

# Mapping mathematical competences across subjects for advanced level qualifications in England

Jennifer Norris  | Andrew Noyes 

School of Education, University of Nottingham, Nottingham, UK

## Correspondence

Jennifer Norris, School of Education, University of Nottingham, Jubilee Campus, Wollaton Road, Nottingham NG8 1BB, UK.  
Email: [jennifer.norris@nottingham.ac.uk](mailto:jennifer.norris@nottingham.ac.uk)

## Funding information

Economic and Social Research Council

## Abstract

Efforts to increase the number of young people in England studying mathematics post-16 have historically focused on participation in standalone mathematics qualifications. However, following the recent A-level reforms, many advanced level students are engaging with some form of mathematics through the mathematical content now formally embedded within other subjects. To offer a more comprehensive view of the mathematics being learned post-16 we present a framework analysis of the subject content documents of the 19 A-level subjects with a required quantitative component, using a recently developed framework of General Mathematical Competences (GMCs). Results are visualised as maps showing the presence of GMC sub-competences for individual A-level subjects and combinations of subjects. The application of the GMC framework in this new context provides a much-needed common language for cross-curricular discussion of the types of mathematics present in different subjects, with implications for interdisciplinary mathematics learning and curriculum alignment between post-16 and higher education across the disciplines. In addition, the framework highlights the non-binary nature of mathematics participation, calling into question what counts as participating in mathematics in the post-16 phase.

## KEYWORDS

interdisciplinary mathematics, mathematical competences, post-16, participation

This is an open access article under the terms of the [Creative Commons Attribution](https://creativecommons.org/licenses/by/4.0/) License, which permits use, distribution and reproduction in any medium, provided the original work is properly cited.

© 2023 The Authors. *The Curriculum Journal* published by John Wiley & Sons Ltd on behalf of British Educational Research Association.

## INTRODUCTION

Upper secondary mathematics education serves multiple purposes, including preparing students for everyday life, employment, and further study in both Science, Technology, Engineering and Mathematics (STEM) subjects and many increasingly data-reliant non-STEM subjects. The question of preparing students mathematically for all disciplines within higher education is certainly of international importance (e.g. Er, 2018; Neumann et al., 2021); however, the peculiarities of the English post-16 education system make this issue particularly pressing. As Hodgen et al. (2010) noted, the four countries of the United Kingdom were unusual in that mathematics was not a compulsory subject within general post-16 education. More recent policy changes have introduced a minimum numeracy requirement for England, beyond which mathematics is still optional. However, in early 2023 the British Prime Minister, Rishi Sunak, announced his ambition ‘to move towards all children studying some form of maths to 18’ (Sunak, 2023). It is, therefore, crucial to understand what form(s) this mathematics could take.

In the current context of (mostly) voluntary mathematics, policy in England has taken a ‘two pronged approach’ to increase post-16 mathematics participation, including both *adding* – encouraging the take-up of standalone mathematics qualifications – and *embedding* mathematics within other subjects (McAlinden & Noyes, 2019b). We first describe how these additional mathematics qualifications fit within existing post-16 pathways in England, before considering the limited research on embedded mathematics within the recently reformed A-level qualifications.

### ‘Added’ mathematics qualifications

Since 2015, young people in England have been required to remain in education or training until the age of 18, meaning that the question of who is studying mathematics in the post-16 phase is more relevant than ever. While there has been political support for compulsory mathematics to 18 for a number of years, it has not been implemented due to feasibility issues, including the provision of appropriate qualifications, supply of qualified teachers and funding (Advisory Committee on Mathematics Education [ACME], 2014; Mathematics in Education and Industry [MEI], 2011; Smith, 2017). A new advanced maths premium does now provide some additional funding for schools and colleges offering advanced mathematics qualifications, with evaluation of the policy still pending (Education Endowment Foundation [EEF], 2018), while a basic maths premium for students who have not yet met the minimum numeracy requirement is only now being piloted (EEF, 2021). It is, therefore, too soon to judge the impact of these funding incentives.

Up to the age of 16, all students follow the national curriculum, culminating in General Certificate of Secondary Education (GCSE) examinations in a range of subjects including English and mathematics. Following GCSEs, young people can broadly choose between academic, technical, and vocational pathways (Table 1). For more detail on the complexity of vocational and level 2 qualifications see Department for Education (DfE, 2019). We focus here on level 3, ‘advanced’ qualifications, as being potential routes to higher education. Students with technical or vocational qualifications generally study one major subject over 2 years, whereas those on the academic route typically choose three or four subjects over 2 years (A-levels) and may also take additional 1-year courses (AS-levels). For clarity, we will use lower case when talking about a subject area in general (e.g. mathematics), and capitalise the word when talking about a specific qualification (e.g. A-level Mathematics).

Mathematics is an option at both AS- and A-level, and some students additionally choose an AS- or A-level in Further mathematics. Alternatively, students on any pathway can opt to take Core Maths as an additional qualification or may be required to resit GCSE Mathematics if they did not achieve a passing grade at age 16 (i.e. the minimum numeracy requirement

TABLE 1 A simplified overview of level 3 post-16 qualification pathways in England.

|        |        | Academic pathway<br>Level 3 |         |         |                       | Technical pathway<br>Level 3 |         | Vocational pathway<br>Level 3 |                                                              | Additional mathematics qualifications |                                    |    |                                    |
|--------|--------|-----------------------------|---------|---------|-----------------------|------------------------------|---------|-------------------------------|--------------------------------------------------------------|---------------------------------------|------------------------------------|----|------------------------------------|
| Age 17 | Age 18 | A-level                     | A-level | A-level | AS-level <sup>a</sup> | or                           | T-level | or                            | Vocational qualification, e.g. BTEC, advanced apprenticeship | +                                     | Core Maths <sup>a</sup><br>Level 3 | or | GCSE Resit <sup>a</sup><br>Level 2 |
|        |        |                             |         |         |                       |                              |         |                               |                                                              |                                       |                                    |    |                                    |

Blue shading indicates qualification pathways, while purple shading shows additional qualifications that can be taken alongside these pathways.

<sup>a</sup>One-year courses may be taken at age 17 or 18.

mentioned above, also called level 2). Core Maths is an umbrella term for a set of applied mathematics qualifications aimed at preparing students for further study in subjects such as geography and business where some application of mathematical and statistical knowledge may be important (DfE, 2013). Comparison of student numbers for these four mathematics qualifications is difficult due to different baselines used in the reporting of publicly available data. However, examination entry numbers for one recent cohort suggest that around 50% of students were entered for some form of post-16 mathematics, with most of these retaking GCSE (Table 2). This tallies with Sunak's (2023) assertion that 'right now, just half of all 16–19-year-olds study any maths at all'. Mathematics remains the most popular A-level subject, although student numbers have stabilised in recent years suggesting that perhaps a saturation point has been reached; meanwhile, entries to Core Maths qualifications have increased only gradually since their introduction in 2014 (Homer et al., 2020). With the current set of available qualifications, it, therefore, seems that it may be difficult to appreciably increase participation numbers in the future. Moreover, 'participation' in this context simply means that a student is entered for a particular qualification and hence does not say anything about their learning experience or engagement with the subject.

Nevertheless, there is some consensus that being entered for, and achieving well in, advanced mathematics qualifications is beneficial for students taking a range of subjects in higher education. A-level Mathematics grade was found to be a predictor of success in first-year science modules (King & Hambrook, 2020), although other studies have suggested that mathematical aptitude is more important than specific qualifications (Adkins & Noyes, 2017). In geography, undergraduates who had taken Mathematics A-level reported that the statistics components had provided helpful preparation for their degree course (Darlington & Bowyer, 2017). However, there can be considerable variation in the highest prior mathematics qualification of students studying the same subjects in different universities (Hodgen et al., 2020), raising questions about equality of opportunity. Hence, ensuring all students have access to appropriate post-16 mathematics education also has implications for social justice.

Mathematics embedded in A-level subjects

Embedding mathematics within other subjects is, therefore, a means of increasing the number of students encountering mathematics in some form, with the potential additional benefits of highlighting the role of mathematics within other disciplines and developing students' ability

**TABLE 2** Mathematics qualification entries in England for the cohort aged 16 in 2019. GCSE, AS- and A-level data from Ofqual (2019, 2020, 2021), Core Maths from MEI (2021).

|                                                                                                        | Age 17,<br>2020     | Age 18,<br>2021     | Total post-16,<br>2020–2021 |
|--------------------------------------------------------------------------------------------------------|---------------------|---------------------|-----------------------------|
| GCSE Mathematics                                                                                       | 81,035              | 99,590 <sup>a</sup> | 180,625 <sup>b</sup>        |
| AS-level Mathematics                                                                                   | 9775                | 1155                | 10,930                      |
| A-level Mathematics                                                                                    | 2490                | 80,615              | 83,105                      |
| Core Maths                                                                                             | 11,792 <sup>c</sup> | –                   | 11,792                      |
| Total post-16 mathematics                                                                              | 105,092             | 181,360             | 286,452                     |
| Percentage of cohort, estimated using age 16 GCSE<br>Mathematics entries in 2019 ( <i>n</i> = 552,340) | 19.0%               | 32.8%               | 51.9%                       |

<sup>a</sup>Entries are for age 18+ and may include students who were entered for GCSE at both age 17 and 18, so is an overestimate.  
<sup>b</sup>Many of these GCSE resit students will be following level 2 pathways, rather than the level 3 pathways shown in Table 1.  
<sup>c</sup>Core Maths data are not split by age; it is assumed here that all students took the qualification at age 17.

to apply mathematics within subject-based contexts. While the embedding approach applies across academic, technical and vocational pathways, in this paper, we focus on A-levels to examine the effects of recent curriculum reforms (from 2015), which formalised the mathematical content of 19 A-levels, including subjects such as Geography and Sociology as well as the sciences (Smith, 2013). Determining the number of students affected is difficult as published entry numbers relate to individual subjects and cannot simply be summed up.

Nevertheless, a key curriculum question is whether embedded mathematics provides appropriate and sufficient mathematical preparation for university study. Prior to the reforms, the quantitative content in both A-level Psychology and Sociology was found to be primarily statistics, taught in the context of research methods, with the Psychology curriculum covering the content in greater depth (Field, 2014; Scott Jones & Goldring, 2014). More recent studies have similarly explored the quantitative research methods embedded within the reformed curricula for A-level Sociology (Hampton, 2018) and A-level Geography (Harris, 2020). However, a fundamental problem with this single-subject approach is that many university subjects do not require students to have taken the same subject at A-level. Hence it is far from guaranteed that a sociology undergraduate will have taken A-level Sociology, or that a psychology undergraduate will have an A-level in Psychology. It is, therefore, important to consider not only the mathematics within individual subjects but also how these mathematics interrelate. In their analysis of sample assessment materials for the reformed A-levels in Biology, Chemistry and Physics, McAlinden and Noyes (2019a) make an important step in this direction by comparing the mathematical content domain, processes and level across the sciences. This paper seeks to extend current knowledge by examining the mathematics present within all quantitative A-level subjects, with the aim of offering a more comprehensive and authentic view of the mathematics being learned post-16 in England, as well as contributing to a broader discussion about meaningful ways to compare mathematical content across subjects. Our focus is on the written, ‘intended’ curriculum. While there will certainly be gaps between the curriculum intended to be taught and that which is actually enacted by teachers or received by students, these are beyond the scope of the current study. This research, therefore, addresses two questions:

1. What mathematics is present within the intended curricula for individual A-level subjects?
2. How does mathematics present within the curriculum interact across combinations of A-level subjects?

Mathematical competences across subjects

Approaching these questions requires a means of comparing mathematics across subjects using a common language that makes sense within each subject, and that attempts to avoid historical hierarchies of knowledge, for example, the privileging of ‘hard’ over ‘soft’ subjects and quantitative over qualitative methods (Williams et al., 2016). One previous attempt to solve this problem was concerned with the mathematics embedded in the new T-level qualifications and, therefore, aimed to provide a common language for the mathematics embedded in technical areas as diverse as construction, finance, education, and health care. This resulted in the set of 10 General Mathematical Competences (GMCs) shown in Table 3 (ACME, 2019). In our experience, these GMCs have been successful in circumventing traditional academic-vocational hierarchies in further education colleges, where they have been used to facilitate cross-curricular conversations about mathematics between vocational areas. In contrast, the Nuffield Foundation’s (2012) comprehensive analysis of the mathematics within A-level assessments used the broad content categories of number, algebra, geometry, graphs, statistics and probability, drawn from the school mathematics curriculum. This risks reinforcing existing hierarchies, where disciplinary mathematical activity may not be recognised as such because it is not couched in the language of the mathematics classroom. Williams and Wake (2006) point out the privileging of school mathematics over workplace mathematics, since ‘everyone goes to school and learns to some extent to speak this genre: it is the commonly accepted cultural model of what ‘mathematics’ is for most people. This in turn inflects our interactions with workplaces, wherein workers say they ‘don’t do mathematics’ (p. 338). The same could easily be true of other school subjects.

The use of the term ‘competences’ is intended to encompass both knowledge and skills as being essential to the application of mathematics within interdisciplinary contexts and, therefore, makes sense within the context of embedded mathematics in A-levels. As Hyland and Johnson (1998) convincingly argued over 20 years ago, all skills are context-bound, so that it is meaningless to attempt to teach a generic skill such as numeracy outside of a specific domain or to assume that skills learnt in one context can be universally applied to all contexts. In contrast, the generality aimed for here acknowledges the contextual nature of mathematics learning and looks for similarities in the kinds of mathematics being used within different contexts. From a communities-of-practice perspective (Wenger, 1998), we could describe the GMCs as boundary objects between two (or more) distinct communities of practice, that is, ‘artefacts that are structured in ways that they have global meaning in general across the boundary but are also sufficiently ‘plastic’ to be locally adaptable’ (ACME, 2019, p. 8). This means that we are also not expecting these general competences

TABLE 3 Mathematical competences identified in different frameworks.

| General Mathematical Competences (GMCs)                  | Competences and the learning of mathematics (KOM) project | PISA 2022 mathematics framework |
|----------------------------------------------------------|-----------------------------------------------------------|---------------------------------|
| Measuring with precision                                 | Thinking mathematically                                   | <i>Content domains</i>          |
| Estimating, calculating and error spotting               | Posing and solving mathematical problems                  | Quantity                        |
| Working with proportion                                  | Modelling mathematically                                  | Uncertainty and data            |
| Using rules and formulae                                 | Reasoning mathematically                                  | Change and relationships        |
| Processing data                                          | Representing mathematical entities                        | Space and shape                 |
| Understanding data and risk                              | Handling mathematical symbols and formalisms              | <i>Mathematical reasoning</i>   |
| Interpreting and representing with mathematical diagrams | Communicating in, with and about mathematics              | Interpret & evaluate            |
| Communicating using mathematics                          | Making use of aids and tools (IT included)                | Formulate                       |
| Costing a project                                        |                                                           | Employ                          |
| Optimising work processes                                |                                                           |                                 |

to be automatically transferrable between contexts for any given student without the additional work of making explicit connections between the contexts in question. In contrast, we could also talk about specific mathematics competences that are unique to only one discipline. Indeed, in the future, this could form an extension to the GMC framework, highlighting the differences between the mathematics of different disciplines alongside the similarities.

The focus on the mathematics common to different vocational areas is a distinguishing feature of the GMCs, compared with other mathematics competence frameworks (Table 3). The Danish Competencies and the Learning of Mathematics (KOM) project aimed to produce a common framework for the different phases of mathematics education, from primary up to higher education (Niss, 2003), while the Programme for International Student Assessment (PISA) 2022 Mathematics Framework was designed for international comparison of mathematical literacy (Organisation for Economic Co-operation and Development, 2018). A common feature of all three frameworks is that competences are independent of the level of difficulty. That is, each competence can be demonstrated at a range of levels, from very basic to very advanced. However, they take different approaches to the relationship between the two dimensions of content (or knowledge) and reasoning (or skills). The KOM framework highlights mathematical reasoning as the thread running through phases of education, whereas primary, secondary and higher education mathematics curricula may well have little subject content in common. This approach would be difficult to apply to mathematics embedded in other subjects because the focus on thinking, modelling and reasoning *mathematically* fails to define what this looks like outside of the mathematics classroom. Indeed, Niss (2003) notes that 'although the competences are formulated in terms that may apply to other subjects as well, these terms are here to be understood in a strict mathematical sense' (p. 9). In contrast, the PISA framework lists both content domains and mathematical reasoning skills, with clear parallels to the GMCs. For example, the *working with proportion* GMC encompasses interpreting, evaluating, formulating and employing the content domain *change and relationships*. Hence in the GMC framework, content and reasoning are interwoven. A further difference between the PISA and GMC frameworks is the range of contexts in which they are intended to operate. While PISA views mathematical literacy as involving 'real-world' applications, this is still from the perspective of the mathematics curriculum, whereas the GMCs were designed to describe the mathematics that takes place outside the mathematics classroom. We do not yet know whether the same set of competences can provide a useful common language between academic subjects as well as technical and vocational ones. Choosing to employ the GMCs in the context of A-levels is, therefore, both exploratory and pragmatic, investigating what this existing framework can reveal in a new context as well as producing an analysis that can subsequently be used to compare the mathematics across both A-level and T-level post-16 pathways.

## RESEARCH DESIGN

### Data collection

Data consisted of the subject content documents for all 19 reformed A-level qualifications with a quantitative element: Accounting, Biology, Business, Chemistry, Computer Science, Design and Technology, Economics, Electronics, Environmental Science, Further Mathematics, Geography, Geology, Mathematics, Music Technology, Physical Education, Physics, Psychology, Sociology and Statistics. These are publicly available on the Gov.UK website (DfE, 2014) and set out minimum content requirements for both AS- and A-level qualifications, although here we only consider the full A-levels.

Based on these minimum requirements, the qualifications themselves are developed by a number of competing awarding bodies, each with their own specification and examinations, accredited by the Office of Qualifications and Examinations Regulation (Ofqual) (Isaacs, 2010). While all subjects will have some variation between curricula offered by different awarding bodies, some of the subject content documents are much more prescriptive than others. Mathematics and Further Mathematics have the longest documents at 25 and 27 pages respectively, while the subject content for Computer Science is covered in four pages and Sociology in only three pages. The minimum mathematics requirements for Computer Science and Sociology are, therefore, likely to be very small. Some awarding bodies may choose to go beyond these minimum requirements; however, comparison of these interpretations of the subject content is beyond the scope of the current, exploratory paper.

## Data analysis

Documents were examined using framework analysis, a form of thematic analysis enabling comparison across and within cases through the use of a spreadsheet or table to organise the data. As with all forms of qualitative analysis, interpretation and transparency are crucial, despite the highly structured outputs which can give the impression of objectivity (Gale et al., 2013). Like Garthwaite et al.'s (2013) study into scientific literacy, our analysis used a pre-determined framework to organise the data, retaining only those elements which fit within the framework of the GMCs. This approach differs from the majority of thematic analyses where all the data must be represented within the themes generated. A limitation of any framework is the subjective selection involved in choosing what to keep and what to leave out. However, the strength of this approach is in making 'useful sense of the infinite detail and nuance of reality' (Jameson, 2016, p. 6) within a given context and for specific purposes.

To remain close to the GMC framework as originally described, each competence was expanded into sub-competences (Table 4), based on their longer descriptions (ACME, 2019). Most competences seemed to naturally divide into four elements (labelled a, b, c, d), and the others were adjusted to ensure equal weighing between competences. An iterative process was used to search for evidence of these sub-competences within the subject content documents. First, for those documents containing an explicit list of mathematical or quantitative skills, each of these skills was matched with an appropriate GMC sub-competence, as well as being used to suggest keywords (Table 4). Mathematics, Further Mathematics, and Statistics were excluded from this first step as not including a list of quantitative skills separate from the main subject content, so areas of mathematics excluded from the analysis were 'pure' topics such as sets, complex numbers, and calculus. Second, the entire documents for all subjects were searched for the keywords, with the first instance only being recorded in a spreadsheet since the aim was to look for the presence of the GMCs, not the frequency of occurrence. Third, all documents were read for any evidence missed by the first two rounds, with new keywords added as appropriate. All documents were then searched again using these additional keywords. This third round was important to surface quantitative skills couched in disciplinary terms.

While the keywords listed may look like a list of subject content, in practice, they were simply a means of locating possible mathematics within the text before applying judgement to decide whether a GMC was present. For example, in Physics, force diagrams were considered to be mathematical due to the integral role of angles but circuit diagrams were considered to be non-mathematical. This often required reference to the wider context of the statement. For example, 'communicate accounting information' was taken to be evidence of *communicating using mathematics* because of the mention of numerical and graphical as well as written forms. Likewise, the sub-competences relating to *moving between graphical, numerical, algebraic, and written forms* were evidenced by reference within the text to two or

**TABLE 4** General Mathematical Competence (GMC) framework expanded to include sub-competences.

| <b>GMC</b>                                               | <b>Sub-competences</b>                                            | <b>Keywords</b>                                                            |
|----------------------------------------------------------|-------------------------------------------------------------------|----------------------------------------------------------------------------|
| Measuring with precision                                 | a. Measuring physical objects and processes                       | Measure, size                                                              |
|                                                          | b. Measuring constructs                                           | Measure, indicator, index                                                  |
|                                                          | c. Units of measurement                                           | Unit                                                                       |
|                                                          | d. Accuracy of measurement                                        | Accuracy, significant figures                                              |
| Estimating, calculating, and error spotting              | a. Estimation                                                     | Estimate/estimation                                                        |
|                                                          | b. Use of number/calculating                                      | Calculate/calculation, number/numerical, numeracy, operations              |
|                                                          | c. Order of magnitude calculations                                | Magnitude, standard form, ordinary form                                    |
|                                                          | d. Recognise sources of error/error spotting                      | Error, outlier                                                             |
| Working with proportion                                  | a. Numerical                                                      | Fraction, ratio, percentage                                                |
|                                                          | b. Graphical                                                      | Graph, chart, linear, rate of change                                       |
|                                                          | c. Algebraic                                                      | Formula, line/linear, correlation                                          |
|                                                          | d. Move between graphical, numerical, algebraic and written forms |                                                                            |
| Using rules and formulae                                 | a. Use appropriate symbols                                        | Symbol, notation                                                           |
|                                                          | b. Substitute numerical values into a formula                     | Formula, calculate (where formula implied), equation                       |
|                                                          | c. Change the subject of a formula                                | Formula, equation, solve                                                   |
|                                                          | d. Select appropriate data and units                              | Data, units                                                                |
| Processing data                                          | a. Identifying suitable data                                      | Qualitative/quantitative data, levels of measurement, variables, data type |
|                                                          | b. Collecting, generating and organising data                     | Record data, collect data/Information, organise data                       |
|                                                          | c. Generating and interpreting graphs                             | Graph, chart                                                               |
|                                                          | d. Using technology to process data                               | ICT, digital, technology                                                   |
| Understanding data and risk                              | a. Avoiding bias and misrepresentation                            | Bias, sampling                                                             |
|                                                          | b. Measures of location and spread                                | Average, mean, median, range                                               |
|                                                          | c. Establishing relationships and making predictions              | Data analysis, predict, statistical test, model                            |
|                                                          | d. Probability                                                    | Probability                                                                |
| Interpreting and representing with mathematical diagrams | a. Move between graphical, numerical, algebraic and written forms |                                                                            |
|                                                          | b. Work with appropriate diagrams                                 | Diagram, draw, coordinates                                                 |
|                                                          | c. 2D and 3D shapes                                               | Shape, 2D, 3D, area, volume, angle                                         |
|                                                          | d. Use of technology to work with diagrams                        | ICT, digital, technology                                                   |

TABLE 4 (Continued)

| GMC                                   | Sub-competences                                                                 | Keywords                                                         |
|---------------------------------------|---------------------------------------------------------------------------------|------------------------------------------------------------------|
| Communicating using mathematics       | a. Reason with mathematics/logic                                                | Logic, reason                                                    |
|                                       | b. Communicate using mathematics (calculations, diagrams, data representations) | Communicate, articulate, present, report                         |
|                                       | c. Draw conclusions from mathematical information in context                    | Interpret, evaluate, significance, context, conclusion, decision |
|                                       | d. Adapt presentation to intended purpose and audience                          | Stakeholder, audience, customer/client, non-specialist           |
| Costing and optimising work processes | a. Calculating costs (financial, resources, labour, space)                      | Cost, time, space                                                |
|                                       | b. Using costs to make decisions, including considering risks                   | Cost, decision, risk                                             |
|                                       | c. Organising the factors involved in a complex process                         | Organise, complex                                                |
|                                       | d. Optimising a process                                                         | Optimise, model                                                  |

more of these forms together, for example as alternative representations, rather than being identified by specific keywords. Coding and analysis were conducted by one of the authors, with the other author acting as a discussant throughout the process.

The GMC framework, including sub-competences, was refined over the course of this iterative process. A notable example was the distinction between measuring physical objects and processes such as mass and speed, and measuring constructs such as efficiency, biodiversity, and anxiety. This was not evident in the original GMC descriptor for *measuring with precision* but arose from the use of the keyword ‘measure’ in the subject content documents. The two competences *costing a project* and *optimising work processes* were the least represented within the A-level documents and so have been combined here, resulting in an overall framework of nine rather than 10 competences.

Limitations

The current analysis is only concerned with the presence or absence of mathematical competences within curriculum documents. Given the sparseness of the subject content documents, additional dimensions such as degree of coverage or difficulty level (Niss, 2003) would perhaps make more sense when applied to the enacted, or perhaps to the assessed, curriculum. For example, awarding body assessment materials were used by the Nuffield Foundation (2012) to explore the extent and complexity of mathematics assessed in six A-level subjects, as well as the types of mathematical content. A key finding was the variability in assessed mathematical content depending on how students respond to a question, for example choosing to refer to a quantitative case study. Hence, a further possible level of analysis would be to consider the attained curriculum by analysing the mathematical content of students’ examination scripts. The present study is, therefore, only a first step to developing a more comprehensive comparative framework to fully describe the mathematical routes through the A-level curriculum as experienced and attained by students.

In addition, while framework analysis has the appearance of quantitative objectivity, it is based on an inevitably subjective content analysis. As mathematics educators and researchers, we bring our own preconceptions about what counts as mathematics, which may well

differ from the perspectives of specialists in other subjects. What we hope to present is a tool to facilitate discussion across these different viewpoints.

## FINDINGS

### What mathematics is present within the intended curricula for individual A-level subjects?

The subjects with the greatest number of GMC sub-competences found in the intended curricula are Environmental Science and Geology, while Music Technology and Sociology have the fewest (Table 5). The total number of sub-competences here should not be taken as the amount or depth of mathematical content but rather indicates the breadth of mathematics by counting the number of different competences within the subject content documents. Hence, Further Mathematics covers fewer sub-competences than either Mathematics or Statistics A-levels, suggesting a narrower focus. There is some variation between the traditional sciences, with Biology and Physics covering more GMCs than Chemistry. Within the social sciences, there are even greater differences, with Psychology A-level covering a broader range of GMCs than Business or Economics, and a great deal more than Sociology. Comparisons can also be made regarding specific GMCs. For example, the A-levels with the greatest presence of *measuring with precision* sub-competences are Environmental Science, Biology, and Geography; while Design and Technology and Computer Science are comparatively strong in *costing and optimising work processes*. The GMCs present in all 19 subjects are *processing data* and *communicating with mathematics*, although within these there are no sub-competences common to all subjects.

### How does mathematics present within the curriculum interact across combinations of A-level subjects?

The most prevalent GMCs across the 19 A-level subjects are *processing data* and *understanding data and risk* (Figure 1). Students are, therefore, reasonably likely to encounter data-related competences in more than one of their subject choices. *Costing and optimising work processes* is the least represented overall (Figure 1), meaning that this competence is unlikely to occur more than once in any given subject combination.

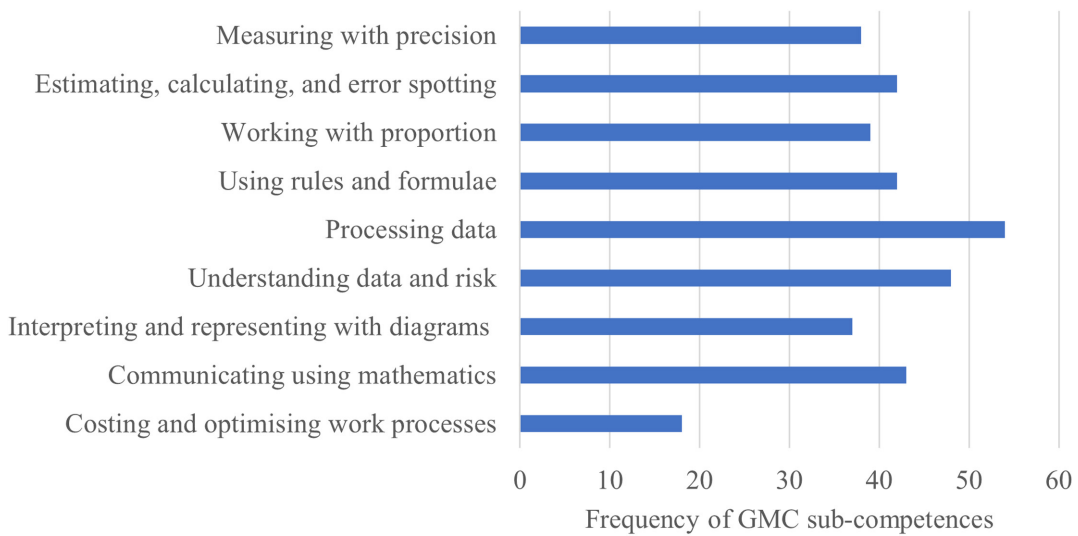
To consider this interaction between subjects further, we present a series of examples. These are in no way comprehensive but are chosen to illustrate the possible diversity of mathematical experiences of students who go on to study similar degree programmes at university. Thus, Tables 6 and 7 show GMC maps for four common subject combinations taken by undergraduate social science students (Vidal Rodeiro, 2019). The overall presence of a sub-competence is determined using an inclusive OR, that is, a sub-competence is marked as present within the overall subject combination if it is present within at least one of the individual subjects. Hence, presence in the subject combination is not a measure of the extent of coverage; it could equally mean that a sub-competence is present in all three subjects or only one.

Remembering that we are discussing curriculum intentions, not learning outcomes, the overall effect of choosing History (a non-quantitative subject) over Geography A-level, alongside Economics and Mathematics, is slightly narrower coverage of the *measuring with precision* and *processing data* GMCs (Table 6). While Geography also has strengths in *understanding data and risk* and *communicating using mathematics*, all these elements

**TABLE 5** GMC sub-competences present in A-level subjects. Blue shading indicates that a sub-competence is present in the subject curriculum.

| General<br>Mathematical<br>Competences      | A-level subject       |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|---------------------------------------------|-----------------------|---------|---------|---------|-------------|------------|-----------|-------------|---------------------|---------------------|------------|------------|-----------|------------------|----------|--------------------|
|                                             | Environmental Science | Geology | Biology | Physics | Electronics | Statistics | Chemistry | Mathematics | Design & Technology | Further Mathematics | Psychology | Accounting | Geography | Computer Science | Business | Physical Education |
| Measuring with precision                    | a                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | b                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | c                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | d                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
| Estimating, calculating, and error spotting | a                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | b                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | c                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | d                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
| Working with proportion                     | a                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | b                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | c                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | d                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
| Using rules and formulae                    | a                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | b                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | c                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | d                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
| Processing data                             | a                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | b                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | c                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | d                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
| Understanding data and risk                 | a                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | b                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | c                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | d                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
| Interpreting and representing with diagrams | a                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | b                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | c                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | d                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
| Communicating using mathematics             | a                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | b                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | c                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | d                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
| Costing and optimising work processes       | a                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | b                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | c                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
|                                             | d                     |         |         |         |             |            |           |             |                     |                     |            |            |           |                  |          |                    |
| Number of sub-competences                   | 29                    | 29      | 28      | 27      | 26          | 26         | 25        | 25          | 20                  | 18                  | 18         | 16         | 15        | 14               | 11       | 11                 |

Note: Subjects are ordered by the number of sub-competences (descending). See Table 4 for the description of letters a, b, c and d.



**FIGURE 1** Frequency of GMC sub-competences across all A-level subjects, out of a maximum of 76 (4 sub-competences × 19 subjects).

are covered within either the Economics or Mathematics curriculum and so are still present within the overall combination.

In contrast, when in combination with Psychology and Sociology, choosing Law instead of Geography has a greater effect on the breadth of GMCs covered (Table 7). The non-overlapping strengths between the Geography and Psychology curricula mean that the GMCs present in Geography are not entirely covered by the other subjects. In addition, the very narrow coverage of GMCs in Sociology means that in the combination of Law + Psychology + Sociology, the mathematical content is concentrated within a single subject, namely Psychology.

As a further example, we present GMC maps for the 10 most common A-level subject combinations for business studies degree students (Table 8). In this instance, there is a clear advantage in the inclusion of A-level Mathematics within a subject combination. This is particularly striking for the two GMCs *working with proportion* and *interpreting and representing with diagrams*. This is not simply down to the Mathematics curriculum, since in all the cases where Mathematics is one of the chosen subjects, the overall GMC coverage includes sub-competences not present within A-level Mathematics. Moreover, combinations without Mathematics can still have strengths in terms of specific GMCs. For example, *using rules and formulae* and *understanding data and risk* are strengths for the combination of Business + Psychology + Economics/Sociology. However, they are also more likely to have competence gaps, with three of the subject combinations having no coverage at all of *using rules and formulae*. Hence, the presence of mathematical competences within pre-university qualifications can vary widely for students on similar degree programmes, purely based on their subject choices.

## DISCUSSION

### Post-16 mathematics participation

Post-16 mathematics participation has to date been discussed primarily in binary terms: either you are taking mathematics or you are not. Thus, political statements consistently

**TABLE 6** GMC sub-competences present in A-level subject combinations:  
Mathematics + Economics + Geography and Mathematics + Economics + History. Blue shading indicates that a sub-competence is present in the subject curriculum, dark blue shading indicates that the sub-competence is present within the subject combination.

| General Mathematical Competences            |   | Mathematics | Economics | Geography | Mathematics + Economics + Geography |
|---------------------------------------------|---|-------------|-----------|-----------|-------------------------------------|
| Measuring with precision                    | a |             |           |           |                                     |
|                                             | b |             |           |           |                                     |
|                                             | c |             |           |           |                                     |
|                                             | d |             |           |           |                                     |
| Estimating, calculating, and error spotting | a |             |           |           |                                     |
|                                             | b |             |           |           |                                     |
|                                             | c |             |           |           |                                     |
|                                             | d |             |           |           |                                     |
| Working with proportion                     | a |             |           |           |                                     |
|                                             | b |             |           |           |                                     |
|                                             | c |             |           |           |                                     |
|                                             | d |             |           |           |                                     |
| Using rules and formulae                    | a |             |           |           |                                     |
|                                             | b |             |           |           |                                     |
|                                             | c |             |           |           |                                     |
|                                             | d |             |           |           |                                     |
| Processing data                             | a |             |           |           |                                     |
|                                             | b |             |           |           |                                     |
|                                             | c |             |           |           |                                     |
|                                             | d |             |           |           |                                     |
| Understanding data and risk                 | a |             |           |           |                                     |
|                                             | b |             |           |           |                                     |
|                                             | c |             |           |           |                                     |
|                                             | d |             |           |           |                                     |
| Interpreting and representing with diagrams | a |             |           |           |                                     |
|                                             | b |             |           |           |                                     |
|                                             | c |             |           |           |                                     |
|                                             | d |             |           |           |                                     |
| Communicating using mathematics             | a |             |           |           |                                     |
|                                             | b |             |           |           |                                     |
|                                             | c |             |           |           |                                     |
|                                             | d |             |           |           |                                     |
| Costing and optimising work processes       | a |             |           |           |                                     |
|                                             | b |             |           |           |                                     |
|                                             | c |             |           |           |                                     |
|                                             | d |             |           |           |                                     |
| Number of sub-competences                   |   |             |           |           | 30                                  |

| General Mathematical Competences            |   | Mathematics | Economics | History | Mathematics + Economics + History |
|---------------------------------------------|---|-------------|-----------|---------|-----------------------------------|
| Measuring with precision                    | a |             |           |         |                                   |
|                                             | b |             |           |         |                                   |
|                                             | c |             |           |         |                                   |
|                                             | d |             |           |         |                                   |
| Estimating, calculating, and error spotting | a |             |           |         |                                   |
|                                             | b |             |           |         |                                   |
|                                             | c |             |           |         |                                   |
|                                             | d |             |           |         |                                   |
| Working with proportion                     | a |             |           |         |                                   |
|                                             | b |             |           |         |                                   |
|                                             | c |             |           |         |                                   |
|                                             | d |             |           |         |                                   |
| Using rules and formulae                    | a |             |           |         |                                   |
|                                             | b |             |           |         |                                   |
|                                             | c |             |           |         |                                   |
|                                             | d |             |           |         |                                   |
| Processing data                             | a |             |           |         |                                   |
|                                             | b |             |           |         |                                   |
|                                             | c |             |           |         |                                   |
|                                             | d |             |           |         |                                   |
| Understanding data and risk                 | a |             |           |         |                                   |
|                                             | b |             |           |         |                                   |
|                                             | c |             |           |         |                                   |
|                                             | d |             |           |         |                                   |
| Interpreting and representing with diagrams | a |             |           |         |                                   |
|                                             | b |             |           |         |                                   |
|                                             | c |             |           |         |                                   |
|                                             | d |             |           |         |                                   |
| Communicating using mathematics             | a |             |           |         |                                   |
|                                             | b |             |           |         |                                   |
|                                             | c |             |           |         |                                   |
|                                             | d |             |           |         |                                   |
| Costing and optimising work processes       | a |             |           |         |                                   |
|                                             | b |             |           |         |                                   |
|                                             | c |             |           |         |                                   |
|                                             | d |             |           |         |                                   |
| Number of sub-competences                   |   |             |           |         | 28                                |

Note: Subjects are ordered by the number of sub-competences (descending). See Table 4 for the description of letters a, b, c and d.

**TABLE 7** GMC sub-competences present in A-level subject combinations: Geography + Psychology + Sociology and Law + Psychology + Sociology. Blue shading indicates that a sub-competence is present in the subject curriculum, dark blue shading indicates that the sub-competence is present within the subject combination.

| General Mathematical Competences            |   | Geography | Psychology | Sociology | Geography + Psychology + Sociology |
|---------------------------------------------|---|-----------|------------|-----------|------------------------------------|
| Measuring with precision                    | a |           |            |           |                                    |
|                                             | b |           |            |           |                                    |
|                                             | c |           |            |           |                                    |
|                                             | d |           |            |           |                                    |
| Estimating, calculating, and error spotting | a |           |            |           |                                    |
|                                             | b |           |            |           |                                    |
|                                             | c |           |            |           |                                    |
|                                             | d |           |            |           |                                    |
| Working with proportion                     | a |           |            |           |                                    |
|                                             | b |           |            |           |                                    |
|                                             | c |           |            |           |                                    |
|                                             | d |           |            |           |                                    |
| Using rules and formulae                    | a |           |            |           |                                    |
|                                             | b |           |            |           |                                    |
|                                             | c |           |            |           |                                    |
|                                             | d |           |            |           |                                    |
| Processing data                             | a |           |            |           |                                    |
|                                             | b |           |            |           |                                    |
|                                             | c |           |            |           |                                    |
|                                             | d |           |            |           |                                    |
| Understanding data and risk                 | a |           |            |           |                                    |
|                                             | b |           |            |           |                                    |
|                                             | c |           |            |           |                                    |
|                                             | d |           |            |           |                                    |
| Interpreting and representing with diagrams | a |           |            |           |                                    |
|                                             | b |           |            |           |                                    |
|                                             | c |           |            |           |                                    |
|                                             | d |           |            |           |                                    |
| Communicating using mathematics             | a |           |            |           |                                    |
|                                             | b |           |            |           |                                    |
|                                             | c |           |            |           |                                    |
|                                             | d |           |            |           |                                    |
| Costing and optimising work processes       | a |           |            |           |                                    |
|                                             | b |           |            |           |                                    |
|                                             | c |           |            |           |                                    |
|                                             | d |           |            |           |                                    |
| Number of sub-competences                   |   |           |            |           | 24                                 |

| General Mathematical Competences            |   | Law | Psychology | Sociology | Law + Psychology + Sociology |
|---------------------------------------------|---|-----|------------|-----------|------------------------------|
| Measuring with precision                    | a |     |            |           |                              |
|                                             | b |     |            |           |                              |
|                                             | c |     |            |           |                              |
|                                             | d |     |            |           |                              |
| Estimating, calculating, and error spotting | a |     |            |           |                              |
|                                             | b |     |            |           |                              |
|                                             | c |     |            |           |                              |
|                                             | d |     |            |           |                              |
| Working with proportion                     | a |     |            |           |                              |
|                                             | b |     |            |           |                              |
|                                             | c |     |            |           |                              |
|                                             | d |     |            |           |                              |
| Using rules and formulae                    | a |     |            |           |                              |
|                                             | b |     |            |           |                              |
|                                             | c |     |            |           |                              |
|                                             | d |     |            |           |                              |
| Processing data                             | a |     |            |           |                              |
|                                             | b |     |            |           |                              |
|                                             | c |     |            |           |                              |
|                                             | d |     |            |           |                              |
| Understanding data and risk                 | a |     |            |           |                              |
|                                             | b |     |            |           |                              |
|                                             | c |     |            |           |                              |
|                                             | d |     |            |           |                              |
| Interpreting and representing with diagrams | a |     |            |           |                              |
|                                             | b |     |            |           |                              |
|                                             | c |     |            |           |                              |
|                                             | d |     |            |           |                              |
| Communicating using mathematics             | a |     |            |           |                              |
|                                             | b |     |            |           |                              |
|                                             | c |     |            |           |                              |
|                                             | d |     |            |           |                              |
| Costing and optimising work processes       | a |     |            |           |                              |
|                                             | b |     |            |           |                              |
|                                             | c |     |            |           |                              |
|                                             | d |     |            |           |                              |
| Number of sub-competences                   |   |     |            |           | 19                           |

Note: See Table 4 for the description of letters a, b, c and d.

**TABLE 8** General Mathematical Competence (GMC) sub-competences present in 10 most common subject combinations for students enrolled in business studies degrees. Blue shading indicates that a sub-competence is present in the subject curriculum, dark blue shading indicates that the sub-competence is present within the subject combination.

|                                             |   | Subject combination      |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|---------------------------------------------|---|--------------------------|-----------------------------------|-------------------------------------|-----------------------------------|-----------------------------------|------------------------------------|-----------------------------------|-----------------------------------|----------------------------------|--------------------------------|-----------------------------------------|
| General Mathematical Competences            |   | A-level Mathematics only | Economics + Mathematics + Physics | Chemistry + Economics + Mathematics | Biology + Economics + Mathematics | Biology + Chemistry + Mathematics | Business + Economics + Mathematics | Business + Psychology + Sociology | Business + Economics + Psychology | Business + Economics + Geography | Business + Economics + History | Business + English Literature + History |
| Measuring with precision                    | a |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | b |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | c |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | d |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
| Estimating, calculating, and error spotting | a |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | b |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | c |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | d |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
| Working with proportion                     | a |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | b |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | c |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | d |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
| Using rules and formulae                    | a |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | b |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | c |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | d |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
| Processing data                             | a |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | b |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | c |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | d |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
| Understanding data and risk                 | a |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | b |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | c |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | d |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
| Interpreting and representing with diagrams | a |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | b |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | c |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | d |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
| Communicating using mathematics             | a |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | b |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | c |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | d |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
| Costing and optimising work processes       | a |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | b |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | c |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
|                                             | d |                          |                                   |                                     |                                   |                                   |                                    |                                   |                                   |                                  |                                |                                         |
| Number of sub-competences                   |   | 25                       | 33                                | 33                                  | 33                                | 32                                | 30                                 | 23                                | 22                                | 20                               | 11                             | 11                                      |

Note: Subject combinations are ordered by the number of sub-competences (descending). A-level mathematics is included for reference. See Table 4 for the description of letters a, b, c, and d.

use the benchmark of participation in standalone qualifications such as A-level and GCSE (e.g. Gove, 2011; Sunak, 2023). This qualification-focused perspective is often reflected in research, for example, comparing university students with A-level Mathematics to those with no post-16 mathematics qualification (Hodgen et al., 2020) or students with A-level Further Mathematics to those with only A-level Mathematics (Lyakhova & Neate, 2021). In contrast, McAlinden and Noyes (2019b) challenged the idea of a one-dimensional 'gap', arguing that it 'ignores the situatedness of knowledge and the challenge of developing expertise in mathematical problem solving and modelling' (p.69). The GMC framework analysis provides evidence for the presence of this situated knowledge within A-level subjects and, furthermore, demonstrates that mathematics participation is about more than Mathematics qualifications. For a student taking a subject combination like Geography + Psychology + Sociology (Table 7), it would be hard to argue that they are not taking 'some maths'. On the other hand, it is easy to view embedded mathematics as primarily benefitting students who do not choose Mathematics as one of their three or four subjects, enabling them to gather enough mathematical content elsewhere to prepare them for future education or employment. However, as Smith and Morgan (2016) point out, application of mathematics to real-world contexts is frequently missing from advanced mathematics curricula, particularly in the upper secondary phase. It may, therefore, be just as important for students who do choose A-level Mathematics to also have the opportunity to apply mathematics in the real-world contexts provided by their other A-level subjects. This 'added value' is illustrated in Table 8, where subject combinations that include A-level Mathematics have a breadth of GMC coverage greater than that of Mathematics alone. Hence, there is a vital need to find ways of describing mathematics participation that capture the diversity of curricular pathways, for students with and without Mathematics A-level.

## Mathematics present and not present in A-level subjects

The GMC maps also provide insights into what kinds of mathematics are considered important within these subjects, or at least, by the subject representatives involved in setting high-level curriculum policy. Our analysis appears to confirm the high value placed on statistical literacy, with *processing data* and *understanding data and risk* being the most common GMCs. Indeed, the GMC map for Sociology (Table 5) shows very few, but primarily data-related, mathematical competences agrees with Hampton's (2018) description of quantitative methods as 'marginalised' within the A-level Sociology curriculum. Similarly, the GMC analysis for Geography aligns with Harris's (2020) finding that the most common mathematical elements in the 2018 Geography examinations involved 'being able to interpret graphically presented information — sometimes to describe it in its own right, sometimes to draw out the relationships (or lack of) between various geographical features or phenomena, and sometimes to combine it with other disciplinary knowledge to reason to a conclusion' (p. 5). In GMC terms, we could describe this as a combination of *processing data*, *understanding data and risk* and *communicating using mathematics*. Interestingly, the GMC analysis also highlights *measuring with precision* as a key competence in Geography, whereas Harris interprets measurement as relating to spatial or fieldwork skills rather than mathematics. While the widespread inclusion of statistical content within A-level subjects overall bodes well for students' preparedness for learning statistics within university disciplines, there remains significant scope for investigating how the intended quantitative curriculum is implemented in practice.

In contrast to the data-related competences, the GMCs *costing a project* and *optimising work processes* are notably absent from most A-level curricula. This is perhaps unsurprising, since these competences were originally placed at the bottom of the list to acknowledge their likely familiarity to vocational teachers and employers rather than to mathematics teachers and policy makers. Hence, the two subjects with strengths in these competences are also at

the more vocational end of the spectrum with explicit links to workplace practices. In Design and Technology, it is the use of materials that need to be optimised, taking into account the costs of materials and manufacturing processes; in Computer Science, optimisation relates to an algorithm's efficiency, taking into account the time and space 'costs' of executing the algorithm. What is curious is why these competences are considered only relevant to vocational subjects and not to academic ones. An academic researcher would certainly need to be able to cost a project when applying for grant funding, as well as optimising the work processes across the lifespan of the project. The GMC framework, therefore, potentially exposes an implicit hierarchy of 'academic' over 'vocational' mathematics in advanced-level curricula.

## Overlapping and non-overlapping mathematics

Applying the GMCs as a common framework for A-levels raises questions about how these competences interact across subjects. One possible use is to identify 'gaps' in particular subject combinations. For example, all the subject combinations shown in Table 8 are missing sub-competence 'd' of *communicating using mathematics*: 'adapt presentation to intended purpose and audience'. Since this would seem to be a relevant competence for the field of business studies, there are implications for lecturers on undergraduate degree programmes, who may otherwise expect students to arrive with prior experience in this kind of mathematical communication. Conversely, many subject combinations have GMCs that overlap between two or more subjects and which could, therefore, potentially reinforce the development of these competences. For example, students could be expected to gain greater competence in *processing data* through the combination of Biology + Geography + Psychology, than by taking any of these subjects individually. However, further research is needed to investigate whether, and how, reinforcement works in practice. Learning transfer research has long since abandoned the idea of knowledge or skills being transferred automatically between contexts (Lobato & Hohensee, 2021), while the communities of practice perspective suggest that students moving between different subjects are experiencing distinct subject-based practices (Wenger, 1998). This could shed light on the recent negative finding that the applied mathematics learnt as part of Core Maths qualifications does not improve students' attainment in quantitative A-level subjects such as Psychology and Geography (Mathieson & Homer, 2021). The study looked at overall grades rather than performance on quantitative questions, potentially diluting the mathematical effect. However, even where the same kinds of mathematics are present, they may be applied and discussed in very different ways. The GMC mapping is, therefore, simply a starting point for further discussion of cross-curricular mathematical connections, both for A-levels and for other phases of education and education systems.

## CONCLUSION

Cross-curricular comparison of embedded mathematics necessitates finding a common language between subjects. The GMC framework is a promising candidate for such a common language, providing insights into the mathematics students are expected to encounter not only in individual A-levels but also in their chosen combination of subjects. One of the original aims of the framework was to inform professional development for vocational teachers, bringing together teachers from different subjects to discuss the pedagogical challenges of applying a particular GMC across contexts. The same approach could certainly be applied to teachers of academic subjects, both within and beyond the English education system. In addition, mapping mathematical competences across typical subject combinations could help university lecturers to understand the diverse mathematical backgrounds of incoming students, both to increase curriculum continuity between the two phases and to provide targeted additional support.

Furthermore, our findings challenge the characterisation of post-16 mathematics participation as binary, suggesting instead a more nuanced spectrum of participation. Not only does the embedding approach enable many students not taking a standalone mathematics qualification to participate in mathematics to some extent, but it also has the potential to enhance the mathematics learning of A-level Mathematics students through application to varied real-world contexts. Hence, even in countries where mathematics study to age 18 is compulsory, the mathematics embedded in other subjects could still provide valuable learning. The concept of participation, therefore, needs expanding to include not only whether students are taking mathematics, or even how much mathematics they are taking, but what kinds of mathematics. This may well necessitate further exploration of ways to describe mathematics pathways qualitatively, in addition to quantitative measures.

The Royal Society (2014), among others, continue to champion a bold vision for a broad, balanced, and coherent post-16 curriculum in England, advocating for a baccalaureate-style qualification. We suggest that the mathematics embedded in the reformed A-levels is a step in the direction of this vision, albeit by stealth and within the existing A-level/T-level framework. There is no reason why the same approach could not be used to embed other important competences such as literacy and creativity and indeed to highlight where these competences are already present within subject curricula. What is essential is to move away from seeing A-level subjects as individual silos and towards a coherent view of how curricula interact across subjects.

### FUNDING INFORMATION

This work was supported by the Economic and Social Research Council under Grant ES/P000711/1.

### CONFLICT OF INTEREST STATEMENT

No financial interest or benefit has arisen from the direct applications of this research.

### DATA AVAILABILITY STATEMENT

Data supporting the analysis presented in the paper are available on the UK government website: <https://www.gov.uk/government/collections/gce-as-and-a-level-subject-content>.

### DATA DEPOSITION

Not applicable.

### ETHICS STATEMENT

This research was conducted within BERA guidelines (2018). Due to the nature of the study, no institutional ethical review was required.

### GEOLOCATION INFORMATION

United Kingdom.

### ORCID

Jennifer Norris  <https://orcid.org/0000-0001-9406-570X>

Andrew Noyes  <https://orcid.org/0000-0002-8147-0105>

### REFERENCES

- Adkins, M., & Noyes, A. (2017). Do advanced mathematics skills predict success in biology and chemistry degrees? *International Journal of Science and Mathematics Education*, 16(3), 487–502. <https://doi.org/10.1007/s10763-016-9794-y>
- Advisory Committee on Mathematics Education. (2014). *Maths for all to 18*. <https://royalsociety.org/-/media/policy/Publications/2014/maths-to-18-maths-snapshot-12-2014.pdf>

- Advisory Committee on Mathematics Education. (2019). *Mathematics for the T level qualifications: A rationale for general mathematical competences (GMCs)*. The Royal Society.
- Darlington, E., & Bowyer, J. (2017). The mathematics needs of prospective geography undergraduates. *Journal of Research in Social Sciences*, 5(2), 11–32.
- Department for Education. (2013). *Introduction of 16 to 18 core maths qualifications: Policy statement*. [https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment\\_data/file/266717/Policy\\_statement\\_on\\_16-18\\_Core\\_Maths\\_qualifications\\_-\\_final\\_\\_3\\_.pdf](https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/266717/Policy_statement_on_16-18_Core_Maths_qualifications_-_final__3_.pdf)
- Department for Education. (2014). *GCE AS and A level subject content*. <https://www.gov.uk/government/collections/gce-as-and-a-level-subject-content>
- Department for Education. (2019). *Review of post-16 qualifications at level 3 and below in England: The current system and the case for change*. [https://consult.education.gov.uk/post-16-qualifications-review-team/post-16-level-3-and-below-qualifications-review/supporting\\_documents/Post%2016%20level%203%20and%20below%20qualifications%20review%20%20Case%20for%20Change.pdf](https://consult.education.gov.uk/post-16-qualifications-review-team/post-16-level-3-and-below-qualifications-review/supporting_documents/Post%2016%20level%203%20and%20below%20qualifications%20review%20%20Case%20for%20Change.pdf)
- Education Endowment Foundation. (2018). *EEF to evaluate trial of new maths premium*. <https://educationendowmentfoundation.org.uk/news/eef-to-evaluate-trial-of-new-maths-premium>
- Education Endowment Foundation. (2021). *Basic maths premium pilot*. <https://educationendowmentfoundation.org.uk/projects-and-evaluation/projects/basic-maths-premium-pilot>
- Er, S. N. (2018). Mathematics readiness of first-year college students and missing necessary skills: Perspectives of mathematics faculty. *Journal of Further and Higher Education*, 42(7), 937–952. <https://doi.org/10.1080/0309877x.2017.1332354>
- Field, A. P. (2014). *Skills in mathematics and statistics in psychology and tackling transition*. The Higher Education Academy.
- Gale, N. K., Heath, G., Cameron, E., Rashid, S., & Redwood, S. (2013). Using the framework method for the analysis of qualitative data in multi-disciplinary health research. *BMC Medical Research Methodology*, 13, 1–8. <https://doi.org/10.1186/1471-2288-13-117>
- Garthwaite, K., France, B., & Ward, G. (2013). The complexity of scientific literacy: The development and use of a data analysis matrix. *International Journal of Science Education*, 36(10), 1568–1587. <https://doi.org/10.1080/09500693.2013.870363>
- Gove, M. (2011). *Michael Gove speaks to the Royal Society on maths and science*. <https://www.gov.uk/government/speeches/michael-gove-speaks-to-the-royal-society-on-maths-and-science>
- Hampton, J. M. (2018). *The nature of quantitative methods in A level sociology*. (Doctoral dissertation). Cardiff University.
- Harris, R. (2020). *An independent review of quantitative methods in A level geography*. Royal Geographical Society.
- Hodgen, J., Adkins, M., & Tomei, A. (2020). The mathematical backgrounds of undergraduates from England. *Teaching Mathematics and its Applications*, 39(1), 38–60. <https://doi.org/10.1093/teamat/hry017>
- Hodgen, J., Pepper, D., Sturman, L., & Ruddock, G. (2010). *Is the UK an outlier? An international comparison of upper secondary mathematics education*. Nuffield Foundation.
- Homer, M., Mathieson, R., Tasara, I., & Banner, I. (2020). *The early take-up of core maths: Successes and challenges*. University of Leeds.
- Hyland, T., & Johnson, S. (1998). Of cabbages and key skills: Exploding the mythology of core transferable skills in post-school education. *Journal of Further and Higher Education*, 22(2), 163–172. <https://doi.org/10.1080/0309877980220205>
- Isaacs, T. (2010). Educational assessment in England. *Assessment in Education: Principles, Policy & Practice*, 17(3), 315–334. <https://doi.org/10.1080/0969594x.2010.491787>
- Jameson, E. (2016). *Roles and limits of curriculum frameworks in mathematics education*. Cambridge Mathematics.
- King, N. C., & Hambrook, G. (2020). Examining the impact of pre-university qualifications on success in interdisciplinary science. *Journal of Further and Higher Education*, 1-14, 1192–1205. <https://doi.org/10.1080/0309877x.2020.1854695>
- Lobato, J., & Hohensee, C. (2021). Current conceptualizations of the transfer of learning and their use in STEM education research. In C. Hohensee & J. Lobato (Eds.), *Transfer of learning: Progressive perspectives for mathematics education and related fields* (pp. 3–25). Springer. <https://doi.org/10.1007/978-3-030-65632-4>
- Lyakhova, S., & Neate, A. (2021). Further mathematics, student choice and transition to university: Part 2 - non-mathematics STEM degrees. *Teaching Mathematics and its Applications: An International Journal of the IMA*, 40(3), 210–233. <https://doi.org/10.1093/teamat/hrab004>
- Mathematics in Education and Industry. (2011). *Mathematics for all post-16: An MEI position paper*. <https://mei.org.uk/reports/mathematics-for-all-post-16/>
- Mathematics in Education and Industry. (2021). *Summary of Core Maths entries and results*. [https://mei.org.uk/app/uploads/2021/08/Summary\\_of\\_Core\\_Maths\\_entries\\_and\\_results.pdf](https://mei.org.uk/app/uploads/2021/08/Summary_of_Core_Maths_entries_and_results.pdf)
- Mathieson, R., & Homer, M. (2021). “I was told it would help with my psychology”: Do post-16 core maths qualifications in England support other subjects? *Research in Mathematics Education*, 24(1), 69–87. <https://doi.org/10.1080/14794802.2021.1959391>

- McAlinden, M., & Noyes, A. (2019a). Assessing mathematics within advanced school science qualifications. *Assessment in Education: Principles, Policy & Practice*, 26(3), 340–355. <https://doi.org/10.1080/0969594x.2017.1321524>
- McAlinden, M., & Noyes, A. (2019b). Mathematics in the disciplines at the transition to university. *Teaching Mathematics and its Applications*, 38(2), 61–73. <https://doi.org/10.1093/teamat/hry004>
- Neumann, I., Rohenroth, D., & Heinze, A. (2021). Studieren ohne Mathe? Welche mathematischen Lernvoraussetzungen erwarten Hochschullehrende für Studienfächer außerhalb des MINT-Bereichs? [Studying without maths? What mathematical learning requirements do university lecturers expect for subjects outside of the STEM area?]. [https://www.ipn.uni-kiel.de/de/das-ipn/abteilungen/didaktik-der-mathematik/forschung-und-projekte/malemint-e/malemint-e\\_web/at\\_download/file](https://www.ipn.uni-kiel.de/de/das-ipn/abteilungen/didaktik-der-mathematik/forschung-und-projekte/malemint-e/malemint-e_web/at_download/file)
- Niss, M. (2003). Mathematical competencies and the learning of mathematics: The Danish KOM project. Paper presented at the 3rd Mediterranean conference on mathematical education, Athens.
- Nuffield Foundation. (2012). *Mathematics in A level assessments: A report on the mathematical content of A level assessments in business studies, computing, economics, geography, psychology and sociology*. Nuffield Foundation.
- Ofqual. (2019). Provisional entries for GCSE, AS and A level: Summer 2019 exam series. <https://www.gov.uk/government/statistics/provisional-entries-for-gcse-as-and-a-level-summer-2019-exam-series>
- Ofqual. (2020). Provisional entries for GCSE, AS and A level: summer 2020 exam series. <https://www.gov.uk/government/statistics/provisional-entries-for-gcse-as-and-a-level-summer-2020-exam-series>
- Ofqual. (2021). Provisional entries for GCSE, AS and A level: summer 2021 exam series. <https://www.gov.uk/government/statistics/provisional-entries-for-gcse-as-and-a-level-summer-2021-exam-series>
- Organisation for Economic Co-operation and Development. (2018). PISA 2022 mathematics framework (draft). <https://pisa2022-maths.oecd.org/files/PISA%202022%20Mathematics%20Framework%20Draft.pdf>
- Scott Jones, J., & Goldring, J. (2014). *Skills in mathematics and statistics in sociology and tackling transition*. The Higher Education Academy.
- Smith, A. (2017). *Report of professor sir Adrian Smith's review of post-16 mathematics*. Department for Education.
- Smith, C., & Morgan, C. (2016). Curricular orientations to real-world contexts in mathematics. *The Curriculum Journal*, 27(1), 24–45. <https://doi.org/10.1080/09585176.2016.1139498>
- Smith, M. E. (2013). *Independent chair's report on the review of current GCE 'specification content' within subject criteria*. Ofqual.
- Sunak, R. (2023). *PM speech on building a better future: 4 January 2023*. <https://www.gov.uk/government/speeches/pm-speech-on-making-2023-the-first-year-of-a-new-and-better-future-4-january-2023>
- The Royal Society. (2014). *Vision for science and mathematics education*. The Royal Society.
- Vidal Rodeiro, C. (2019). *Popularity of A level subjects among university students*. Cambridge Assessment.
- Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge University Press.
- Williams, J., Roth, W.-M., Swanson, D., Doig, B., Groves, S., Omuvwie, M., Ferri, R. B., & Mousoulides, N. (2016). *Interdisciplinary mathematics education: A state of the art*. Springer. <https://doi.org/10.1007/978-3-319-42267-1>
- Williams, J., & Wake, G. (2006). Black boxes in workplace mathematics. *Educational Studies in Mathematics*, 64(3), 317–343. <https://doi.org/10.1007/s10649-006-9039-z>

**How to cite this article:** Norris, J., & Noyes, A. (2023). Mapping mathematical competences across subjects for advanced level qualifications in England. *The Curriculum Journal*, 34, 613–632. <https://doi.org/10.1002/curj.204>