Simple Near Exact Image Solution for Vertical Antenna Above Lossy Ground Using Stationary Phase Approximation

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Abstract: **In this paper, the traditional problem of field computation for a vertical radiating antenna above a lossy infinite ground is discussed and revisited. To simplify and solve the traditional solutions consist of complicated integrals of sommerfeld type, phase stationary approximation is used. First, an infinitesimal dipole is considered over the ground and it's attenuation versus distance from the antenna is derived. Then, to verify the results, every vertical monopole antenna is supposed to be made of the large number of the infinitesimal dipoles and so it's charactristics such as pattern can be easily calculated. Agreement between our results and previous ones shows acceptable accuracy of this method. This method is valid for any vertical arbitrary source alignment or observation position.**

Index Terms—**Dipole antennas, phase Stationary approximation .**

I. INTRODUCTION

THE accurate and efficient prediction of natural features effects on radio-wave propagation is essential in the development and design of a communications system. In 1909, Sommerfeld [1] solved the general problem of the ground effect with the finite conductivity on the radiation from a grounded condenser antenna. However, after Sommerfeld paper, some works published and complete him solution such as [2],[3],[4],[5]. One of the most important works in this field was done by Norton[6] ,[7] who presented concise formulation of the problem. The Sommerfeld analytic solution is presented in terms of integrals of the Sommerfeld type, which cannot be calculated in closed form and because of their nature of being highly oscillatory are difficult to evaluate numerically. Huge number of techniques have been used to approximately evaluate the Sommerfeld integrals. To calculate the Sommerfeld integrals analytically asymptotic techniques, such as the method of saddle-point, are employed [8]–[10]. However, In this paper we will reduce the complicated equations of the Sommerfeld theory by means of stationary phase approximation to the form of simple formulas which may readily be used by the engineer and to show their limitations by comparing them to the available experimental data.

II. Theoretical Background

As shown in Fig.1, the first desired problem is an infinitesimal dipole at a height of h over a lossy ground with $k' - jk''$ as it's complex permeability . Following the equation derived in [11], to find the adequate image for the proposed problem, we start from following equation:

$$\widehat{\psi} = \frac{\mu_0}{8\pi^2} \iint_{-\infty}^{\infty} \frac{1}{\gamma_0} (1 - e^{-2\gamma_0 h} \frac{\gamma - \kappa\gamma_0}{\gamma + \kappa\gamma_0}) e^{-j\beta_x x - j\beta_y y - \gamma_0 (z-h)} d\beta_x d\beta_y \tag{1}$$

Where $\widehat{\psi}$ is the Fourier transform of potential function, and

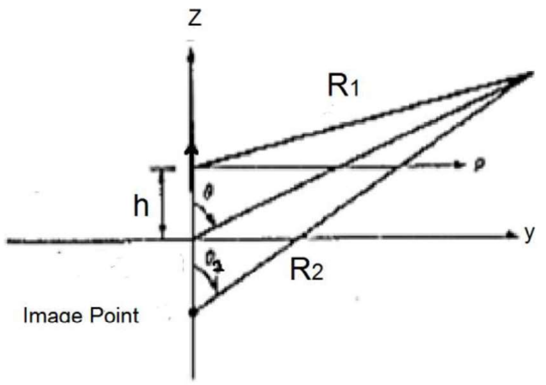


Fig.1. An infinitesimal dipole over lossy ground

$$\omega^2 = \beta_x^2 + \beta_y^2 \tag{2}$$

$$\gamma_0 = j\sqrt{k_0^2 - \omega^2} \tag{3}$$

$$\gamma = j\sqrt{k^2 - \omega^2} \tag{4}$$

$$\kappa = k' - jk'' \tag{5}$$

Now, we should evaluate the above integral by using stationary phase method. So, we define:

$$F = \beta_x x + \beta_y y + \gamma_0(z+h) \tag{6}$$

Or,

$$F = \beta_x x + \beta_y y + \sqrt{k_0^2 - \beta_x^2 - \beta_y^2}\,(z+h) \tag{7}$$

After transforming to spherical coordinate and applying phase stationary condition, we have:

$$\frac{\partial \mathrm{F}}{\partial \beta_x} = 0$$

$$\rightarrow Rsin\theta cos\varphi\sqrt{k_0^2 - \beta_x^2 - \beta_y^2} - \beta_x(Rcos\theta + h) = 0$$

$$\beta_x = \frac{k_0 Rsin\theta cos\varphi}{\sqrt{R^2+2hRcos\theta+h^2}} \tag{8}$$

$$\frac{\partial \mathrm{F}}{\partial \beta_y} = 0$$

$$\rightarrow Rsin\theta sin\varphi\sqrt{k_0^2 - \beta_x^2 - \beta_y^2} - \beta_y(Rcos\theta + h) = 0$$

$$\beta_y = \frac{k_0 Rsin\theta sin\varphi}{\sqrt{R^2 + 2hRcos\theta + h^2}} \tag{9}$$

Also, Fresnel reflection coefficient is given by

$$\Gamma_\upsilon = \frac{\gamma - \kappa\gamma_0}{\gamma + \kappa\gamma_0} = \frac{\sqrt{\beta_x^2+\beta_y^2-\kappa k_0^2} - \kappa\sqrt{\beta_x^2+\beta_y^2-k_0^2}}{\sqrt{\beta_x^2+\beta_y^2-\kappa k_0^2} + \kappa\sqrt{\beta_x^2+\beta_y^2-k_0^2}} \tag{10}$$

After inserting (8) and (9) into (10), we have:

$$\Gamma_\upsilon = \frac{\sqrt{\mathrm{R}^2\sin^2\theta - \kappa(R^2+2hRcos\theta+h^2)} - \kappa\sqrt{\mathrm{R}^2\sin^2\theta - (R^2+2hRcos\theta+h^2)}}{\sqrt{\mathrm{R}^2\sin^2\theta - \kappa(R^2+2hRcos\theta+h^2)} + \kappa\sqrt{\mathrm{R}^2\sin^2\theta - (R^2+2hRcos\theta+h^2)}} \tag{11}$$

By a little mathematical simplification, we drive:

$$\Gamma_\upsilon = -\frac{\kappa^2(A-B) + \kappa A - B - 2\kappa\sqrt{B^2 - (1+\kappa)AB + \kappa A^2}}{\kappa A - B - \kappa^2(A-B)} \tag{12}$$

Where,

$$A = R^2 + 2hRcos\theta + h^2$$
$$\mathrm{B} = R^2\sin^2\theta$$

By looking at (1), it can be inferred that the electromagnetic fields over the ground are the summation of two distinct parts appeared in the parenthesis, namely, 1 and $e^{-2\gamma_0 h}\frac{\gamma-\kappa\gamma_0}{\gamma+\kappa\gamma_0}$ that represent the effect of the dipole and the reflection from the ground, respectively. Therefore, we can conclude the second term in the parenthesis as the effect of the image antenna in the absent of the ground. It should be noted that the exponential term compensates the phase difference due to the distance of 2h between the dipole and it's image. So, the ratio of the image current, I', and the dipole current, I, can be expressed as:

$$\frac{\mathrm{I}'}{\mathrm{I}} = \frac{\gamma - \kappa\gamma_0}{\gamma + \kappa\gamma_0} = \Gamma_\upsilon \tag{13}$$

Now, the original problem shown in Fig.1 can be replaced by the problem of two vertical infinitesimal dipoles with current ratio of Γ_υ which placed 2h meters far from each other. Certainly, this analysis should lead to correct results in two following special cases:

1. Perfectly conducting ground ($\sigma \rightarrow \infty$): in this case, $\kappa \rightarrow \infty$, therefore, according to (12), the ratio of the image current, I', and the dipole current, I, equals to 1, as expected.

2. $\kappa \rightarrow 1$ (an infinitesimal dipole in free space):

In this case, because of absence of the lossy ground, the image current should be zero. After applying Hopital's rule to (12) when $\kappa \to 1$, it can be seen that $I' = 0$.

III. Simulation Results

To verify the derived results in previous section, first, we evaluate the attenuation factor versus numerical distance(p) and compare our results against the ones derived in [11]. According to expressed definitions in [11], Fig.2. shows the attenuation factor versus numerical distance in comparison with previous results derived from numerical solution of Sommerfeld integrals in [11]. Good agreement between these results confirm the exactness of the applied image theory in this paper.

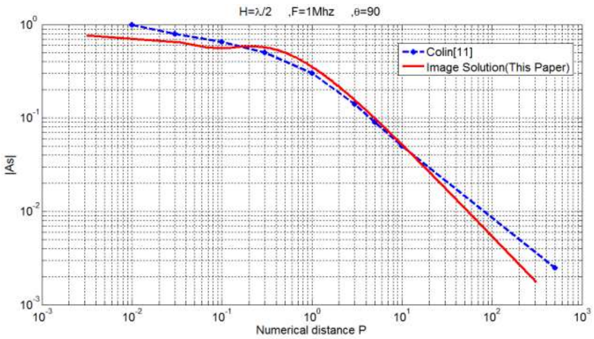


Fig.2. The attenuation factor versus numerical distance

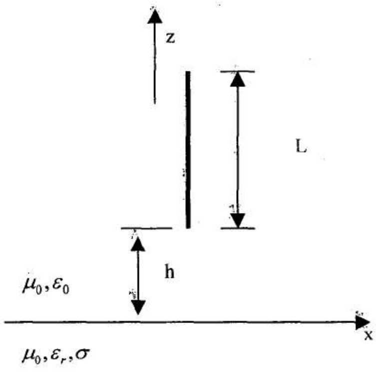
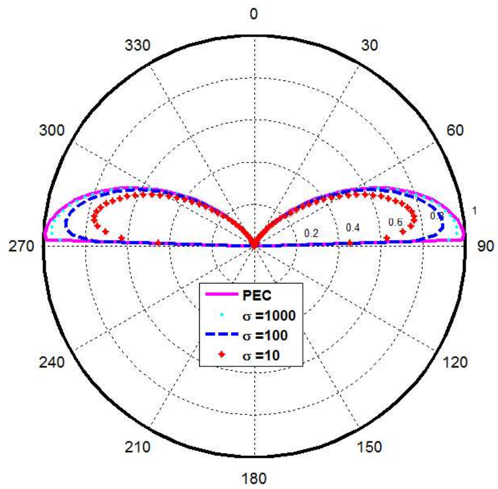
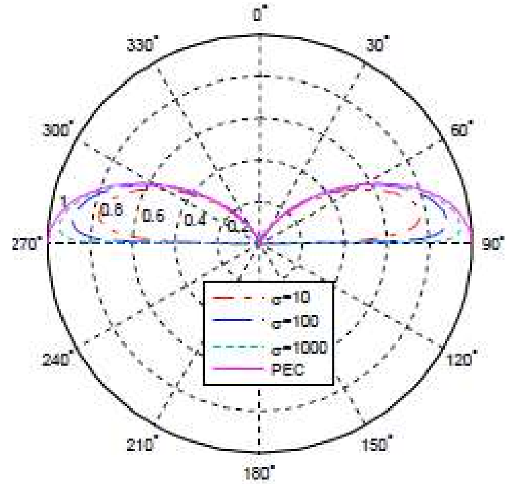


Fig.3. Monopole antenna with finite length over lossy ground

In addition, to consider more practical problem, a vertical monopole antenna with finite length is supposed over the lossy ground as shown in Fig.3. Assuming the monopole antenna composed of infinite number of the infinitesimal dipoles, we can plot it's pattern for different parameters, namely, antenna height, antenna length, conductivity of the ground using presented theory as shown in Fig. 4-a and Fig. 5-a. Again, for comparison, Fig.4-b and Fig.5-b shows similar results from the work of [12].

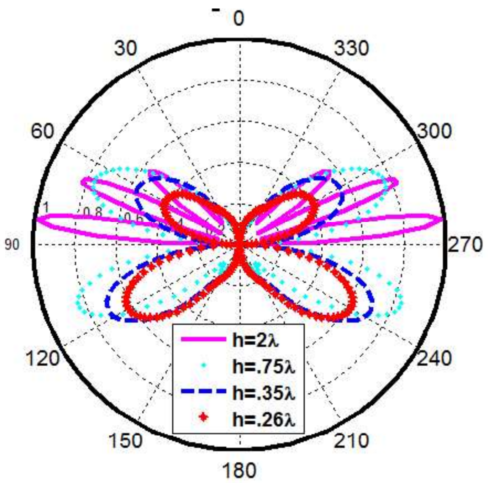


(a)

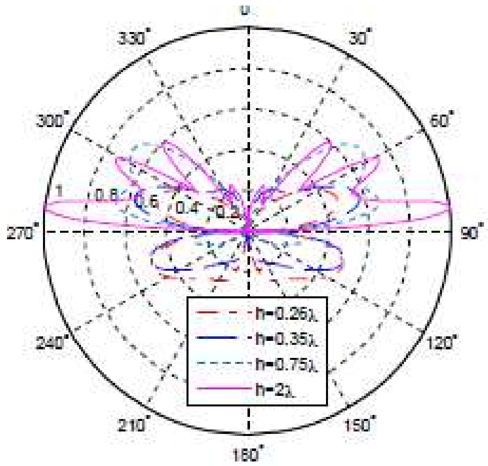


(b)

Fig.4. Monopole antenna's pattern for different conductivity of the ground, a) this work, b) work of [12]. (L=0.5λ, $\varepsilon_r = 1.001, h = .26\lambda, f = 300Mhz$)



(a)



(b)

Fig.5. Monopol antenna's pattern for different height of antenna, a) this work, b) work of [12].(L=0.5λ, $\eta = 0.3 + j0.1$)

III. Conclusion

In this paper, we found a simple image for a vertical antenna above infinite lossy ground. After checking this image experssion for some especial cases, we derived attenuation factor versus distance from antenna. In addition, we plotted pattern of typical monopole antennas by means of this theory and acceptable results confirm correctness of the our work. This study concentrate on the vertical antennas, so, it should be noted for analyzing every antenna using this method, the image theory for horizontal antenna should be applied, too.

IV. References

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