiwa (2009).

Joint profit maximising R&D investments with licensing (CL): Here, like NCL, firms do not commit before R&D about sharing R&D outcomes and may engage in technology licensing ex-post R&D if both firms are not successful in R&D. However, unlike NCL, here firms choose their R&D investments to maximise joint profits. It is like R&D cartel in Miyagiwa and Ohno (2002) with the possibility of licensing ex-post R&D. Like CO, we have ignored synergic benefits from cooperation in R&D investments, in order to avoid influencing our results by the synergic effect.

There could be another possibility where firms commit to share the R&D outcomes but determine the R&D investments to maximise their own profits. This situation is similar to the case of perfect knowledge spillover of R&D outcomes under non-cooperative R&D. We will discuss below that CO will dominate this arrangement. Hence, we ignore this case in our main analysis.

Hence, we consider three types of cooperation. CO, which is a commitment before R&D for cooperating on R&D investments and R&D outcomes. NCL, which considers technological cooperation after R&D through technology licensing. CL, which is a mix of cooperation before and after R&D – cooperation on R&D investments and technological cooperation after R&D through technology licensing.

We contribute to the literature by deriving the following results. First, we show that the firms may prefer CO compared to NC in the presence of convex production costs, even if there is no knowledge spillover. If the convex cost makes the duopoly industry profit higher than the monopoly profit, CO is always better than NC (Proposition 4(iii)). Thus, we provide a new reason for cooperative R&D. Consumer surplus and the expected welfare can be higher under CO compared to NC (Propositions 5(iii) and 6(iii)).

Second, we show that convex production costs create the incentive for technology licensing ex-post R&D. In the presence of licensing ex-post R&D, the firms prefer CL compared to other arrangements (Proposition 4(i, ii)) while the consumers and the society may prefer CO compared to other arrangements (Propositions 5 and 6).

Hence, the presence of convex costs may make the firms' preferred R&D organisation significantly different from the consumers' and the society's preferred R&D organisation by encouraging technology licensing ex-post R&D.[2] Therefore, a proper distribution scheme, such as a tax/subsidy policy, might be required to encourage firms to undertake CO. Since the total surplus can be higher under CO compared to other arrangements, the government may impose a lump-sum tax on the consumers to subside the firms to encourage them to undertake CO.

Since the presence of technology licensing affects our results, it may worth mentioning the factors that may make licensing (un)profitable. It is well documented that there are costs of technology licensing (Teece, 1976; Arora, Fosfuri and Gambardella, 2001). Hence, if the cost of technology licensing is significantly high, it will make licensing ex-post R&D unprofitable. Further, we consider that the licensing contract involves a non-negative up-front fixed-fee and a per-unit output royalty. However, if the licensee can imitate the licensed technology costlessly or the output of the licensee is not verifiable, the output royalty may not be a feasible option (see, e.g., Rockett, 1990). In this situation, technology licensing through a fixed-fee only

[2] Although licensing of innovation is widely observed, the literature on cooperative R&D did not pay much attention to this aspect. For example, Arundel and Kabla (1998) show 35.9% licensing contracts are related to product innovations and 24.8% are related to process innovations. Cohen, Nelson and Walsh (2000) show more than 50% licensing contracts are related to product innovation.

may not be profitable. If licensing ex-post R&D is not profitable, CO and NC will be the only feasible options for our analysis.

The remainder of the paper is organised as follows. Section 2 reviews the relevant literature. Section 3 describes the model and derives the equilibrium values. Sections 4-7 compare respectively the equilibrium R&D investments, total profits, consumer surplus and welfare for the four arrangements considered. Section 9 concludes.

2 Literature Review

The literature on cooperative R&D got momentum with Katz (1986), and d'Aspremont and Jacquemin (1988), which considered deterministic R&D, where success in R&D is certain and the marginal costs of production reduce with higher R&D investments. This literature is developed in different directions by Kamien, Muller and Zang (1992), Suzumura (1992), Kesteloot and Veugelers (1995), Beath, Poyago-Theotoky and Ulph (1998), Salant and Saffer (1998), Amir and Wooders (1999, 2000), Amir (2000), Hinloopen (2000), Amir, Evstigneev and Wooders (2003), Gil Moltó, Georgantzís and Orts (2005) and Karbowski (2019), to name a few. In contrast to those papers, we consider a stochastic R&D process with no knowledge spillover. Further, those papers do not consider technology licensing ex-post R&D. Hence, CL, which is the firms' preferred choice in our analysis, cannot occur in those papers.

There is another set of papers considering stochastic R&D process, where success in R&D is uncertain and the probability of success in R&D increases with higher R&D investment (see, e.g., Marjit, 1991; Choi, 1993; Gandal and Scotchmer, 1993; Miyagiwa and Ohno, 2002; Miyagiwa, 2009; Erkal and Piccinin, 2010;

Besanko and Wu, 2013).[3] Like these papers, we consider a stochastic R&D process, yet our structure and the results differ from these papers in some important ways.

Except Miyagiwa and Ohno (2002) and Miyagiwa (2009), the papers with stochastic R&D process did not consider technology licensing ex-post R&D. Hence, CL, which is the firms' preferred choice in our analysis, cannot occur in these papers. Therefore, the different preferred choices of the firms and the society shown in our analysis, i.e., firms prefer CL and the society prefers CO, do not occur in those papers.

Miyagiwa and Ohno (2002) considered the possibility of technology licensing ex-post R&D. However, in their paper, the equilibrium values are the same under CO and CL, while, in our paper, the firms prefer CL and the society can prefer CO.[4] This difference is due to the different licensing contracts considered in their paper and in this paper. They considered the licensing payment as a share of profit, and treated it effectively like a fixed-fee.[5] In contrast, we consider the widely used two-part tariff licensing contract consisting of a fixed-fee and an output royalty.[6]

Miyagiwa (2009) compared the effects of cooperative R&D and "non-cooperative R&D with licensing ex-post R&D" on the sustainability of tacit collusion. It shows that cooperative R&D increases the possibility of collusion and may reduce

[3] Some papers in this area consider exogenously given probability of success in R&D and R&D requires certain amount of investment.

[4] Marjit and Shi (1995), Marjit, Mukherjee and Shi (2001), Mukherjee and Marjit (2004), and Mukherjee (2005) consider technology licensing ex-post R&D. However, these papers neither considered endogenous R&D investments nor considered CL. Bandyopadhyay and Mukherjee (2014) show the effects of entry on the incentive for cooperative R&D. They neither considered endogenous R&D investments nor considered technology licensing ex-post R&D. Gallini and Winter (1985) consider licensing of different technologies before and after non-cooperative R&D. In our case, there is no possibility of licensing before innovation. Brocas (2004) considers a situation where a regulator offers a cooperative contract to two firms to develop a research project when the firms have private information about their skills. In our case, the regulator is not offering a contract for the research project.

[5] Although they wrote the licensing payment as a share of profit, they effectively treated it as a fixed-fee. Otherwise, the licensing payment as the share of profit will generate a more collusive output or price choice of the firms, which will affect the gross profits. They avoided this complication by ignoring strategic licensing. See their footnote 15 for ignoring strategic licensing.

[6] See, e.g., Rockett (1990), for an early work with this type of licensing contract.

welfare. Further, cooperative R&D and non-cooperative R&D with licensing may be substitutable or the firms may prefer cooperative R&D than non-cooperative R&D with licensing due to the former's higher ability to sustain collusion than the latter. In contrast, there is no tacit collusion in our analysis and cooperative R&D in our analysis can always be welfare improving compared to other arrangements. Further, there is no possibility of CL in that paper.

We differ from the papers on stochastic R&D in some other important ways also. Unlike Gandal and Scotchmer (1993), Miyagiwa and Ohno (2002), Miyagiwa (2009), Erkal and Piccinin (2010) and Besanko and Wu (2013), we do not consider a patent race model where the winner takes all, but consider the possibility of multiple successes, which is relevant for many industries, such as the smartphone industry, where multiple firms innovate and produce smartphones. Similarly, different research firms got the approval for COVID-19 vaccines.

Unlike Choi (1993), Miyagiwa and Ohno (2002), Erkal and Piccinin (2010), and Besanko and Wu (2913), we do not consider spillover, thus considering a scenario with strong patent protection that may prevent knowledge spillover. This may also occur when firms target significantly different technologies for the product.[7]

Although we considered a stochastic non-tournament R&D process like Choi (1993), we differ from that paper in some important ways. Cooperative R&D in Choi (1993) helped to internalise the adverse effects of knowledge spillover on non-cooperative R&D. Further, he assumed that sharing of R&D outcomes under cooperative R&D tends to reduce the industry profit ex-post R&D by intensifying competition in the product market. As the sharing of R&D outcomes reduces the

[7] As an example, when different research firms were trying to find vaccines for COVID-19, many of them were trying to innovate different types of vaccines, such as RNA vaccine, Viral vector vaccine and Protein subunit vaccine. As another example, iOS and Android are two different operating systems used by smartphone companies.

industry profit ex-post R&D, licensing ex-post R&D was not profitable in that paper. Hence, in Choi (1993), cooperative R&D is unprofitable compared to non-cooperative R&D if knowledge spillover is not very high, and NCL and CL were not the options in that paper.

In contrast, convex production costs in our analysis may make the monopoly profit lower than the duopoly industry profit. On the one hand, this effect helps to make cooperative R&D preferred than non-cooperative R&D without knowledge spillover, and on the other hand, it helps to make licensing profitable ex-post R&D. Hence, the convex production cost is an important aspect of our analysis.

Further, unlike Choi (1993), the consumers and the society can be better off in our analysis under CO even if there is no knowledge spillover, while the firms are better off under CL in the presence of licensing and under CO or NC in the absence of licensing.

As mentioned above, the R&D process considered in this paper is similar to the R&D process considered in Federico, Langus and Valletti (2017, 2018) and Denicolò and Polo (2018). However, they examined how merger, i.e., cooperation in both R&D and product market, affect product innovation, while the firms in our paper always compete in the product market.

In sum, our paper differs from the extant literature in the following ways. First, our paper is different from the literature on deterministic R&D due to the consideration of a stochastic R&D process, technology licensing and no knowledge spillover. Second, in contrast to the papers on patent race models, we consider multiple successes, and technology licensing ex-post R&D. Although Miyagiwa and Ohno (2002) and Miyagiwa (2009) considered the possibility of technology licensing, they considered a patent race model where the winner takes all, while we consider multiple successes. Further, unlike this paper, the equilibrium values are the same under CO and CL in Miyagiwa and Ohno (2002). While there is no CL in Miyagiwa (2009), there is no tacit collusion in our analysis. Third, unlike the stochastic non-tournament R&D model of Choi (1993), we consider technology licensing ex-post R&D with no knowledge spillover. Finally, we find that the firms prefer CL and the society can prefer CO, which do not occur in those papers.

Gallini and Winter (1985) considered technology licensing before and after R&D. While the firms can engage in licensing the old technology before R&D, they can engage in licensing the new technology after R&D. Although CO and CL may look like licensing contracts before and after R&D respectively, they involve the same technologies. Under CO, the firms commit before R&D about sharing the new technology. Under CL, the firms decide after R&D about sharing the new technology. Hence, CO and CL considered in this paper have different implications than licensing before and after R&D in Gallini and Winter (1985).

3 The Model and the Equilibrium Values

Assume that there are two firms, called firm 1 and firm 2. Both firms invest in R&D to find a new product. Assume that the ith firm's probability of success in R&D is $z(x_i)$ with $z' > 0$, $z'' < 0$, $z'(0) = \infty$ and $z'(\infty) = 0$, where x_i is firm i's investment in R&D, $i = 1, 2$. This assumption is similar to Federico, Langus and Valletti (2017, 2018), Denicolò and Polo (2018) and Choi (1993). This implies that there is no knowledge spillover during the R&D phase.[8]

If the ith firm is successful in R&D, it can produce the product at the total cost $TC_i = c(q_i)$, with $c' > 0$ and $c'' > 0$, where q_i is the output of the ith firm. The

[8] For an earlier work on knowledge spillover during R&D phase, one may look at Kamien, Muller and Zang (1992).

use of a convex production cost represents soft capacity constraints (Cabon-Dhersin and Drouhin, 2020). The capacity constraint effect increases as c'' increases.

The inverse demand function for the product is $P = P(q)$, with $q = q_1 + q_2$, $P' < 0$ and $P'' \leq 0$.

We will consider the four arrangements – NC, CO, NCL and CL – mentioned in the introduction.

3.1 Non-cooperation (NC)

We consider the following game under NC. At stage 1, firms invest in R&D simultaneously to maximise their own profits. At stage 2, conditional on the R&D outcomes, the outputs are determined simultaneously by the firms if both firms are successful in R&D or by the successful firm under a unilateral success in R&D, and the profits are realised. We solve the game through backward induction.

We allow for multiple successes. Hence, given the R&D investments of the firms, there are three possibilities – (i) both firms are successful in R&D, (ii) only one firm is successful in R&D, and (iii) neither firm is successful in R&D.

If both firms are successful in R&D, the ith firm, *i = 1, 2*, maximises $P(q)q_i - c(q_i)$ to determine its output. Hence, the equilibrium output of the ith firm is determined by $P + P'q_i - c' = 0$. In this situation, we denote the equilibrium profit of the ith firm ex-post R&D by *D*. Hence, if both firms are successful in R&D, the profit of the ith firm ex-post R&D is $\pi_i^{ss} = D$, *i = 1, 2*, where the first (second) superscript denotes the ith (jth) firm's R&D outcome, $i \neq j$, and *s* stands for success in R&D.

If only the ith firm is successful in R&D, it maximises $P(q_i)q_i - c(q_i)$ to determine its output. We denote the equilibrium monopoly profit of the ith firm ex-post R&D by M. Hence, if only the ith firm is successful in R&D, the profit of the successful ith firm ex-post R&D is $\pi_i^{sf} = M$, i = *1, 2*, and the profit of the unsuccessful jth firm ex-post R&D is $\pi_j^{sf} = 0$, j = *1, 2*, $i \neq j$, where s and f stand for success and failure in R&D respectively. Since we are considering product innovation and the unsuccessful firm does not have any product to produce, the profit of the unsuccessful firm is zero.

If neither firm is successful in R&D, the profit of each firm ex-post R&D is *0*.

Although we consider that the firms produce homogeneous products when both firms are successful in R&D, it is intuitive that the presence of convex production costs makes the monopoly profit greater (less) than the duopoly industry profit, i.e., $M >(<) 2D$, if convexity of the production cost is low (high).

Internalising the profits ex-post R&D, the ith firm chooses its R&D investment to maximise the following expression:

$$(1) \qquad \underset{x_i}{Max}\, \pi_i^{NC} = \underset{x_i}{Max}\; z(x_i)z(x_j)D + z(x_i)(1 - z(x_j))M - x_i \,,$$

where $i, j = 1, 2,\ i \neq j$.

The equilibrium R&D investment of the ith firm is given by

$$(2) \qquad z'(x_i)z(x_j)D + z'(x_i)(1 - z(x_j))M = 1 \,.$$

The second-order condition for maximization holds. The equilibrium R&D investments of the firms can be found by solving equation (2). Symmetry of the firms create the same equilibrium R&D investments, $x_1^{NC} = x_2^{NC} = x^{NC}$.

The equilibrium expected total profits of firms 1 and 2, expected consumer surplus and expected welfare are

$$(3) \qquad \pi^{NC} = \pi_1^{NC} + \pi_2^{NC} = z^2(x^{NC})2D + 2z(x^{NC})(1 - z(x^{NC}))M - 2x^{NC}$$

(4) $CS^{NC} = z^2(x^{NC})\mathrm{CS}^D + 2z(x^{NC})(1 - z(x^{NC}))\mathrm{CS}^M$

(5) $W^{NC} = z^2(x^{NC})W^D + 2z(x^{NC})(1 - z(x^{NC}))W^M - 2x^{NC}$,

where CS^D (CS^M) shows consumer surplus ex-post R&D when both firms are successful (only one firm is successful) in R&D and $W^D = 2D + CS^D$ ($W^M = M + CS^M$) shows welfare ex-post R&D when both firms are successful (only one firm is successful) in R&D. The expected welfare is the summation of expected total profits of firms 1 and 2 and expected consumer surplus.

3.2 Cooperative R&D (CO)

Under cooperative R&D, firms commit before R&D about sharing R&D outcomes and choosing the R&D investments to maximise their joint profits. However, they compete in the product market ex-post R&D. Hence, even if only one firm is successful in R&D, both firms can produce the product. Like the previous subsection, R&D investments are determined in stage 1, and the outputs and profits are determined in stage 2. As mentioned in the introduction, like Choi (1993), we do not consider synergic benefits under cooperative R&D, which will trivially increase the preference for cooperative R&D.

Internalising the profits ex-post R&D, the ith firm chooses its investment to maximise the following expression:

(6) $$\underset{x_i}{Max}\sum_{i=1}^{2}\pi_i^{CO} = \underset{x_i}{Max}\sum_{\substack{i,j=1\\ i\neq j}}^{2}\left[\left(\mathrm{z}(x_i)\,\mathrm{z}(x_j) + z(x_i)(1 - z(x_j)) + (1 - z(x_i))\,\mathrm{z}(x_j)\right)D - x_i\right].$$

The equilibrium R&D investment of the ith firm is given by

(7) $z'(x_i)(1 - \mathrm{z}(x_j))2D = 1$.

Symmetry of the firms creates the same equilibrium R&D investments, $x_1^{CO} = x_2^{CO} = x^{CO}$.

The symmetric solutions $x_1^{CO} = x_2^{CO} = x^{CO}$ satisfy the second order condition of maximisation if the second derivative of (6) with respect to x_i is negative, which holds true, and the Hessian matrix to be positive, i.e., $[z'(x^{CO})]^2 + z''(x^{CO})(1 - z(x^{CO})) < 0$, which is assumed to hold.

The equilibrium expected total profits of firms 1 and 2, expected consumer surplus and expected welfare are

(8) $\pi^{CO} = \pi_1^{CO} + \pi_2^{CO} = z(x^{CO})(2 - z(x^{CO}))2D - 2x^{CO}$

(9) $CS^{CO} = z(x^{CO})(2 - z(x^{CO}))\,\mathrm{CS}^D$

(10) $W^{CO} = z(x^{CO})(2 - z(x^{CO}))W^D - 2x^{CO}$.

3.3 Non-cooperative R&D with Technology Licensing (NCL)

We have considered cooperative R&D in subsection 3.2 where firms commit before R&D about sharing R&D outcomes. Hence, each firm sacrifices the option to become a monopolist under a unilateral success in R&D. In contrast, we consider in this subsection the possibility of technological cooperation ex-post R&D through technology licensing, while the firms do R&D non-cooperatively, i.e., determining the R&D investments to maximise own profits and not committing to share R&D outcomes.

Hence, we consider the following game in this subsection. At stage 1, firms invest in R&D simultaneously to maximise their own profits and do not commit to share R&D outcomes. If there is a unilateral success in R&D in stage 1, the successful firm decides at stage 2 whether to license its technology to the unsuccessful firm. In case of licensing, the successful firm offers the unsuccessful firm a take-it-or-leave-it two-part tariff contract, consisting of a non-negative up-front fixed-fee and a royalty per-unit of output, and the unsuccessful firm accepts the offer if it is not worse off under licensing compared to no licensing. At stage 3, the outputs are determined and the profits are realised. We solve the game through backward induction.

Before determining the R&D investments, first examine whether licensing occurs under a unilateral success in R&D.

3.3.1 Licensing Decision

Without a loss of generality, assume that only firm 1 was successful in R&D. Similar analysis will follow if only firm 2 was successful in R&D.

If there is no licensing under a unilateral success in R&D, we know from subsection 3.1 that the profit of firm 1 ex-post R&D is $\pi_1^{sf} = M$ and the profit of firm 2 ex-post R&D is *0*.

Now consider the situation under licensing ex-post R&D. Under licensing, firm 1 offers a non-negative up-front fixed-fee, *F*, and a royalty per-unit of output, *r*. Hence, under licensing, firms 1 and 2 maximise respectively $P(q)q_1 - c(q_1) + rq_2 + F$ and $P(q)q_2 - c(q_2) - rq_2 - F$ simultaneously to determine their outputs. The equilibrium outputs are determined respectively by the expressions:

(11) $P + P'q_1 - c'(q_1) = 0$ and $P + P'q_2 - c'(q_2) - r = 0$.

Hence, the total output is given by

(12) $2P + P'q - c'(q_1) - c'(q_2) - r = 0$.

Differentiating the expressions in (11) with respect to r and solving those expressions, we get $\frac{\partial q_1}{\partial r} > 0$, $\frac{\partial q_2}{\partial r} < 0$ and $\frac{\partial q}{\partial r} < 0$ (see appendix A.1).

Since firm 1 will choose the fixed-fee, F, to extract the entire benefit of licensing from firm 2, the fixed-fee will be equal to $F^* = P(q^*(r))q_2^*(r) - c(q_2^*(r)) - rq_2^*(r)$.

Hence, firm 1 will determine the equilibrium royalty to maximise

(13) $$\begin{aligned} &P(q^*(r))q_1^*(r) - c(q_1^*(r)) + rq_2^*(r) + P(q^*(r))q_2^*(r) - c(q_2^*(r)) - rq_2^*(r) \\ &= P(q^*(r))(q_1^*(r) + q_2^*(r)) - c(q_1^*(r)) - c(q_2^*(r)), \end{aligned}$$

i.e., firm 1 wants to set the royalty rate to maximise the total profits of firms 1 and 2.

The equilibrium royalty rate is determined by (using (11))

(14) $$r\frac{\partial q_2^*(r)}{\partial r} + P'\left(q_1\frac{\partial q_2^*(r)}{\partial r} + q_2\frac{\partial q_1^*(r)}{\partial r}\right) = 0,$$

(14') or $$r^* = \frac{-P'\left(q_1\frac{\partial q_2^*(r^*)}{\partial r} + q_2\frac{\partial q_1^*(r^*)}{\partial r}\right)}{\frac{\partial q_2^*(r^*)}{\partial r}},$$

where $\left(q_1\frac{\partial q_2^*(r)}{\partial r} + q_2\frac{\partial q_1^*(r)}{\partial r}\right) = \frac{\left(q_1^*P' + (q_1^* - q_2^*)(P' + q_1^*P'') - q_1^*c''(q_1^*)\right)}{Z} < 0$, since

$$Z = 3P'^2 + c''(q_1)c''(q_2) - \left(q_2c''(q_1) + q_1c''(q_2)\right)P'' + P'\left(-2\left(c''(q_1) + c''(q_2)\right) + qP''\right) > 0$$

due to the stability condition in output determination[9] and $q_1^* > q_2^*$, which follows from (11).[10]

The left hand side (LHS) of (14) is positive at $r = 0$, implying that the equilibrium royalty rate, r^*, is positive. We also get $r^* < r^{\max}$, where $r^{\max}$ is the

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[9] Under licensing, the stability condition for output determination is given by $\frac{\partial^2\pi_i}{\partial x_i^2}\frac{\partial^2\pi_j}{\partial x_j^2} > \frac{\partial^2\pi_i}{\partial x_i\partial x_j}\frac{\partial^2\pi_j}{\partial x_j\partial x_i}$, where $i = 1,2$, $i \neq j$, $\pi_i = P(q)q_i - c(q_i) + rq_j + F$ and $\pi_j = P(q)q_j - c(q_j) - rq_j - F$.

[10] If we have $q_1^* = q_2^*$, it follows from (11) that $P + P'q_1^* - c'(q_1^*) = 0$ and $P + P'q_2^* - c'(q_2^*) - r < 0$, suggesting that for the interior solutions, both conditions in (11) hold if $q_1^* > q_2^*$.

royalty rate that makes zero output as the licensee's profit maximising output choice ex-post licensing, i.e., $r^{\max} = (P - c'(0))$, which follows from firm 2's (i.e., the licensee's) first order condition for output determination with $q_2^* = 0$. Hence, the equilibrium royalty is $r^* \in (0, r^{\max})$.

It is now worth explaining the importance of convex production costs for our analysis. We get from firm 2's (i.e., the licensee's) first order condition for output determination $r^{\max} = (P - c'(0))$, since $q_2^* = 0$ at $r^{\max}$. Evaluating LHS of (14) at $r = (P - c'(0))$ and $q_2^* = 0$, we get $r\frac{\partial q_2^*(r)}{\partial r} + P'\left(q_1\frac{\partial q_2^*(r)}{\partial r} + q_2\frac{\partial q_1^*(r)}{\partial r}\right) = \frac{\partial q_2^*(r)}{\partial r}(c'(q_1) - c'(0)) < 0$, since $P'q_1 = c'(q_1) - P$ from firm 1's output determination condition, and $c'(q_1) > c'(0)$ because $c'' > 0$ and $q_1 > 0$. Hence, a royalty lower than $r^{\max}$ increases the total profits of the firms shown in (13). If we have considered $c'' = 0$, implying $c'(q_1) = c'(0)$, LHS of (14) will be equal to zero when evaluating at $r = (P - c'(0))$ and $q_2^* = 0$. Hence, if $c'' = 0$, $r^{\max}$ is the royalty rate that maximises the total profits shown in (13), implying that the successful firm has no incentive to sacrifice its monopoly status through licensing if we have considered constant marginal cost of production in our analysis. That was the reason why licensing ex-post R&D was not profitable in Choi (1993). In other words, convex production costs are important to make licensing ex-post R&D profitable in our analysis, and licensing will have no real effects for our analysis if we have considered constant marginal cost of production.

Denote $P(q^*(r^*))(q_1^*(r^*) + q_2^*(r^*)) - c(q_1^*(r^*)) - c(q_2^*(r^*))$ by *L*. Hence, under licensing, the equilibrium profit of firm 1 ex-post R&D is *L* and the net equilibrium profit of firm 2 ex-post R&D is *0*. Since $r^* < r^{\max}$, it is intuitive that licensing is profitable compared to no licensing, i.e., $L > M$.[11] Since $r^* > 0$, it is also easy to understand that $L > 2D$.[12]

We summarise the above discussion in the following proposition.

PROPOSITION 1 *If the production costs are convex then under a unilateral success in R&D, the successful firm will license its technology to the unsuccessful firm and the total profit under licensing, L, will be greater than both M and 2D.*

The reason for the above result is as follows. In the presence of convex production costs, on the one hand, productions by two firms rather than one firm help to increase the total profits by saving costs, but on the other hand, competition between the firms in the product market tends to reduce the total profits. Since an appropriately chosen royalty rate helps to soften competition between the firms, productions by two firms increase the total profits compared to the situation where only one firm produces the product.[13]

Although licensing creates a duopoly market structure ex-post R&D, since $r^* > 0$ and $\frac{\partial q}{\partial r} < 0$, the total outputs are lower under licensing compared to the

[11] The licenser, i.e., the firm that is successful in R&D and licenses the technology, can always charge $r^{\max}$ to generate *M* as its equilibrium profit ex-post licensing. Since the licenser's profit maximising royalty, r^*, is different from $r^{\max}$, it implies that the licenser's net profit under licensing, *L*, is higher than *M*.

[12] We get $L = P(q^*(r^*))(q_1^*(r^*) + q_2^*(r^*)) - c(q_1^*(r^*)) - c(q_2^*(r^*)) > P(q^*(0))(q_1^*(0) + q_2^*(0)) - c(q_1^*(0)) - c(q_2^*(0)) = 2D$. This happens because, under licensing, the licenser could get *2D* by charging zero royalty. Since the equilibrium profit maximising royalty is positive, i.e., $r^* > 0$, the licenser's net profit from licensing, *L*, must be greater than *2D*. Hence, the licenser's power to control the effective marginal cost of the licensee through output royalty makes *L* higher than *2D*.

[13] Using a linear demand curve, Mukherjee (2014) shows that a monopolist producer finds it profitable to license its technology to the potential competitors. Sen and Stamatopoulos (2016) also show that a monopolist producer licenses its technology to a competitor in the presence of convex production costs.

situation where both firms are successful in R&D and the market structure ex-post R&D is duopoly without licensing.

If there is no licensing ex-post R&D and only one firm is successful in R&D, the total output in this situation is lower than the situation where both firms are successful in R&D and the market structure ex-post R&D is duopoly without licensing. This happens since the total output under the former situation can be found by maximising $P(q)q - c(q)$, which gives the equilibrium total output $q = \frac{(P - c')}{-P'}$, while the equilibrium total output under the latter situation can be found by summing the two first order conditions $P + P'q_1 - c' = 0$ and $P + P'q_2 - c' = 0$,[14] which gives $q = \frac{2(P - c')}{-P'} (> \frac{(P - c')}{-P'})$. This is due to the usual monopoly distortion.

Since a higher total output increases consumer surplus, the following result is immediate from the above discussion.

PROPOSITION 2 *Irrespective of licensing ex-post R&D under a unilateral success in R&D, consumer surplus ex-post R&D is lower under a unilateral success in R&D compared to the situation where both firms are successful in R&D.*

3.3.2 R&D Investments

Internalising that there will be licensing ex-post R&D under a unilateral success in R&D, the ith firm maximises the following expression to determine the R&D investment:

(15) $$\underset{x_i}{Max}\, \pi_i^{NCL} = \underset{x_i}{Max}\ z(x_i)z(x_j)D + z(x_i)(1 - z(x_j)L - x_i ,$$

[14] These two first order conditions come from firm 1’s and firm 2’s profit maximising outputs under duopoly with no licensing, as mentioned in subsection 3.1.

where $i, j = 1, 2$, $i \neq j$.

The equilibrium R&D investment of the ith firm is given by

(16) $z'(x_i)\,z(x_j)D + z'(x_i)(1 - z(x_j))L = 1$.

The second-order condition for maximization is satisfied. Symmetry of the firms creates the same equilibrium R&D investments, $x_1^{NCL} = x_2^{NCL} = x^{NCL}$.

The equilibrium expected total profits of firms 1 and 2, expected consumer surplus and expected welfare are

(17) $\pi^{NCL} = \pi_1^{NCL} + \pi_2^{NCL} = z^2(x^{NCL})2D + 2z(x^{NCL})(1 - z(x^{NCL}))L - 2x^{NCL}$

(18) $CS^{NCL} = z^2(x^{NCL})\,\mathrm{CS}^D + 2z(x^{NCL})(1 - z(x^{NCL}))\,\mathrm{CS}^L$

(19) $W^{NCL} = z^2(x^{NCL})W^D + 2z(x^{NCL})(1 - z(x^{NCL}))W^L - 2x^{NCL}$,

where CS^L ($W^L = L + CS^L$) shows consumer surplus (welfare) ex-post R&D when only one firm is successful in R&D and the successful firm licenses its technology to the unsuccessful firm.

3.4 Joint Profit Maximising R&D Investments with Licensing (CL)

Now we consider a game similar to subsection 3.3 with the exception that the firms determine the R&D investments cooperatively to maximise their joint profits. Hence, like subsection 3.3, they do not commit before R&D about sharing R&D outcomes but can engage in technology licensing ex-post R&D. As mentioned in the introduction, we do not consider synergic benefits from cooperation in R&D investments.

As shown in subsection 3.3.1, technology licensing will occur ex-post R&D if there is a unilateral success in R&D. Internalising that there will be licensing ex-post R&D under a unilateral success in R&D, the ith firm maximises the following expression to determine the R&D investment:

(20) $$\underset{x_i}{Max} \sum_{i=1}^{2} \pi_i^{CL} = \underset{x_i}{Max} \sum_{\substack{i,j=1 \\ i \neq j}}^{2} \left[\left(z(x_i)\, z(x_j) D + z(x_i)(1 - z(x_j)) L \right) - x_i \right].$$

The equilibrium R&D investment of the ith firm is given by

(21) $$z'(x_i)\, z(x_j) 2D + z'(x_i)(1 - 2z(x_j)) L = 1.$$

Symmetry of the firms creates the same equilibrium R&D investments, $x_1^{CL} = x_2^{CL} = x^{CL}$.

Under this arrangement, the firms do not commit before R&D about sharing R&D outcomes under a unilateral success in R&D, but they engage in licensing ex-post R&D. Hence, both firms will operate their labs under this arrangement because otherwise, the net profit of the firm, which is not using its lab, will be zero.

The equilibrium expected total profits of firms 1 and 2, expected consumer surplus and expected welfare are

(22) $$\pi^{CL} = \pi_1^{CL} + \pi_2^{CL} = z^2(x^{CL}) 2D + 2z(x^{CL})(1 - z(x^{CL}) L - 2x^{CL}$$

(23) $$CS^{CL} = z^2(x^{CL}) CS^D + 2z(x^{CL})(1 - z(x^{CL}) CS^L$$

(24) $$W^{CL} = z^2(x^{CL}) W^D + 2z(x^{CL})(1 - z(x^{CL}) W^L - 2x^{CL}.$$

4 Comparison of R&D Investments

Now we are in a position to compare the equilibrium outcomes under NC, CO, NCL and CL. First, compare the R&D investments under different arrangements.

PROPOSITION 3 *(i)* $x^{NCL} > x^{CL}$, *(ii)* $x^{NCL} > x^{NC}$, *(iii)* $x^{NCL} > x^{CO}$, *(iv)* $x^{CL} > x^{CO}$, *and (v)* $x^{NC} > x^{CO}$ *if* $M > 2D$ *but* $x^{NC} > (<) x^{CO}$ *for*

$z(x^{NC}) \in (z(x^{NC*}),1]$ $(z(x^{NC}) \in [0, z(x^{NC*}))$ *if* $M < 2D$ *where* $z(x^{NC*})D + (1 - z(x^{NC*}))(M - 2D) = 0$.

PROOF See appendix A.2. *Q.E.D.*

The reasons for the above results are as follows. The game structure under NCL and CL are the same with the exception that the firms determine the R&D investments non-cooperatively under NCL to maximise their own profits while they determine the R&D investments cooperatively under CL to maximise their joint profits. Hence, the firms reduce R&D investments under CL compared to NCL as they internalise the competitive effect of a firm's R&D investment on the other firm.

The possibility of licensing ex-post R&D under a unilateral success in R&D generates higher R&D investments under NCL compared to NC.

Both NCL and CO allow the firms to share the R&D outcomes under a unilateral success in R&D. However, under a unilateral success in R&D, the successful firm earns more under NCL compared to CO, which encourages the firms to invest more in R&D under NCL compared to CO.

The firms choose the R&D investments under CL and CO to maximise their joint profits. However, the benefit from licensing under CL creates higher R&D investments under CL compared to CO.

Finally, compare the R&D investments under NC and CO. Under a unilateral success in R&D, NC allows the successful firm to earn the monopoly profit, *M*, while CO allows it to earn *2D*. CO also allows the firms to internalise the competitive effect of a firm's R&D investment on the other firm. If *M > 2D*, the monopoly benefit under NC and the benefit from internalising the competitive effect of a firm's R&D investment on the other firm under CO encourage the firms to invest more in R&D

under NC compared to CO. If $M < 2D$, the higher profit under CO compared to NC tends to increase the R&D investments under CO compared to NC, while the incentive to internalise the competitive effect of a firm's R&D investment on the other firm under CO tends to reduce the R&D investments under CO compared to NC. Hence, in this situation, the R&D investments are higher under CO (NC) compared to NC (CO) if the benefit from internalising the competitive effect of a firm's R&D investment on the other firm is small (large), which happens if the equilibrium R&D investments and therefore, the equilibrium probabilities of success in R&D are low (high).

5 *Comparison of Total Profits*

Now we compare the expected total profits of the firms under different arrangements, which will show the firms' incentive for cooperation.

PROPOSITION 4 *We get (i)* $\pi^{CL} > \pi^{NCL} > \pi^{NC}$*, (ii)* $\pi^{CL} > \pi^{CO}$*, and (iii)* $\pi^{CO} > \pi^{NC}$ *for* $2D > M$ *but* $\pi^{CO} < \pi^{NC}$ *may happen for* $2D < M$*.*

PROOF See appendix A.3. *Q.E.D.*

The possibility of licensing ex-post R&D makes NCL better than NC for the firms.

NCL allows licensing ex-post R&D when there is a unilateral success in R&D but the firms determine the R&D investments to maximise their own profits. CO allows the firms to determine their joint profit maximising R&D investments but does not keep the option for licensing ex-post R&D. However, CL allows the firms to incorporate both the aspects – the firms cooperate on R&D investments to maximise their joint profits and keep the option for licensing ex-post R&D under a unilateral success in R&D. As a result, CL is preferable than NCL and CO.

Since NCL is preferable than NC, as mentioned above, we get CL as the most preferred arrangement for the firms if technology licensing ex-post R&D is an option.

Now compare CO with NC. There are two effects in this comparison. First, CO allows the firms to earn $2D$ under a unilateral success in R&D, but NC allows the firms to earn M under a unilateral success in R&D. Second, CO helps the firms to internalise the competitive effect of a firm's R&D investment on the other firm. Hence, both these effects create the incentive for CO if $2D$ is greater than M. However, if $2D$ is less than M, the first effect tends to create the incentive for NC but the second effect tends to create the incentive for CO. The firms prefer NC if the first effect is stronger, which can happen if M is sufficiently higher than $2D$.

The above discussion suggests that if technology licensing is an option ex-post R&D, the firms would be better off under CL compared to other arrangements. However, if technology licensing is not an option ex-post R&D, the firms prefer CO for $M < 2D$ and they may prefer NC for $M > 2D$.

6 Comparison of Consumer Surplus

It is intuitive that *consumer surplus ex-post R&D* cannot be lower under CO compared to other arrangements. Consumer surplus ex-post R&D is the same under all the arrangements when both firms are successful in R&D. Under a unilateral success in R&D, both firms can produce the products under CO, NCL and CL. However, the presence of a positive royalty under NCL and CL reduce the consumer surplus ex-post R&D under NCL and CL compared to CO. When comparing CO and NC under a unilateral success in R&D, higher competition in the product market under CO compared to NC creates higher consumer surplus ex-post R&D under CO compared to NC.

Even if the consumer surplus ex-post R&D cannot be lower under CO compared to other arrangements, it is not immediate that the *expected consumer surplus*, i.e., the consumer surplus ex-ante R&D, cannot be lower under CO compared to other arrangements. This is because the R&D investments and therefore, the probabilities of getting the products are lower under CO compared to NCL and CL and maybe compared to NC.

Hence, there is a trade off when we compare the expected consumer surplus. Higher or the same consumer surplus ex-post R&D under CO tends to provide a higher expected consumer surplus under CO compared to other arrangements, while lower R&D investments under CO tend to provide a lower expected consumer surplus under CO compared to other arrangements.

The following result shows the conditions under which CO provides a higher equilibrium expected consumer surplus for a general demand function. We then use widely used specific functional forms for the probability function, demand function and convex production cost to show that the equilibrium expected consumer surplus can be higher under CO irrespective of the possibility of licensing ex-post R&D.

PROPOSITION 5 *(i)* $CS^{CO} > CS^{NCL}$ *for* $CS^{D} >> CS^{L}$ *or* $x^{NCL} \to x^{CO}$ *but* $CS^{CO} < CS^{NCL}$ *for* $CS^{D} \to CS^{L}$ *or* $x^{NCL} >> x^{CO}$,

(ii) $CS^{CO} > CS^{CL}$ *for* $CS^{D} >> CS^{L}$ *or* $x^{CL} \to x^{CO}$ *but* $CS^{CO} < CS^{CL}$ *for* $CS^{D} \to CS^{L}$ *or* $x^{CL} >> x^{CO}$,

(iii) $CS^{CO} > CS^{NC}$ if $x^{CO} > x^{NC}$. If $x^{CO} < x^{NC}$, $CS^{CO} > CS^{NC}$ for $CS^{D} >> CS^{M}$ or $x^{NC} \to x^{CO}$ but $CS^{CO} < CS^{NC}$ for $CS^{D} \to CS^{L}$ or $x^{NC} >> x^{CO}$.

PROOF See appendix A.4. *Q.E.D.*

In the above proposition, except for the comparison between CO and NC with higher R&D investments under CO, whether the expected consumer surplus is higher or lower under CO compared to other arrangements depends on the trade-off created by the higher or same consumer surplus ex-post R&D under CO and the lower probability of getting the product under CO.

It follows from Propositions 4 and 5 that the firms prefer CL compared to other arrangements but the consumers may prefer CO compared to other arrangements. Now we use an example to show a situation where the expected consumer surplus is higher under CO compared to other arrangements. Consider the widely used probability function $z(x_i) = \sqrt{x_i}$ (see, e.g., Delbono and Denicolò, 1990; Hartwick, 1991; Denicolò and Polo, 2018, for similar probability functions), a linear demand function $P = 1 - q$ and a convex cost of production $TC_i = cq_i^2$ for the ith firm with $c > 0$. We find $CS^{CO} > CS^{NCL}$, $CS^{CO} > CS^{CL}$ and $CS^{CO} > CS^{NC}$ (see appendix A.5). It is worth mentioning that the results might change with other functional forms and as mentioned in the above proposition, the results are ambiguous in general.

Hence, we can conclude from the discussion in Section 5 and the analysis in this section with the specific functional forms that while the consumers may always prefer CO, the firms prefer CL in the presence of licensing and CO or NC in the absence of licensing.

7 Comparison of Welfare

Finally, compare the equilibrium expected welfare under different arrangements.

It is clear from Propositions 4 and 5 that there is a conflict of interest between the firms and the consumers. It is intuitive that the equilibrium expected welfare is higher under CO compared to other arrangements if the effects of consumer surplus are stronger than the effects of the total profits of firms 1 and 2. We mention this in the following result. We then use the specific functional forms considered in the previous section to show that the equilibrium expected welfare can be higher under CO compared to other arrangements.

PROPOSITION 6 *(i)* $W^{CO} > W^{NCL}$ for $(\mathrm{CS}^{CO} - CS^{NCL}) > (\pi^{NCL} - \pi^{CO})$, (ii) $W^{CO} > W^{CL}$ *for* $(\mathrm{CS}^{CO} - CS^{CL}) > (\pi^{CL} - \pi^{CO})$, and (iii) $W^{CO} > W^{NC}$ *for* $(\mathrm{CS}^{CO} - CS^{NC}) > (\pi^{NC} - \pi^{CO})$.

It follows from Propositions 4 and 6 that the firms prefer CL compared to other arrangements but the expected welfare may be higher under CO compared to other arrangements. Now we use the specific functional forms considered in the previous section to show a situation where the expected welfare is higher under CO compared to other arrangements. For the specific functional forms considered in the previous section, we get $W^{CO} > W^{NCL}$, $W^{CO} > W^{CL}$ and $W^{CO} > W^{NC}$ (see appendix A.6). It is worth mentioning that the results might change with other functional forms and the results differ in general if the conditions in the above proposition do not hold.

Hence, we can conclude from the discussion in Section 5 and the analysis in this section with the specific functional forms that while the society may always prefer CO, the firms prefer CL in the presence of licensing and CO or NC in the absence of licensing.

8 *Implications of Some Assumptions*

8.1 *Sharing R&D Outcomes with Non-cooperative R&D Investments*

We will now discuss the implications of commitment before R&D to share R&D outcomes while the R&D investments are determined non-cooperatively. This situation is similar to the case of perfect knowledge spillover of R&D outcomes under non-cooperative R&D.

It is easy to understand that if the firms commit to share R&D outcomes but determine the R&D investments to maximise their own profits, CO will dominate this arrangement. This happens for the following reason. If the firms commit before R&D to share R&D outcomes, the total profits ex-post R&D will be either 2D (if at least one firm is successful in R&D) or 0 (if neither firm is successful in R&D) whether or not the firms determine the R&D investments to maximise their joint profits. Hence, if they determine the R&D investments to maximise their joint profits, by definition, the expected total profits of firms 1 and 2 are higher in this situation compared to the situation where they determine the R&D investments to maximise their own profits.

It is also easy to check that the R&D investments are higher under CO compared to the situation where the firms commit to share R&D outcomes but determine the R&D investments to maximise their own profits. If the firms share R&D outcomes but determine the R&D investments to maximise their own profits, the ith firm maximises the following expression to determine its R&D investment:

$$\underset{x_i}{Max}\,[z(x_i)\,z(x_j)+z(x_i)(1-z(x_j))+(1-z(x_i))\,z(x_j)]D-x_i\,,\ i,j=1,2\,,\ i\neq j\,.$$

The equilibrium R&D investment of the ith firm is given by

(22) $z'(x_i)(1-\mathrm{z}(x_j))D = 1$.

If evaluating the LHS of (7) and LHS of (22) at $x_1^{CO} = x_2^{CO} = x^{CO}$, we get $z'(x^{CO})(1-\mathrm{z}(x^{CO}))2D > z'(x^{CO})(1-\mathrm{z}(x^{CO}))D$, implying that the equilibrium R&D investments are higher under CO (given by (7)) compared to (22).

Since the R&D investments are higher under CO compared to the situation where the firms commit to share R&D outcomes but determine the R&D investments to maximise their own profits, we get the equilibrium expected consumer surplus higher under the former than the latter. This happens because if the firms commit to share R&D outcomes, the equilibrium expected consumer surplus is $z(x)(2-z(x))\,\mathrm{CS}^D$, which increases with higher R&D investments since $2z'(x)(1-z(x))\,\mathrm{CS}^D > 0$.

Since the equilibrium expected total profits of firms 1 and 2 and the equilibrium expected consumer surplus are higher under CO compared to the situation where the firms commit to share R&D outcomes but determine the R&D investments to maximise their own profits, the equilibrium expected welfare is higher under the former than the latter.

Since commitment before R&D to share R&D outcomes while investing in R&D non-cooperatively is similar to the case of perfect knowledge spillover of R&D outcomes under non-cooperative R&D, the analysis of this subsection suggests that CO is preferred than non-cooperative R&D with perfect knowledge spillover.

8.2 Asymmetric Firms, Oligopoly and Knowledge Spillover

We showed our results in a duopoly model with symmetric firms. Further, we assume away knowledge spillover to show the implications of convex costs. Future research can focus on oligopoly models with asymmetric firms (created by asymmetric production costs and/or asymmetric R&D costs), and knowledge spillover. However, let us briefly mention some of the implications of knowledge spillover, asymmetric firms and oligopoly models.

The implication of knowledge spillover is relatively easy to understand. Choi (1993) shows that knowledge spillover creates the incentive for cooperative R&D under stochastic non-tournament R&D. In contrast, we show that the presence of convex production costs may create the incentive for cooperation even if there is no knowledge spillover. Hence, it is intuitive that the inclusion of knowledge spillover increases the incentive for cooperative R&D in our analysis. In this respect, one may look at two types of knowledge spillovers – during the R&D phase and after the R&D outcome.[15]

The situations will be more complicated for asymmetric firms and oligopolistic market structure. First, consider the case of asymmetric cost firms. Asymmetric costs will create asymmetric equilibrium R&D investments, and it will bring the issue of production inefficiency. On the one hand, like our analysis, cooperation (either through cooperative R&D or through licensing ex-post R&D) will help to reduce the total cost in the industry by using two production plants than one plant. On the other hand, cooperation will tend to increase the total cost in the industry by allocating outputs to the higher cost firm if only the low-cost firm is successful in R&D. Given the equilibrium values are continuous in cost parameters, our results will go through if the extent of cost asymmetry is not large. If the cost asymmetry is large, it may not encourage licensing by the low-cost firm, and cooperative R&D may impose a negative effect on welfare as cooperation may

[15] See, e.g., d'Aspremont and Jacquemin (1988) for knowledge spillover after R&D, Kamien, Muller and Zang (1992) for knowledge spillover during R&D, and Amir (2000) for a comparison between these two types of spillovers.

increase the total cost in the industry significantly by allowing the high-cost firm to produce even if it is unsuccessful in R&D.

If the firms have asymmetric R&D costs, it will create asymmetric equilibrium R&D investments and therefore, different probabilities of success. Due to continuity, our results will go through if the asymmetry is not large. If the R&D costs are significantly different, the equilibrium R&D investments by the firms and the corresponding probabilities of success will differ significantly. In the extreme case, we will get one innovating firm and one non-innovating firm. However, the main reasons for our results (i.e., the innovator's incentive to keep the option for licensing open and the society's incentive for CO to avoid the output royalty) are expected to hold.

If we consider an oligopoly model with n innovating firms, it will bring at least two new features. First, there will be the possibilities where k number of firms are successful in R&D, where $1 < k < n$. This will imply that there can be potentially more than one licenser and more than one licensee. However, following La Manna (1993) and Creane, Ko and Konishi (2013) and because licensing helps to reduce the total costs by allocating total outputs in multiple plants, technology licensing to all licensees are expected to occur ex-post R&D if there are some firms which are not successful in R&D and the cost of technology licensing is not very high. If the cost of technology licensing is high, there will not be any licensing. If the cost of technology licensing is moderate, there may be licensing to a subset of firms.

Second, the presence of n firms will create the possibility of a *single group* created by all or a subset of firms or *multiple groups* under cooperation, depending on the way increased competition due to knowledge sharing affects the total profits in the industry. Following our analysis, it is expected that the society will prefer all firms to do cooperative R&D, since it will help all firms to use the new technology without incurring any output royalty. However, if the cost of technology licensing is low, the firms will prefer joint profit maximising R&D investments with licensing ex-post R&D. If the cost of technology licensing is high, the firms may not prefer licensing and may prefer complete non-cooperation, or a single group with all or a subset of firms or multiple groups, depending on the way increased competition due to knowledge sharing affects the total industry profits. If the cost of technology licensing is moderate, we may get joint profit maximising R&D investments with licensing ex-post R&D to a subset of firms. Hence, while our main conclusion is likely to remain under oligopoly for certain parametric configurations, such as the low cost of technology licensing and increasing industry profit ex-post R&D due to knowledge sharing, the oligopolistic market structure will generate new findings in terms of the number of licensing and the number of research groups created. In this respect, it will also contribute to the literature on R&D network (see, e.g., Billand et al., 2019 for a recent work on R&D network and the references therein).

9 Conclusion

We show that the firms may prefer cooperative R&D compared to non-cooperative R&D in the presence of convex production costs, even if there is no knowledge spillover. Thus, we provide a new rationale for cooperative R&D. Consumer surplus and the expected welfare can be higher under cooperative R&D compared to non-cooperative R&D.

The presence of convex production costs create the incentive for technology licensing ex-post R&D when there is a unilateral success in R&D. In the presence of licensing, the firms prefer joint profit maximising R&D investments with the option for licensing ex-post R&D, while the consumers and the society may prefer cooperative R&D. Hence, the firms' preferred R&D organisation can be significantly different from the consumers' and the society's preferred R&D organisation and a proper distribution scheme, such as a tax/subsidy policy, could be designed to encourage the firms to undertake cooperative R&D.

Following the literature (e.g., Choi, 1993), we have assumed a linear R&D cost. However, our results will hold even if we consider convex R&D costs. This will happen for the following reason. If we have considered convex R&D costs, $R(x_i)$, the right hand sides of the first order conditions will be $R'(x_i)$ instead of 1, and all the comparisons will follow in the same way. Due to continuity, the results under the special functional forms will also hold true, at least for lower convexity, which will be enough to prove the conflict of interest between the firms, and the consumers and the society.

Our analysis may have some implications for the COVID-19 vaccine research. Multiple successes and convex production costs are two important assumptions for our analysis, which may be relevant for the COVID-19 vaccine research.

Different research groups were trying to invent COVID-19 vaccines at the same time and multiple research groups, such as Pfizer-BioNTech, Oxford-AstraZeneca and Moderna-NIH, got the approval for their vaccines. Hence, it is similar to a non-tournament stochastic R&D process considered in this paper. CO will then imply an agreement between Pfizer-BioNTech, Oxford-AstraZeneca and Moderna-NIH.

The convex production cost represents soft capacity constraints. Hence, it can capture the possibility that producing vaccines within a short time period and distributing them quickly to different countries may impose capacity constraints or creates diseconomies of scale, which could make licensing of the vaccine technology profitable.

For the case of COVID-19 vaccines, the demand curve may reflect different country's willingness to pay for vaccines. Although, rather than Cournot competition, there could be other ways to sell the product, such as through bargaining between the research groups and the governments of different countries, it is intuitive that the way we model the product market competition is not important for our results. The trade-off between keeping the monopoly option of the research groups ex-post R&D and their commitment before R&D for sharing the R&D outcomes are the important factors for our results.

Some other assumptions considered in our analysis might also be relevant for the COVID-19 vaccine research. For example, no knowledge spillover might be a reasonable assumption, since all research groups were not trying to invent similar vaccine technologies, and travel restrictions were creating some barriers for the researchers to share their experiences. Similarly, besides the second order condition for maximisation, the travel restrictions, social distancing and the short time period within which the research groups were trying to innovate new vaccines, might justify the use two research labs under CO by making quick capacity expansion difficult.[16]

Of course, some of our assumptions might be restrictive for the case of COVID-19 vaccine research. For example, as mentioned in the subsection 8.2, in the presence of more than two research groups, there could be other possibilities, such as cooperative agreement among a subset of research groups or creation of multiple cooperative groups, which our analysis did not capture.

[16] If licensing is not a feasible option due to the restrictions created by the pandemic, we need to compare between CO and NC. Similarly, if it is difficult to monitor the R&D investments to implement joint profit maximising R&D investments, we need to compare NC and NCL. In this situation, the firms prefer NCL compared to NC, and as shown in appendix A.7 with the functional forms considered in Sections 6 and 7, the consumers and the society may also prefer NCL than NC.

However, given the relevance of some of our important assumptions with the COVID-19 vaccine research, our analysis may at least raise the following question. God forbid, if the world faces similar problems in the future, would it be better for the governments to work with the firms to create a system that encourages research groups to undertake cooperative R&D?

Appendix

A.1 Proof of $\frac{\partial q_1}{\partial r} > 0$, $\frac{\partial q_2}{\partial r} < 0$ and $\frac{\partial q}{\partial r} < 0$

Differentiating the first order conditions in (11) with respect to *r*, we get

(A1) $P' q_1' + P'\left(q_1' + q_2'\right) - q_1' c''(q_1) + q_1\left(q_1' + q_2'\right)P'' = 0$

(A2) $-1 + P' q_2' + P'\left(q_1' + q_2'\right) - q_2' c''(q_2) + q_2\left(q_1' + q_2'\right)P'' = 0$,

where $q_1' = \frac{\partial q_1}{\partial r}$ and $q_2' = \frac{\partial q_2}{\partial r}$.

Solving (A1) and (A2), we get $\frac{\partial q_1}{\partial r} = \frac{-\left(P' + q_1 P''\right)}{Z} > 0$ and $\frac{\partial q_2}{\partial r} = \frac{\left(2P' - c''(q_1) + q_1 P''\right)}{Z} < 0$, where $Z > 0$ (as mentioned in the text). We also get $\frac{\partial q}{\partial r} = \frac{\partial q_1}{\partial r} + \frac{\partial q_2}{\partial r} = \frac{\left(P' - c''(q_1)\right)}{Z} < 0$. *Q.E.D.*

A.2 Proof of Proposition 3

(i) The equilibrium R&D investments under NCL and CL are respectively given by (16), which is $z'(x^{NCL})\,\mathrm{z}(x^{NCL})D + z'(x^{NCL})(1 - z(x^{NCL}))L = 1$, and by (21), which is $z'(x^{CL})\,\mathrm{z}(x^{CL})2D + z'(x^{CL})(1 - 2z(x^{CL}))L = 1$. If we evaluate LHS of both (16) and (21) at x^{NCL}, we get LHS of (16) is greater than LHS of (21) since $L > D$, implying that the firms invest more in R&D under NCL compared to CL, i.e., $x^{NCL} > x^{CL}$. We follow the same process for the other parts of this Proposition.

(ii) The equilibrium R&D investments under NCL and NC are given by (16) and (2) respectively. If we evaluate LHS of both (16) and (2) at x^{NCL}, we get LHS of (16) is greater than LHS of (2), since $L > M$, implying $x^{NCL} > x^{NC}$.

(iii) The equilibrium R&D investments under NCL and CO are given by (16) and (7) respectively. If we evaluate LHS of both (16) and (7) at x^{NCL}, we get LHS of (16) is greater than LHS of (7), since $L > 2D$, implying $x^{NCL} > x^{CO}$.

(iv) The equilibrium R&D investments under CL and CO are given by (21) and (7) respectively. If we evaluate LHS of both (21) and (7) at x^{CL}, we get LHS of (21) is greater than LHS of (7), since $L > 2D$, implying $x^{CL} > x^{CO}$.

(v) The equilibrium R&D investments under NC and CO are given by (2) and (7) respectively. If we evaluate LHS of both (2) and (7) at x^{NC}, we get LHS of (2) >(<) LHS of (7) if $z(x^{NC})D + (1 - z(x^{NC}))(M - 2D) > (<)0$. Hence, if x^{NC} is such that $z(x^{NC}) \to 1$, we get $z(x^{NC})D + (1 - z(x^{NC}))(M - 2D) \to D > 0$. However, if x^{NC} is such that $z(x^{NC}) \to 0$, we get $z(x^{NC})D + (1 - z(x^{NC}))(M - 2D) \to (M - 2D) > (<)0$ for $M > (<)2D$.

Since $z(x^{NC})D + (1 - z(x^{NC}))(M - 2D)$ is linear and continuous in $z \in [0,1]$, if $M > 2D$, we get $x^{NC} > x^{CO}$ for $z(x^{NC}) \in [0,1]$ but if $M < 2D$, there exists a x^{NC}, say, x^{NC*}, such that $z(x^{NC*})D + (1 - z(x^{NC*}))(M - 2D) = 0$ and $x^{NC} > (<) x^{CO}$ for $x^{NC} \in (x^{NC*}, x^{NC\max}]$ $(x^{NC} \in [0, x^{NC*}))$ or $z(x^{NC}) \in (z(x^{NC*}),1]$ $(z(x^{NC}) \in [0, z(x^{NC*})))$, where $z(x^{NC\max}) = 1$. *Q.E.D.*

A.3 Proof of Proposition 4

(i) The total profits of firms 1 and 2 ex-post R&D are the same under *CL* and *NCL*, since (a) if both firms are successful in R&D, the total profits ex-post R&D are *2D*, but (b) if only one firm is successful in R&D, the total profits ex-post R&D are *L*. Hence, π^{CL} must be greater than π^{NCL}, since the market outcomes under both CL and NCL are the same ex-post R&D but the former determines the R&D investments to maximise the joint profits of the firms, while that is not the case under the latter.

Since x^{NCL} maximises the profits of the firms under NCL, we get from (17) and (3)

(A3)
$$\begin{aligned}\pi^{NCL} &= z^2(x^{NCL})2D + 2z(x^{NCL})(1-z(x^{NCL}))L - 2x^{NCL} > \\ &z^2(x^{NC})2D + 2z(x^{NC})(1-z(x^{NC}))L - 2x^{NC} > \\ &z^2(x^{NC})2D + 2z(x^{NC})(1-z(x^{NC}))M - 2x^{NC} = \pi^{NC},\end{aligned}$$

where the first inequality in (B1) follows from the definition that x^{NCL} is the equilibrium R&D investment under *NCL* and the second inequality in (A3) holds since $L > M$.

(ii) Since x^{CL} maximises the total profits of the firms under CL, we get from (22) and (8)

(A4)
$$\begin{aligned}\pi^{CL} &= z^2(x^{CL})2D + 2z(x^{CL})(1-z(x^{CL})L - 2x^{CL} > \\ &z^2(x^{CO})2D + 2z(x^{CO})(1-z(x^{CO}))L - 2x^{CO} > \\ &z^2(x^{CO})2D + 2z(x^{CO})(1-z(x^{CO}))2D - 2x^{CO} = \\ &z(x^{CO})(2-z(x^{CO}))2D - 2x^{CO} = \pi^{CO},\end{aligned}$$

where the first inequality in (A4) follows from the definition that x^{CL} is the equilibrium R&D investment under *CL* and the second inequality in (A4) holds since $L > 2D$.

(iii) We get from (8) and (4)

$$\text{(A5)}\quad \begin{aligned}\pi^{CO} &= z(x^{CO})(2-z(x^{CO}))2D-2x^{CO} = \\ &z^2(x^{CO})2D+2z(x^{CO})(1-z(x^{CO}))2D-2x^{CO} > \\ &z^2(x^{NC})2D+2z(x^{NC})(1-z(x^{NC}))2D-2x^{NC} > \\ &z^2(x^{NC})2D+2z(x^{NC})(1-z(x^{NC}))M-2x^{NC} = \pi^{NC},\ if\ 2D > M,\end{aligned}$$

where the first inequality in (A5) follows from the definition that x^{CO} is the equilibrium R&D investment under *CO*.

Now consider the case of $2D < M$. If $2D = M$, we get

$$\text{(A6)}\quad \begin{aligned}\pi^{CO} &= z^2(x^{CO})2D+2z(x^{CO})(1-z(x^{CO}))2D-2x^{CO} > \\ &= z^2(x^{NC})2D+2z(x^{NC})(1-z(x^{NC}))M-2x^{NC} = \pi^{NC},\end{aligned}$$

since the profits ex-post R&D are the same for the respective R&D outcomes and the firms choose R&D investments under CO to maximise their joint profits.

Since higher *M* increases the equilibrium R&D investments under NC (which follows from (2)) and π^{NC} is continuous in *M* and the R&D investments, it is clear from (A6) that there can be a value of *M (> 2D)*, such that the higher value of *M* and the corresponding higher R&D investments under NC can make $\pi^{NC} > \pi^{CO}$. *Q.E.D.*

A.4 Proof of Proposition 5

(i) We get from (18)

$$\begin{aligned}CS^{NCL} &= z^2(x^{NCL})\mathrm{CS}^D+2z(x^{NCL})(1-z(x^{NCL}))(\mathrm{CS}^D-\mathrm{CS}^D+\mathrm{CS}^L) \\ &= z(x^{NCL})(2-z(x^{NCL}))\mathrm{CS}^D-2z(x^{NCL})(1-z(x^{NCL}))(\mathrm{CS}^D-\mathrm{CS}^L).\end{aligned}$$

Hence,

$$\begin{aligned}CS^{CO}-CS^{NCL} = {}&[z(x^{CO})(2-z(x^{CO}))-z(x^{NCL})(2-z(x^{NCL}))]\mathrm{CS}^D \\ &+2z(x^{NCL})(1-z(x^{NCL}))(\mathrm{CS}^D-\mathrm{CS}^L).\end{aligned}$$

Since $z(x)(2-z(x))$ increases with x and $x^{NCL} > x^{CO}$, we get $[z(x^{CO})(2-z(x^{CO}))-z(x^{NCL})(2-z(x^{NCL}))]\mathrm{CS}^{D} < 0$. However, $2z(x^{NCL})(1-z(x^{NCL}))(\mathrm{CS}^{D}-\mathrm{CS}^{L}) > 0$ since $CS^{D} > CS^{L}$, which follows from Proposition 2.

Hence, $CS^{CO} > CS^{NCL}$ for $CS^{D} >> CS^{L}$, which happens if $r^{*} >> 0$, or $x^{NCL} \to x^{CO}$ but $CS^{CO} < CS^{NCL}$ for $CS^{D} \to CS^{L}$ or $x^{NCL} >> x^{CO}$.

(ii) We get from (23)

$$\begin{aligned} CS^{CL} &= z^{2}(x^{CL})CS^{D} + 2z(x^{CL})(1-z(x^{CL})(\mathrm{CS}^{D}-CS^{D}+CS^{L}) \\ &= z(x^{CL})(2-z(x^{CL}))\,\mathrm{CS}^{D} - 2z(x^{CL})(1-z(x^{CL})(CS^{D}-CS^{L}). \end{aligned}$$

Hence,

$$\begin{aligned} CS^{CO} - CS^{CL} = {} & [z(x^{CO})(2-z(x^{CO}))-z(x^{CL})(2-z(x^{CL}))]\mathrm{CS}^{D} \\ & + 2z(x^{CL})(1-z(x^{CL})(CS^{D}-CS^{L}). \end{aligned}$$

Since $x^{CL} > x^{CO}$ and $z(x)(2-z(x))$ increases with x, $[z(x^{CO})(2-z(x^{CO}))-z(x^{CL})(2-z(x^{CL}))]\mathrm{CS}^{D} < 0$. However, $2z(x^{CL})(1-z(x^{CL})(CS^{D}-CS^{L}) > 0$ since $CS^{D} > CS^{L}$, which follows from Proposition 2.

Hence, $CS^{CO} > CS^{CL}$ if $CS^{D} >> CS^{L}$ or $x^{CL} \to x^{CO}$ but $CS^{CO} < CS^{CL}$ if $CS^{D} \to CS^{L}$ or $x^{CL} >> x^{CO}$.

(iii) We get from (4)

$$\begin{aligned} CS^{NC} &= z^{2}(x^{NC})\,\mathrm{CS}^{D} + 2z(x^{NC})(1-z(x^{NC}))(\mathrm{CS}^{D}-\mathrm{CS}^{D}+\mathrm{CS}^{M}) \\ &= z(x^{NC})(2-z(x^{NC}))\,\mathrm{CS}^{D} - 2z(x^{NC})(1-z(x^{NC}))(\mathrm{CS}^{D}-\mathrm{CS}^{M}). \end{aligned}$$

Hence,

$$CS^{CO} - CS^{NC} = [z(x^{CO})(2 - z(x^{CO})) - z(x^{NC})(2 - z(x^{NC}))]\text{CS}^D$$
$$+ 2z(x^{NC})(1 - z(x^{NC}))(\text{CS}^D - \text{CS}^M).$$

Since $CS^D > CS^M$, which follows from Proposition 2, and $z(x)(2 - z(x))$ increases with *x*, we get $CS^{CO} > CS^{NC}$ if $x^{CO} > x^{NC}$.

If $x^{CO} < x^{NC}$, we get $[z(x^{CO})(2 - z(x^{CO})) - z(x^{NC})(2 - z(x^{NC}))]\text{CS}^D < 0$ but $2z(x^{NC})(1 - z(x^{NC}))(\text{CS}^D - \text{CS}^M) > 0$ since $z(x)(2 - z(x))$ increases with x and $CS^D > CS^M$.

Hence, if $x^{CO} < x^{NC}$, $CS^{CO} > CS^{NC}$ if $CS^D >> CS^M$ or $x^{NC} \to x^{CO}$ but $CS^{CO} < CS^{NC}$ if $CS^D \to CS^L$ or $x^{NC} >> x^{CO}$. *Q.E.D.*

A.5 Comparison of consumer surplus for the specific functional forms considered in Section 6

Given the functional forms considered, we can find the following equilibrium total outputs and profits ex-post R&D depending on the R&D outcomes. If both firms use the technology, the total equilibrium output ex-post R&D is $q = \frac{2}{3 + 2c}$, *i = 1, 2*. If only one firm is successful in R&D and there is no licensing ex-post R&D, the total equilibrium output ex-post R&D is $q^m = \frac{1}{2(1 + c)}$. If only one firm is successful in R&D and the successful firm licenses the technology to the unsuccessful firm ex-post R&D, the total equilibrium output ex-post R&D is $q^l = \frac{1 + 2c(5 + 4c)}{4(1 + c)(1 + 4c(2 + c))}$. If neither firm is successful is R&D, the total equilibrium output ex-post R&D is *0*.

If both firms use the technology, the equilibrium profit of the ith firm ex-post R&D is $\pi_i = \frac{1 + c}{(3 + 2c)^2}$, *i = 1, 2*. If only one firm is successful in R&D and there is no licensing ex-post R&D, the equilibrium profit of the successful firm ex-post R&D is $\pi^m = \frac{1}{4(1+c)}$ and the equilibrium profit of the unsuccessful firm ex-post R&D is *0*. If only one firm is successful in R&D and the successful firm licenses the technology to the unsuccessful firm ex-post R&D, the equilibrium profit of the successful firm ex-post R&D is $\pi^l = \frac{1+8c(1+c)}{4(1+c)(1+4c(2+c))}$ and the equilibrium profit of the unsuccessful firm ex-post R&D is *0*. If neither firm is successful is R&D, the equilibrium profit of each firm ex-post R&D is *0*.

Given the above-mentioned equilibrium profits, we get the equilibrium R&D investments under NC, CO, NCL and CL as $x_1^{NC} = x_2^{NC} = x^{NC} = \frac{(3+2c)^4}{[77+4c(43+8c(4+c))]^2}$, $x_1^{CO} = x_2^{C0} = x^{CO} = \frac{(1+c)^2}{(2+c)^2(5+4c)^2}$, $x_1^{NCL} = x_2^{NCL} = x^{NCL} = \frac{(3+2c)^4(1+8c(1+c))^2}{[77+4c(197+2c(231+2c(112+c(49+8c))))]^2}$ and

$$x_1^{CL} = x_2^{CL} = x^{CL} = \frac{\left(3+2c\right)^4\left(1+8c\left(1+c\right)\right)^2}{4\left[41+8c\left(52+c\left(121+2c\left(58+c\left(25+4c\right)\right)\right)\right)\right]^2}.$$

We get the equilibrium expected consumer surplus as $CS^{NC} = \frac{\left(3+2c\right)^2\left(19+c\left(25+8c\right)\right)}{\left(1+c\right)\left(77+4c\left(43+8c\left(4+c\right)\right)\right)^2}$, $CS^{CO} = \frac{2\left(1+c\right)\left(19+c\left(25+8c\right)\right)}{\left(2+c\right)^2\left(3+2c\right)^2\left(5+4c\right)^2}$,

$$CS^{NCL}=\frac{(3+2c)^2(1+8c(1+c))\left(19+c\left(397+4c\left(656+c\left(1473+16c\left(93+44c+8c^2\right)\right)\right)\right)\right)}{(1+c)(1+4c(2+c))\left(77+4c\left(197+2c\left(231+2c\left(112+c(49+8c)\right)\right)\right)\right)^2}$$ and

$$CS^{CL}=\frac{(3+2c)^2\left(1+8c+8c^2\right)\begin{bmatrix}81+2416c+27248c^2+145120c^3\\+387088c^4+574976c^5+501120c^6\\+255488c^7+70656c^8+8192c^9\end{bmatrix}}{(16(1+c)^2\left(1+8c+4c^2\right)^2\left[41+416c+968c^2+928c^3+400c^4+64c^5\right]^2}.$$

We find

$$CS^{CO}-CS^{NCL}=\frac{\begin{pmatrix}71402+1903406c+20890243c^2+127183005c^3+493315472c^4+\\1314902228c^5+2517782480c^6+3555877616c^7+3757351168c^8+\\2984520256c^9+1774245376c^{10}+777651200c^{11}+243890176c^{12}+\\51783680c^{13}+6668288c^{14}+393216c^{15}\end{pmatrix}}{(1+c)(2+c)^2(3+2c)^2(5+4c)^2\left(1+8c+4c^2\right)\left(77+788c+1848c^2+1792c^3+784c^4+128c^5\right)^2}>0,$$

for $c > 0$,

$$CS^{CO}-CS^{CL}=\frac{(1+2c)^2\begin{pmatrix}365948+11766156c+156838215c^2\\+1144970108c^3+5165830900c^4+15605721984c^5\\+33310656336c^6+52045754688c^7+60855031488c^8\\+53890392576c^9+36259937280c^{10}+18434983936c^{11}\\+6970163200c^{12}+1900593152c^{13}+353370112c^{14}\\+40108032c^{15}+2097152c^{16}\end{pmatrix}}{16(1+c)^2(2+c)^2(3+2c)^2(5+4c)^2\left(1+8c+4c^2\right)^2\left(41+416c+968c^2+928c^3+400c^4+64c^5\right)^2}>0,$$

for $c > 0$,

$$CS^{CO} - CS^{NC} = \frac{\left(19+25c+8c^2\right)\begin{pmatrix} 3758+34032c+118473c^2 \\ +220112c^3+245112c^4+169888c^5 \\ +72176c^6+17280c^7+1792c^8 \end{pmatrix}}{\left(1+c\right)\left(2+c\right)^2\left(3+2c\right)^2\left(5+4c\right)^2 \left(77+172c+128c^2+32c^3\right)^2} > 0,$$

for $c > 0$.

A.6 Comparison of welfare for the specific functional forms considered in Section 6

Given the equilibrium values shown in Appendix A.5, we get the equilibrium expected welfare as $W^{NC} = \frac{(3+2c)^2[37+c(67+8c(5+c))]}{(1+c)[77+4c(43+8c(4+c))]^2}$,

$$W^{CO} = \frac{2\left(1+c\right)\left(29+c\left(48+c\left(25+4c\right)\right)\right)}{\left(2+c\right)^2\left(3+2c\right)^2\left(5+4c\right)^2},$$

$$W^{NCL} = \frac{(3+2c)^2(1+8c(1+c))[37+c(727+4c(1174+c(2833+8c(410+c(245+8c(9+c))))))]}{(1+c)(1+4c(2+c))[77+4c(197+2c(231+2c(112+c(49+8c))))]^2}$$ and

$$W^{CL} = \frac{\left(3+2c\right)^2\left(1+8c+8c^2\right)\begin{bmatrix} 163+4642c+50872c^2+270536c^3 \\ +751152c^4+1201568c^5+1170560c^6 \\ +704384c^7+254464c^8+50176c^9+4096c^{10} \end{bmatrix}}{(16\left(1+c\right)^2\left(1+8c+4c^2\right)^2\left[41+416c+968c^2+928c^3+400c^4+64c^5\right]^2}.$$

We find

$$W^{CO} - W^{NCL} = \frac{\begin{pmatrix} 44182+1181700c+12955499c^2+78724565c^3+ \\ 305623922c^4+818145732c^5+1577818872c^6+ \\ 2248681936c^7+2400518464c^8+1927464256c^9+ \\ 1158481152c^{10}+513321984c^{11}+162716160c^{12}+ \\ 34908160c^{13}+4540416c^{14}+270336c^{15} \end{pmatrix}}{(1+c)\left(2+c\right)^2\left(3+2c\right)^2\left(5+4c\right)^2\left(1+8c+4c^2\right)\left(77+788c+1848c^2+1792c^3+784c^4+128c^5\right)^2} > 0$$ for $c > 0$,

$$W^{CO}-W^{CL}=\frac{(1+2c)^2\begin{pmatrix}239668+7802492c+105180253c^2+775926418c^3\\+3535846252c^4+10783254368c^5+23224714800c^6\\+36597960576c^7+43141696448c^8+38502565888c^9\\+26100841472c^{10}+13366330880c^{11}+5089396736c^{12}\\+1397317632c^{13}+261554176c^{14}+29884416c^{15}+1572864c^{16}\end{pmatrix}}{\begin{matrix}16(1+c)^2(2+c)^2(3+2c)^2(5+4c)^2(1+8c+4c^2)^2\\(41+416c+968c^2+928c^3+400c^4+64c^5)^2\end{matrix}}>0$$

for $c > 0$ and

$$W^{CO}-W^{NC}=\frac{\begin{pmatrix}44182+672132c+3447731c^2+9379089c^3\\+15867778c^4+17947736c^5+14017456c^6\\+7610384c^7+2825216c^8+684160c^9+97280c^{10}\\+6144c^{11}\end{pmatrix}}{\begin{matrix}(1+c)(2+c)^2(3+2c)^2(5+4c)^2\\(77+172c+128c^2+32c^3)^2\end{matrix}}>0 \text{ for } c > 0.$$

A.7 NCL – NC

Given the equilibrium values shown in Appendices A.5 and A.6, we get

$$CS^{NCL}-CS^{NC}=\frac{4c(3+2c)^2(17+23c+8c^2)\begin{pmatrix}5929+144914c+1383208c^2+6792832c^3\\+19509280c^4+35436576c^5+42535040c^6\\+34331456c^7+18494976c^8+6394112c^9\\+1286144c^{10}+114688c^{11}\end{pmatrix}}{\begin{matrix}(1+c)(1+8c+4c^2)(77+172c+128c^2+32c^3)^2\\(77+788c+1848c^2+1792c^3+784c^4+128c^5)^2\end{matrix}}>0,$$

for $c > 0$, and

$$W^{NCL}-W^{NC}=\frac{4c(3+2c)^2(17+23c+8c^2)\begin{pmatrix}5929+156002c\\+1622344c^2+8750160c^3+27534752c^4\\+54849312c^5+72735872c^6+65882304c^7\\+40955904c^8+17177856c^9\\+4634624c^{10}+722944c^{11}+49152c^{12}\end{pmatrix}}{\begin{matrix}(1+c)(1+8c+4c^2)(77+172c+128c^2+32c^3)^2\\(77+788c+1848c^2+1792c^3+784c^4+128c^5)^2\end{matrix}}>0$$

for $c > 0$.

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