A Continuum Damage Model for Transverse Cracking in UD Composites of Linear Viscoelastic Behaviour

Fei Xu[a], I. Arthur Jones[a] and Shuguang Li[a]

[a]Institute of Aerospace Technology, Faculty of Engineering,
University of Nottingham, Nottingham, NG7 2RD, UK

Email addresses:
xufeiasri@hotmail.com (F. Xu), arthur.jones@nottingham.ac.uk (I. A. Jones),
shuguang.li@nottingham.ac.uk (S. Li)

*Corresponding author.
Email: arthur.jones@nottingham.ac.uk Tel: +44 (0)115 951 3784

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Abstract

A damage model for linear viscoelastic unidirectional (UD) composites undergoing transverse matrix cracking is proposed. The damage representation for the corresponding elastic UD composite with an array of dispersed matrix cracks was derived from Li's work based on continuum damage mechanics (CDM). The elastic-viscoelastic correspondence principle (CP) was used to obtain the damage representation for corresponding linear viscoelastic UD composites in the Laplace domain, and re-expressed the time domain by taking the inverse Laplace transformation. A damage evolution law was constructed using the Weibull distribution of defects which will develop into cracks as a result of deformation. The time-temperature superposition principle (TTSP) approach has been incorporated into this model. Applications of this damage model are described in detail, and the predictions are compared with experimental data.

Keywords: Damage, Linear viscoelasticity, Correspondence principle, Transverse matrix cracks, Weibull distribution, Time-temperature superposition principle (TTSP).

1. Introduction

The mechanical behaviour of cured polymer resins exhibits time and temperature dependence under certain conditions, known as viscoelastic behaviour. In aerospace applications of polymeric composites, it is sometimes necessary to estimate the properties of the composites taking account of viscoelasticity. Unlike metals, the timescales cannot be validly ignored when considering the loading regimes in which viscoelastic behaviour becomes significant. In particular, airframes have long lifetimes (up to 50 years) during which they will be subjected to complex loading cycles having frequencies as low as around 10^{-5} Hz (one cycle per flight) as well as much higher frequencies. Direct replication of these timescales within tests is clearly not practical. Fatigue tests at increased frequencies (such as those used with metals) cannot be used directly as they do not account for the creep taking place within each cycle. The present paper is a step towards undertaking accelerated fatigue and long term creep tests for unidirectional (UD) composites with due consideration of viscoelastic effects.

By contrast with the time-independent linear elastic case, the constitutive equations for viscoelastic materials are written in an integral form using the well-known Boltzman superposition principle. In order to describe the degradation of properties, it is also desirable to incorporate the effects and evolution of damage into the constitutive relations, since fatigue and long-term creep processes often involve the material undergoing damage. This will be achieved in the present paper by using the theoretical framework of continuum damage mechanics (CDM) in conjunction with a damage representation and damage evolution law combined with viscoelasticity.

1.1. Damage representation

Talreja [1] proposed a damage representation for composites based on the concept of CDM for unidirectional composites, in the form of a vectorial internal state variable. The constitutive relationship of the material under a fixed damaged state was derived from the constraints of the second law of thermodynamics for an isothermal process. For composites with small damage, the degraded elastic properties can be expressed as linear functions of an appropriate damage parameter, defined as the length squared of the vectorial damage variable. However, a considerable number of damage related material constants need to be determined before the theory can be applied to a real material. Based on Talreja's work, Li et al [2] determined the damage related constants for transversely isotropic UD composites, primarily by using well

justified virtual experiments. In their work, the effects of damage caused by matrix cracks (with crack surfaces perpendicular to axis 2, taken to be a direction perpendicular to the fibre direction and aligned with the transverse stress) were described in terms of a damage parameter which could be directly associated with the reduction in effective Young's modulus E_2 of the damaged material. It was also found that the degradations of E_2 and G_{12} are synchronised. However, these developments apply only to the elastic regime and extension is required in order to cover their viscoelastic applications.

To combine viscoelastic behaviour with damage in composite materials, Schapery [3, 4] introduced the concepts of pseudo stress and pseudo strain which are single integral forms of the viscoelastic constitutive equations. The concepts of pseudo strain energy density and pseudo complementary strain energy density were then introduced in terms of pseudo strains and damage as an analogy to the form of the strain energy in the elasticity theory [3]. Damage can be represented as internal state variables. A constitutive model for uniaxial strain-stress behaviour of viscoelastic materials with time-dependent damage growth was then introduced based on this pseudo energy. However, the model obtained was limited to materials in which all the properties were expressed in term of a single function of time, whereas for anisotropic materials, the time dependencies in different directions may be different.

Kumar and Talreja [5, 6] presented a damage representation for a linear viscoelastic cross-ply laminated composite based on new functions for pseudo strain energy and pseudo complementary strain energy in the Laplace domain, which are extended from the elastic-viscoelastic correspondence principle. Here the pseudo strain energy is a function of Laplace transformed strains and the pseudo complementary strain energy is a function of Laplace transformed stresses. These pseudo energies are different from the pseudo energies defined by Schapery [3] since they are based on Laplace transformed pseudo strains or pseudo stresses. Kumar's model was limited to cross-ply laminates with transverse matrix cracks in which damage is represented through internal variables taking the form of second rank tensors.

1.2. Damage evolution law

Damage representation describes the effects of damage at a given level on the constitutive relationship in terms of residual properties; however, in order to describe the behaviour of the material over the complete process of deformation, damage evolution should also be

incorporated into the model. One way of constructing an approximate damage evolution law is by the interpolation of experimental stress-strain curves using empirical functions to incorporate the effects of damage and damage evolution. However, this type of damage evolution law is limited in its ability to describe interactions between different effective properties.

There are two major systematic approaches to the formulation of damage evolution laws: one follows the concept of a damage surface which is similar to the concept of a yield surface in plasticity, and the other is based on the derivation of a damage driving force which is a concept analogous to energy release rate in fracture mechanics [7, 8]. Damage surface expressions are often derived from damage initiation criteria so that, as damage evolves, the damage surface will be updated continuously. An incremental damage evolution law based on a damage surface can then be devised in a similar way to that in the incremental theory of plasticity. Li et al. [9] developed a CDM model for characterising transverse matrix cracks in laminates, by employing the concept of a damage surface in order to formulate a damage evolution law. Damage driving force expressions are normally derived from energy functions of damaged materials. In the CDM model proposed by Daghia and Ladeveze [10], a strain energy density function for the damaged UD composite was used to derive the damage driving force. The Helmholtz free energy function was chosen by Talreja [1] as a generally applicable approach to derivation of the damage driving force. Yu [11] took it further by presenting a damage evolution law also based on the concept of damage driving force for modelling the evolution of matrix damage in UD composites. This particular damage evolution law led to the damage driving force being expressed in terms of three naturally partitioned components directly associated with the corresponding stress components. However the damage evolution laws described above were all based on elastic theory. The damage evolution law required in the present work will be based on viscoelastic theory and applicable to viscoelastic materials.

Several other types of damage evolution law have been proposed for composite materials. Akshantala and Talreja [12] proposed a mechanistic model for fatigue damage evolution in composite laminates. In that paper, the changes in the Young's modulus of the laminate due to matrix cracking and delamination were found by calculating the changes in the average strain of the laminate, and the damage evolution was described in terms those changes. Lemaitre et al. [13] presented a damage evolution law for fatigue, where the coupling of damage with elastic properties was expressed in terms of a tensor associated with the deviatoric strain and a

scalar associated with the hydrostatic strain. The kinetic law of damage evolution was an extension from the isotropic case.

There is clearly a need for a damage evolution law to complement the viscoelastic damage representation, and it is the objective of this paper to propose both the damage representation and damage evolution for UD composites of a linear viscoelastic matrix while incorporating the time-temperature superposition principle (TTSP). Some of the predictions produced from the model will be verified against experimental data.

2. Correspondence Principle

In the viscoelasticity literature, the similarity in form between the linear elastic problem and the Carson transformation of the linear viscoelastic problem is normally referred to as the viscoelastic correspondence principle (CP).

For elastic materials, the constitutive equation is a linear relationship between stresses and strains and the generalized Hooke's law relating stresses to strains can be expressed in contracted notation [14] as

$$\sigma_i = C_{ij}\epsilon_j \quad i,j = 1,\dots,6 \tag{1}$$

where σ_i and ϵ_j are the stress and strain components, respectively, C_{ij} is the stiffness matrix. For a linear viscoelastic and non-aging material, the constitutive equations can be given in an integral form using the well-known Boltzman superposition principle [6] as

$$\sigma_{ij} = C_{ijkl}(t)\epsilon_{kl}^0 + \int_0^t C_{ijkl}(t-\tau)\frac{\partial \epsilon_{kl}(\tau)}{\partial \tau}d\tau \tag{2}$$

or, inversely

$$\epsilon_{ij} = S_{ijkl}(t)\sigma_{kl}^0 + \int_0^t S_{ijkl}(t-\tau)\frac{\partial \sigma_{kl}(\tau)}{\partial \tau}d\tau \tag{3}$$

Here $C_{ijkl}(t)$ and $S_{ijkl}(t)$ are the relaxation modulus tensor and creep compliance tensor, respectively. Taking the Laplace transform of both sides of (2) and (3) and applying the convolution theorem [15], one obtains

$$\begin{aligned}\bar{\sigma}_{ij} &= \bar{C}_{ijkl}\epsilon^0_{kl} + \bar{C}_{ijkl} \cdot \overline{\left(\frac{\partial \epsilon_{kl}(\tau)}{\partial \tau}\right)} \\ &= \bar{C}_{ijkl}\epsilon^0_{kl} + \bar{C}_{ijkl} \cdot (-\epsilon_{kl}(0) + s\bar{\epsilon}_{kl}) \\ &= \tilde{C}_{ijkl}\bar{\epsilon}_{kl}\end{aligned} \tag{4}$$

and

$$\begin{aligned}\bar{\epsilon}_{ij} &= \bar{S}_{ijkl}\sigma^0_{kl} + \bar{S}_{ijkl} \cdot \overline{\left(\frac{\partial \sigma_{kl}(\tau)}{\partial \tau}\right)} \\ &= \bar{S}_{ijkl}\sigma^0_{kl} + \bar{S}_{ijkl} \cdot (-\sigma_{kl}(0) + s\bar{\sigma}_{kl}) \\ &= \tilde{S}_{ijkl}\bar{\sigma}_{kl}\end{aligned} \tag{5}$$

where $\epsilon^0_{kl} = \epsilon_{kl}(\tau)|_{\tau=0}$, $\sigma^0_{kl} = \sigma_{kl}(\tau)|_{\tau=0}$, the Laplace transform $\bar{f}(s)$ of a function $f(t)$ is defined as

$$\bar{f}(s) \equiv \mathcal{L}\{f(t)\} \equiv \int_0^\infty e^{-st} f(t) dt \tag{6}$$

$\tilde{C}_{ijkl}$ and $\tilde{S}_{ijkl}$ are the Carson transforms of $C_{ijkl}(t)$ and $S_{ijkl}(t)$ defined as

$$\tilde{C}_{ijkl} \equiv s\bar{C}_{ijkl} \tag{7}$$

and

$$\tilde{S}_{ijkl} \equiv s\bar{S}_{ijkl} \tag{8}$$

respectively.

After taking the Laplace transformation, the integral constitutive equations transform to purely algebraic equations in the Laplace domain. (4) and (5) are analogous to the linear elastic constitutive equations except that they are now given as relationships between the Laplace transformed stresses and strains. The constitutive equation of a linear viscoelastic material is time dependent. Since the Laplace transformation affects time but not spatial parameters, the corresponding viscoelastic constitutive relationship in the Laplace domain is analogous to that of the elastic counterpart in the time domain [16]. If the solution for an elastic problem is available, then the solution in the Laplace domain for the corresponding viscoelastic problem can be obtained by replacing all the material properties appearing in the elastic solution by their Carson transforms. This solution is then inverted to obtain the time domain solution. This is the well known correspondence principle of linear viscoelasticity theory [17]. In the next section, it will be shown how the constitutive relationship of elastic materials with damage is extended to cover the viscoelastic case.

3. Damage Representation

3.1. Damage representation of the elastic UD composite with matrix cracks

Based on continuum damage mechanics (CDM), Li et al. [2] obtained an expression for the stiffness modulus of UD composites with matrix cracks (the crack surface perpendicular to axis 2). There are three assumptions in their work, which are commonly employed in damage theories, either explicitly or implicitly.

(1) The virgin material is homogeneous so that the heterogeneity between reinforcing fibres and the matrix can be neglected;

(2) The virgin material is transversely isotropic. The damage to it takes a form of a single array of cracks, small in size but large in number, with a common orientation such that the damaged material exhibits homogeneous orthotropic behaviour;

(3) The matrix cracks concerned are all mathematical cracks having completely flat and closed crack surfaces under an unloaded condition and the material around the cracks is free from any initial stresses.

The compliance matrix of a virgin UD composite is expressed as

$$[S^0] = \begin{bmatrix} 1/E_1^0 & & & & & \\ -\nu_{12}^0/E_1^0 & 1/E_2^0 & & & \text{Symm.} & \\ -\nu_{12}^0/E_1^0 & -\nu_{23}^0/E_2^0 & 1/E_2^0 & & & \\ 0 & 0 & 0 & 1/G_{23}^0 & & \\ 0 & 0 & 0 & 0 & 1/G_{12}^0 & \\ 0 & 0 & 0 & 0 & 0 & 1/G_{12}^0 \end{bmatrix} \tag{9}$$

where $G_{23}^0 = \frac{E_2^0}{2(1+\nu_{23}^0)}$.

For the particular damage mode concerned, i.e. where matrix cracks have a common orientation with crack surfaces perpendicular to axis 2, damage is characterized in terms of variable D defined as the reduction in stiffness in direction 2.

$$D = 1 - E_2/E_2^0 \tag{10}$$

and the compliance matrix of the composite damaged in this way can be given as [2]

$$[S] = \begin{bmatrix} 1/E_1^0 & & & & & \\ -\nu_{12}^0/E_1^0 & 1/[E_2^0(1-D)] & & & \text{Symm.} & \\ -\nu_{12}^0/E_1^0 & -\nu_{23}^0/E_2^0 & 1/E_2^0 & & & \\ 0 & 0 & 0 & 1/\left[G_{23}^0\left(1-\frac{1}{2(1+\nu_{23}^0)}D\right)\right] & & \\ 0 & 0 & 0 & 0 & 1/G_{12}^0 & \\ 0 & 0 & 0 & 0 & 0 & 1/[G_{12}^0(1-kD)] \end{bmatrix} \quad (11)$$

where k is a damage related material constant which can be determined through virtual testing [2]. Its magnitude usually falls between 0 and 1.

The UD composite becomes orthotropic in the presence of damage and its effective properties can be expressed as

$$\begin{gathered} E_1 = E_1^0, \quad E_2 = E_2^0(1-D), \quad E_3 = E_3^0\ (= E_2^0) \\ \nu_{12} = \nu_{12}^0, \quad \nu_{13} = \nu_{13}^0\ (= \nu_{12}^0), \quad \nu_{32} = \nu_{32}^0\ (= \nu_{23}^0) \\ G_{13} = G_{13}^0\ (= G_{12}^0), \quad G_{23} = G_{23}^0\left(1-\frac{1}{2(1+\nu_{23}^0)}D\right) \\ G_{12} = G_{12}^0\ (1-kD). \end{gathered} \quad (12)$$

3.2. Damage representation of a viscoelastic UD composite

According to the correspondence principle, the damage representation for an elastic UD composite can be extended to the viscoelastic case in the Laplace domain by replacing the engineering constants with the Carson transformation of the relaxation modulus in the corresponding creep compliance.

$$[\tilde{S}] = \begin{bmatrix} 1/\tilde{E}_1^0 & & & & & \\ -\tilde{\nu}_{12}^0/\tilde{E}_1^0 & 1/\left[\tilde{E}_2^0(1-\widehat{D})\right] & & & \text{Symm.} & \\ -\tilde{\nu}_{12}^0/\tilde{E}_1^0 & -\tilde{\nu}_{23}^0/\tilde{E}_2^0 & 1/\tilde{E}_2^0 & & & \\ 0 & 0 & 0 & 1/\left[\tilde{G}_{23}^0\left(1-\frac{1}{2(1+\tilde{\nu}_{23}^0)}\widehat{D}\right)\right] & & \\ 0 & 0 & 0 & 0 & 1/\tilde{G}_{12}^0 & \\ 0 & 0 & 0 & 0 & 0 & 1/\left[\tilde{G}_{12}^0(1-k\widehat{D})\right] \end{bmatrix} \quad (13)$$

where $\widehat{D} = 1 - \tilde{E}_2/\tilde{E}_2^0$.

It is noted that $[\tilde{S}]$ is the creep compliance in the Laplace domain. Then the creep compliance in the time domain taking account of damage with damage, $[S(t)]$, can be obtained by taking the inverse Laplace transformation of $[\tilde{S}]$. The expression for $[\tilde{S}]$ was chosen for expressing the damage in terms of the Carson transformed engineering constants as it is much simpler than $[\tilde{C}]$. The commonly used relaxation modulus $[C(t)]$ can be obtained through the inverse Laplace transformation of $[\tilde{C}]$ which is the inverse matrix of $[\tilde{S}]$. The relaxation modulus

incorporating damage $[C(t)]$ can also be considered as the representation of this damaged viscoelastic UD composite.

The Carson transformed engineering constants such as $\tilde{E}_1^0$, $\tilde{E}_2^0$, $\tilde{\nu}_{12}^0$, $\tilde{\nu}_{23}^0$ and $\tilde{G}_{12}^0$ can be obtained from the relaxation moduli $E_1^0(t)$, $E_2^0(t)$, $\nu_{12}^0(t)$, $\nu_{23}^0(t)$ and $G_{12}^0(t)$ which in turn can be obtained from stress relaxation tests. The damage representation for viscoelastic UD composites with matrix cracks can then be obtained in the Laplace domain for a given damage state $\widehat{D}$.

4. Damage Evolution Law

The damage representation for a transversely isotropic viscoelastic UD composite is obtained from the damage representation for the elastic case, extended using the correspondence principle (CP). Unfortunately, the CP can only be used in cases where the state of damage, is fixed [18]. A damage evolution law based on viscoelastic theory is required to determine the growth of damage and its simplest form can be obtained in a one-dimensional idealisation as follows, starting with a general definition of a viscoelasticity model.

4.1. The Wiechert model

The most common mathematical model used to describe viscoelasticity in a one-dimensional form is the Wiechert model [16] (Figure 1), which represents viscoelasticity using a series of springs (elasticity) and dashpots (viscosity) connected in parallel.
The relaxation modulus for this model is

$$E_{rel}(t) = k_e + \sum_j k_j e^{\left(-\frac{t}{\tau_j}\right)} \tag{14}$$

where k_e is the elastic modulus of the main spring, k_j is the elastic modulus of spring connected with a dashpot, τ_j is their relaxation time and

$$\tau_j = \frac{\eta_j}{k_j} \tag{15}$$

η_j being the viscosity of the dashpot.

In the CDM theory, the effects of damage are often represented in terms of the reduction in stiffness [19-21], as shown in equation (10) in the case of matrix cracking in direction 2. For

viscoelastic materials represented by the Wiechert model, it will be assumed that the damage affects all its parts (springs k_j and dashpots η_j) equally.

The relaxation modulus with damage can then be defined as

$$E_D(t) = (1 - D)\left[k_e + \sum_j k_j e^{\left(-\frac{t}{\tau_j}\right)}\right] \tag{16}$$

where $E_D(t)$ is interpreted as $E_2(t)$ (transverse modulus) in the present representation of damage in three dimensions.

4.2. Damage evolution law for the transverse direction behaviour of UD composites

To describe matrix cracking in terms of CDM, a damage evolution law will be formulated here, based on the assumption that the damage evolution is controlled by the transverse tensile strain in line with the work in [22]. The Weibull distribution can be employed to characterise the defects in the matrix, in which damage (cracking) is expressed as a function of the strain in the transverse direction. Transverse tensile tests will be conducted on UD composites to obtain the necessary material properties.

The concept of a representative volume element (RVE) in a UD composite will be employed to introduce damage. It will be assumed that the RVE contains a large number of microscopic defects of different sizes as shown in Figure 2 (a). These defects develop into cracks as the strain increases as illustrated in Figure 2 (b). These discrete cracks are the physical manifestation of the idealised, continuous damage. The effects of these cracks are expressed as the damage parameter D, the value of which is in the range 0 and 1. When D equal to 0, the material is undamaged. When D becomes equal to 1, the material has completely failed. The probability density of these defects can be described by the Weibull distribution which is defined here as a function of strain.

$$\rho(\epsilon; \lambda, h) = \begin{cases} \frac{h}{\lambda}\left(\frac{\epsilon}{\lambda}\right)^{h-1} e^{-\left(\frac{\epsilon}{\lambda}\right)^h} & \sigma \geq 0 \\ 0 & \sigma < 0 \end{cases} \tag{17}$$

where h and λ are constants. Then damage can be defined as

$$D = \int_0^{\epsilon} \rho \, \mathrm{d}x = 1 - e^{-\left(\frac{\epsilon}{\lambda}\right)^h} \tag{18}$$

This definition of the damage ratio D is consistent with the previous definition of D as stiffness reduction in (10).

Using the viscoelastic constitutive relations in (2) or (3) and treating the relaxation modulus as a function of damage as expressed in (16), the constitutive equation of the damaged viscoelastic material is written as

$$\begin{aligned}
\sigma(t) &= E_D(t)\epsilon_0 + \int_0^t E_D(t-\tau)\frac{\partial \epsilon(\tau)}{\partial \tau}\mathrm{d}\tau \\
&= (1-D)E_{rel}(t)\epsilon_0 + (1-D)\int_0^t E_{rel}(t-\tau)\frac{\partial \epsilon(\tau)}{\partial \tau}\mathrm{d}\tau \\
&= e^{-\left(\frac{\epsilon}{\lambda}\right)^h} E_{rel}(t)\epsilon_0 + e^{-\left(\frac{\epsilon}{\lambda}\right)^h}\int_0^t E_{rel}(t-\tau)\frac{\partial \epsilon(\tau)}{\partial \tau}\mathrm{d}\tau \\
&= e^{-\left(\frac{\epsilon}{\lambda}\right)^h}\left[k_e + \sum_j k_j e^{\left(-\frac{t}{\tau_j}\right)}\right]\epsilon_0 \\
&+ e^{-\left(\frac{\epsilon}{\lambda}\right)^h}\int_0^t \left[k_e + \sum_j k_j e^{\left(-\frac{t-\tau}{\tau_j}\right)}\right]\frac{\partial \epsilon(\tau)}{\partial \tau}\mathrm{d}\tau
\end{aligned} \tag{19}$$

where $\epsilon = \epsilon_0 + \int_0^t \frac{\partial \epsilon(\tau)}{\partial \tau}\mathrm{d}\tau$ and $\epsilon = \epsilon_0$ when $t = 0$. Therefore (19) presents the damage evolution law for the viscoelastic material.

In the case of a constant strain rate test at a strain rate a, (19) will transform into

$$\begin{aligned}
\sigma(t) &= ae^{-\left(\frac{at}{\lambda}\right)^h}\int_0^t \left[k_e + \sum_j k_j e^{\left(-\frac{t-\tau}{\tau_j}\right)}\right]\mathrm{d}\tau \\
&= ae^{-\left(\frac{at}{\lambda}\right)^h}\left\{k_e t + \sum_j \left[k_j\tau_j - k_j\tau_j e^{\left(-\frac{t}{\tau_j}\right)}\right]\right\}
\end{aligned} \tag{20}$$

In Kumar and Talreja [6] and Koyanagi et al. [23], the purely elastic part of the Wiechert model is absent for the cases of pure resin or the transverse properties of the UD composite, so that (20) can be further simplified as

$$\sigma(t) = ae^{-\left(\frac{at}{\lambda}\right)^h}\cdot\sum_j \left[k_j\tau_j - k_j\tau_j e^{\left(-\frac{t}{\tau_j}\right)}\right] \tag{21}$$

The strength under constant strain rate can be determined as the maximum value of (21) corresponding to its first derivative with respect to time becoming zero:

$$\frac{\mathrm{d}\sigma(t)}{\mathrm{d}t} = ae^{-\left(\frac{at}{\lambda}\right)^h} \sum_j k_j \left[e^{-\frac{t}{\tau_j}} - \frac{ah}{\lambda}\tau_j \left(\frac{at}{\lambda}\right)^{h-1} \left(1 - e^{-\frac{t}{\tau_j}}\right)\right] = 0 \tag{22}$$

Incidentally, this damage evolution law obtained for a UD composite under transverse tensile can be also used, with different parameter values, to describe the damage evolution for purely homogenous viscoelastic materials.

5. Incorporation of the time-temperature superposition principle (TTSP)

The mechanical behaviour of viscoelastic composites can be attributed to the rearrangement of long chain molecules in polymers, which exhibit time and temperature dependence [24]. This double dependence exacerbates the difficulty in describing this kind of material, but also brings an opportunity to accelerate fatigue and long-term creep tests. In general, viscoelastic moduli increase with loading rate but decrease with rising temperature [25]. Furthermore, if these properties are plotted as functions of logarithmic time, it is observed that the profiles of the resulting curves retain the same shape at different temperatures but are shifted along the logarithmic time axis. This implies that a master curve at a given temperature can be used as a template to generate curves at other temperatures by applying an appropriate shift operation. This so-called time-temperature superposition principle (TTSP) of linear viscoelasticity is based on the above observation [17], and can be is employed to determine temperature-dependent mechanical properties of linear viscoelastic materials at a given time and temperature from the known properties at a reference temperature.

5.1. The time-temperature shift factor defined in Miyano's work

Miyano and Nakada conducted a large body of research on accelerating fatigue tests and predicting fatigue life by using TTSP [22, 26-29] where the TTSP was used to transform the effects of temperature by replacing the time t with a shifted or reduced time t' scaled by the TTSP at the reference temperature.

The time-temperature shift factor $a_{T_0}(T)$ is defined as

$$a_T = a_{T_0}(T) = \frac{t}{t'} \tag{23}$$

and in its the integral form as

$$t' = \int_0^t \frac{d\tau}{a_T} \tag{24}$$

with t' being the reduced time to failure at temperature T_0. In this way, the reduced time t' (a long time scale) at the reference temperature T_0 (room temperature) can be obtained by conducting the test over actual time t at temperature T and scaling the time by using the time-temperature shift factor $a_{T_0}(T)$.

In Miyano's work [22], time and temperature were transformed simply by the time-temperature shift factor a_T from the creep compliance of the resin without any further manipulation. The shift factor a_T is determined by the Arrhenius equation as

$$\log a_T = \frac{Q}{2.303R}\left(\frac{1}{T} - \frac{1}{T_0}\right) \tag{25}$$

where Q is the activation energy as a property of a specific resin which should be determined before the theory can be applied, T is the absolute temperature and R is the gas constant, 8.314×10^{-3}[kJ/s(K×mol)].

5.2. The time-temperature shift factor defined from viscosity

The straightforward application of a time-temperature shift factor to obtain reduced time for a UD composite may be viewed as an arbitrary approach lacking any clear theoretical basis. The present work aims to combine the TTSP with the viscoelastic damage model presented earlier in this paper. According to (15), τ_j can be replaced with η_j and k_j in (21), so that

$$\sigma\,(t) = \; a e^{-\left(\frac{at}{\lambda}\right)^h} \cdot \sum_j \left[\eta_j - \eta_j \, exp\left(-\frac{t}{\eta_j} k_j\right)\right] \tag{26}$$

and $\epsilon = at$, then (26) can be further transformed to

$$\sigma\,(t) = \; \epsilon e^{-\left(\frac{\epsilon}{\lambda}\right)^h} \cdot \sum_j \left[\frac{\eta_j}{t} - \frac{\eta_j}{t} exp\left(-\frac{t}{\eta_j} k_j\right)\right] \tag{27}$$

It is known that the viscosity η_j is temperature dependent. From Ojovan's work [30, 31], if the temperature is significantly lower than the glass transition temperature, $T \ll T_g$, then the viscosity can be expressed via an Arrhenius relationship as

$$\eta = A_L T \cdot e^{Q_H/RT} \tag{28}$$

with

$$Q_H = H_d + H_m \tag{29}$$

where H_d is the enthalpy of formation of broken bonds and H_m is the enthalpy of their motion. When the temperature approaches the glass transition temperature but is still below it, the activation energy of viscosity is high because most of the joining bonds are intact when amorphous materials are in glassy state.

If the temperature is significantly higher than the glass transition temperature, i.e. $T \gg T_g$, then the viscosity can be expressed as

$$\eta = A_H T \cdot e^{Q_L/RT} \tag{30}$$

with

$$Q_L = H_m \tag{31}$$

When the temperature is marginally higher than the glass transition temperature, the activation energy of viscosity is low because amorphous materials are melted and have most of their joining bonds broken, facilitating flow.

The definition of the time-temperature shift factor is the same as (23) which implies the time effect on the test can be replaced by the temperature effect. In order to distinguish it from the time-temperature shift factor defined by Miyano [22], the time-temperature shift factor in this paper is defined as

$$b_T = b_{T_0}(T) = \frac{t}{t'} \tag{32}$$

As shown in (27), as the temperature increases, the viscosity reduces. This is equivalent to increasing the test time according the damage model presented earlier. Then the time-temperature shift factor b_T can be described temperature-dependent viscosity changes as

$$b_T = b_{T_0}(T) = \frac{t}{t'} = \frac{\eta}{\eta'} \tag{33}$$

where η' is the viscosity at the reference temperature T_0 and b_T can be obtained from (28) or (30) as

$$b_T = \frac{\eta}{\eta'} = \frac{T}{T_0} e^{\frac{Q}{R}\left(\frac{1}{T} - \frac{1}{T_0}\right)} \tag{34}$$

with Q taking the value Q_H or Q_L according to the temperature. Thus

$$\log b_T = \frac{Q}{2.303R}\left(\frac{1}{T} - \frac{1}{T_0}\right) + \log\frac{T}{T_0} \tag{35}$$

The term $\log\frac{T}{T_0}$ arises from the temperature dependency of the pre-exponential factor in (28) and (30) compared with the constant pre-exponential factor assumed by Miyano [22]. In this paper the time-temperature shift factor is defined by (34) and (35) whilst incorporating the damage model (27) and the temperature effect upon viscosity (28). This is a further interpretation and extension of TTSP which was originally developed for use with the stress relaxation test [32]. By comparing the time-temperature shift factor a_T defined by Miyano (25) [22] with (35), it can be seen that the values of a_T and b_T are approximately equal as the value of $\log\frac{T}{T_0}$ is close to 0.

6. Application

6.1. Determination of the parameters in the Wiechert model

Measurements were undertaken of the time and temperature dependence of the relaxation modulus and tensile strength in the transverse direction of the UD composite of T700 carbon fibre and VTM 264-1 resin produced by Cytec Industries Inc. The volume fractions of the fibre and resin are 65% and 35% respectively. The curing cycle was 80°C for 5 hours followed by 120°C for 1 hour. The glass transition temperature T_g is 120°C. The specimen for the stress relaxation tests is illustrated in Figure 3 (ASTM D3039/D3039M – 14).

In order to obtain the relaxation modulus, tensile stress relaxation tests were carried out from 50°C to 120°C in steps of 10°C. The results are summarized in Figure 4 (a). The overlapped curves in Figure 4 (b) depict the relaxation modulus when the reference temperature is 50°C, and are plotted by taking the shift factor determined by (35). The activation energy is $298.4\ \mathrm{kJ/mol}$ which is obtained by calculating the shift factor from the testing times for the same stress relaxation test carried out at 80°C and 90°C respectively.

The parameters of the relaxation modulus can be obtained from Figure 4 (b) in terms of the elasticity and viscosity coefficients of the Wiechert model. The relaxation times τ_j are set from 10^{-3} to 10^{11} in steps of one order of magnitude to cover the full time scale in Figure 4 (b). Then the Young's moduli of the springs k_j are extracted from the relaxation curve using a simple least squares solution (Appendix A) or by using constrained least squares (MATLAB function lsqnonneg) so that none of the stiffnesses or viscosities in the Wiechert model is negative. The

resulting parameters are shown in Table 1 and the master curve of the resulting relaxation modulus is plotted in Figure 4 (b) by using the Wiechert model and compared against with the master curve plotted from the raw data.

The time-shifted experimental relaxation modulus curves in Figure 4(b) generally show very good overall agreement with the master curve, especially in the range 70-90°C, which includes the range of temperatures used for obtaining the activation energy used for predicting the master curve. Indeed, it is very encouraging that the master curve fits the time-shifted data well over most of the wide temperature range, given that it uses a single activation energy to make its predictions over that range. However there are some puzzling differences both at low temperatures e.g. 50°C and 60°C (where there is a physically unrealistic increase in the measured value of modulus over long timescales) and at higher temperatures e.g. 110°C and 120°C (where the fit is less good and the modulus does not drop as much as expected with time). It is believed that the unexpected increase in experimental modulus at 50°C and 60°C is due to hygrothermal shrinkage as moisture is lost during the test at elevated temperature. This effect will also be present at higher temperatures, but will be less significant compared with the viscoelastic effects. At temperatures approaching the final curing temperature (120°C), in addition to possible imperfect modelling of the temperature dependency with a single Arrhenius equation and activation energy which may account for imperfect fit in that region, it is likely that the high temperature tests lasting 1.5 hours will take the composite closer to complete cure. This will involve completing any reaction that was left incomplete at the end of the one-hour 120°C curing phase, and any such curing will lead to a small amount of shrinkage which appears to make the relaxation-related extensions smaller than expected.

6.2. Determination of the parameters for the damage evolution law

Damage evolution in the transverse direction of the UD composites is represented by (21). The damage parameters h and λ can be determined from the stress-strain curve at a given temperature by making use of the Arrhenius equation. A constant strain rate test was designed at 0.1mm/min for transverse UD composite at 90°C in order to obtain the stress-strain curve shown in Figure 5. The damage parameters h and λ can then be obtained by linearizing the difference between prediction without damage evolution (the dashed line in Figure 5) and experimental data (red line in Figure 5) as shown in Appendix B. The parameters h and λ are

obtained by fitting the experimental data to equation (21) based on a least squares approximation as 2.7381 and 0.0117 respectively.

6.3. Stress-strain curves prediction and comparison

After obtaining the purely viscoelastic properties described by the parameters in the Wiechert model in Table 1, the damage evolution parameters h, λ and the activation energy Q, the stress-strain curve may be extracted for any loading rate and any temperature using equations (21) and (35). Figures 6 and 7 show sample stress-strain curves comparing the model predictions (solid line) with the experimental data. If the specimens fail at the maximum stress, then the transverse strength of the UD composites can be predicted by (21) and (22). Figure 8 shows the strength of transverse UD composite compared with the predictions against experiments for a range of loading rates at 50°C. Limitations on availability of testing resources mean that only a single specimen was tested at each value of shifted time in order to plot Figures 6 and 7, so the results in Figure 8 (corresponding to the failure points in Figures 6 and 7) should be regarded only as illustrative of the trend shown, rather than for quantitative comparison between theory and experiment or to produce definitive data for design. A much more comprehensive programme of tests would clearly be needed to establish the degree of consistency of the trend noted.

The strain range of the prediction is obtained by assuming that the material fails at the maximum stress which is determined by (22). It can be seen from Figures 6 and 7 that, when the temperature is below 110°C, the specimens failed before the strain reached the value associated with the stress determined by (22). With increasing temperature, the strain range and the strength start to approach the predicted value. This is due to the fact that increased temperature reduces the brittleness of materials and hence their sensitivity to defects. On the other hand, the experimental data fit the prediction very well in the range of actual testing strain. This demonstrates that the damage model presented in this paper satisfactorily predicts the behaviour of viscoelastic material during deformation. However, as shown in Figure 8, this damage model still does not seem to predict the final failure of the specimens well since the specimens usually tend to fail before the maximum stress predicted by (22). It should be noted that this is foreseeable due to the brittle nature of transverse direction of UD composites, the failure of which is dictated by the weakest link. Fortunately, transverse failure usually features

local failure in laminated composites where the presence of such local failure does not usually result in global failure of the laminate.

When the temperature approached 110 °C, the prediction did not fit the experimental data, implying that the Arrhenius equation does not apply well for the VTM 264_1 resin system (Figure 4) at a temperature close to the glass transition temperature T_g (120 °C). Furthermore, the damage evolution law will also be affected by the low viscosity at elevated temperatures. These factors limit the temperatures for reliable application of the theory.

7. Conclusion

A matrix cracking damage model including damage representation and damage evolution for linear viscoelastic UD composites has been formulated in this paper. The CDM damage representation of Li et al. [2] has been extended to incorporate the effects of damage and viscoelasticity. A damage evolution law for viscoelastic materials has been proposed using a one-dimensional Wiechert model. The TTSP approach can then be employed to associate the effects of temperature on the viscosity with the time reduction, laying the foundations for accelerated testing on fatigue or creep. The whole process of using this damage model to predict the properties of a UD composite during deformation is demonstrated in this paper through an example, and the results have been compared with experimental data. The stress-strain curve predictions fit the experimental data very well in the range of actual testing strain with one exception which is discussed and justified. In terms of failure and strength prediction, a noticeable discrepancy is present without fundamentally undermining the value of the present model.

Acknowledgements

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Data Availability

The raw data required to reproduce these findings are available to download from http://dx.doi.org/10.17639/nott.368, file “Raw data for Xu Jones Li.zip”. The processed data

required to reproduce these findings are available to download from http://dx.doi.org/10.17639/nott.368, file “Results for Xu Jones Li.zip”.

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Appendices

A. The Identification of the Relaxation Modulus

When modelling a viscoelastic material using experimental data, the Wiechert model can be employed as an approximation as presented in equations (14) and (15). The constants involved can be obtained by fitting the experimental data to the function of $E_{rel}(t)$ as defined in (14). The procedure is illustrated through a relaxation problem as follows.

a) In equation (14), choose $k_e = E_{rel}(t = 0) = 0$ since the overlapped curve (Figure 4 (b)) approaching 0 when the testing time is more than 10^{11} sec.
b) Choose sufficient number of values of τ_j (j=1, 2,…, n) as sampling point across the range from 10^{-3} to 10^{11} sec. Here $n = 15$.
c) For each sampling point, equation (14) gives an equation for the spring constants k_j involved in the Wiechert model. Considering all m equations so obtained, an overdetermined system is obtained, which can be solved using least squares as illustrated below.

$$\sum_{j=1}^{15} k_j e^{\left(-\frac{t}{\tau_j}\right)} = E_{rel}(t_i) - k_e, \quad i = 1, \dots 15 \tag{A-1}$$

For the system $Ax = b$ the least squares formula is obtained from the problem

$$\min_x \|Ax - b\| \tag{A-2}$$

the solution of which can be written with the normal equations

$$x = (A^T A)^{-1} A^T b \tag{A-3}$$

where T indicates a matrix transpose, provided $A^T A$ is non-singular.

Applying it to the present problem, one has

$$k_j = (Q^T Q)^{-1} Q^T E \tag{A-4}$$

where

$$Q = \begin{bmatrix} e^{\left(-\frac{t_1}{\tau_1}\right)} & e^{\left(-\frac{t_1}{\tau_2}\right)} & e^{\left(-\frac{t_1}{\tau_3}\right)} & e^{\left(-\frac{t_1}{\tau_4}\right)} & e^{\left(-\frac{t_1}{\tau_5}\right)} & e^{\left(-\frac{t_1}{\tau_6}\right)} & e^{\left(-\frac{t_1}{\tau_{15}}\right)} \\ e^{\left(-\frac{t_2}{\tau_1}\right)} & e^{\left(-\frac{t_2}{\tau_2}\right)} & e^{\left(-\frac{t_2}{\tau_3}\right)} & e^{\left(-\frac{t_2}{\tau_4}\right)} & e^{\left(-\frac{t_2}{\tau_5}\right)} & e^{\left(-\frac{t_2}{\tau_6}\right)} & e^{\left(-\frac{t_2}{\tau_{15}}\right)} \\ e^{\left(-\frac{t_3}{\tau_1}\right)} & e^{\left(-\frac{t_3}{\tau_2}\right)} & e^{\left(-\frac{t_3}{\tau_3}\right)} & e^{\left(-\frac{t_3}{\tau_4}\right)} & e^{\left(-\frac{t_3}{\tau_5}\right)} & e^{\left(-\frac{t_3}{\tau_6}\right)} & e^{\left(-\frac{t_3}{\tau_{15}}\right)} \\ e^{\left(-\frac{t_4}{\tau_1}\right)} & e^{\left(-\frac{t_4}{\tau_2}\right)} & e^{\left(-\frac{t_4}{\tau_3}\right)} & e^{\left(-\frac{t_4}{\tau_4}\right)} & e^{\left(-\frac{t_4}{\tau_5}\right)} & e^{\left(-\frac{t_4}{\tau_6}\right)} \cdots\cdots & e^{\left(-\frac{t_4}{\tau_{15}}\right)} \\ e^{\left(-\frac{t_5}{\tau_1}\right)} & e^{\left(-\frac{t_5}{\tau_2}\right)} & e^{\left(-\frac{t_5}{\tau_3}\right)} & e^{\left(-\frac{t_5}{\tau_4}\right)} & e^{\left(-\frac{t_5}{\tau_5}\right)} & e^{\left(-\frac{t_5}{\tau_6}\right)} & e^{\left(-\frac{t_5}{\tau_{15}}\right)} \\ e^{\left(-\frac{t_6}{\tau_1}\right)} & e^{\left(-\frac{t_6}{\tau_2}\right)} & e^{\left(-\frac{t_6}{\tau_3}\right)} & e^{\left(-\frac{t_6}{\tau_4}\right)} & e^{\left(-\frac{t_6}{\tau_5}\right)} & e^{\left(-\frac{t_6}{\tau_6}\right)} & e^{\left(-\frac{t_6}{\tau_{15}}\right)} \\ & & \vdots & & & & \vdots \\ & & \vdots & & & & \vdots \end{bmatrix} \tag{A-5}$$

and

$$E = \begin{bmatrix} E_{rel}(t_1) - k_e \\ E_{rel}(t_2) - k_e \\ E_{rel}(t_3) - k_e \\ E_{rel}(t_4) - k_e \\ E_{rel}(t_5) - k_e \\ E_{rel}(t_6) - k_e \\ \vdots \\ \vdots \end{bmatrix} \tag{A-6}$$

B. Identification of Parameters of Damage Evolution Law

In the damage evolution law for the matrix of the UD composites, the damage is driven by the strain. The Weibull distribution of the defects is given as (17). Then

$$\sigma = e^{-\left(\frac{\epsilon}{\lambda}\right)^h} \sigma_e \tag{B-1}$$

where

$$\sigma_e = \sum_j \left[k_j \tau_j - k_j \tau_j \, exp\left(-\frac{t}{\tau_j}\right) \right] \tag{B-2}$$

Applying a logarithm transformation to both sides of (B-1) twice,

$$h\ln\epsilon - h\ln\lambda - \ln\left(\ln\frac{\sigma_e}{\sigma}\right) = 0 \tag{B-3}$$

Substituting experimental values for $\ln\epsilon$ and $\ln\left(\ln\frac{\sigma_e}{\sigma}\right)$ in (B-3), one obtains a series of simultaneous equations for h and $h\ln\lambda$ as the unknowns. The least squares method can be employed to determine them before the value of h and λ can be obtained.

As an example of application, σ in (B-3) can be extracted from the nonlinear experimental curve (red) and σ_e can be extracted from the straight line (blue) in Figure 5.

$$\begin{pmatrix} \ln\epsilon_1 & -1 \\ \vdots & \vdots \\ \ln\epsilon_m & -1 \end{pmatrix} \begin{pmatrix} h \\ h\ln\lambda \end{pmatrix} = \begin{pmatrix} \ln\left(\ln\dfrac{\sigma_{e_1}}{\sigma_1}\right) \\ \vdots \\ \ln\left(\ln\dfrac{\sigma_{e_m}}{\sigma_m}\right) \end{pmatrix} \tag{B-4}$$

The least square solution for h and $h\ln\lambda$ is obtained as

$$\begin{pmatrix} h \\ h\ln\lambda \end{pmatrix} = \begin{pmatrix} 2.7381 \\ -12.1721 \end{pmatrix} \tag{B-5}$$

Then the solution of h and λ is

$$\begin{pmatrix} h \\ \lambda \end{pmatrix} = \begin{pmatrix} 2.7381 \\ 0.0117 \end{pmatrix} \tag{B-6}$$

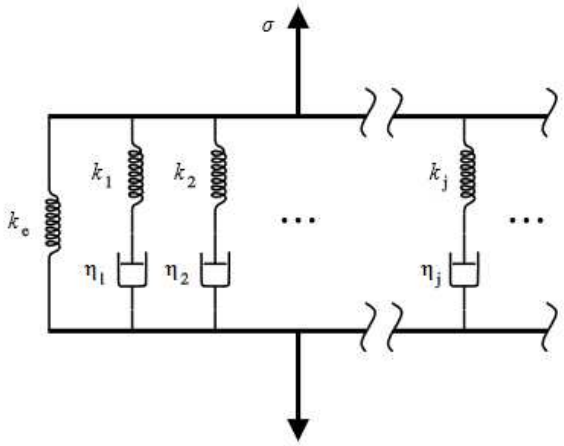


Figure 1: The Wiechert model (adapted from [16]).

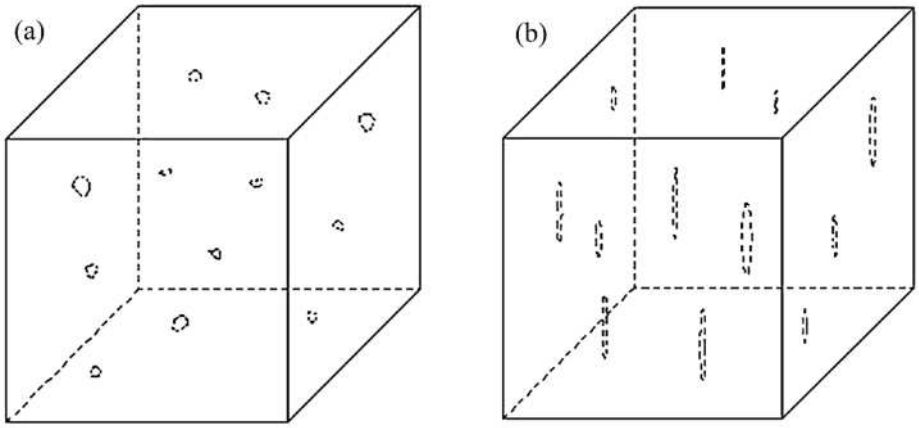


Figure 2: The representative volume element showing (a) defects and (b) cracks into which the defects develop

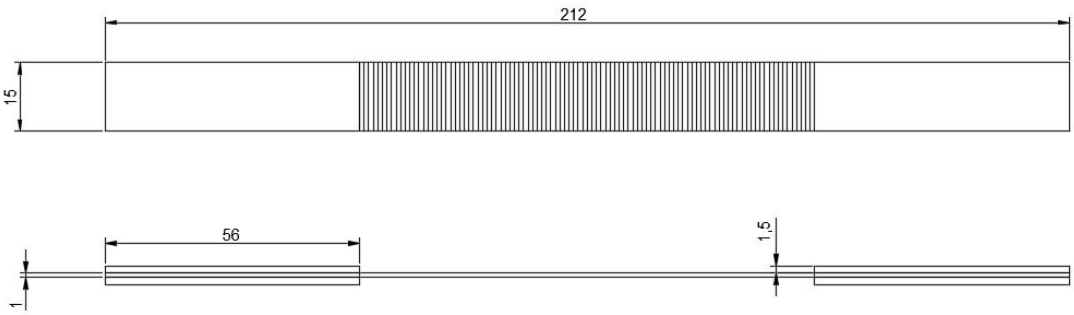
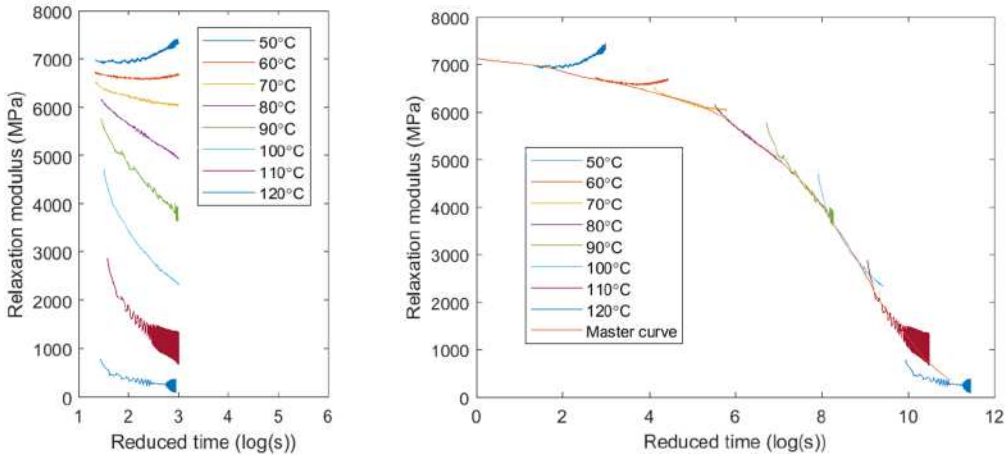


Figure 3: Dimensioned drawing of stress relaxation test specimen.



(a) (b)

Figure 4: Relaxation modulus at the reference temperature (50 °C) (a) measured as a function of time and (b) described by the Wiechert model compared with master curve of raw data plotted against reduced time

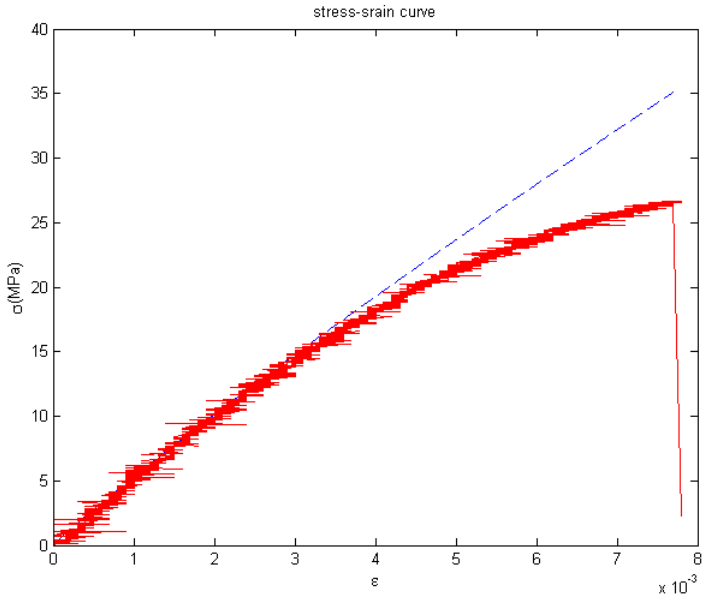


Figure 5: Stress-strain curve of UD composite at 90 °C compared with prediction without damage evolution

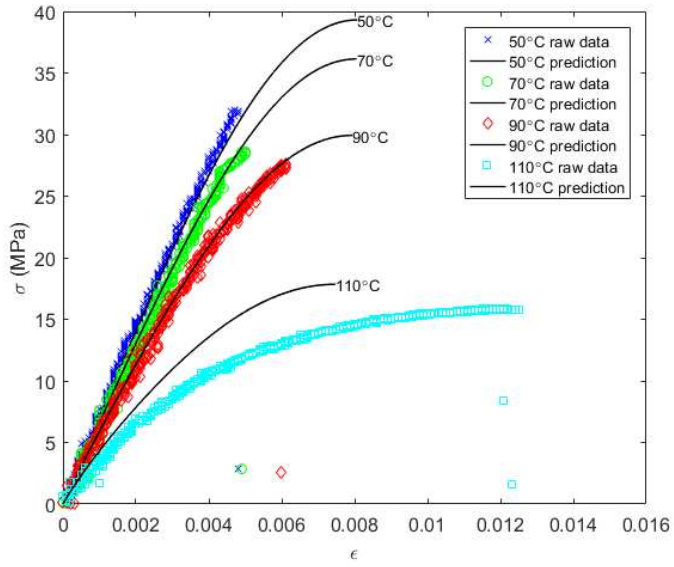


Figure 6: Sample comparisons of model with damage evolution vs. experiment at 1mm/min

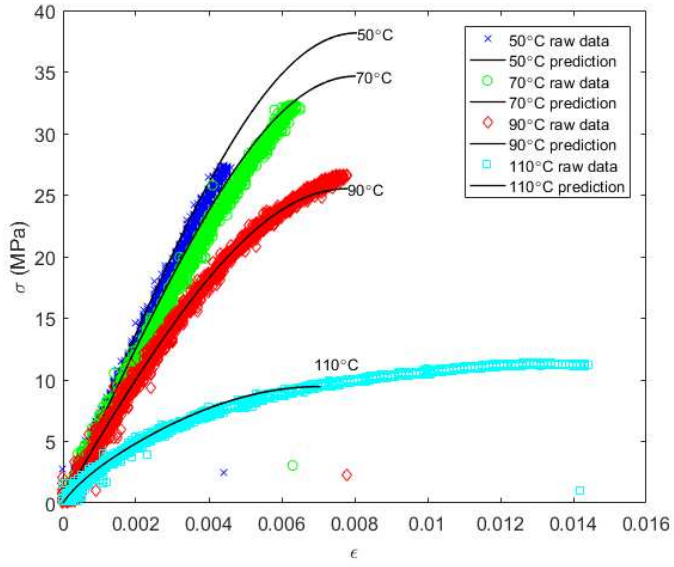


Figure 7: Sample comparisons of model with damage evolution vs. experiment at 0.1mm/min.

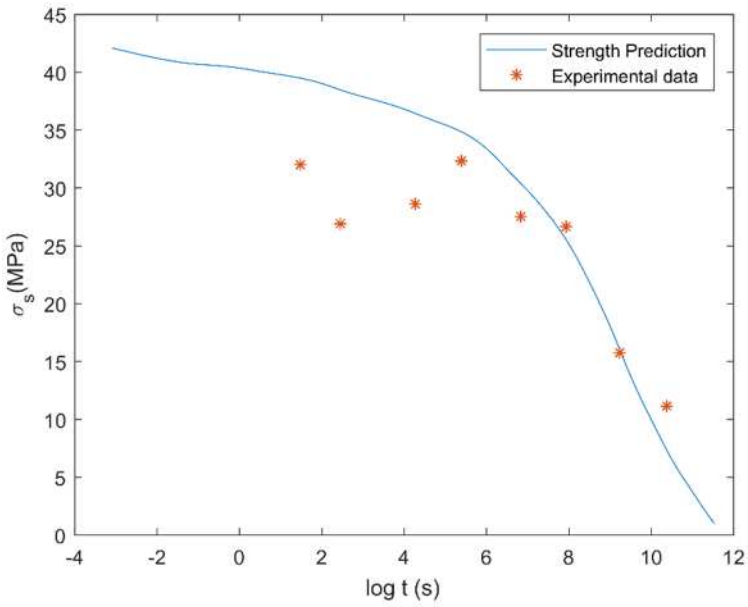
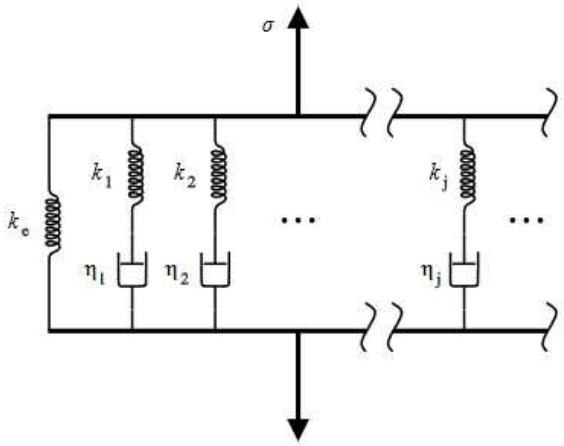


Figure 8: Strength of transverse UD composites at various loading rates plotted against shifted time.

Table 1: values of $\boldsymbol{k_j}$ and $\boldsymbol{\tau_j}$

	k_j(MPa)	$\tau_j(s)$
1	166.16	10^{-3}
2	134.45	10^{-2}
3	0	10^{-1}
4	127.59	1
5	60.331	10
6	270.04	10^{2}
7	125.21	10^{3}
8	278.8	10^{4}
9	140.82	10^{5}
10	725.03	10^{6}
11	586.34	10^{7}
12	1116.9	10^{8}
13	1687.6	10^{9}
14	1190.2	10^{10}
15	921.53	10^{11}



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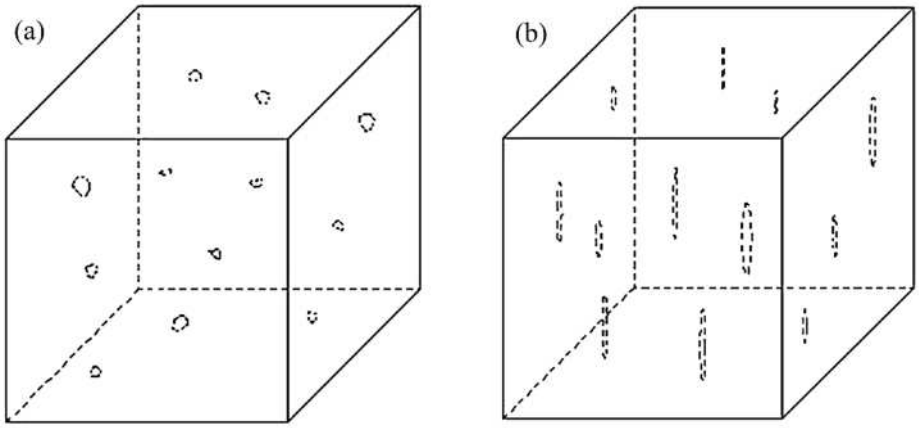

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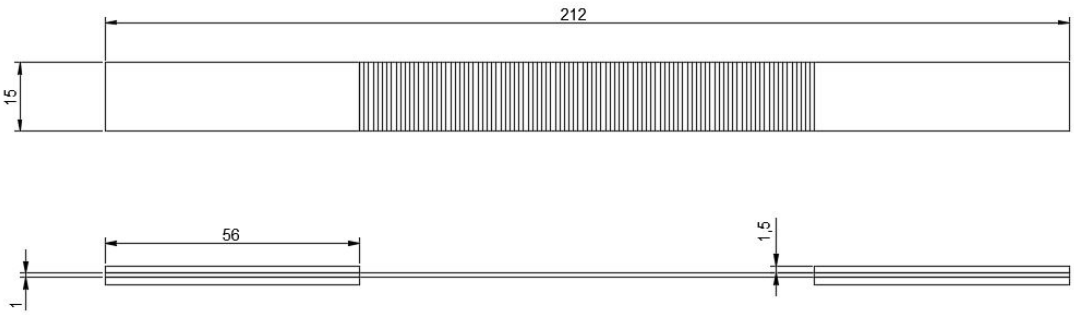

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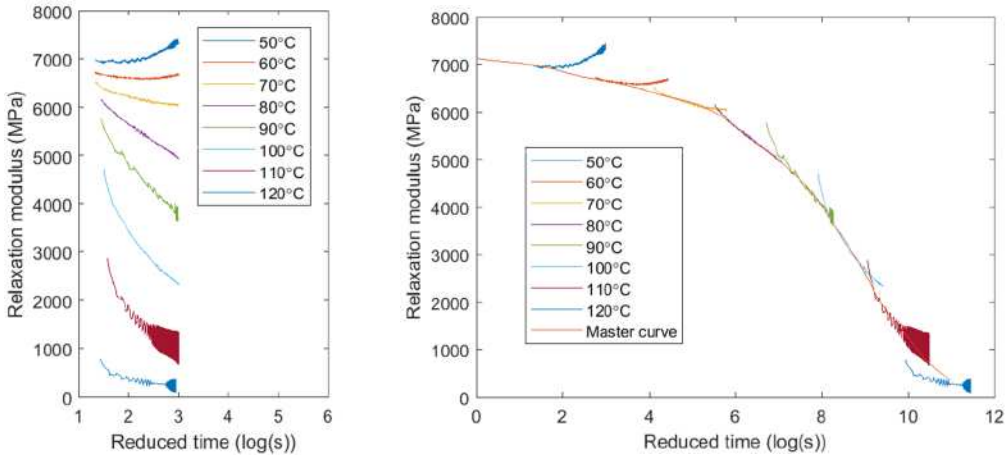

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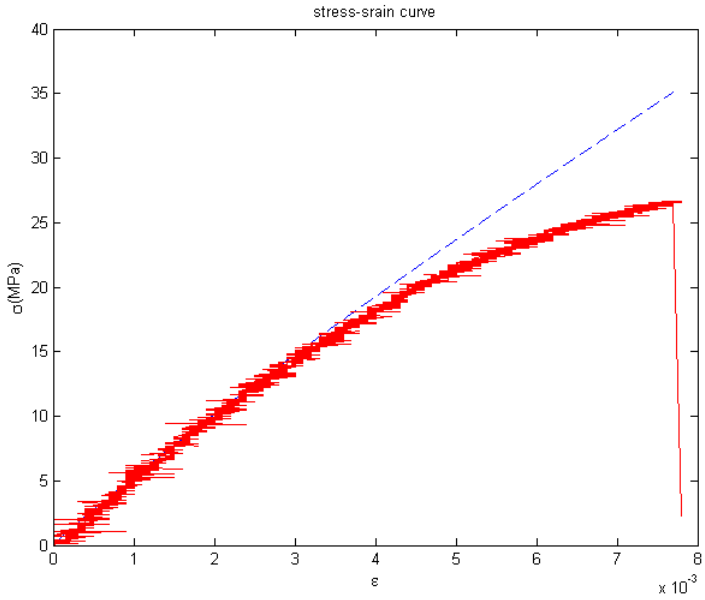

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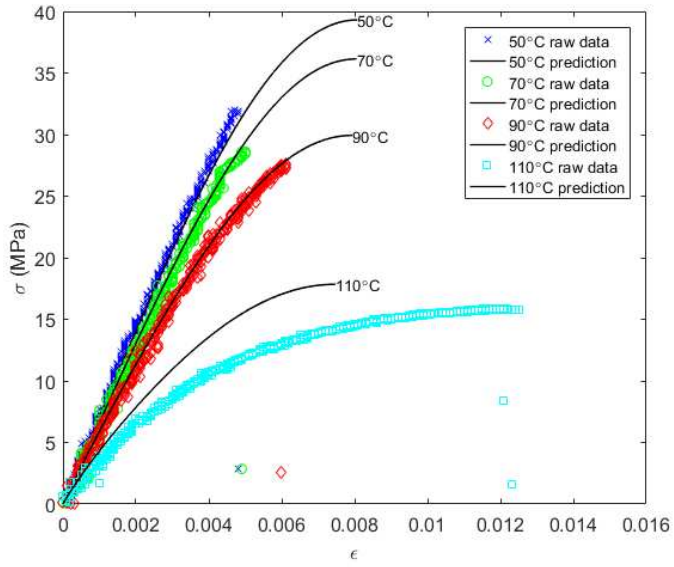


Figure 6: Sample comparisons of model with damage evolution vs. experiment at 1mm/min

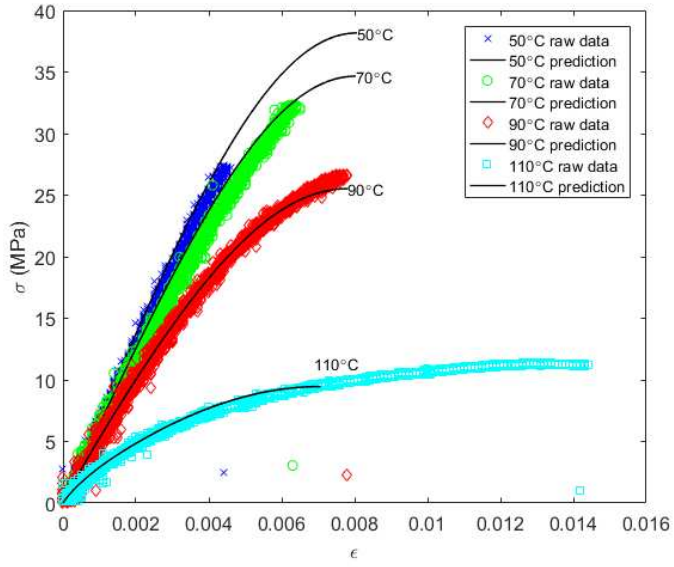
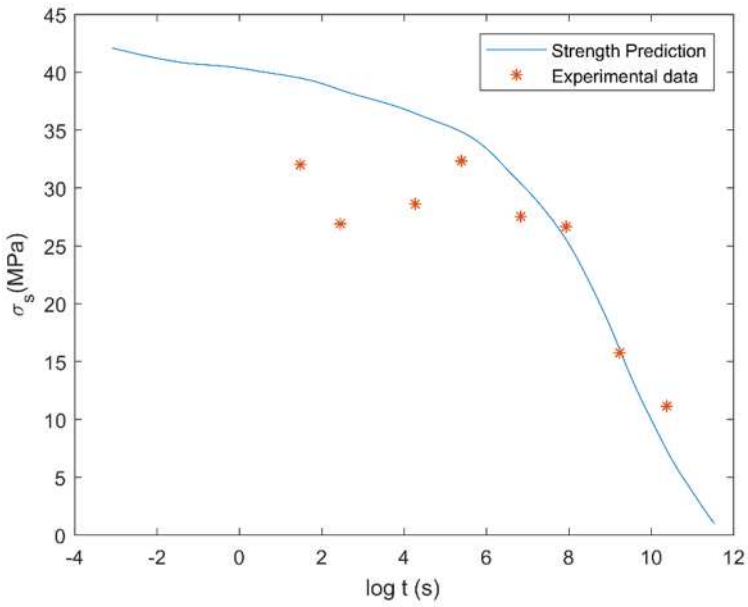


Figure 7: Sample comparisons of model with damage evolution vs. experiment at 0.1mm/min.



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