Pioneer, Early Follower or Late Entrant: Entry Dynamics with Learning and Market Competition*

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Abstract. Timing of market entry is one of the most important strategic decisions a firm must make, but its decision process becomes convoluted with information and payoff externalities. The threat of competition pushes firms to enter earlier to preempt their rivals while the possibility of learning makes them cautiously wait for others to take action. This combination amounts to a new class of timing games where a first-mover advantage first emerges as in preemption games but a second-mover advantage later prevails as in wars of attrition. Our model identifies under what conditions a firm becomes a pioneer, early follower or late entrant and provides efficiency implications by highlighting an elusive link between static market competition and dynamic entry competition.

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1. Introduction

It is often emphasized that timing of market entry is one of the most critical strategic decisions a firm must make whenever there is a new (geographical) market, a new product, or a new technology becoming available.[1] Market entry strategies are in fact highly complicated, as the benefit of market entry depends not only on the profitability of the market, which is often uncertain, but also on potential responses of other rival firms. Should a firm take the initiative in opening up a market and be a "pioneer," or more cautiously wait for others to take action? In case there emerges a pioneer, should a firm follow immediately or take some time to see how the market develops over time? Understanding this strategic decision process is of first-order importance, not only for potential entrants but also for policymakers, as it leads to immense welfare and policy implications: valuable resources are wasted if firms are rushed to enter a failed market while potential gains must be sacrificed if they wait too long to enter a successful one.

One important factor which underlies this problem is the tradeoff between becoming a leader and a follower. On this tradeoff, Lieberman and Montgomery (1988)—one of the most influential articles on first-mover advantage—descriptively raise two strategic considerations, among some others, as crucial forces in shaping market entry outcomes. On one hand, in the presence of market competition, there arises a benefit of *preemption*, which urges potential entrants to enter the market before their rivals do in order to seize market power (p.44-47). On the other hand, there is also a benefit of *learning from rivals* when there is uncertainty over potential benefits of market entry, as they argue "Late movers can gain an edge through resolution of market or technological uncertainty" (p.47). These two considerations generate counteracting incentives and a dynamic tradeoff of our focus.

In this paper, we build on this broad yet somewhat informal insight and develop a stylized model of market entry which sheds light on determinants and efficiency consequences of entry dynamics in a tractable manner. The primary purpose of the paper is to provide a unified framework that simultaneously captures the two aforementioned incentives; main applications of our model include new product markets, technology adoption and foreign direct investment among others. We consider an environment with two potential entrants, each of which independently decides whether and when to enter a new market. The profitability of the market is determined by the market condition (e.g., market size, production cost, quality of labor force) which is not known to anyone initially. Each firm thus privately

[1] There is a voluminous literature on the effect of timing and order of entry in the fields of strategic management and marketing. For instance, Lilien and Yoon (1990) note that "the choice of market-entry time is one of the major reasons for new product success or failure."

investigates whether the market is profitable enough over time and enters when it becomes sufficiently confident about the market.

The strategic nature of our model is determined by two external effects of market entry which stem from learning and market competition, as summarized below.

- *Information externalities from private learning.* Since each firm privately collects information about the market condition, a firm's entry serves as a signal of the firm's confidence in the market. The information externality generates a strong incentive to be the second mover to learn from the rival firm's action.
- *Payoff externalities from market competition.* The payoff from entry (in case the market turns out to be good) is decreasing in the number of firms in the market. A firm's entry thus reduces the residual demand and makes the rival firm's subsequent entry less profitable. The payoff externality generates a strong incentive to be the first mover to preempt the rival firm.

In this setup, the first mover can potentially capture a larger monopoly rent by entering early and consequently preempting its rival, but loses information that it could have obtained from its rival, thereby generating a dynamic tradeoff.

Due to this dynamic tradeoff, the game is divided into two distinct phases, called *preemption* and *waiting*, depending on the amount of information revealed by a market entry. A key insight of the model is that since each firm accumulates more information as time passes, and the signaling effect of a market entry strengthens over time, the preemption phase generally precedes the waiting phase. In the preemption phase, an entry reveals less information and the incentive to preempt its rival is the dominant concern, as in preemption games. In the waiting phase, an entry reveals enough information to induce the rival firm to follow immediately, and the incentive to wait and learn from the rival's entry is the dominant concern, as in wars of attrition. The combination of these counteracting forces thus amounts to a new class of timing games where a first-mover advantage first emerges as in preemption games but a second-mover advantage later prevails as in wars of attrition. For the sake of exposition, we say that a market entry is "pioneering" if it occurs in the preemption phase.[2]

We highlight three main results. First, we obtain a characterization of symmetric equilibrium in the environment described above.[3] We show that the model admits two classes

[2] We call it "pioneering" rather than "preemptive" because, as we will see later, it generates valuable information to the other firm and is generally socially beneficial, despite the fact that it is driven by preemption motives.

[3] Symmetric equilibria are more relevant in our context because asymmetric equilibria inherently require ex ante

of equilibria—one in which a pioneering entry occurs with positive probability on the equilibrium path and the other in which it never does—and derive a necessary and sufficient condition for a pioneering entry to occur in equilibrium. When this condition is satisfied, the firms enter the market at some positive rate in the preemption phase until it reaches a "saturation point" where the amount of information revealed by a market entry becomes too much. Market entry then ceases to occur past this point, with neither firm taking any action, as the net value of entry becomes strictly negative. After a while, though, a firm that has accumulated more favorable information becomes confident enough and willing to enter the market again, even without the chance to earn the monopoly rent. Our model thus exhibits on-and-off dynamics of market entry where the firms gradually enter the market at early and late stages, with a period of no entry in between.[4]

Second, we examine the timing of entry in comparison to the cooperative benchmark (where the firms cooperatively choose the timing of entry to maximize the joint profit, thus eliminating the externalities created by learning and market competition) to evaluate efficiency implications of entry dynamics. An important observation is that the timing of entry can be either too early or too late, depending crucially on the realized time of the first entry. Specifically, the efficiency of entry dynamics is determined by whether the first entry occurs in the preemption phase. If it does, the timing of entry is excessively early because: (i) in addition to the preemption motive which pushes the timing of entry forward, the first mover fails to take into account the positive information externality; and (ii) the second mover fails to take into account the negative payoff externality (or the "business-stealing effect"). If no entry occurs in the preemption phase, the timing of entry tends to be excessively delayed because each firm has a strong incentive to wait and see the rival's action. Drawing on this result, we argue, somewhat paradoxically, that the timing of entry tends to be too early when there is a late entrant while it tends to be too late when there is an early follower.

Finally, we note a policy implication of our analysis by arguing that consumer inertia, arising from various factors such as brand loyalty, habit formation, switching costs, and

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coordination, which is often difficult to attain. This is especially the case when the identify of potential entrants is not known ex ante, as is often the case in newly emerging markets—markets of our focus.

[4]Our on-and-off entry dynamics are reminiscent of trading dynamics described by Daley and Green (2012). In their setting, the market dries up when the belief is in some intermediate range, because the high type has no incentive to accept any offer, and the low type is willing to pool with the high type in the hope that good news will arrive in future. In contrast, in our model, a period of no entry emerges when the incentive structure is about to flip—from the preemption phase where each firm would like to enter slightly earlier than the rival firm to the waiting phase where each firm would like to enter slightly later—caused by the signaling effect that gets intensified over time. Bulow and Klemperer (1994) also provide a model of market frenzies and crashes in an auction-like environment.

slow diffusion of product information, can work to alleviate the inefficiencies mentioned above. For most of our analysis, we assume that there is no market friction and the first-mover advantage dissipates immediately after the arrival of a follower. In reality, however, we often find instances where first-mover advantage persists over time, especially once the first mover has established its presence in the market. In an extended version of our model, we incorporate this type of consumer inertia and examine how it affects entry dynamics. We argue that consumer inertia, which biases the allocation of surplus in favor of the first mover, can be efficiency-enhancing in our environment because it raises the benefit of becoming the first mover and facilitates information sharing between the competing firms. More broadly, this argument points to an elusive link between static market competition and dynamic entry competition: market competition on equal footing may be beneficial from the *ex post* point of view (once all the entry decisions are made), but it may distort the timing of market entry by limiting the benefits of becoming the first mover that are not fully internalized by potential entrants.

1.1. Related literature

Our paper builds on canonical preemption games such as Reinganum (1981) and Fudenberg and Tirole (1985).[5] They consider an environment where firms independently determine the timing of technology adoption with payoff externalities. The payoff to a firm depends on and is decreasing in the number of firms adopting the technology while the cost of doing so varies over time. Their models are pure preemption games in which there are no strategic incentives to delay adoption.[6] We extend this setup by incorporating the possibility of learning along with post-entry market competition. Two aspects of our learning process are particularly important: it is *dynamic*, where each firm gradually updates its belief via the arrival of a private signal, and *observational*, where each firm can also learn from the actions of the rival firm. Combined with payoff externalities that arise from market competition, these two aspects of learning qualitatively change the strategic nature of the problem and provides a new angle to address the question of when first-mover advantage prevails.

There are several works which incorporate dynamic learning and information externalities into a timing game. Decamps and Mariotti (2004) consider a similar learning process

[5] See Hopenhayn and Squintani (2016) and Bobtcheff et al. (2017) for some recent examples of preemption games. Bobtcheff et al. (2021) allow for an exit option in a preemption game, where exit may or may not be publicly observed.

[6] In canonical preemption games, the only reason to delay adoption is because the cost of doing so decreases over time; otherwise, there would only be a trivial equilibrium in which all players adopt immediately. The benefit of adoption delay is thus exogenous and non-strategic in this framework.

to ours in a model of strategic investment.[7] As in our model, they assume that the quality of the project, which is common to both players, is not known *ex ante* and gradually revealed over time via the arrival of a bad (public) signal. Their model is one of public learning where all the information is publicly observed, as a consequence of which there is no possibility of observational learning.[8] Our model also exhibits a phase which is effectively a war of attrition, and is in this sense related to Chen and Ishida (2021) who analyze a war of attrition, as formulated by Fudenberg and Tirole (1986), with learning about the unobserved state of nature via exponential bandits. In their model, each player may receive a signal indicating that the underlying state is good for sure, in which case it is optimal to stay in the game indefinitely. A crucial difference is that the state is individual-specific and independent across players, again eliminating any possibility of observational learning.

Several works introduce both dynamic and observational learning into a timing game.[9] Rosenberge et al. (2007) and Murto and Välimäki (2011) study a bandit problem where each player decides whether and when to stop experimenting, and the decision to exit is publicly observable. Kirpalani and Madsen (2021) analyze a situation where two firms engage in costly information acquisition (called prospecting) and decide whether and when to invest in a project, with the decision to invest publicly observable. In those models, the payoff one player can earn from a successful project is independent of the other player's action, and the type of externality considered is thus purely informational. Awaya and Krishna (2021) consider an R&D race between an established firm and a startup, where the established firm is assumed to be better informed about the feasibility of the innovation. The R&D race in their model is a winner-take-all contest such that the first one to succeed wins the race and earns the monopoly rent; as such, their model features both payoff and information externalities.[10] Their focus is on the asymmetry between the firms. They

[7]As we will detail below, we consider "no news is good news" whereby a firm observes a bad signal at some random time if the market condition is bad. Block et al. (2015, 2020) also incorporate private learning into a model of market entry but consider different learning processes where a firm can perfectly identify the true state of nature with some probability in each period or learn nothing at all ("no news is no news").

[8]The optimal timing of investment with strategic interactions is investigated actively also in the real options literature (Trigeorgis, 1991; Grenadier, 1996; Weeds, 2002; Pacheco de Almeida and Zemsky, 2003; Shackleton et al., 2004; Pawlina and Kort, 2006). This strand of literature generally assumes public learning with no information asymmetry among agents as in Decamps and Mariotti (2004).

[9]Other notable examples include Chamley and Gale (1994), Grenadier (1999), and Rasmusen and Yoon (2012). In those works, the information structure is fixed at the outset and there is no experimentation on the part of agents.

[10]In Awaya and Krishna (2021), since the game ends immediately when one of the firms succeeds in R&D, the information generated by a success is irrelevant. This is a crucial difference from our setting which incorporates post-entry market competition.

show that because the less informed startup has more to learn from its rival, there is an equilibrium in which it wins more often and earns a higher expected profit.

In the field of industrial organization, models of market entry tend to place more emphasis on market competition but less on dynamic learning.[11] For instance, Levin and Peck (2003) analyze a duopoly model of market entry in which each firm privately observes its entry cost at the outset of the game. The market environment is similar to ours in that the first mover can earn monopoly rents until the second mover arrives. Aside from the fact that there is no learning, a crucial difference is that the cost uncertainty in their model is firm-specific and hence a firm's entry does not reveal any useful information to the other firm. Rasmusen and Yoon (2012) analyze a duopoly model of market entry which incorporate both market competition and signaling. They consider a two-period model in which one of the firms is better informed about the market size than the other, and market entry by the informed firm hence becomes a signal of its private information. As in Levin and Peck (2003), however, the information structure is exogenously fixed at the outset of the game, which rules out the possibility of learning over time.

Finally, there are several attempts to endogenize the timing of moves in the more traditional branch of industrial organization (Gal-Or, 1985, 1987; Hamilton and Slutsky, 1990; Mailath, 1993). This strand of literature considers a situation where firms choose whether to move simultaneously and sequentially and analyzes under what conditions a Stackelberg leader emerges. In those models, it is the timing of decision making (i.e., how much to produce) that is endogenously determined, but actual production takes place simultaneously even when the firms choose to move sequentially. The analytical focus of this literature is generally on the strategic commitment effect of expanding production capacity.

2. Model

2.1. Setup

We consider a dynamic game of market entry with two firms, indexed by $i = 1, 2$, which contemplate to enter a market of unknown profitability. Time is continuous and extends

[11] Profit uncertainty also plays an eminent role in the context of foreign direct investment. Horstmann and Markusen (1996, 2018) consider a model in which a producer is unsure of the potential customer size and chooses either to contract with a local sales agent or to establish an owned local sales operation. While contracting with the local sales agent, the producer gains information about the customer size and switches to an owned sales operation if this option is found to be profitable. In their models, however, there is only one producer and hence no market competition.

from zero to infinity,[12] and each firm decides whether to enter the market at each point in time. For expositional purposes, we say that a firm is *active* if it has entered the market and *inactive* otherwise. When a firm decides to enter, it must incur a fixed entry cost $c > 0$. As a key feature of our model, we assume that the entry decision of a firm is immediately observed by the rival firm, so that a market entry serves as a signal of its private information.[13]

The market condition, which is common to both firms, is either good or bad, and each firm can "test the waters" before it makes an entry decision. Both firms start with a common prior that the market is good with probability p_0 and gradually acquire information via the arrival of a signal. Specifically, conditional on the market being bad, an inactive firm privately observes a signal with probability $\lambda \mathrm{d}t$ for an interval of time $[t, t+\mathrm{d}t)$. Note that a signal arrives only if the market is bad and hence that the arrival of a signal indicates that the market is bad for sure in our setup. This type of information structure is commonly assumed in the literature on strategic experimentation and is assumed here to highlight the fundamental forces with as much clarity.[14] We say that an inactive firm is *informed* if it has observed a signal and *uninformed* otherwise.

Given this structure, at any point in time, we can classify each firm into three distinct categories: *active*, *informed*, and *uninformed*. In particular, when we refer to a firm as either informed or uninformed, it implies that it is inactive at the moment. Once a firm enters, there are no further decisions to make, and the game effectively ends for that firm; we later extend our analysis and briefly discuss the case with potential market exits in section 5.2. Note also that an informed firm has no incentive to enter, knowing that the market is bad for sure. As such, we generally focus on the problem of a firm that is currently uninformed.

[12] We choose a continuous-time framework because of its greater tractability. In order to capture our on-and-off dynamics, we would need at least a three-period model in discrete time, which is in general far more complicated than a two-period model. Added to the complication is the possibility that the firms may enter simultaneously in discrete time—the possibility that vanishes in continuous time. Although this possibility of coordination failure can be interesting in some contexts, it does not add much to our analysis given our focus.

[13] We assume that each firm can observe the rival firm's entry timing but not the realized profit (or equivalently the market condition). We make this assumption because the eventual profit that a firm can achieve in the long run is typically realized after some time lag. In most cases of our interest, what is immediately inferrable at the time of entry is not the eventual gain of market entry but the expectation held by the entrant

[14] This type of structure emerges in experimentation with exponential bandits. Due to its tractability, the approach to model learning by exponential bandits, pioneered by Keller et al. (2005), has become a workhorse specification in the literature and offered many applications such as Strulovici (2010), Bonatti and Hörner (2011), Keller and Rady (2010, 2015), Guo (2016), Che and Hörner (2018), Chen and Ishida (2018), and Margaria (2020), just to name a few.

The net profit a firm can earn is determined by the market condition and the number of firms in the market. If the market is good, an active firm earns a flow payoff of $\pi+m$ if it is the only active firm and of π if both of them are active. We call m the monopoly premium, which could depend on the extent of market competition, and in general assume $m>0$. If the market is bad, on the other hand, an active firm invariably earns zero profit. The net profit for an inactive firm is also normalized at zero. Each firm maximizes the discounted sum of payoffs with common discount rate $r>0$. Throughout the analysis, we assume that the entry cost is sufficiently small and $\pi > rc$ holds to rule out a trivial equilibrium in which no firm ever enters.

2.2. Beliefs and strategies

The strategic nature of the problem changes once one of the firms enters the market. We thus divide the game into two stages, before and after one of the firms enters the market. For clarity, we say that the game is in the *pre-entry stage* if both firms are inactive and is in the *post-entry stage* if only one of the firms is inactive.

Since an informed firm never enters, we only define an uninformed firm's belief and strategy for ease of notation. From the viewpoint of an uninformed firm in the pre-entry stage, there are two unknowns that are relevant for its entry decisions: the market condition (good or bad) and the rival firm's state of knowledge (informed or uninformed). Since the rival firm is by construction uninformed if the market is good, we have three possible states of the economy as described below:

1. The market is good (state G);
2. The market is bad, and the rival firm is uninformed (state BU);
3. The market is bad, and the rival firm is informed (state BI).

Given this formulation, the belief in the pre-entry stage (hereafter, simply the belief) is defined in two dimensions and denoted by (p_t, q_t) where: (i) p_t is the conditional probability that the market is good (state G); (ii) q_t is the conditional probability that the market is bad and the rival firm is uninformed (state BU).[15] By definition, $1-p_t-q_t$ is the conditional probability that the market is bad and the rival firm is informed (state BI). All the probabilities are conditional on the history such that the firm has observed no signal (i.e., the firm is uninformed) and the rival firm has not entered the market (i.e., the game is in the pre-entry stage). In the post-entry stage, the only relevant belief is the probability that

[15] Although the private beliefs can differ between the two firms, the belief of an uninformed firm is unique and publicly known, so we suppress the firm subscript i for brevity.

r	Common discount rate
c	Market entry cost
λ	Rate of signal arrival
π	Duopoly profit
m	Monopoly premium
p_0	Prior belief that the market is good
p_t	Pre-entry belief that the market is good
q_t	Pre-entry belief that the market is bad and the rival firm is uninformed
$\tilde{p}_t$	Post-entry belief that the market is good
σ_i	Pre-entry strategy
δ_i	Post-entry strategy

Table 1. *Parameters and notations*

the market is good conditional on the firm being uninformed. We denote this belief by $\tilde{p}_t$ and specifically refer to it as the *post-entry belief*.

In what follows, we focus on symmetric perfect Bayesian equilibria in Markov strategies that satisfy the Intuitive Criterion (hereafter, simply the equilibrium), where we use the belief (p_t, t) as the state variable.[16] A strategy of an uninformed firm is defined by a pair of functions (σ_i, δ_i) where $\sigma_i : [0,1] \times [0, \infty) \to \mathbb{R}_+$ is the pre-entry strategy and $\delta_i : [0,1] \times [0,\infty) \to \mathbb{R}_+$ is the post-entry strategy.[17] The first component $\sigma_i(p_t, t)$ is the rate at which firm i enters the market in the pre-entry stage. The second component indicates the delay time in the post-entry stage: given that the rival firm enters at some time τ with belief p_τ, firm i waits until time $\tau + \delta_i(p_\tau, \tau)$ and enters if it is still uninformed at that time. It is without loss of generality to specify the strategy in this way because our characterization result (Proposition 1) suggests that the firms always adopt a mixed strategy and enter gradually over time in the pre-entry stage while they always adopt a pure strategy in the post-entry stage. Note that the strategy in general depends also on q_t but we omit this dependence because q_t can be derived from (p_t, t) as we will see below.

[16]Note that an informed firm would never enter the market at any point (knowing that the market is bad for sure), no matter what belief the remaining firm assigns to the entry. Any unexpected entry is therefore regarded as coming from the uninformed type under the Intuitive Criterion. Our equilibrium construction is based on this restriction on off-path beliefs.

[17]We adopt this multi-stage approach to allow for the possibility that a firm enters the market *immediately* after the rival firm's entry in continuous time, which corresponds to $\delta_i(p_t, t) = 0$ for a given (p_t, t). See Murto and Välimäki (2013) and Awaya and Krishna (2021) for this approach.

3. Equilibrium characterization

This section provides an equilibrium characterization of the model described above. In what follows, we let (σ, δ) denote the (symmetric) equilibrium strategy. Note that since we only need to look at the problem of an uniformed firm, there is only one relevant history of the game in the pre-entry stage, which is the one in which no signal has been observed (provided that the rival firm has not entered). To simplify notation, we often denote by s_t the pre-entry strategy in this history.

To obtain a characterization, it is important to note that there are two sources of information in our model: on one hand, each firm may privately observe a signal of the market condition which arrives stochastically over time; on the other hand, the entry decision of each firm is publicly observable and hence serves as an additional signal. The fact that a firm can observe the rival firm's entry implies a benefit of waiting, giving rise to a second-mover advantage stemming from information externalities. However, the presence of payoff externalities generates a tradeoff: as the profitability of each firm depends negatively on the number of firms in the market, the first one to enter can delay the rival firm's entry by reducing the residual demand while enjoying the monopoly profit. The equilibrium dynamics of our model are shaped by this tradeoff.

Our model admits two types of equilibrium depending on the way the remaining firm reacts to the first entry: in some cases, a market entry induces the remaining firm to follow immediately, i.e., $\delta(p_t, t) = 0$; in others, the remaining firm takes some time before it enters, i.e., $\delta(p_t, t) > 0$. For expositional clarity, we refer to a market entry that occurs when $\delta(p_t, t) > 0$ as a *pioneering entry*. Moreover, we say that an equilibrium is a *pioneer equilibrium* if $s_t > 0$ for any (p_t, t) such that $\delta(p_t, t) > 0$, i.e., if a pioneering entry occurs with positive probability on the equilibrium path; otherwise, we say that it is a *no-pioneer equilibrium*. This distinction is important because there is a qualitative difference between these two classes of equilibria.

If the prior belief p_0 is too high, learning is not essential and both firms may enter immediately at time 0. To focus on more relevant cases, we for now assume that the prior belief p_0 is small so that the firms never enter at time 0; a more precise condition for this will be provided later in section 3.3 (as Assumption 1). The following result provides a characterization of symmetric equilibrium in this model.

Proposition 1. *In any symmetric equilibrium, one of the following properties holds.*

(a) In a no-pioneer equilibrium, the firms wait until the belief p_t reaches the threshold $p^* := \frac{(\lambda+r)c}{\pi+\lambda c}$. When p_t reaches p^*, the firms enter gradually at a rate to keep p_t at p^*.

Once one of the firms enters, the remaining firm follows immediately if it is uninformed.

(b) *In a pioneer equilibrium, there exist* $\underline{\tau}$, $\overline{\tau}$ *and* τ^* *such that: (i) for* $t \in (\underline{\tau}, \overline{\tau})$, *the firms enter gradually, and once one of the firms enter at some* τ, *the remaining firm follows with some delay* $\delta(p_\tau, \tau) > 0$ *if it is uninformed at time; (ii) for* $t \in (\overline{\tau}, \tau^*)$, *the firms never enter; (iii) at time* τ^*, *the belief* p_t *reaches* p^*, *after which the firms again enter gradually as described in (a).*

Proof. See section 3.1 and Appendix A. ∎

In a no-pioneer equilibrium, the firms wait until p_t reaches p^* and start entering at some rate past that point. Otherwise, we have a pioneer equilibrium in which entry occurs in two disjoint intervals. In this class of equilibria, the firms start entering at some rate from $\underline{\tau}$ but stops at $\overline{\tau}$. This is followed by an interval of no entry $(\overline{\tau}, \tau^*)$ where neither firm takes any action. After a while, though, the belief p_t eventually reaches the threshold p^* at time τ^*, at which point the firms start entering again as in a no-pioneer equilibrium.

3.1. A sketch of the proof

In this subsection, we provide a sketch of the proof to illustrate the underlying intuition behind our characterization result. The technical details are relegated to Appendix A.

To solve the model backwards, we begin with the post-entry stage. The problem in this stage is straightforward, given that there is only one decision maker left in the game. It is easy to show that the post-entry belief follows

$$\dot{\tilde{p}}_t = \lambda \tilde{p}_t (1 - \tilde{p}_t), \tag{1}$$

once there is a market entry, as long as no signal is observed. Observe that a signal arrives only if the state is bad, and hence by Bayes' rule, the post-entry belief $\tilde{p}_t$ increases monotonically over time as long as the firm observes no signal (i.e., "no news is good news"). We can show that there is a unique threshold $p^* := \frac{(\lambda+r)c}{\pi+\lambda c}$ such that the firm enters once $\tilde{p}_t$ reaches p^*. Note that $p^* < 1$ by assumption, which rules out a trivial equilibrium in which no firm ever enters. Also, as we will see below, $\underline{\tau} > 0$ implies $p^* > p_0$.

Lemma 1. *In the post-entry stage, the remaining firm enters the market if and only if the post-entry belief exceeds the threshold, i.e.,* $\tilde{p}_t \geq p^* := \frac{(\lambda+r)c}{\pi+\lambda c}$.

Proof. See Appendix A. ∎

If one of the firms enters the market, the remaining firm naturally gains more confidence about the market because it necessarily implies that the first mover has observed no signal. Let ϕ_t be the post-entry belief that would prevail if there were a market entry at time t, i.e, $\tilde{p}_t = \phi_t$. By Bayes' rule, this is obtained as

$$\phi_t = \frac{p_t}{p_t + q_t},$$

which indicates the amount of information revealed by a market entry. Note in particular that since no firm is informed at time 0 with $q_0 = 1 - p_0$ and $\phi_0 = p_0$, a firm's immediate entry reveals no additional information.

Suppose that a firm enters at some time τ. The rival firm's post-entry belief $\tilde{p}_\tau$ is then given by ϕ_τ and follows (1) for $t > \tau$. It follows from Lemma 1 that: (i) if (p_τ, τ) is such that ϕ_τ is low enough and $\phi_\tau < p^*$, the remaining firm would take some time before it enters the market, i.e., $\delta(p_\tau, \tau) > 0$; (ii) otherwise, the remaining firm would immediately follow suit, i.e., $\delta(p_\tau, \tau) = 0$. Simple computation then gives

$$\delta(p_t, t) = \begin{cases} \frac{1}{\lambda} \ln \frac{p^* q_t}{(1-p^*)p_t} & \text{if } p^* > \phi_t, \\ 0 & \text{if } \phi_t \geq p^*, \end{cases} \tag{2}$$

which is the optimal strategy in the post-entry stage.

Given this, we now turn to the problem in the pre-entry stage. A major technical complication of this model arises from the fact that the evolution of the belief during this stage depends on the pre-entry strategy σ. Fortunately, while (p_t, q_t) may follow a highly complicated path, it is relatively straightforward to compute ϕ_t as it is independent of the pre-entry strategy σ. With some algebra, we obtain

$$\phi_{t+\mathrm{d}t} := \frac{p_{t+\mathrm{d}t}}{p_{t+\mathrm{d}t} + q_{t+\mathrm{d}t}} = \frac{p_t}{p_t + q_t e^{-2\lambda \mathrm{d}t}} > \phi_t,$$

which indicates that ϕ_t monotonically increases over time for any given strategy σ; the details of this derivation are placed in Appendix B. It is also important to note that for any σ,

$$\frac{q_t}{p_t} = \frac{1-p_0}{p_0} e^{-2\lambda t},$$

suggesting that q_t can be uniquely identified from (p_t, t) as we noted at the outset. Then, combined with (2), $\delta(p_t, t)$ depends only on t and can be written as

$$\delta_t = \delta(p_t, t) = \begin{cases} \frac{1}{\lambda} \ln \frac{p^*(1-p_0)}{(1-p^*)p_0} - 2t & \text{if } p^* > \phi_t, \\ 0 & \text{if } \phi_t \geq p^*. \end{cases} \tag{3}$$

This is an essential technical property of our model which enables us to simplify the analysis substantially while preserving the substance of the issue at hand.

The strategic nature of the problem depends crucially on whether the updated belief ϕ_t is above or below the threshold p^*. We consider these two cases in turn.

Waiting phase: $\phi_t \geq p^*$. Suppose $\phi_t \geq p^*$, so that an uniformed firm would immediately follow the rival firm. Note that this is the "winner's curse range" where the first mover can monopolize the market only if the market condition is bad. An uninformed firm thus prefers its rival to move first. As this is a phase where the second-mover advantage dominates, we call it the *waiting phase*.

It is easy to see that any continuation equilibrium in the waiting phase is in mixed strategies. To see this, suppose that the firms adopt a pure strategy of entering at some τ. Then, if a firm does not enter at τ, it signals that it is informed and hence the market condition is bad for sure. This creates an incentive for each firm to deviate and wait slightly more because the firm can gain this extra information at almost no cost. In the proof of Proposition 1 (Lemma 2), we show that in this phase, there is a unique symmetric continuation equilibrium in which: (i) neither firm enters until the belief p_t reaches the threshold p^*; (ii) when p_t reaches p^*, the two firms start entering at a rate to keep $p_t = p^*$; and (iii) once a firm enters, the remaining firm immediately follows at the next instant.

It is important to emphasize that the game in this phase is not a preemption game where each firm has an incentive to enter slightly earlier than its rival. This is a departure from the canonical preemption game such as Fudenberg and Tirole (1985) where there is no strategic benefit of becoming the second mover. To illustrate the key difference, it is worth emphasizing that the equilibrium identified above is not a "joint-adoption equilibrium" of Fudenberg and Tirole (1985): the entry times of the two firms in our model are arbitrarily close but not simultaneous—the type of equilibrium that does not exist in their framework (or more generally in preemption games).[18] In fact, in the waiting phase, neither firm wants to move first, because there is apparently no benefit of becoming the first mover. The game in this phase thus resembles a war of attrition where each firm waits for the rival to move first.

Preemption phase: $p^* > \phi_t$. The strategic nature of the problem flips if ϕ_t is below the threshold p^*. As entry in this phase is driven by preemption motives, we call it the

[18]Here, we assume no reaction delay for expositional simplicity, but entries in the waiting phase do occur in sequence in that one entry triggers the other. We could alternatively introduce a small observation lag Δ such that an entry at time t is observed by the rival firm at time $t + \Delta$. Our current setup assumes $\Delta = 0$.

preemption phase.[19] In the preemption phase, if a firm enters at some time τ, the remaining firm's belief jumps up but is still lower than p^*. As such, the remaining firm will not enter immediately, and the first mover can monopolize the market for some duration δ_τ given by (3), which gives rise to a first-mover advantage. Again, any equilibrium in the preemption phase must involve mixed strategies. To see this, suppose that the firms adopt a pure strategy of entering at some time τ. Then, there arises an incentive for each firm to deviate and enter slightly before time τ so as to secure the first-mover advantage while collecting as much information as possible. As in the waiting phase, the firms must randomize over time although incentives now point to the opposite direction. As noted in Proposition 1, the firms enter smoothly over some interval $(\underline{\tau}, \overline{\tau})$. Moreover, $\overline{\tau} < \tau^*$, which implies that there must be a period of no entry in between.

3.2. Entry dynamics

Our analysis sheds light on the underlying mechanism behind entry dynamics and offers some empirical implications. To see this more clearly, note that each realized equilibrium allocation is characterized by a pair of entry times (τ_1, τ_2) where τ_1 (τ_2) denotes the entry time of the first (second) mover. Our characterization result suggests that our model yields four classes of entry dynamics that are observationally distinguishable.

1. *No entry* $(\tau_1 = \tau_2 = \infty)$: Neither firm chooses to enter, and the market never materializes.
2. *Only one entry* $(\tau_1 < \tau_2 = \infty)$: Only one firm enters while the other firm chooses not to follow. This is the case of premature entry.
3. *Early follower* $(\tau_1 \approx \tau_2 < \infty)$: A firm enters in the waiting phase and is immediately followed by the rival firm. The two entries are clustered together in time.
4. *Late entrant* $(\tau_1 < \tau_2 < \infty)$: A firm enters in the preemption phase and is followed by the rival firm with some time lag. The two entries are spaced apart in time.

The first two cases occur only when the market condition is bad, reflecting the obvious fact that no successful market can be monopolized forever. The latter two cases admit two entrants and are the focus of attention in the existing literature. The distinction between these last two cases is economically meaningful and crucial, because they represent totally different mechanisms: the case of early follower is driven by the forces of war of attrition whereas the case of late entrant is driven by the forces of preemption game. Moreover,

[19] If a firm chooses not to enter for some $[\tau, \tau + \mathrm{d}t)$, the rival firm may enter with positive probability, in which case the firm's eventual entry is delayed by δ_τ. It is in this sense that we call this phase preemptive.

since a pioneer equilibrium always entails an in-between period of no entry, these two cases can be clearly separated by a discontinuity in entry times. These properties point to a qualitative, rather than quantitative, difference between the case of early follower and that of late entrant. Although many existing studies (Robinson and Fornell, 1985; Lambkin, 1988) classify entrants into three broad categories—pioneer, early follower, and late entrant—it is not necessarily clear why and in what sense this distinction is important. Our analysis provides a theoretical foundation for this seemingly *ad hoc* classification and suggests that the time lag between entries contains a wealth of information about the underlying mechanism of an observed entry pattern.

The reason why we have this period of no entry pertains to the amount of information revealed by an entry which increases over time. Although the firms have less private information and face more uncertainty early on, the fact that they have less private information means that there is less to learn from the rival firm's action, thereby making the preemption effect stronger. When this first-mover advantage dominates the cost of entering prematurely with insufficient information, the firms enter with some positive probability in the preemption phase. As each firm accumulates more information over time, however, the signaling effect of entry becomes stronger and the game reaches a point where the net payoff of becoming the first mover is negative. In any pioneer equilibrium, therefore, there must be an in-between phase where market entry ceases to occur.

3.3. Constrained problem

We have thus far established that there are two forms of equilibrium, depending on whether pioneering entry occurs on the equilibrium path. We now derive a precise condition for this to occur in equilibrium and also establish the existence of a symmetric equilibrium in the process. To this end, we first consider a hypothetical situation in which a firm, say firm 2, never enters in the pre-entry stage. As it turns out, this constrained version of the problem, which excludes the possibility of entry competition, provides enough information to see when pioneering entry occurs in the original (unconstrained) problem.

Under the restriction that firm 2 must be the second mover, the problem faced by firm 1 is substantially simpler: firm 1 simply decides when to enter conditional on having observed no signal. As a consequence, the expected payoff of entering at t can also be written as a function of t. Let $\hat{\Pi}(t)$ denote the expected (average) payoff of entering at t, evaluated at time 0, under the restriction that firm 2 must be the second mover. If firm 1 enters at time τ, firm 2 will wait until p_t reaches p^*. We thus obtain

$$\hat{\Pi}(\tau) = e^{-r\tau}[p_0(\pi + M_\tau - rc) - e^{-\lambda\tau}(1-p_0)rc], \tag{4}$$

where $M_t := (1-e^{-r\delta_t})m$ and δ_t is as given by (3). If $\phi_t \geq p^*$, then $\delta_t = 0$. In contrast, if $p^* > \phi_t$, firm 2 must wait to collect more information, giving firm 1 some time to monopolize the market. The incentive for pioneering entry thus hinges crucially on δ_t.

Suppose first that there is no pioneering entry. In this case, the earliest possible entry occurs when the belief p_t reaches p^*. Define τ^{NP} as the time at which p_t equals p^* when there is no pioneering entry, which must solve

$$e^{-\lambda\tau^{\mathrm{NP}}} = \frac{p_0(1-p^*)}{(1-p_0)p^*} = \frac{p_0(\pi - rc)}{(1-p_0)(\lambda+r)c}.$$

With this definition, (3) is now reduced to

$$\delta_t = \max\{\tau^{\mathrm{NP}} - 2t, 0\}.$$

Let $\Pi^{\mathrm{NP}} := \max_{t\in(\frac{\tau^{\mathrm{NP}}}{2},\infty)} \hat{\Pi}(t)$ denote the expected payoff without pioneering entry, which can be written as

$$\Pi^{\mathrm{NP}} = \hat{\Pi}(\tau^{\mathrm{NP}}) = e^{-r\tau^{\mathrm{NP}}}[p_0(\pi - rc) - e^{-\lambda\tau^{\mathrm{NP}}}(1-p_0)rc] = e^{-r\tau^{\mathrm{NP}}}\frac{p_0\lambda(\pi - rc)}{\lambda + r}.$$

The problem is meaningful only if $\tau^{\mathrm{NP}} > 0$, which we maintain throughout the analysis and is ensured by Assumption 1 provided below.

Now suppose that firm 1 enters in the preemption phase. If firm 1 enters at some time τ, firm 2's belief jumps up to ϕ_τ, but firm 2 still needs to wait until the post-entry belief $\tilde{p}_t$ reaches p^*, which allows firm 1 to monopolize the market for a duration δ_τ of time. Therefore, the expected payoff of entering at $\tau \in [0, \frac{\tau^{\mathrm{NP}}}{2}]$ is given by

$$\hat{\Pi}(\tau) = e^{-r\tau}[p_0(\pi + M_\tau - rc) - e^{-\lambda\tau}(1-p_0)rc].$$

Let τ^{P} denote the optimal timing of entry in the preemption phase, which can be found by maximizing this function over $\tau \in [0, \frac{\tau^{\mathrm{NP}}}{2}]$. In the proof of Proposition 2, we show that this maximization problem is well defined and always admits a unique τ^{P} in $[0, \frac{\tau^{\mathrm{NP}}}{2}]$. Let $\Pi^{\mathrm{P}} := \max_{t\in[0,\frac{\tau^{\mathrm{NP}}}{2}]} \hat{\Pi}(t)$ denote the expected payoff under the restriction that firm 2 must be the second mover, which can be written as

$$\Pi^{\mathrm{P}} = \hat{\Pi}(\tau^{\mathrm{P}}) = e^{-r\tau^{\mathrm{P}}}[p_0(\pi + M_{\tau^{\mathrm{P}}} - rc) - e^{-\lambda\tau^{\mathrm{P}}}(1-p_0)rc].$$

As the monopoly premium m becomes larger, it becomes more costly to wait and collect more information. As a consequence, the optimal timing of pioneering entry moves forward with an increase in the monopoly premium.

Finally, we have thus far assumed, somewhat loosely, that learning is essential and $\underline{\tau} > 0$. To ensure this, the expected payoff of entering immediately at time 0 must be smaller than Π^{NP}, i.e.,

$$\Pi^{\mathrm{NP}} > \hat{\Pi}(0) \iff e^{-r\tau^{\mathrm{NP}}}\frac{p_0\lambda(\pi - rc)}{\lambda + r} > p_0[\pi + (1 - e^{-r\tau^{\mathrm{NP}}})m] - rc, \tag{5}$$

where

$$e^{-r\tau^{\mathrm{NP}}} = \left[\frac{p_0(\pi - rc)}{(1 - p_0)(\lambda + r)c}\right]^{\frac{r}{\lambda}}.$$

Assumption 1. $\Pi^{\mathrm{NP}} > \hat{\Pi}(0)$.

Although the restriction imposed by this assumption is somewhat complicated, it is important to note that as $p_0 \to 0$, the left-hand side of (5) converges to zero while the right-hand side dips below zero, so that (5) holds for any given (λ, π, c, m, r) such that $\pi > rc$ if p_0 is sufficiently small. Note also that for all $p_0 \geq p^*$, $\tau^{\mathrm{NP}} = 0$ and (5) is reduced to

$$\frac{p_0\lambda(\pi - rc)}{\lambda + r} > p_0\pi - rc.$$

Since $\Pi^{\mathrm{NP}} = \hat{\Pi}(0)$ when $p_0 = p^*$ by construction, this condition cannot be satisfied for any $p_0 \geq p^*$, meaning that Assumption 1 implies $p^* > p_0$.

3.4. Necessary and sufficient condition for pioneering entry

Under the restriction that firm 2 must be the second mover, it is optimal for firm 1 to enter once and for all at τ^{P} if $\Pi^{\mathrm{P}} > \Pi^{\mathrm{NP}}$. Clearly, though, this does not constitute an equilibrium when firm 2 is also an active player who can enter the market at any point in time. As Proposition 1 indicates, the firms must adopt mixed strategies when they compete to be the first mover. Even then, these payoffs are still useful as they provide a necessary and sufficient condition for a market pioneer to emerge in equilibrium.

Proposition 2. *(i) There always exists a symmetric equilibrium that satisfies the Intuitive Criterion. (ii) There exists a pioneer equilibrium if and only if* $\Pi^{\mathrm{P}} > \Pi^{\mathrm{NP}}$.

Proof. See Appendix A.

The main statement of Proposition 2 is part (ii) which pins down a necessary and sufficient condition for pioneering entry. To see the intuition behind this result, observe that in a no-pioneer equilibrium, the firms wait until time τ^{NP} and then gradually enter past that point; hence, the expected payoff in this equilibrium is Π^{NP} as in the constrained problem. Given this, the sufficiency is obvious, because if $\Pi^{\mathrm{P}} > \Pi^{\mathrm{NP}}$, a firm must have an incentive to deviate and enter with some positive probability in the preemption phase. On the other hand, the necessity comes from the fact that the entry competition can only lower the benefit of pioneering entry while it raises the benefit of waiting due to the signaling effect (see the next section for more detail). As such, if there is no incentive to enter in the preemption phase in the constrained problem, then there is certainly no incentive to do so in the original problem.

Part (ii) of Proposition 2 implies that as $\Pi^{\mathrm{P}} - \Pi^{\mathrm{NP}}$ becomes larger, the first-mover advantage becomes more salient, rendering pioneering entry more likely. The following proposition clarifies under what conditions a market pioneer is more likely to emerge, which offers crucial efficiency and policy implications.

Proposition 3. *There exist $\hat{m}$ and $\hat{p} \in (0, p^*)$ such that there is a pioneer equilibrium if and only if $m > \hat{m}$ or $p^* > p_0 > \hat{p}$.*

Proof. See Appendix A.

The proposition suggests that two factors are particularly crucial as determinants of entry dynamics. First, it is clear that the monopoly premium m, which measures the extent of market competition, has a decisive impact on the timing of entry. Since an increase in m only raises the value of preemption, it generally favors pioneering entry. See figures 1 and 2 which depict $\hat{\Pi}(\cdot)$ for different values of π and m (with $\pi + m$ fixed). Second, the prior belief p_0, which measures the extent of uncertainty faced by the firms, also plays an important role in shaping entry dynamics. Although the effect of p_0 is less clear, as an increase in p_0 can raise both Π^{P} and Π^{NP}, a high p_0 tends to favor pioneering entry. To see why, observe that the expected payoff is larger for the first mover; a firm can thus enter with more confidence earlier while revealing less information. From these findings, we can conclude that pioneering entry is more likely when: (i) market competition is intense; and/or (ii) there is less uncertainty regarding the eventual likelihood of the market being good.

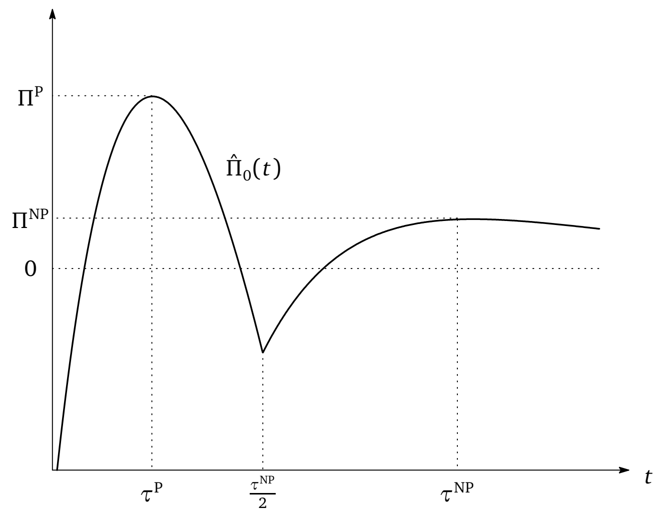

Figure 1. *The emergence of a market pioneer ($\lambda = 0.1, r = 0.1, p_0 = 0.3, c = 3, \pi = 0.5, m = 0.5$)*

4. Discussion

4.1. Equilibrium payoff bounds

Let Π^* denote the expected equilibrium payoff for each firm. In a no-pioneer equilibrium, the earliest possible entry occurs at time τ^{NP}, and $\Pi^* = \Pi^{\mathrm{NP}}$ as we have seen in section 3.3. In a pioneer equilibrium, on the other hand, the expected equilibrium payoff is given by

$$\Pi^* = e^{-r\underline{\tau}}[p_0(\pi + M_{\underline{\tau}} - rc) - e^{-\lambda\underline{\tau}}(1 - p_0)rc],$$

which is in general different from Π^{P}. More precisely, we have $\Pi^* < \Pi^{\mathrm{P}}$ (if $\Pi^{\mathrm{P}} > \Pi^{\mathrm{NP}}$) because the entry competition is self-defeating and shifts the timing of entry forward, inducing the firms to start entering before time τ^{P}. The question is then whether this competition drives the value of the first-mover advantage down to zero, i.e., $\Pi^* \to \Pi^{\mathrm{NP}}$. As it turns out, this is not the case because the second mover can benefit from the information revealed by the first mover's entry. The following result characterizes the equilibrium payoff bounds when $\Pi^{\mathrm{P}} > \Pi^{\mathrm{NP}}$.

Proposition 4. *Suppose that $\Pi^{\mathrm{P}} > \Pi^{\mathrm{NP}}$ so that pioneering entry occurs with positive probability. Then, each firm's expected payoff is between Π^{NP} and Π^{P}, i.e.,*

$$\Pi^* = e^{-r\underline{\tau}}[p_0(\pi + M_{\underline{\tau}} - rc) - e^{-\lambda\underline{\tau}}(1 - p_0)rc] \in (\Pi^{\mathrm{NP}}, \Pi^{\mathrm{P}}).$$

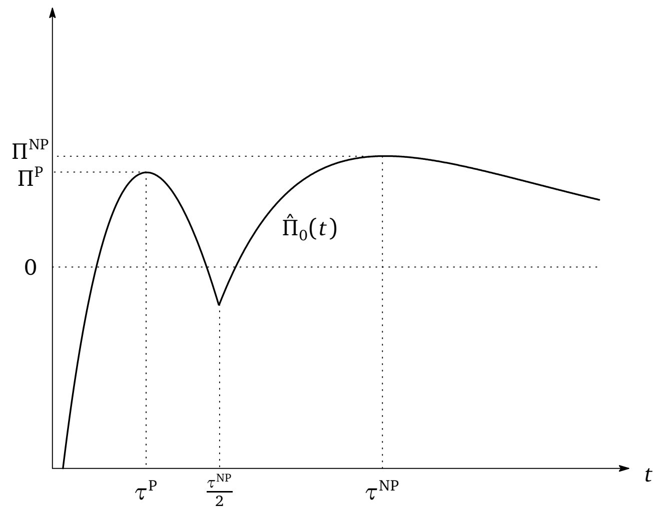

Figure 2. *No market pioneer* ($\lambda = 0.1, r = 0.1, p_0 = 0.3, c = 3, \pi = 0.6, m = 0.4$)

Proof. See Appendix A.

To understand this result, especially why the expected payoff is not driven down to Π^{NP}, it is important to understand the roles of the two types of externality that are present in this setting. On one hand, there is a negative payoff externality via market competition which is captured by m. The payoff externality is clearly the source of the entry competition. This is most clearly seen by supposing $m = 0$, in which case

$$\hat{\Pi}(t) = e^{-rt}[p_0(\pi - rc) - e^{-\lambda t}(1 - p_0)c],$$

for all $t \in [0, \infty)$. Since the problem is equivalent to the case where $\delta_t = 0$ for all $t \in [0, \infty)$, there is no preemption phase when $m = 0$.

In contrast, as m increases, the first-mover advantage becomes more salient, giving each firm an incentive to become a market pioneer. This entry competition forces the firms to enter earlier than the optimal timing τ^{P}, which necessarily lowers the expected payoff of becoming a market pioneer. In equilibrium, this expected payoff must be driven down to the expected payoff of becoming a follower which is strictly larger than Π^{NP} because of the information externality: with the pioneer's entry providing additional information, the follower can enter earlier than τ^{NP} and hence on average achieve a higher payoff. The presence of pioneering entry thus accelerates the learning process at the industry level.

4.2. Cooperative problem

To derive efficiency properties of the model, we now consider a social planner who attempts to maximize the joint payoff of the firms, which corresponds to the case where the firms determine when to enter cooperatively to maximize their joint profit. The solution to this problem gives the efficient allocation of our model as it eliminates any inefficiencies arising from the two forms of externality—the only sources of distortion in our model. Since the efficient allocation under complete information is rather trivial in this setting,[20] here we focus on the situation where the social planner is subject to the same informational constraints as the firms. Specifically, we consider an environment in which the social planner specifies the entry times (τ_1, τ_2) such that firm i enters the market at time τ_i if it is uninformed at the time.

Let $W(\tau_1, \tau_2)$ denote the joint payoff for a given pair (τ_1, τ_2), which can be written as

$$\begin{aligned} W(\tau_1, \tau_2) = e^{-r\tau_1}[p_0(\pi + m - rc) - e^{-\lambda\tau_1}(1-p_0)rc] \\ + e^{-r\tau_2}[p_0(\pi - m - rc) - e^{-\lambda(\tau_1+\tau_2)}(1-p_0)rc]. \end{aligned}$$

Without loss of generality, we assume $\tau_1 \le \tau_2$.[21] The social planner's problem is defined as

$$\max_{(\tau_1,\tau_2)} W(\tau_1, \tau_2),$$

subject to $\tau_1 \le \tau_2$.

Two remarks are in order regarding the two types of externality in this setting. First, firm 2's belief at τ_2 is $\frac{p_0}{p_0+(1-p_0)e^{-\lambda(\tau_1+\tau_2)}}$, rather than $\frac{p_0}{p_0+(1-p_0)e^{-\lambda\tau_2}}$, because of the positive information externality of the first entry. Second, the second entry contributes only $\pi - m$ to the joint profit (while its private gain is π) due to the negative payoff externality, which corresponds to what is often referred to as the "business-stealing effect" in standard static oligopoly models.

It is also important to note that because of the payoff externality, there may arise a case where it is socially optimal to have only one firm in the market. This is the case if

$$rc \ge \pi - m,$$

[20]The problem is trivial when the market condition is known to the social planner: if the market is good, the firms should enter immediately at time 0; if not, they should never enter.

[21]In this setup, $\tau_1 = \tau_2$ indicates "almost simultaneous entries" where firm 1 enters first and then firm 2 follows with no delay.

in which case the social planner would allow only one firm to enter the market ($t_2 = \infty$). Since this case is relatively straightforward, we restrict our attention to the case where it is socially optimal to have two firms whenever the market condition is good.

Assumption 2. $\pi - m > rc$.

Define $(\tau_1^{**}, \tau_2^{**})$ as the efficient timing of entry, and let $T^{**} = \tau_1^{**} + \tau_2^{**}$. The following proposition yields some important implications regarding the timing of entry which we will discuss in depth below.

Proposition 5. *(i) $T^{**} > \tau^{\mathrm{NP}}$. (ii) $\tau_1^{**} > \tau^{\mathrm{P}}$ if $\tau^{\mathrm{P}} > 0$. (iii) $\tau^{\mathrm{NP}} > \tau_1^{**}$ if p_0 is sufficiently small.*

Proof. See Appendix A.

Parts (i) and (ii) of Proposition 5 concern the case with a late entrant and suggest that the firms enter the market too early compared to the social optimum.

- Part (i) states that the equilibrium timing of second entry is earlier than the efficient timing.[22] This is due to the negative payoff externality: when the second entry occurs, the firm's average net payoff when the market is good is $\pi - rc$ while its contribution to the joint profit is only $\pi - m - rc$ due to the business-stealing effect.[23] In the efficient allocation, therefore, the entry threshold is higher and the firm should wait longer to collect more information.

- Part (ii) states that the equilibrium timing of first entry is also earlier than the efficient timing. This is mainly due to the positive information externality: if the first entry occurs later, it reveals more information and benefits the rival firm. The first entrant not only ignores this external benefit of information sharing ($\tau_1^{**} > \tau^{\mathrm{P}}$), but in equilibrium enters even earlier so as to reveal less information to the rival firm and delay the subsequent entry ($\tau^{\mathrm{P}} > \underline{\tau}$).

[22] Suppose that the first entry occurs at time τ_1^*. Then, in the unique continuation equilibrium, the remaining firm waits for a duration $\tau^{\mathrm{NP}} - 2\tau_1^*$ and hence enters at time $\tau^{\mathrm{NP}} - \tau_1^*$. In contrast, under the social optimum, the second entry occurs at time $T^{**} - \tau_1^* = \tau_2^*$.

[23] The celebrated excess entry theorem (Mankiw and Whinston, 1986; Suzumura and Kiyono, 1987; Lahiri and Ono, 1988) generally builds on this effect and demonstrates that the number of firms in a market can be too many in static oligopoly models. Our analysis complements this literature by extending this argument to a dynamic context, showing that market entry is too early with this same effect.

In contrast, part (iii) of the proposition concerns the case with an early follower. This case emerges when p_0 is relatively small, in which case the firms tend to enter too late. The intuition behind this is relatively clear. Once the game reaches the waiting phase, the clear winner is the one that becomes the follower as it can minimize the risk of wrong entry while losing almost no monopoly rent. This incentive to wait for the rival's action is often excessively strong, preventing the firms from entering the market at an opportune time.

Note that the timing of first entry is often not observable to the econometrician who typically lacks exact knowledge of calender time (namely, of time 0). Even in this case, we can make inference about whether a pioneering entry occurs or not from the temporal distribution of entry times. Note that we have a late entrant only when there is a pioneering entry. The fact that entries are spaced apart in time hence suggests that the first entry indeed occurs in the preemption phase. Combined with the earlier discussion in section 3.2, Proposition 5 implies a paradoxical fact which is worth emphasizing: the firms enter too early when there is a late entrant and too late when there is an early follower.

4.3. Welfare with consumers

The efficiency analysis in the previous subsection illuminates how the two forms of externality affect and distort entry dynamics. To derive useful policy recommendations in some contexts, however, it is also important to consider the welfare of consumers as well. In this subsection, we extend the efficiency analysis to incorporate consumer surplus and discuss how this additional factor affects the efficient timing of entry. Note that the importance of consumer surplus varies depending on the underlying context: of the three possible applications we raised in the introduction, consumer surplus is more likely to be important for new product markets but less so for foreign direct investment.[24]

To deliver clearer predictions, we assume throughout this subsection that there is no consumer surplus when the market is bad. Let CS_n be the consumer surplus when the number of firms in the market is $n = 1, 2$, where $CS_1 = \nu CS$ and $CS_2 = CS \geq 0$. We define the social welfare as the sum of the firms' profits and the consumer surplus and denote it by $W^*(\tau_1, \tau_2)$ for a given pair of (τ_1, τ_2). Under this specification, the social planner

[24] In the context of foreign direct investment, domestic consumers are not affected, and consumer surplus is typically not a factor of consideration for policymakers. The case of technology adoption falls somewhere in-between, as the impact on consumers depends crucially on the market structure and the appropriability of the gains from new technology adoption.

chooses (τ_1, τ_2) to maximize

$$W^*(\tau_1, \tau_2) = e^{-r\tau_1}[p_0(\pi + m + \nu CS - rc) - e^{-\lambda\tau_1}(1-p_0)rc] \\ + e^{-r\tau_2}\{p_0[\pi - m + (1-\nu)CS - rc] - e^{-\lambda(\tau_1+\tau_2)}(1-p_0)rc)\}.$$

As can be seen from this, the maximization problem with consumer surplus is not qualitatively different from the joint-profit maximization problem (without consumer surplus) and can be analyzed in essentially the same way. Note that the conflict between the firms and consumers arises from the fact that the firms must bear the cost of entry and thus care about the market condition.[25]

Clearly, all the statements of Proposition 5 hold as they are if CS is sufficiently small. Here, we thus focus on the case where CS is relatively large; the technical details are relegated to Appendix C. In this setup, the consumer surplus is just another social benefit that is not internalized by the firms. As a consequence, both τ_1^{**} and T^{**} move forward as CS increases. With respect to Proposition 5, we can make the following observations.

- Part (i) holds if and only if $m > (1-\nu)CS$, i.e., the monopoly premium is larger than the marginal social benefit of the second entry.
- Part (ii) holds for any $CS > 0$ if ν is sufficiently small. When monopoly is bad for consumers, the social benefit of the first entry is limited, and the interests of the firms and consumers are better aligned as a consequence.
- Part (iii) always holds because τ_1^{**} is decreasing in CS while τ^{NP} is independent of it.

From these observations, we argue that part (i) may not hold in environments where the welfare of consumers plays a major role (such as in new product markets). In contrast, part (ii) holds in many competitive environments where ν is expected to be small.

5. Extensions

5.1. Benefits of consumer inertia

As we have seen, efficiency properties of the model depend crucially on the monopoly premium m which measures the extent of the payoff externality. More precisely, the possibility of business stealing generates two important forces which generally induce the firms

[25] When there is uncertainty about the value of the product, and the value is small (or even negative) when the market is bad, the consumer surplus may take a negative value. In this case, consumers also care about the market condition, and the interests are better aligned as a consequence. If the consumer surplus is always positive as we assume here, consumers have no interests in the market condition and always prefer the firms to enter immediately at time 0.

to enter too early: first, it gives the first mover an additional strategic incentive to enter early so as to reveal less information to the rival firm; second, it also induces the second mover to enter early as it fails to internalize the loss to the first mover.

In reality, however, first movers can benefit from establishing their presence early on due to consumer inertia, which could arise from various factors such as brand loyalty, habit formation, switching costs, patent protection, and slow diffusion of product information. It is hence more realistic to assume that the monopoly premium m decays only slowly over time. To capture this aspect in a simple way, we now suppose that the payoff to the first mover is $\pi + \eta m$ and to the second mover is $\pi - \eta m$ instead of both receiving π if entry times are bounded away from each other.[26] Note that in this specification, $\eta \in [0, 1]$ measures the extent of consumer inertia.[27]

Since detailed analysis of this extended case is out of the scope of this paper, we briefly describe important forces that are generated by consumer inertia; throughout this subsection, we restrict our attention to a pioneering equilibrium. To illustrate how consumer inertia affects the equilibrium allocation, we consider the same constrained problem as in section 3.3. Since the expected flow payoff when the market is good is now $\pi - \eta m$, the second mover waits until the belief reaches $p^{\mathrm{F}} := \frac{(\lambda+r)c}{\pi-\eta m+\lambda c}$,[28] which is larger than p^* for any $\eta > 0$. Define τ^{F} such that

$$p_0(\pi - \eta m - rc) = (1 - p_0)(\lambda + r)ce^{-\lambda\tau^{\mathrm{F}}}.$$

Given this, we redefine

$$\hat{\Pi}(\tau; \eta) = e^{-r\tau}\{p_0[\pi + m - (1-\eta)me^{-r(\tau^{\mathrm{F}}-2\tau)} - rc] - e^{-\lambda\tau}(1-p_0)rc\},$$

for $\tau \in [0, \frac{\tau^{\mathrm{F}}}{2}]$ and $\Pi^{\mathrm{P}}(\eta) := \max_\tau \hat{\Pi}(\tau; \eta)$ as a function of η. Clearly, $\Pi^{\mathrm{P}}(\cdot)$ is increasing in η because: (i) the duopoly payoff increases from π to $\pi + \eta m$; and (ii) it reduces the payoff for the follower and hence delays its arrival. Observe, on the other hand, that Π^{NP}

[26] More precisely, letting d be the time lag between the two entries, the payoff to the first mover is $\pi + \eta(d)$ and to the second mover is $\pi - \eta(d)$ where $\eta(d) = \eta$ if $d > \overline{d}$ for some $\overline{d} > 0$ and $\eta(d) = 0$ otherwise. This assumption reflects the idea that persistent first-mover advantage arises only when the first mover establishes its presence as a pioneer in the market. Here, we consider an extreme case where $\overline{d} \to 0$.

[27] Since only the discounted sum of payoffs matters after both firms enter, η constitutes a sufficient statistic for our purpose. For instance, suppose that the payoff for the first mover when the second mover enters at some τ is $\pi + me^{-\xi(t-\tau)}$ for $t > \tau$, in which case $\eta = \frac{r}{\xi+r}$. Our baseline model then corresponds to the limit case of this specification where $\xi \to \infty$.

[28] For this to be optimal, the expected payoff of waiting until the belief reaches p^{F} must be higher than the payoff of immediately following, which holds if η is sufficiently small. In this subsection, we assume that this condition generally holds.

is independent of η because we assume no inertia when entries occur in the waiting phase with no time lag. Now suppose $\Pi^{P}(0) = \Pi^{NP}$, so that there is (almost) no pioneering entry and the equilibrium payoff is Π^{NP} when $\eta = 0$. If there is an increase in η, $\Pi^{P}(\eta)$ also increases from $\Pi^{P}(0)$. Then, the joint profit necessarily increases because the equilibrium profit must be in $(\Pi^{NP}, \Pi^{P}(\eta))$ by Proposition 4.

This argument suggests that there are possible efficiency gains from consumer inertia. As discussed, although pioneering entry is socially beneficial, the firms fail to internalize this benefit. Biasing the allocation of surplus in favor of the pioneer alleviates this inefficiency by making pioneering entry more attractive. From a broader perspective, our argument offers crucial policy implications by highlighting how the extent of market competition shapes entry dynamics. Consider a regulatory authority who has policy tools to manipulate η in some ways, e.g., any policies to affect consumer switching costs or entry barriers. If the authority is concerned only about *ex post* static gains, it may be tempted to reduce consumer inertia (lower η) as much as possible, in order to intensify market competition among incumbent firms. Although these *ex post* gains, which are assumed away in our analysis, are certainly important, the extent of market competition can have huge impacts on the way firms enter a new market or adopt a new technology.

It is also worth noting that most theoretical models argue that the presence of switching costs, which is one primary reason why the first-mover advantage may exhibit persistence, is welfare-reducing (Klemperer, 2008) in contrast to our finding here. Our analysis sheds light on a different aspect of switching costs and provides a new mechanism through which the presence of switching costs improves efficiency by inducing the firms to enter earlier, thereby facilitating information sharing between the firms and expediting collective experimentation.

5.2. A model with market exits

Our baseline model assumes that a firm is indifferent between staying in the market and exiting when the market turns out to be bad; as such, a firm would have no incentive to exit the market even if it later finds out that the market is actually bad. This assumption is clearly at odds with reality where we frequently observe market exits. Here, we briefly discuss how this aspect can be incorporated into our framework and argue that our results would hold in a qualitative sense even with the possibility of market exits.

Now suppose that the expected payoff to a firm is slightly negative when the market is bad (while the outside payoff of no entry is still 0). Suppose further that a firm can privately observe the market condition at some rate $\omega > 0$ while it operates in the market.

Then, under this set of assumptions, an entrant would choose to exit the market as soon as it finds out that the market is bad. If market exit is publicly observable, as it is most likely in reality, a firm's exit serves as an additional signal that the market is bad for sure, and no firm will henceforth enter.

This specification is equivalent to assuming that a firm observes a bad signal at rate λ^0 in the pre-entry stage and at $\lambda^1 := \lambda^0 + \omega$ in the post-entry stage. This extended model can thus be analyzed in essentially the same way, only with slight technical complications. The only significant difference is that in our baseline model, the threshold belief in the pre-entry stage—the belief at which the firms start entering at some rate—coincides with the threshold belief in the post-entry stage. More precisely, in the baseline model, we define $p^* := \frac{(\lambda+r)c}{\pi+\lambda c}$, which is the belief at which a firm is indifferent between waiting and entering when it can only earn a duopoly profit π even if the market is good. This is therefore the belief at which the remaining firm enters in the post-entry stage. Also, in situations where the rival firm is expected to follow immediately, this is the belief at which the firms are indifferent between entering and waiting.

These threshold beliefs, however, differ in the extended setup. Because the rate of learning is λ^1 in the post-entry stage, the threshold belief is now $p^{*1} := \frac{(\lambda^1+r)c}{\pi+\lambda^1 c}$ if the rival firm has already entered. In contrast, in the pre-entry stage, the threshold is given by $p^{*0} := \frac{(\lambda^0+r)c}{\pi+\lambda^0 c}$. Since $\lambda^1 > \lambda^0$, we have $p^{*1} > p^{*0}$, i.e., the remaining firm has more incentive to wait in the post-entry stage. Now suppose that p_t reaches p^{*0} at some time τ^{*0}. If $\phi_{\tau^{*0}} \geq p^{*1}$, which is automatically satisfied if $\lambda^1 = \lambda^0$, nothing really changes, and all of our results apply as they are. If $\phi_{\tau^{*0}} < p^{*1}$, on the other hand, the rival firm would wait for some time even if a firm enters at time τ^{*0}. In a no-pioneer equilibrium, therefore, the firms have an incentive to wait slightly more: there would be a threshold belief which is bounded between p^{*0} and p^{*1}.

This latter case occurs when ω is significantly large. We argue, however, that this is not a particularly interesting case to consider. To see this, suppose that ω is infinitely large, so that the entrant can observe the market condition almost immediately. In this case, there is no benefit of preemption, and the game essentially becomes a war of attrition where each firm just waits for the other firm to enter (no pioneering entry). Aside from this, the assumption that ω is very large is not very realistic as it often takes a substantial amount of time to gauge the true profitability of the market in question.

6. Conclusion

In the existing literature, the roles of pre-entry learning and post-entry market competition have been investigated extensively but almost independently. To provide a more comprehensive description of the tradeoff faced by potential market entrants, this paper constructs a dynamic model of market entry which features these two elements in a unified framework. We fully characterize symmetric equilibrium of this game and identify a necessary and sufficient condition for the first-mover advantage to dominate. This condition is more likely to hold if the market is more competitive (so that the monopoly premium is large) or if the prior belief that the market is good is relatively high. We also argue that consumer inertia is generally efficiency-enhancing, which highlights an elusive link between static market competition and dynamic entry competition and suggests some policy implications.

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Appendix A: Proofs

Proof of Lemma 1. It is straightforward to derive the optimal strategy for the remaining firm who, with no strategic concerns, simply enters if the current belief $\tilde{p}_t$ is high enough. Let $v(T;\tilde{p}_t)$ be the expected (average) profit of waiting for a duration T of time when the post-entry belief is $\tilde{p}_t$.[29] We then obtain

$$v(T;\tilde{p}_t) = \tilde{p}_t(\pi - rc)e^{-rT} - (1-\tilde{p}_t)rce^{-(r+\lambda)T}.$$

The first-order condition is obtained as

$$-\tilde{p}_t(\pi - rc) + (r+\lambda)(1-\tilde{p}_t)ce^{-\lambda T} = 0.$$

Observe that $v(\cdot;\tilde{p}_t)$ is strictly concave for any $\tilde{p}_t$ and admits a unique optimum. Let p^* be the critical belief such that $v'(0;p^*) = 0$. Simple computation then yields

$$p^* = \frac{(r+\lambda)c}{\pi + \lambda c}.$$

meaning that it is optimal to enter now if $\tilde{p}_t \geq p^*$. The remaining firm thus enters as soon as the current belief reaches p^*. ■

Proof of Proposition 1. In what follows, we let β_t be the probability that the firms enter in $[t, t+\mathrm{d}t)$. Given this, we also let $\gamma_t := \beta_t(p_t + q_t)$ be the probability that the rival firm enters in $[t, t+\mathrm{d}t)$ and let $G_t := \int_0^t \gamma_{t'}\mathrm{d}t'$ be the cumulative distribution. Observe that G_t may or may not be continuous in t, although we will show below that G_t is always continuous in equilibrium. For expositional purposes, we say that the firms enter with strictly positive probability (with an infinite density) at time τ if G_t is discontinuous at $t = \tau$; otherwise, we say that the firms enter smoothly.

In Lemma 1, we have characterized the optimal strategy in the post-entry stage. Let $\tilde{V}(p_t, t)$ be the value function in the post-entry stage when the first entry occurs at (p_t, t). This can be written as

$$\tilde{V}(p_t, t) = e^{-r\delta_t}\phi_t(\pi - rc) - e^{-(\lambda+r)\delta_t}(1-\phi_t)rc, \tag{6}$$

where δ_t is as given by (3). Note that the value function is defined over (p_t, t) because q_t can be uniquely pinned down from (p_t, t).

[29]Throughout the analysis, we often say "wait for a duration T" (or until time $t+T$ where t denotes the current time) to refer to the following entry strategy: (i) the firm does not enter until $t+T$; (ii) the firm enters if it is still uniformed at time $t+T$.

Define $V(p_t, t)$ as the value function in the pre-entry stage; again, the value function is defined over (p_t, t) for the same reason as above. The value function can be written as

$$V(p_t, t) = \max\{p_t[\pi + (1-\beta_t)M_t] - rc, E[U(p_t, t) \mid \Omega_t]\},$$

where $\Omega_t := (p_t, t, G_t)$ and

$$E[U(p_t, t) \mid \Omega_t] := \gamma_t \tilde{V}(p_t, t) + e^{-r\mathrm{d}t}(1-\gamma_t)E[V(p_{t+\mathrm{d}t}, t+\mathrm{d}t) \mid \Omega_t].$$

The first term of the right-hand side is the expected payoff when the rival firm enters, in which case the game immediately enters the post-entry stage. The second term is the expected payoff when it does not. As such, $E[U(p_t, t) \mid \Omega_t]$ denotes the value of waiting. For $\beta_t > 0$, the expected payoff of entering now must be weakly larger than the value of waiting:

$$\begin{aligned} p_t[\pi + (1-\beta_t)M_t] - rc &\geq E[U(p_t, t) \mid \Omega_t] \\ &= \gamma_t \tilde{V}(p_t, t) + e^{-r\mathrm{d}t}(1-\gamma_t)E[V(p_{t+\mathrm{d}t}, t+\mathrm{d}t) \mid \Omega_t], \end{aligned} \tag{7}$$

which must hold with equality if the firms adopt a mixed strategy.

Waiting phase: We have $\delta_t = M_t = 0$ in this phase, where an uninformed firm follows immediately once the rival firm enters. Now suppose that the rival firm does not enter, and further that the firm adopts a strategy of entering at the next instant if it is still uninformed. Since it is always possible to adopt this strategy, this gives us a lower bound for the continuation payoff in this contingency:

$$E[V(p_{t+\mathrm{d}t}, t+\mathrm{d}t) \mid \Omega_t] \geq B_t[p_{t+\mathrm{d}t}(\pi - rc) + (1-p_{t+\mathrm{d}t})rc],$$

where

$$B_t := \frac{(p_t + q_t e^{-\lambda \mathrm{d}t})(1-\beta_t) + (1-p_t-q_t)e^{-\lambda \mathrm{d}t}}{1-\gamma_t}$$

is the probability that an uninformed firm observes no signal in $[t, t+\mathrm{d}t)$ conditional on the rival firm not entering. Combined with (7), a necessary condition for $\beta_t > 0$ is obtained as

$$\begin{aligned} p_t\pi - rc \geq\ & \beta_t[p_t(\pi - rc) - q_t rc] \\ & + e^{-r\mathrm{d}t}[p_t(1-\beta_t)(\pi - rc) - e^{-\lambda \mathrm{d}t}(1-p_t-\beta_t q_t)rc]. \end{aligned} \tag{8}$$

Given this, we can make the following statement.

Lemma 2. *For $\phi_t \geq p^* > p_t$ (i.e., when the game is in the waiting phase), there exists a unique symmetric continuation equilibrium in which:*

1. *Neither firm enters until the belief p_t reaches the threshold p^*;*
2. *When p_t reaches p^*, the two firms start entering at a rate to keep $p_t = p^*$;*
3. *Once a firm enters, the remaining firm immediately follows at the next instant.*

Proof. We first claim that it is never optimal to adopt a pure strategy in the waiting phase. Suppose on the contrary that the firms adopt a pure strategy of entering at some time τ. Then, if a firm does not enter at time τ, it means that the firm is informed and the state is bad for sure. Therefore, $p_{\tau+\mathrm{d}t} = 0$ and $E[V(p_{\tau+\mathrm{d}t}, \tau + \mathrm{d}t) \mid \Omega_t] = 0$. Given this, (7) can be written as

$$p_\tau \pi - rc \geq p_\tau(\pi - rc) - q_\tau rc,$$

which is reduced to $p_\tau + q_\tau \geq 1$. This is a contradiction because this condition holds only when $p_\tau = 1$ but p_t cannot reach 1 for any finite t.

We now consider an interval of t such that the firms enter smoothly and $\beta_t = 1 - e^{-s_t \mathrm{d}t}$. In this case, (8) must hold with equality. Plugging $\beta_t = 1 - e^{-s_t \mathrm{d}t}$ into (8), we obtain

$$\begin{aligned} p_t \pi - rc = {} & (1 - e^{-s_t \mathrm{d}t})[p_t(\pi - rc) - q_t rc] \\ & + e^{-r\mathrm{d}t}\{p_t e^{-s_t \mathrm{d}t}(\pi - rc) - e^{-\lambda \mathrm{d}t}[1 - p_t - (1 - e^{-s_t \mathrm{d}t})q_t] rc\}. \end{aligned} \tag{9}$$

In the limit $\mathrm{d}t \to 0$, this condition is reduced to

$$p_t(\pi - rc) = (\lambda + r)(1 - p_t)c \iff p_t = p^*.$$

If the firms enter with strictly positive probability at some τ, we directly take the limit of (8) and obtain $p_\tau \geq p^*$.

This argument shows that the firms enter in the waiting phase only if $p_t \geq p^*$. We argue that there is no equilibrium in which $p_t > p^*$ for any t. To this end, we first note that it is never optimal to wait if $p_t \geq p^*$. Suppose on the contrary that $p_\tau \geq p^*$ and there is an interval $[\tau, \tau')$ such that $\beta_t = 0$ for $t \in [\tau, \tau')$ but $\beta_{\tau'} > 0$. Note that $p_{\tau'} > p_\tau \geq p^*$ since the belief must be increasing in this interval. Then, for t sufficient close to τ', (9) holds with strict inequality, but this is a contradiction because it must be weakly optimal to enter at time τ'. This means that if there is such an interval, it must be that $\beta_t = 0$ for all $t \geq \tau$. This is, however, a contradiction again because the expected payoff of this strategy is zero which is lower than the expected payoff of entering at τ. This suggests that $\beta_t > 0$ if $p_t \geq p^*$.

This observation implies that if $p_\tau > p^*$, then $p_{\tau+\mathrm{d}t} < p^*$, for otherwise (9) would hold with strict inequality, and the firms would have to adopt a pure strategy of entering at time τ for sure. This means that the firms must enter with strictly positive probability at time τ, so that the belief jumps down below p^*. If this is the case, however, the firms must also enter with strictly positive probability slightly before time τ because p_t can only go up continuously and $p_{\tau-\varepsilon} > p^*$ for a sufficiently small ε. As this argument holds for any $p_t > p^*$, p_t cannot go above p^* on the equilibrium path.

Given this, the only remaining possibility is to keep p_t at p^* by choosing s_t to satisfy $(1-p^*)\lambda = (1-p^*-q_t)s_t$ as long as the game is in the pre-entry stage. It is straightforward to verify that this indeed constitutes an equilibrium. ■

Preemption phase: In the preemption phase, we have $\delta_t > 0$. We first argue that the firms always enter smoothly in the preemption phase if they ever choose to do so. Suppose on the contrary that the firms enter at some τ with probability β_τ. The expected payoff of entering at time τ is then given by $p_\tau[\pi + (1-\beta_\tau)M_\tau] - rc$. If a firm deviates and enter slightly earlier at $\tau - \varepsilon$, on the other hand, the expected payoff is $p_{\tau-\varepsilon}(\pi + M_\tau) - rc$. For any $\beta_\tau > 0$, we can find a $\tilde{p} < p_\tau$ such that

$$p_\tau[\pi + (1-\beta_\tau)M_\tau] - rc = \tilde{p}(\pi + M_\tau) - rc.$$

Since ε can be made arbitrarily small, we can always find a $p_{\tau-\varepsilon}$ such that $\tilde{p} < p_{\tau-\varepsilon} < p_\tau$, which is a contradiction.

Since ϕ_t is strictly increasing, it will reach p^* sooner or later, and the firms choose not to enter until p_t reaches p^* as shown in Lemma 2. This means that there must be an interval $(\overline{\tau}, \tau^*)$ with $s_t = 0$, followed by (τ^*, ∞) with $s_t \in (0, \infty)$. Note that if $\phi_t = p^* > p_t$, it is strictly optimal for a firm to wait until p_t reaches p^*. Therefore, shortly before ϕ_t reaches p^*, it is still optimal to wait, meaning that $\phi_{\overline{\tau}} < p^*$.

Before time $\overline{\tau}$, the firms may enter at some positive rate. We now argue that the set $\{t : s_t \in (0, \infty), \phi_t < p^*\}$ must be connected. Suppose on the contrary that there exists an interval $(\underline{a}, \overline{a})$ such that $s_t = 0$ for $t \in (\underline{a}, \overline{a})$ but $s_t > 0$ for $t \in (\underline{a} - \varepsilon, \underline{a}) \cup (\overline{a}, \overline{a} + \varepsilon)$ where $\varepsilon > 0$ is some small number. This implies that a firm obtains a weakly higher payoff by entering at $\underline{a}$ or at $\overline{a}$ than at any time in $(\underline{a}, \overline{a})$. Given that $s_t = 0$ for $t \in (\underline{a}, \overline{a})$, the expected payoff of entering in this interval is

$$e^{-r(t-\underline{a})} p_{\underline{a}}(\pi + M_t - rc) - e^{-(\lambda+r)(t-\underline{a})}(1 - p_{\underline{a}})rc.$$

Taking derivative with respect to t and multiplying by $\frac{e^{r(t-\underline{a})}}{r}$, we obtain

$$e^{-\lambda(t-\underline{a})}(1-p_{\underline{a}})(\lambda+r)c-p_{\underline{a}}[\pi+(1+e^{-r\delta_t})m-rc],$$

which is decreasing in t. This means that the expected payoff is strictly quasi-concave in $(\underline{a},\overline{a})$, and hence we cannot have the payoff maximized at $\underline{a}$ and $\overline{a}$ in this interval. Therefore, if $s_t>0$ in the preemption phase, there must be intervals $(\underline{\tau},\overline{\tau})$ and (τ^*,∞) such that $s_t\in(0,\infty)$ for all $t\in(\underline{\tau},\overline{\tau})\cup(\tau^*,\infty)$, and $s_t=0$ otherwise. ■

Proof of Proposition 2. We first establish that there always exists a unique $\tau^{\mathrm{P}}\in[0,\frac{\tau^{\mathrm{NP}}}{2}]$ that maximizes $\hat{\Pi}(\cdot)$ over $t\in[0,\frac{\tau^{\mathrm{NP}}}{2}]$. To see this, note that the first-order condition for the maximization problem is obtained as

$$\hat{\mu}(t):=-p_0(\pi+m-rc)-e^{-r\delta_t}p_0m+e^{-\lambda t}(1-p_0)(\lambda+r)c=0.$$

It is straightforward to verify that $\hat{\mu}(\cdot)$ is strictly decreasing on $[0,\frac{\tau^{\mathrm{NP}}}{2}]$, meaning that there exists at most one $\hat{\tau}$ such that $\hat{\mu}(\hat{\tau})=0$. The optimal timing of pioneering entry, denoted by τ^{P}, is given by

$$\tau^{\mathrm{P}}=\begin{cases}0 & \text{if } 0\geq\hat{\mu}(0),\\ \hat{\tau} & \text{if } \hat{\mu}(0)>0>\hat{\mu}(\frac{\tau^{\mathrm{NP}}}{2}),\\ \frac{\tau^{\mathrm{NP}}}{2} & \text{if } \hat{\mu}(\frac{\tau^{\mathrm{NP}}}{2})\geq 0,\end{cases}$$

where $\hat{\tau}$ is the solution to $\hat{\mu}(\hat{\tau})=0$. This suggests that Π^{P} is always well defined.

Given this, we now show that pioneering entry occurs if and only if $\Pi^{\mathrm{P}}>\Pi^{\mathrm{NP}}$. We then construct an equilibrium to show its existence.

Necessary and sufficient condition: The sufficiency is obvious. If $\Pi^{\mathrm{P}}>\Pi^{\mathrm{NP}}$, there is an incentive for a firm to enter when $p^*>\phi_t$. Pioneering entry must occur with some probability.

To establish the necessity, suppose that pioneering entry occurs. Then, if a firm enters at $\underline{\tau}$, the expected continuation payoff is

$$p_{\underline{\tau}}(\pi+M_{\underline{\tau}}-rc)-(1-p_{\underline{\tau}})rc,$$

which equals the payoff of waiting until any $t\in(\underline{\tau},\overline{\tau})\cup(\tau^*,\infty)$. Now suppose that the firm instead waits until τ^{NP} regardless of what the other firm does, in which case the expected continuation payoff at $\underline{\tau}$ is

$$e^{-r(\tau^{\mathrm{NP}}-\underline{\tau})}[p_{\underline{\tau}}(\pi-rc)-e^{-\lambda(\tau^{\mathrm{NP}}-\underline{\tau})}(1-p_{\underline{\tau}})rc].$$

Observe that this is the payoff when a firm does not utilize any information from the rival firm, which implies that

$$p_{\underline{\tau}}(\pi + M_{\underline{\tau}} - rc) - (1 - p_{\underline{\tau}})rc > e^{-r(\tau^{\mathrm{NP}} - \underline{\tau})}[p_{\underline{\tau}}(\pi - rc) - (1 - p_{\underline{\tau}})rce^{-\lambda(\tau^{\mathrm{NP}} - \underline{\tau})}]$$
$$= e^{r\underline{\tau}} \frac{p_{\underline{\tau}}}{p_0} \Pi^{\mathrm{NP}}.$$

Since

$$\Pi^{\mathrm{P}} \geq e^{-r\underline{\tau}} \frac{p_0}{p_{\underline{\tau}}}[p_{\underline{\tau}}(\pi + M_{\underline{\tau}} - rc) - (1 - p_{\underline{\tau}})rc\},$$

it follows that $\Pi^{\mathrm{P}} > \Pi^{\mathrm{NP}}$.

Equilibrium existence: The existence of an equilibrium is obvious when there is no pioneering entry. We thus focus on the case where pioneering entry occurs on the equilibrium path.

Given our characterization, we need to find an interval $(\underline{\tau}, \overline{\tau})$ such that $s_t \in (0, \infty)$ for $t \in (\underline{\tau}, \overline{\tau})$. During this interval, the firms are indifferent between entering and waiting, which means that (7) must hold with equality for $t \in (\underline{\tau}, \overline{\tau})$:

$$\begin{aligned} p_t[\pi + (1 - \beta_t)M_t] - rc = e^{-r\delta_t}\beta_t[p_t(\pi - rc) - e^{-\lambda\delta_t}q_t rc] \\ + e^{-r\mathrm{d}t}(1 - \gamma_t)E[V(p_{t+\mathrm{d}t}, t + \mathrm{d}t) \mid \Omega_t], \end{aligned} \tag{10}$$

where

$$E[V(p_{t+\mathrm{d}t}, t + \mathrm{d}t) \mid \Omega_t] = B_t\{p_{t+\mathrm{d}t}[\pi + (1 - \beta_{t+\mathrm{d}t})M_{t+\mathrm{d}t} - rc] - (1 - p_{t+\mathrm{d}t})rc\}.$$

Plugging $\beta_t = 1 - e^{-s_t \mathrm{d}t}$ into (7) and taking the limit $\mathrm{d}t \to 0$, this condition can be written as

$$s_t \alpha(p_t, t) = r\mu(p_t, t), \tag{11}$$

where

$$\begin{aligned} \alpha(p_t, t) &:= p_t(\pi + M_t - rc) - q_t rc - e^{-r\delta_t}[p_t(\pi - rc) - e^{-\lambda\delta_t}q_t rc], \\ \mu(p_t, t) &:= -p_t(\pi + m - rc) - e^{-r\delta_t}p_t m + (1 - p_t)(\lambda + r)c. \end{aligned}$$

Define

$$\Sigma := \{t \in [0, \tau^{\mathrm{P}}] : \hat{\Pi}(t) > \Pi^{\mathrm{NP}}\},$$

which is nonempty whenever $\Pi^{\mathrm{P}} > \Pi^{\mathrm{NP}}$. If there is a pioneer equilibrium, the starting point $\underline{\tau}$ must belong to this set. Let

$$\hat{p}_t = \frac{p_0}{p_0 + (1-p_0)e^{-\lambda t}},\ \hat{q}_t = \frac{(1-p_0)e^{-2\lambda t}}{p_0 + (1-p_0)e^{-\lambda t}},$$

be the belief when the firms never enter up to that point, where $p_{\underline{\tau}} = \hat{p}_{\underline{\tau}}$ by definition. Since $\hat{p}_t$ depends only on t, define $\hat{\alpha}(t) := \alpha(\hat{p}_t, t)$. Observe that

$$\frac{p_0}{\hat{p}_t}\mu(\hat{p}_t, t) = \hat{\mu}(t) > 0,$$

for all $t \in \Sigma$. This means that at $\hat{\alpha}(\underline{\tau}) > 0$ must be satisfied to ensure $s_t > 0$.

We now argue $\hat{\alpha}(t) > 0$ for any $t \in \Sigma$. To this end, note that

$$\begin{aligned}\hat{\alpha}(t) &= \hat{p}_t(\pi + M_t - rc) - \hat{q}_t rc - e^{-r\delta_t}[\hat{p}_t(\pi - rc) - e^{-\lambda\delta_t}\hat{q}_t rc] \\ &= e^{rt}\frac{\hat{p}_t}{p_0}\hat{\Pi}(t) + (1-\hat{p}_t-\hat{q}_t)rc - e^{rt}\frac{\hat{p}_t}{p_0}\Pi^{\mathrm{NP}} - e^{-r\delta_t}(1-e^{-rt})\frac{\hat{p}_t}{p^*}(p^*\pi - rc).\end{aligned}$$

Since $\hat{q}_t = (1-\hat{p}_t)e^{-\lambda t}$ and $\hat{\Pi}(t) > \Pi^{\mathrm{NP}}$ for any $t \in \Sigma$ by definition, it suffices to show that

$$(1-e^{-\lambda t})(1-\hat{p}_t)rc > e^{-r\delta_t}(1-e^{-rt})\frac{\hat{p}_t}{p^*}(p^*\pi - rc).$$

Note that this condition is equivalent to

$$\int_0^t e^{-\lambda\tau}\lambda(1-\hat{p}_t)rc\mathrm{d}\tau > \int_0^t e^{-r(\delta_t+\tau)}\frac{\hat{p}_t}{p^*}r(p^*\pi - rc)\mathrm{d}\tau.$$

A sufficient condition for $\hat{\alpha}(t) > 0$ for $t \in \Sigma$ is that

$$e^{-\lambda\tau}\lambda(1-\hat{p}_t)c > e^{-r(\delta_t+\tau)}\frac{\hat{p}_t}{p^*}(p^*\pi - rc), \tag{12}$$

for $t \in \Sigma$, where $\delta_t + t \geq \tau^{\mathrm{P}} \geq t > \tau$. The fact that $\delta_t + t > \tau$ implies

$$e^{-\lambda\tau}\lambda(1-\hat{p}_t)c > e^{-\lambda(\delta_t+t)}\lambda(1-\hat{p}_t)c.$$

Note also that since $p^* = \frac{(\lambda+r)c}{\pi+\lambda c}$ and $p^*\pi - rc = \lambda(1-p^*)c$,

$$\frac{\hat{p}_t}{p^*}(p^*\pi - rc) = e^{-\lambda(\delta_t+t)}\lambda(1-\hat{p}_t)c > e^{-r(\delta_t+\tau)}\frac{\hat{p}_t}{p^*}(p^*\pi - rc),$$

which ensures that (12) holds.

Given this, we can also show that $\hat{\alpha}(t) > 0$ for $t \in [\tau^{\mathrm{P}}, \frac{\tau^{\mathrm{NP}}}{2})$. Observe first that

$$\frac{\alpha(p_t, t)}{p_t} = \pi + M_t - rc - \frac{q_t}{p_t} rc - e^{-r\delta_t}\left(\pi - rc - e^{-\lambda\delta_t}\frac{q_t}{p_t} rc\right)$$

is independent of the pre-entry strategy. Therefore, the fact that $\hat{\alpha}(t) > 0$ for $t \in \Sigma$ implies that $\alpha(p_t, t) > 0$ for $t \in [\underline{\tau}, \tau^{\mathrm{P}}]$. Next observe that $\alpha(p_t, t)$ has the same sign as

$$p_0[\pi + (1 - e^{-r(\frac{\tau^{\mathrm{NP}}}{2} - 2t)})m - rc] - e^{-2\lambda t} q_0 rc - e^{-r(\frac{\tau^{\mathrm{NP}}}{2} - 2t)}[p_0(\pi - rc) - e^{-\lambda\frac{\tau^{\mathrm{NP}}}{2}} q_0 rc],$$

which is first increasing and then decreasing in t. Therefore, $\alpha(p_t, t) > 0$ if and only if t is in some interval containing $\underline{\tau}$. Since $\alpha(p_{\frac{\tau^{\mathrm{NP}}}{2}}, \frac{\tau^{\mathrm{NP}}}{2}) = 0$, it follows that $\alpha(p_t, t) > 0$ for $t \in [\underline{\tau}, \frac{\tau^{\mathrm{NP}}}{2})$.

We are now ready to pin down the equilibrium starting point $\underline{\tau}$. Starting with some $\underline{\tau} \in \Sigma$, both $\hat{\alpha}(\underline{\tau})$ and $\hat{\mu}(\underline{\tau})$ are positive, and we can derive s_t and (p_t, q_t) for $t > \underline{\tau}$. Let $s_t(\underline{\tau})$ and $(p_t(\underline{\tau}), q_t(\underline{\tau}))$ denote the strategy and the belief so obtained. We continue this process until we find some $\overline{\tau} = \overline{\tau}(\underline{\tau}) \leq \frac{\tau^{\mathrm{NP}}}{2}$ such that $\mu(p_{\overline{\tau}}, \overline{\tau}) = 0$; if $\mu(p_t, t) > 0$ for all $t \leq \frac{\tau^{\mathrm{NP}}}{2}$, we let $\overline{\tau}(\underline{\tau}) = \frac{\tau^{\mathrm{NP}}}{2}$. For $t > \overline{\tau}(\underline{\tau})$, $s_t(\underline{\tau}) = 0$ until the induced belief $p_t(\underline{\tau})$ reaches p^*. Define $\tau^*(\underline{\tau})$ such that $p_{\tau^*(\underline{\tau})}(\underline{\tau}) = p^*$. By construction, under strategy $\{s_t(\underline{\tau})\}_{t\geq 0}$, the expected payoff of entering at any $t \in [\underline{\tau}, \overline{\tau}(\underline{\tau})]$ stays constant at $\hat{\Pi}(\underline{\tau})$, and the firms have no incentive to deviate. For this strategy to be part of an equilibrium, we thus only need to ensure that the firms are also indifferent between entering at $t \in [\underline{\tau}, \overline{\tau}(\underline{\tau})]$ and at $t \in [\tau^*(\underline{\tau}), \infty)$. Now suppose that a firm adopts a strategy of waiting until $p_t(\underline{\tau})$ reaches p^* if no entry has occurred while the rival firm plays $\{s_t(\underline{\tau})\}_{t\geq 0}$,[30] and let $\Pi^+(\underline{\tau})$ be the expected payoff of this strategy. An equilibrium exists if there is some $\underline{\tau} \in \Sigma$ such that $\hat{\Pi}(\underline{\tau}) = \Pi^+(\underline{\tau})$. Observe that at $\underline{\tau} = \tau^{\mathrm{P}} = \sup \Sigma$, we have $\hat{\mu}(\underline{\tau}) = 0$ which implies $\overline{\tau}(\tau^{\mathrm{P}}) = \tau^{\mathrm{P}}$, $\Pi^+(\underline{\tau}) = \Pi^{\mathrm{NP}}$, and hence $\hat{\Pi}(\underline{\tau}) > \Pi^+(\underline{\tau})$. At $\underline{\tau} = \inf \Sigma$, since $\hat{\Pi}(\underline{\tau}) = \Pi^{\mathrm{NP}}$ and $\Pi^+(\underline{\tau}) > \Pi^{\mathrm{NP}}$,[31] it follows that $\hat{\Pi}(\underline{\tau}) < \Pi^+(\underline{\tau})$. Since both $\hat{\Pi}(\underline{\tau})$ and $\Pi^+(\underline{\tau})$ change continuously with $\underline{\tau}$, there must exists a $\underline{\tau} \in \Sigma$ such that $\hat{\Pi}(\underline{\tau}) = \Pi^+(\underline{\tau})$.

Off-path deviations: It is straightforward to verify that the equilibrium described above satisfies the Intuitive Criterion. To this end, it is important to recall that we have derived the equilibrium under the presumption that any entry is made by the uninformed type. Now suppose that the informed type deviates and enter at some $t \in [0, \underline{\tau}) \cup (\overline{\tau}, \tau^*)$, in

[30] When the rival firm adopts $\{s_t(\underline{\tau})\}_{t\geq 0}$, it enters with some probability in $[\underline{\tau}, \overline{\tau}(\underline{\tau})]$, in which case the firm follows with delay δ_t. Otherwise, the firm waits until time $\tau^*(\underline{\tau})$ when $p_t(\underline{\tau})$ reaches p^*.

[31] $\Pi^+(\underline{\tau}) > \Pi^{\mathrm{NP}}$ holds because $\Pi^+(\underline{\tau})$ is the payoff when a firm utilizes the information reveals by the rival firm's entry while Π^{NP} is the payoff when a firm receives no additional information.

which case the expected payoff is $-c$ regardless of the belief assigned by the rival firm. Since the equilibrium payoff of never entering is invariably zero, any off-path entry is equilibrium dominated for the informed type, and the Intuitive Criterion assigns probability 1 to the uninformed type for any such deviation. This means that any entry is regarded as coming from the uniformed type, both on and off the equilibrium path under the Intuitive Criterion. We have already shown that under this belief, the uninformed type has no incentive to enter at any $t \in [0, \underline{\tau}) \cup (\overline{\tau}, \tau^*)$, meaning that there are no profitable off-path deviations. ■

Proof of Proposition 3. Note that Π^{NP} and Π^{P} as functions of p_0 are given by

$$\Pi^{\mathrm{NP}}(p_0) = e^{-r\tau^{\mathrm{NP}}}[p_0(\pi - rc) - e^{-\lambda\tau^{\mathrm{NP}}}(1-p_0)rc], \tag{13}$$

$$\Pi^{\mathrm{P}}(p_0) = e^{-r\tau^{\mathrm{P}}}[p_0(\pi + M_{\tau^{\mathrm{P}}} - rc) - e^{-\lambda\tau^{\mathrm{P}}}(1-p_0)rc]. \tag{14}$$

It is clear that (14) can only increase with m by the envelope theorem while (13) is independent of it. Therefore, there must be a threshold $\hat{m}$ such that the condition for pioneering entry is satisfied if and only if $m > \hat{m}$.

For the effect of p_0, define $p_0'' := \frac{p_0'}{p_0' + (1-p_0')e^{-\lambda\varepsilon}}$ for a given $p_0' < p^*$ where ε is a positive number which is small enough to ensure $p^* > p_0'' > p_0'$. Also, we write τ^{NP} and τ^{P} both as functions of p_0. Since

$$\Pi^{\mathrm{NP}}(p_0) = e^{-r\tau^{\mathrm{NP}}(p_0)}\frac{p_0}{p^*}(p^*\pi - rc),$$

we have

$$\Pi^{\mathrm{NP}}(p_0'') = \frac{e^{-r\tau^{\mathrm{NP}}(p_0'')}p_0''}{e^{-r\tau^{\mathrm{NP}}(p_0')}p_0'}\Pi^{\mathrm{NP}}(p_0') = e^{r\varepsilon}\frac{p_0''}{p_0'}\Pi^{\mathrm{NP}}(p_0').$$

As for Π^{P}, observe that

$$\begin{aligned}\lim_{p_0 \uparrow p^*} \hat{\mu}(0) &= -p_0(\pi + m - rc) - p_0 m e^{-r\tau^{\mathrm{NP}}} + (1-p_0)(\lambda + r)c \\ &= -\frac{(\lambda + r)c}{\pi + \lambda c}(\pi + m - rc) - \frac{(\lambda + r)c}{\pi + \lambda c} m e^{-r\tau^{\mathrm{NP}}} + \frac{\pi - rc}{\pi + \lambda c}(\lambda + r)c < 0,\end{aligned}$$

suggesting that there exists some threshold $\overline{p} < p^*$ such that $\tau^{\mathrm{P}} = 0$ if and only if $p_0 \in [\overline{p}, p^*)$. Also, define $p^+ := \frac{p_0'}{p_0' + (1-p_0')e^{-\lambda\tau^{\mathrm{P}}(p_0')}}$.

First, consider the case where $\overline{p} > p_0'' > p_0'$. We then have

$$\begin{aligned}\Pi^{\mathrm{P}}(p_0') &= e^{-r\tau^{\mathrm{P}}(p_0')}\{p_0'[\pi + (1 - e^{-r[\tau^{\mathrm{NP}}(p_0') - 2\tau^{\mathrm{P}}(p_0')]})m - rc] - e^{-\lambda\tau^{\mathrm{P}}(p_0')}(1 - p_0')rc\} \\ &= e^{-r\tau^{\mathrm{P}(p_0')}}\frac{p_0'}{p^+}\{p^+[\pi + (1 - e^{-r[\tau^{\mathrm{NP}}(p_0') - 2\tau^{\mathrm{P}}(p_0')]})m - rc] - (1 - p^+)rc\}.\end{aligned}$$

Similarly,

$$\begin{aligned}\Pi^{\mathrm{P}}(p_0'') &= e^{-r\tau^{\mathrm{P}}(p_0'')}\{p_0''[\pi + (1 - e^{-r[\tau^{\mathrm{NP}}(p_0'') - 2\tau^{\mathrm{P}}(p_0'')]})m - rc] - e^{-\lambda\tau^{\mathrm{P}}(p_0'')}(1 - p_0'')rc\} \\ &> e^{-r(\tau^{\mathrm{P}}(p_0') - \varepsilon)}\frac{p_0''}{p^+}\{[p^+[\pi + (1 - e^{-r[\tau^{\mathrm{NP}}(p_0'') - 2(\tau^{\mathrm{P}}(p_0') - \varepsilon)]})m - rc] - (1 - p^+)rc\} \\ &> e^{-r(\tau^{\mathrm{P}}(p_0') - \varepsilon)}\frac{p_0''}{p^+}\{[p^+[\pi + (1 - e^{-r[\tau^{\mathrm{NP}}(p_0') - 2\tau^{\mathrm{P}}(p_0')]})m - rc] - (1 - p^+)rc\} \\ &= e^{r\varepsilon}\frac{p_0''}{p_0'}\Pi^{\mathrm{P}}(p_0').\end{aligned}$$

Here, the second line shows the payoff when a firm enters at time $\tau^{\mathrm{P}}(p_0') - \varepsilon$ at which point the belief reaches p^+. If $\tilde{p} < p_0' < p_0''$, then

$$\Pi^{\mathrm{P}}(p_0'') = p_0''[\pi + (1 - e^{-r\tau^{\mathrm{NP}}(p_0'')})m] - rc,$$

and

$$\begin{aligned}\Pi^{\mathrm{P}}(p_0') &= p_0'[\pi + (1 - e^{-r\tau^{\mathrm{NP}}(p_0')})m] - rc \\ &> e^{-r\varepsilon}\frac{p_0'}{p_0''}\{p_0''[\pi + (1 - e^{-r\tau^{\mathrm{NP}}(p_0'')})]m - rc\} \\ &= e^{-r\varepsilon}\frac{p_0'}{p_0''}\Pi^{\mathrm{P}}(p_0''),\end{aligned}$$

where the second line shows the payoff when a firm enters at time ε.

It follows from above that if $\Pi^{\mathrm{NP}}(p_0') = \Pi^{\mathrm{P}}(p_0')$, then $\Pi^{\mathrm{NP}}(p_0'') < \Pi^{\mathrm{P}}(p_0'')$ for $\overline{p} > p_0'' > p_0'$ and $\Pi^{\mathrm{NP}}(p_0'') < \Pi^{\mathrm{P}}(p_0'')$ for $p_0'' > p_0' > \overline{p}$. This suggests that Π^{NP} and Π^{P} intersect at most twice for $p_0 \in (0, p^*]$. Note also that $p^* > \overline{p}$ and $\Pi^{\mathrm{NP}}(p^*) = \Pi^{\mathrm{P}}(p^*)$. This proves that there is a threshold $\hat{p} \in (0, p^*)$ such that the condition for pioneering entry is satisfied if and only if $p_0 \in (\hat{p}, p^*)$. ■

Proof of Proposition 4. In the proof of Proposition 2, we observe that

$$p_{\underline{\tau}}(\pi + M_{\underline{\tau}} - rc) - (1 - p_{\underline{\tau}})rc > e^{r\underline{\tau}}\frac{p_{\underline{\tau}}}{p_0}\Pi^{\mathrm{NP}}.$$

Since $\frac{1-p_{\underline{\tau}}}{p_{\underline{\tau}}} = \frac{1-p_0}{p_0} e^{-\lambda \underline{\tau}}$, we obtain

$$e^{-r\underline{\tau}} \frac{p_0}{p_{\underline{\tau}}} [p_{\underline{\tau}}(\pi + M_{\underline{\tau}} - rc) - (1 - p_{\underline{\tau}})rc] = \Pi^* > \Pi^{\mathrm{NP}}.$$

We can also show that the firms start entering earlier than τ^{P}, i.e., $\tau^{\mathrm{P}} > \underline{\tau}$, if $\Pi^{\mathrm{P}} > \Pi^{\mathrm{NP}}$. To see this, suppose on the contrary that $\underline{\tau} \geq \tau^{\mathrm{P}}$. If a firm deviates and enters unilaterally at time $\underline{\tau} - \mathrm{d}t$, the expected payoff is

$$e^{-r(\underline{\tau}-\mathrm{d}t)}\{p_0[\pi + (1 - e^{-r(\delta_{\underline{\tau}}+2\mathrm{d}t)})m - rc] - e^{-\lambda(\underline{\tau}-\mathrm{d}t)}(1 - p_0)rc\}. \tag{15}$$

Now suppose that the firm enters at time $\underline{\tau}$ (if it is still uninformed), in which case the expected payoff is computed as

$$e^{-r\underline{\tau}}\{p_0[\pi + e^{-s_{\underline{\tau}}\mathrm{d}t}(1 - e^{-r\delta_{\underline{\tau}}})m - rc] - e^{-\lambda\underline{\tau}}(1 - p_0)rc\}. \tag{16}$$

By comparing (15) and (16), it is better to deviate if

$$p_{\underline{\tau}} s_{\underline{\tau}} M_{\underline{\tau}} > (1 - p_{\underline{\tau}})(\lambda + r)c - e^{-r\delta_{\underline{\tau}}} p_{\underline{\tau}} m - p_{\underline{\tau}}(\pi + m - rc).$$

Observe that the right-hand side is proportional to $\hat{\mu}(\underline{\tau})$ and takes a non-positive value for $\underline{\tau} \geq \tau^{\mathrm{P}}$ if $\frac{\tau^{\mathrm{NP}}}{2} > \tau^{\mathrm{P}}$. This means that the firms always have an incentive to deviate by entering slightly before time $\underline{\tau}$, a contradiction. This means that $\tau^{\mathrm{P}} > \underline{\tau}$ and hence $\Pi^* < \Pi^{\mathrm{P}}$, giving the payoff bounds. ■

Proof of Proposition 5. Taking derivative of the joint payoff W, the first-order conditions are obtained as

$$\frac{\partial W}{\partial \tau_1} = -e^{-r\tau_1} r p_0(\pi + m - rc) + e^{-\lambda\tau_1}(1 - p_0)[(\lambda + r)e^{-r\tau_1} + \lambda e^{-(\lambda+r)\tau_2}]rc = 0, \tag{17}$$

$$\frac{\partial W}{\partial \tau_2} = -e^{-r\tau_2} r p_0(\pi - m - rc) + e^{-r\tau_2 - \lambda(\tau_1+\tau_2)}(1 - p_0)(\lambda + r)rc = 0. \tag{18}$$

From (18), we obtain

$$e^{-\lambda T^{**}} = \frac{p_0(\pi - m - rc)}{(1 - p_0)(\lambda + r)c},$$

which depends only on the primitives of the model. Since $\tau_2 \geq \tau_1$, for $\tau_1 \in [0, \frac{T^{**}}{2})$, (17) can be written as

$$\begin{aligned} p_0(\pi + m - rc) &= (1 - p_0)[(\lambda + r)e^{-\lambda\tau_1} + \lambda e^{-\lambda T^{**} - r(\tau_2 - \tau_1)}]c \\ &= (1 - p_0)(\lambda + r)ce^{-\lambda\tau_1} + p_0 \frac{\lambda(\pi - m - rc)}{\lambda + r} e^{-r(\tau_2 - \tau_1)}. \end{aligned} \tag{19}$$

If there is no $\tau_1 < \frac{T^{**}}{2}$ that can satisfy (23), the social optimal timing is "almost simultaneous," i.e., $\tau_1 = \tau_2$. The first-order condition in this case is reduced to

$$p_0(\pi - rc) = \frac{1-p_0}{2}[\lambda + r + (2\lambda + r)e^{-\lambda\tau}]ce^{-\lambda\tau}, \tag{20}$$

which is independent of m.

Given this, we are now ready to prove statements (i)-(iii) of the proposition in turn.

(i) Note that τ^{NP} must solve

$$p_0(\pi - rc) = e^{-\lambda\tau^{\text{NP}}}(1-p_0)(\lambda + r)c.$$

The only difference from (18) is that the left-hand side is $p_0(\pi - rc)$ instead of $p_0(\pi - m - rc)$, which implies $T^{**} > \tau^{\text{NP}}$.

(ii) Since $\frac{T^{**}}{2} > \frac{\tau^{\text{NP}}}{2} \geq \tau^{\text{P}}$, $\tau_1^{**} > \tau^{\text{P}}$ holds if $\tau_1^{**} \geq \frac{T^{**}}{2}$. This means that we can focus on $\tau_1^{**} < \frac{T^{**}}{2}$ (with an interior optimum satisfying the second-order condition). In this case, τ_1^{**} must solve (23), which implies that

$$e^{-\lambda\tau_1^{**}} < \frac{p_0(\pi + m - rc)}{(1-p_0)(\lambda + r)c}.$$

On the other hand, if $\tau^{\text{P}} > 0$, it must satisfy

$$p_0(\pi + m - rc) = e^{-\lambda\tau^{\text{P}}}(1-p_0)(\lambda + r)c - e^{-r(\tau^{\text{NP}} - 2\tau^{\text{P}})}p_0 m.$$

As this implies that

$$e^{-\lambda\tau^{\text{P}}} > \frac{p_0(\pi + m - rc)}{(1-p_0)(\lambda + r)c},$$

we have $\tau_1^{**} > \tau^{\text{P}}$.

(iii) Observe that $\tau_1^{**} \geq \frac{T^{**}}{2}$ if

$$p_0(\pi + m - rc) < e^{-\lambda\tau_1}(1-p_0)(\lambda + r)c + e^{-r(T^{**} - 2\tau_1)}p_0\frac{\lambda(\pi - m - rc)}{\lambda + r},$$

for all $\tau_1 \in [0, \frac{T^{**}}{2})$. This obviously holds if p_0 is sufficiently small, in which case $\tau_1 = \tau_2 = \tau$ where τ is the solution to

$$\max_{\tau}\ W(\tau, \tau) = e^{-r\tau}[2p_0(\pi - rc) - e^{-\lambda\tau}(1 + e^{-\lambda\tau})(1-p_0)rc].$$

The first-order condition is obtained as

$$p_0(\pi - rc) = e^{-\lambda\tau}\frac{1-p_0}{2}[\lambda + r + (2\lambda + r)e^{-\lambda\tau}]c.$$

Plugging $\tau = \tau^{\mathrm{NP}}$ into the right-hand side, we obtain

$$e^{-\lambda\tau^{\mathrm{NP}}}\frac{1-p_0}{2}\left[(\lambda+r)+(2\lambda+r)\frac{p_0(\pi-rc)}{(1-p_0)(\lambda+r)c}\right]c,$$

which is smaller than $(1-p_0)(\lambda+r)ce^{-\lambda\tau^{\mathrm{NP}}} = p_0(\pi-rc)$ if p_0 is small enough to satisfy $\frac{p_0(\pi-rc)}{(1-p_0)(\lambda+r)c} < \frac{\lambda+r}{2\lambda+r}$. We then have

$$2p_0(\pi-rc) > e^{-\lambda\tau_1}(1-p_0)[(\lambda+r)+e^{-\lambda\tau_1}(2\lambda+r)]c,$$

for all $\tau_1 \geq \tau^{\mathrm{NP}}$ and hence $\tau_1^{**} < \tau^{\mathrm{NP}}$. ■

Appendix B: State transition

Here, we describe the state-transition process and derive the laws of motion for the belief (p_t, q_t). As noted in the main text, there are three states in the model: G, BU and BI. In states G and BU, the rival firm enters the market with probability $1-e^{-s_t\mathrm{d}t}$; in state BI, knowing that the market is bad, the rival firm never enters. Moreover, the firm observes a signal with probability $1-e^{-\lambda\mathrm{d}t}$ if the market is bad (in states BU and BI). Finally, the state changes from BU to BI when the rival firm observes a signal, which occurs also with probability $1-e^{-\lambda\mathrm{d}t}$. Then, given current belief (p_t, q_t) and strategy s_t, the belief at the next instant is computed as

$$p_{t+\mathrm{d}t} = \frac{p_t e^{-s_t\mathrm{d}t}}{(p_t+q_te^{-\lambda\mathrm{d}t})e^{-s_t\mathrm{d}t}+(1-p_t-q_t)e^{-\lambda\mathrm{d}t}}, \tag{21}$$

$$q_{t+\mathrm{d}t} = \frac{q_t e^{-(s_t+2\lambda)\mathrm{d}t}}{(p_t+q_te^{-\lambda\mathrm{d}t})e^{-s_t\mathrm{d}t}+(1-p_t-q_t)e^{-\lambda\mathrm{d}t}}, \tag{22}$$

with the initial prior given by $q_0 = 1-p_0$. It thus follows from this that

$$\phi_{t+\mathrm{d}t} := \frac{p_{t+\mathrm{d}t}}{p_{t+\mathrm{d}t}+q_{t+\mathrm{d}t}} = \frac{p_t}{p_t+q_te^{-2\lambda\mathrm{d}t}}.$$

Finally, taking the limit gives the laws of motion which clarify how the belief evolves over time:

$$\dot{p}_t = p_t\left[(1-p_t)\lambda-(1-p_t-q_t)s_t\right],$$
$$\dot{q}_t = -q_t\left[(1+p_t)\lambda+(1-p_t-q_t)s_t\right].$$

Figure 3, which graphically summarizes the state-transition process, helps illustrating how we obtain these equations.

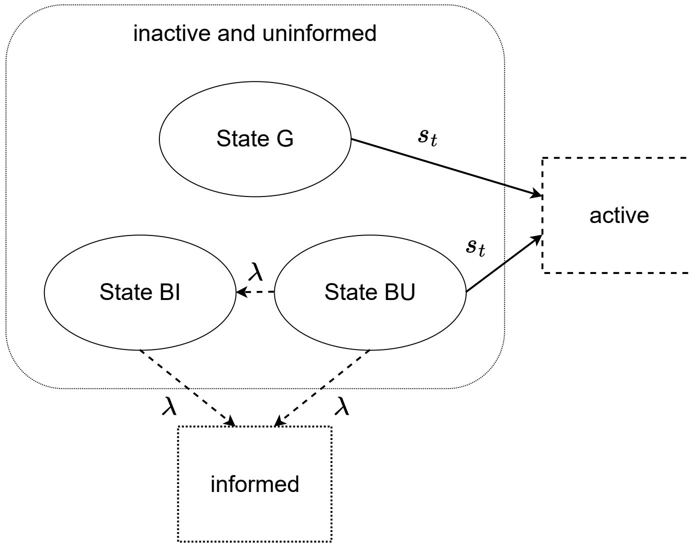

Figure 3. *State transition. There are three possible states $\{G, BI, BU\}$ when a firm is inactive and uninformed. Solid arrows indicate transition by the arrival of information; dotted arrows indicate transition by the firm's strategic entry choice.*

Appendix C: Welfare with consumers

Letting $CS_1^G = \nu CS$, $CS_2^G = CS$, and $CS_1^B = CS_2^B = 0$, the first-order conditions now become

$$\frac{\partial W^*}{\partial \tau_1} = -e^{-r\tau_1} r p_0(\pi + m + \nu CS - rc) \\ + e^{-\lambda\tau_1}(1-p_0)[(\lambda + r)e^{-r\tau_1} + \lambda e^{-(\lambda+r)\tau_2}]rc = 0, \tag{23}$$

$$\frac{\partial W^*}{\partial \tau_2} = -e^{-r\tau_2} r p_0[\pi - m + (1-\nu)CS - rc] + e^{-r\tau_2 - \lambda(\tau_1+\tau_2)}(1-p_0)(\lambda + r)rc = 0. \tag{24}$$

From (24), we obtain

$$e^{-\lambda T^{**}} = \frac{p_0[\pi - m + (1-\nu)CS - rc]}{(1-p_0)(\lambda + r)c}.$$

It follows from this that an increase in CS unambiguously lower T^{**}. Clearly, part (a) of Proposition 5 holds if and only if $m \geq (1-\nu)CS$. On the other hand, the effect on τ_1^{**} is less clear, as (23) now becomes

$$p_0(\pi + m + \nu CS - rc) = (1-p_0)(\lambda + r)ce^{-\lambda\tau_1} + p_0\frac{\lambda[\pi - m + (1-\nu)CS - rc]}{\lambda + r}e^{-r(\tau_2-\tau_1)}. \tag{25}$$

Taking derivative of (25) with respect to CS on both side, we obtain

$$\Gamma \frac{\mathrm{d}\tau_1^{**}}{\mathrm{d}CS} = -p_0 \nu \left(1 + \frac{\lambda e^{-r(\tau_2^{**}-\tau_1^{**})}}{\lambda + r} \right), \tag{26}$$

where

$$\Gamma := \lambda(1-p_0)(\lambda+r)ce^{-\lambda\tau_1^{**}} - p_0 r \frac{\lambda[\pi - m + (1-\nu)CS - rc]}{\lambda + r} e^{-r(\tau_2^{**}-\tau_1^{**})}.$$

Since the right-hand side of (26) is negative, τ_1^{**} is decreasing in CS if $\Gamma > 0$. Observe that $\Gamma > 0$ implies

$$(\lambda + r)e^{-\lambda\tau_1^{**}} > re^{-\lambda T^{**}} e^{-r(\tau_2^{**}-\tau_1^{**})},$$

which holds since $e^{-\lambda\tau_1^{**}} > e^{-\lambda T^{**}}$ and $1 > e^{-r(\tau_2^{**}-\tau_1^{**})}$. This shows that part (c) of the proposition holds for any $CS \geq 0$. Finally, to see when part (b) of the proposition holds, note that

$$\frac{p_0(\pi + m + \nu CS - rc)}{(1-p_0)(\lambda+r)c} > e^{-\lambda\tau_1^{**}}.$$

We can then find a ν that is small enough to satisfy

$$e^{-\lambda\tau^{\mathrm{P}}} > \frac{p_0(\pi + m + \nu CS - rc)}{(1-p_0)(\lambda+r)c} > e^{-\lambda\tau_1^{**}},$$

which implies $\tau_1^{**} > \tau^{\mathrm{P}}$.