

Multi-Frequency Averaging (MFA) Model of Electric-Hybrid Powertrain Suitable for Variable Frequency Operation Applied in Geographically-Distributed Power Hardware-in-the-Loop (GD-PHiL) Simulation

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Abstract—Project aims to develop capability for OEMs and suppliers to ‘virtually-connect’ multiple prototype powertrain components (engine, motor-drive etc.) and engage in real-time system simulation, thereby reducing cost by eliminating co-location dependency. Technical challenge faced is to duplicate test-hardware state & performance at remote location(s) real-time via an indeterminate internet connection characteristic of latency and jitter. MFA-based remote model is a potential solution but applied research till date has been for fixed-frequency operation. This paper explores variable frequency operation of MFA and outlines the implementation of generic electric powertrain intended for remote control/operation, indicating key performance improvements over time-domain switching models.

Keywords— *multi frequency averaging (MFA), generalized average modeling (GAM), dynamic phasors (DP), variable frequency, frequency domain, power hardware in the loop (PHIL), geographically distributed, simulation, automotive, electric, vehicle, powertrain, modeling*

I. INTRODUCTION

Recent changes in global politics towards environmental sustainability has led to disruptive changes in the industry. Rise of digitalization with induction of Silicon Valley giants (Google, Apple etc.) in the industry has forced legacy manufacturers to submit to the post-modern automotive world of a cheap, feature-loaded and highly-connected car. Consequently, electronics and software have become critical to automotive R&D. While this evolution has increased system complexity, it has also opened many avenues to reduce costs. One such opportunity is increased *virtual prototyping* in product development. While the need for a real prototype is still unavoidable (digital models cannot be 100% accurate), this step can be pushed further back in the development cycle; dropping the overall cost of designing. Virtually-Connected Hybrid Vehicle (VCHV) aims to do just that by attempting to build

a virtual test-bed for co-simulating multiple HiL prototypes in real-time distributed across geographies; e.g. GM can pre-test its battery, engine and motor physical prototypes of its future Volt powertrain in a system-level simulation without moving them from their respective R&D centers, reducing time and cost of development. Working with physical devices entails real-time hardware interfacing at both ends of a ‘virtual-connection’ that requires accurate and common clock-synchronized duplication of hardware at each end. The degree of ‘virtual-coupling’ required depends on the physical system and directly exchanging state/output variable values over internet (characteristic of indeterminate amount of latency and jitter) may not be feasible.

It was quickly understood that signals should be exchanged in the frequency-domain (i.e. amplitudes of harmonics of interest as function of time, also called Dynamic Phasors) since although variables are fast, their component phasors (of interest) are slow-varying. Secondly, different variables can be required in different degrees of accuracy, e.g. only DC component of torque demand signal but several higher ‘harmonics of interest’ of inverter current output may be required to estimate switching losses. Lastly, real-time simulation requires model complexity to be minimal. The three main requirements led to identify MFA (interchangeably used with Generalized-Average Modeling and Dynamic Phasors by the research community) as the underlying foundation for the modeling framework.

Most applications of MFA in literature were found to be for stationary/fixed-frequency uses [1] [2] [3] which brought forth a technical challenge. *This paper demonstrates a generic electric vehicle powertrain (remote-operable battery and input filter, full-bridge converter and round-rotor electric machine) simulation implemented with MFA and operating at variable frequency.*

At the global system level, the key challenge is to determine a virtual coupling among hardware components which would

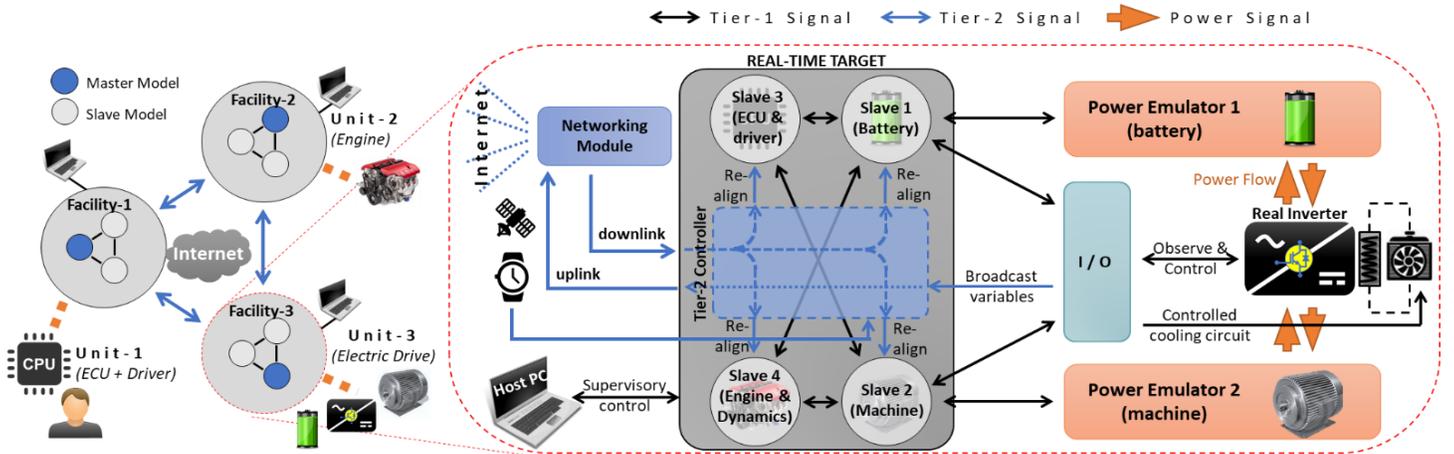


Fig. 1. Global and local architecture schematic simplified. System-wide co-simulation distributed across different facilities having individual processing infrastructure and interacting among themselves only via stream of real-time interface variables. Locally, multi-tier architecture ensures local solver impervious to imperfections in global communications. Tier-2 (blue boxes) responsible for relaying information among facilities (broadcasting timestamped packets, rearranging received packets) and updating slave models based on past DuT behaviour and keeping the system converged.

overcome internet latency/jitter/packet drop and bi-directionally reproduce hardware outputs in the remote locations; ensuring system-wide stability. This is ideated in a ‘master-slave’ methodology working on two tiers (Fig. 1): *tier-1* being the local HiL simulation where the device-under-test (DuT) interacts with ‘slave’ models of the remote DuTs completing the local loop; *tier-2* ties every location globally by updating the ‘slave’ models across internet based on corresponding DuT outputs keeping the system converged. *Dependent on use case, each ‘virtual coupling’ would require a certain degree of fidelity, which would be delivered by a ‘slave’, modeling high frequency behavior (overcoming internet bandwidth issues) and a robust updating mechanism for one-off events like shifting operating region, outliers etc.*

II. BACKGROUND AND LITERATURE

Primary motivation is to reduce development costs and time-to-market of novel hybrid powertrains. The current practices usually require moderately early testing on real prototype that raises cost of subsequent design changes. Moreover, there is a logistic barrier to bring all prototype components together at one place since research centers/suppliers of these components are usually dispersed geographically. The proposed solution would tackle both these problems by allowing multiple R&D centers to ‘hook-up’ their prototype and co-simulate with other physical/virtual prototype sub-systems in a virtual system simulation.

A. Global Powertrain Architecture Proposed

A two-tier architecture is proposed (Fig. 1):

- *Tier-1 (Local HiL simulation)*: Physical Electrical Power Converter (EPC) DuT is connected to the simulator rig. Local simulation loop completed by ‘Slave’ models of other components of the system (e.g. machine, battery, ECU), some of which are remote DuTs in other locations. The EPC DuT

can see only the local slave models which ensures complete isolation from internet errors (safeguards expensive equipment from run-away scenarios).

- *Tier-2 (Global Slave-model updating)*: Blue-colored blocks and arrows represent tier-2. Two functions: constantly re-align the local slave models ensuring tier-1 is on track; observing EPC DuT and broadcasting re-alignment packets to keep the EPC slave/proxy models (in other locations) in check. The re-alignment packets (uplink and downlink) are time-stamped to be checked against a GPS clock at receiver end to ensure against causality.

Challenge is setting up deterministic and fast communication between locations for sustainable real-time co-simulation. *The quality of communication required (in terms of chunk size, frequency of exchange etc.) depends on the physical coupling we are trying to replicate over internet.* In case of power electronics, this problem is pronounced since inverter-controller unit runs on very fast variables (very small time constant). Consequently, there is need for certain level of fast and accurate prediction by the proxy model working in a ‘quasi-islanded mode’ at the remote facilities. Discussions on global architecture shall be continued in future publications and hereon forth this paper would focus on MFA methodology and variable frequency implementation.

B. Electrical Power Converter (EPC) Modeling

Modern EPCs use PWM to control the switching action of devices, which makes modeling and simulation more difficult than other physical systems. PWM today involves flipping switches at very high speeds (up to few hundred kHz for IGBT and several MHz for MOSFET) making real-time switching models very difficult even with the fastest processing speeds of today. Accurate representation of the switching transients is important for many thermal and electromagnetic applications. Secondly, switching devices are inherently non-linear and time-variant in nature. Much of modern control theory and application is based on linear time-

invariant (LTI) systems which makes modelers compelled to select either relatively new non-linear/hybrid methods (which are slow) or approximating to LTI systems (by linearizing that is only accurate enough in and around an operating point).

The research community has been steadily building on non-linear modeling strategies as is evident from the literature. An excellent review in EPC modeling is provided in [4] for Smart DC microgrids. *Non-linear and hybrid methods* are important to consider for very detailed switching models, though they suffer from slow run-time and complexity, which limits their usage to specialty cases (especially for real-time applications). *Averaged models* on the other hand are quick, owing to their simplicity [5]. The idea behind *State-Space-Averaging* (SSA) is to describe the state-space of the system for each switch configuration (i.e. 8 configurations for a standard 6 switch full-bridge converter) and average the variables of interest over one switching cycle (significantly larger than time periods of interest, implying only DC behavior observability possible) by weighing the modes with respective duty cycles. SSA performs very well for conditions where the switching speeds are much higher compared to fundamental frequencies of interest, state variables have insignificant ripple, and slowly-changing duty cycle. PWM-controlled stationary electrical machines like generators are prime examples.

The above conditions are not comfortably met in high-speed high-dynamics applications since frequencies of interest are comparable to switching frequencies and the duty cycle is required to be very fast-changing due to dynamic torque demand. Automotive traction is a classic example. *Multi-Frequency-Averaging* (MFA) is good option if it is preferable to keep computation requirements to minimum [1]. This method is based on Fourier expansion (1-2) of the waveforms of interest and using the Fourier coefficients of selected significant frequencies to build a state-space model. MFA offers a robust and modular format for modeling AC behavior in waveforms with ripple content with the freedom to choose however many harmonics to improve the accuracy at the cost of incremental run-time.

$$x(t) = \sum_{k=-n}^n \langle x \rangle_k(t) e^{jk\omega t} \quad (1)$$

Where index-k refers to the k^{th} harmonic content.

$$\langle x \rangle_k(t) = \frac{1}{T} \int_{t-T}^t x(\tau) e^{-jk\omega t} d\tau \quad (2)$$

MFA has been successfully used in many EPC modeling projects in the recent past, which have shown significant match with switching model outputs while having a fraction of computational cost. An LTI MFA model has been derived in [2] for a 3-phase full-bridge generic inverter with output impedance and incorporated dead-time effect. The authors have demonstrated multiple worked-examples using different sets of harmonics for averaging, based on requirement. In another recent work from Kentucky [3], the authors have derived MFA models of single and

3-phase generic inverters and observed that including certain significant sidebands to carrier signal harmonics have a desirable effect, albeit at the cost of slightly higher simulation run-time. Specifically, including the sideband has demonstrated truer representation of the variation in switching ripple magnitude that occurs in the steady-state. *The mathematical framework developed [3] has been used in this paper with addition of variable frequency capability and inclusion of an input LC filter to converter.*

III. CHALLENGES AND PROPOSED SOLUTION

As already hinted in the background section, the electric powertrain slave model needs to faithfully reproduce DuT (EPC, battery, motor) baseline behavior and be compliant of their erratic behavior as one-off outlier events every now and then through re-alignment packets. Model complexity has a ceiling restricted by real-time operation. Time for propagation of information of the outlier events is restricted by finite latency of internet communication. Sequential realignment packets are also subject to severe jitter characteristic of the internet.

MFA modeling approach has been explored for this application because of its unique advantages:

- ‘Averaging’ over a desired period offers unparalleled simulation speed,
- One-touch complexity scaling: different slave models can be quickly configured to ‘average’ different number of harmonics to produce waveforms of desired accuracy. E.g. EPC slave at machine location can be configured to reproduce output ripple behavior whereas the one at battery location may only produce DC characteristics.
- Since the model is based on dynamic phasors (Fourier coefficients) which is on frequency-domain, effects of latency and jitter is significantly defused.

IV. METHODOLOGY

A generic electric powertrain has been formulated (Fig. 2). A battery (ideal voltage source and internal resistance) in-line with LC filter constitutes the input side, which is separated from the machine (round rotor type) by a full-bridge power converter. This system can be defined through following equations:

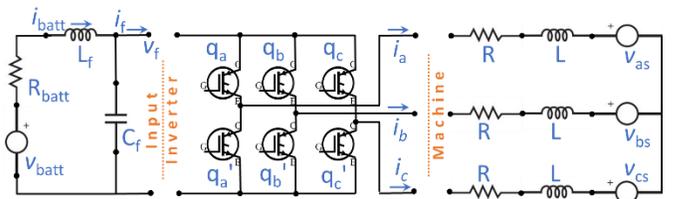


Fig. 2. Electrical system of battery, input LC filter, full-bridge inverter, round rotor electrical machine. Q is gate signal 1/0 for upper-switch on/off.

$$\frac{d}{dt} i_{batt} = \frac{1}{L_f} v_{batt} - \frac{R_{batt}}{L_f} i_{batt} - \frac{1}{L_f} v_f \quad (3)$$

$$\frac{d}{dt} v_f = \frac{1}{C_f} i_{batt} - \frac{1}{C_f} i_f \quad (4)$$

$$i_f = q_a i_a + q_b i_b + q_c i_c \quad (5)$$

$$v_{a,b,c} = v_f \left(\frac{2}{3} q_{a,b,c} - \frac{1}{3} q_{b,a,c} - \frac{1}{3} q_{c,a,b} \right) \quad (6)$$

$$\frac{d}{dt} i_{a,b,c} = \frac{1}{L} v_{a,b,c} - \frac{R}{L} i_{a,b,c} - \frac{1}{L} v_{as,bs,cs} \quad (7)$$

$$\mathbf{V}_{a,b,c} = \mathbf{V}_f \otimes \left(\frac{2}{3} \mathbf{Q}_{a,b,c} - \frac{1}{3} \mathbf{Q}_{b,a,c} - \frac{1}{3} \mathbf{Q}_{c,a,b} \right) \quad (13)$$

$$\frac{d}{dt} \mathbf{I}_{a,b,c} = \frac{1}{L} \mathbf{V}_{a,b,c} - \left(\frac{R}{L} + \mathbf{T} \right) \mathbf{I}_{a,b,c} - \frac{1}{L} \mathbf{V}_{as,bs,cs} \quad (14)$$

Where a vector \mathbf{X} is defined as:

$$\mathbf{X} = [X_{0,0} \ X_{0,1c} \ X_{0,1s}]^T$$

and,

$$\mathbf{X} \in \{ \mathbf{I}_{batt}, \mathbf{V}_{batt}, \mathbf{V}_f, \mathbf{I}_f, \mathbf{V}_{a/b/c}, \mathbf{Q}_{a/b/c}, \mathbf{I}_{a/b/c}, \mathbf{V}_{as/bs/cs} \}$$

A. MFA methodology

This approach relies on using the same system equations to solve frequency-domain variables (i.e. coefficients of the constituent sinusoids). So, a time-domain variable x becomes \mathbf{X} , a vector of Fourier coefficients. The translation is not so straightforward for derivation and multiplication operations.

Say a time-domain waveform $x(t)$ is defined as follows:

$$\begin{aligned} x(t) &\approx [1 \ \cos \bar{\omega}t \ \sin \bar{\omega}t] * \begin{bmatrix} X_{0,0} \\ X_{0,1c} \\ X_{0,1s} \end{bmatrix} \\ &= \mathbf{C}(t) * \mathbf{X}(t) \end{aligned} \quad (8)$$

Differentiating the above leads to:

$$\begin{aligned} \frac{dx}{dt} &= \frac{d\mathbf{C}(t)}{dt} \mathbf{X} + \mathbf{C}(t) \frac{d\mathbf{X}(t)}{dt} \\ &[1 \ \cos \bar{\omega}t \ \sin \bar{\omega}t] * \\ &\approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \bar{\omega} \\ 0 & -\bar{\omega} & 0 \end{pmatrix} * \begin{bmatrix} X_{0,0} \\ X_{0,1c} \\ X_{0,1s} \end{bmatrix} + \begin{bmatrix} \dot{X}_{0,0} \\ \dot{X}_{0,1c} \\ \dot{X}_{0,1s} \end{bmatrix} \\ &\approx \mathbf{C}(t) \left(\mathbf{T}\mathbf{X}(t) + \frac{d\mathbf{X}(t)}{dt} \right) \end{aligned} \quad (9)$$

We can compute the **T-matrix** for higher orders in a similar fashion and deduce a general formula. A similar process can be followed to understand how multiplication in time-domain converts to convolution in frequency domain.

An interesting thing about convolving two vectors is that new frequencies emerge in the form of sums and differences of pairs of existing frequencies. It is up to the user to select which new frequencies are of interest and should be retained for the system solving.

Using the mathematics developed above, the system equations (3-7) can be transformed as follows:

$$\frac{d}{dt} \mathbf{I}_{batt} = \frac{1}{L_f} \mathbf{V}_{batt} - \left(\frac{R_{batt}}{L_f} + \mathbf{T} \right) \mathbf{I}_{batt} - \frac{1}{L_f} \mathbf{V}_f \quad (10)$$

$$\frac{d}{dt} \mathbf{V}_f = \frac{1}{C_f} \mathbf{I}_{batt} - \frac{1}{C_f} \mathbf{I}_f - \mathbf{T}\mathbf{V}_f \quad (11)$$

$$\mathbf{I}_f = \mathbf{Q}_a \otimes \mathbf{I}_a + \mathbf{Q}_b \otimes \mathbf{I}_b + \mathbf{Q}_c \otimes \mathbf{I}_c \quad (12)$$

B. Switching Function in MFA

Stated in [3] (and ratified independently) that the predominant frequency components in the spectrum of a PWM switching (gate) signal are the DC, fundamental ($\bar{\omega}$), switching frequency ($\hat{\omega}$), and some sidebands to the switching frequency ($n\hat{\omega} + i\bar{\omega}$). The gate signal (for each leg of inverter) is essentially a binary 0 or 1 at any time, upper and lower switches complimentary to each other. Mathematically, the switching signal (gate signal) can be expressed as:

$$\begin{aligned} q(t) &= q_{0,0} + q_{0,1c} \cos \bar{\omega}t + q_{0,1s} \sin \bar{\omega}t \\ &+ \sum_{n=1}^{\infty} \sum_{i=-\infty}^{\infty} q_{n,ic} \cos(n\hat{\omega}t + i\bar{\omega}t) \\ &+ \sum_{n=1}^{\infty} \sum_{i=-\infty}^{\infty} q_{n,is} \sin(n\hat{\omega}t + i\bar{\omega}t) \end{aligned} \quad (15)$$

Coefficients in (15) and derivation can be found in [3]. The DC component is 0.5 with the frequency components broken into pairs of cosine and sine. Indices n, i signifies the order of the carrier and fundamental frequency in the harmonic. The MFA model is set up based on this indexing methodology: a pair of index values (n, i) identify a frequency component (k) and the modeler is free to choose any number of pairs in the model based on requirement, at the cost of increased computation time.

C. Implementing variable frequency: Modified T-Matrix

An approach is to modify the T-matrix (see 1st expression of RHS of 9, apply Chain and Product rule) to account for variable frequency:

$$\begin{aligned} \frac{d\mathbf{C}(t)}{dt} &= \mathbf{C}(t)\mathbf{T}(t) \\ &[1 \ \cos \bar{\omega}t \ \sin \bar{\omega}t]^* \\ \begin{bmatrix} 0 \\ -(\bar{\omega} + \dot{\bar{\omega}}t) \sin \bar{\omega}t \\ (\bar{\omega} + \dot{\bar{\omega}}t) \cos \bar{\omega}t \end{bmatrix}' &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & (\bar{\omega} + \dot{\bar{\omega}}t) \\ 0 & -(\bar{\omega} + \dot{\bar{\omega}}t) & 0 \end{bmatrix} \end{aligned} \quad (16)$$

A generic formula for **modified T-matrix** can be computed as follows: if k represents the frequency component in the model, then modified T-matrix is a zero matrix of size $(2k+1, 2k+1)$ with exceptions in the cells around the diagonal. The cell (row= k , col= $k+1$) is populated with $n\hat{\omega} + i\bar{\omega} + i\dot{\bar{\omega}}t$ and the cell (row= $k+1$, col= k) is the same value with negative sign, as below:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & (\bar{\omega} + \dot{\bar{\omega}}t) & \dots & 0 \\ 0 & -(\bar{\omega} + \dot{\bar{\omega}}t) & 0 & & 0 \\ & \vdots & & \ddots & n\hat{\omega} + i(\bar{\omega} + \dot{\bar{\omega}}t) \\ 0 & 0 & 0 & -(\hat{n}\bar{\omega} + i(\bar{\omega} + \dot{\bar{\omega}}t)) & 0 \end{bmatrix}$$

Next section shows good reconciliation with detailed time-domain switching model as benchmark. *This approach makes the system time-variant that needs to be assessed.*

V. RESULTS

Three MFA configuration models (below) have been benchmarked against detailed switching model in time-domain using parameters (below). Fixed-step integrator is used in MATLAB/Simulink with 1 μ s step for time-domain model and 1ms step for MFA and run for 0.1s (steady state start) in one go with input modulation frequency profile as in Fig. 6.

Parameter	Value	Parameter	Value
Carrier freq. $\hat{\omega}$	2kHz	L_f/C_f (filter)	1mH/1mF
Modulation freq. $\bar{\omega}$	50Hz \pm	R/L (machine)	1 Ω /5mH
V_{batt}/R_{batt}	100V/1 Ω	K_{BEMF}	10V/kRPM
MFA-1	{(0,1)}: DC & fundamental only (SSA equivalent)		
MFA-2	{(0,1), (1,-2), (1,0), (1,2)}: included carrier and sideband		
MFA-3	{(0,1), (1,-3), (1,-2), (1,0), (1,2), (1,3)}: extra sideband		

A. Fully-customizable configuration of MFA model

One key advantage of adopting MFA methodology is the capability to choose whichever harmonics for required waveform fidelity in the simulation (Fig. 3). MFA output of stator currents is compared against benchmark. RMS error is indicated in the figure but it's important to observe that error is inflated because of slight phase shift which is inconsequential in practical use. MFA waveform with 1ms phase advance is also indicated. It is observed that while MFA-2 waveform is clearly more representative of the switching waveform (inset), there is no significant improvement in error in comparison with MFA-1. Upon closer inspection, it was identified that error value in different segments of a whole wavelength varies, i.e. highest in zero-crossing regions (~ 0.38 A).

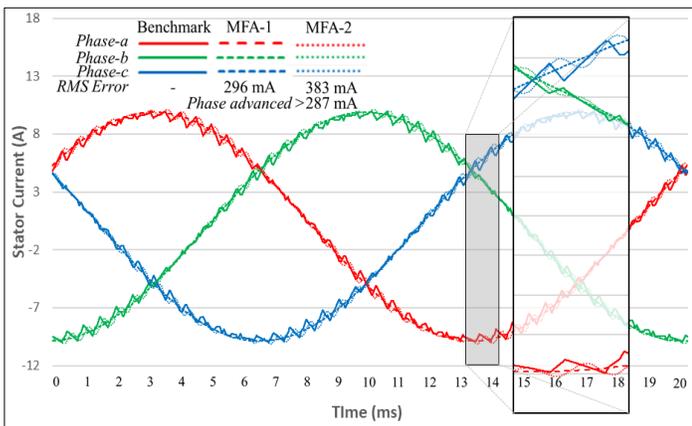


Fig. 3. Customizable output-accuracy of MFA by including certain harmonics

Filter output current (Fig. 4) is produced by convolution of stator currents and gate signals which brings attention to an implementation challenge: *new harmonics may become predominant*. Herein, only the DC component is remaining in the MFA-2 configuration since after convolution the carrier and sideband harmonics become zero. If harmonic content is required for these waveforms, additional sidebands need to be computed for the whole system. MFA-3 configuration includes the 3rd sideband to carrier which improves the waveform.

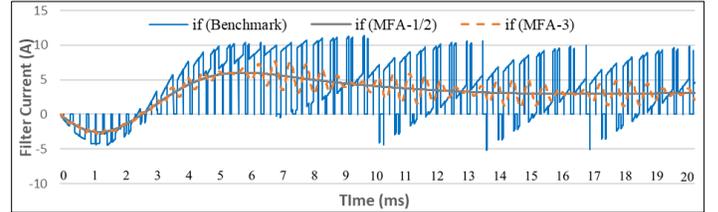


Fig. 4. DC source (battery + filter) output current

B. Simulation Time (variable frequency operation)

MFA-1	28.8%
MFA-2	29.1%
MFA-3	30.8%
Benchmark	

Fig. 5. Comparison of simulation time of MFA with switching model

Figure above shows there is appreciable reduction in simulation time compared with switching model, and only minor increment for a higher accuracy MFA configuration. Equivalent results in [3] showed much greater time advantage ($\sim 5\%$ - 20% compared to 29%-31% here) over switching model and higher segregation among MFA models themselves, *which suggests there is high processing overhead from the convolution steps (which is missing in [3])*. Another interesting point to highlight here is that if the user requires waveforms in time-domain, significant time advantage would be lost since the time-stepping would have to be reduced to adequately resolve the waveforms. Nyquist's rule dictates time-step to be of at-most half of the wavelength of the fastest harmonic being computed.

C. Variable Frequency operation

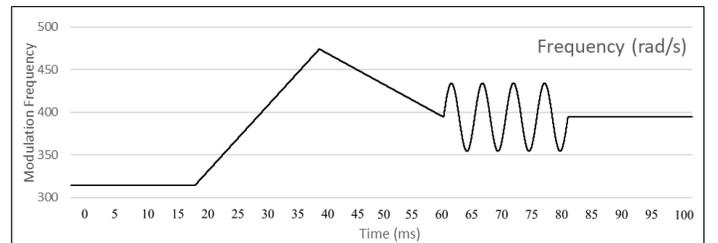


Fig. 6. Modulation frequency input profile applied to MFA model

The variable frequency profile (Fig. 6) was applied to the MFA model. A frequency ramp of 8000 rad/s² and -4000 rad/s²

was applied at 20 and 40 ms respectively, followed by sine profile. The back EMF from the rotor increases with increased rotation accordingly as per the motor constant.

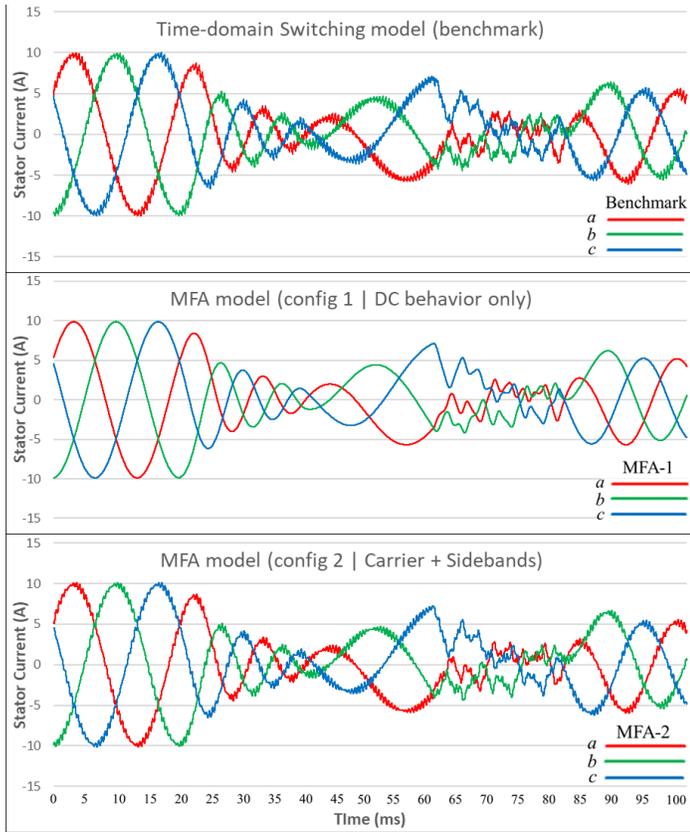


Fig. 7. Stator current profile from modified MFA model in different configurations benchmarked against time-domain switching model

As seen above (Fig. 7), the MFA models can capture the variations introduced by changing frequency adequately well. Remote operation is facilitated by freedom to change any of the battery (V_{bat} , R_{bat}), filter (V_f , L_f) and machine parameters (R , L) on-the-fly. The idea is to do all the time-indeterminate communications (internet) on the frequency domain and it is imperative to have the slave model based on this.

VI. CONCLUSIONS

A schematic for enabling geographically-distributed multiple HiL automotive powertrain simulation test-bed has been envisioned after thorough literature review. A remotely-operable model of a generic electric powertrain has been developed based on MFA methodology. Variable frequency operation with MFA has been explored in a suitable application for the first time with appreciable results. One-touch complexity-scaling of output

waveform capability has been quantitatively depicted although the RMSE advantage is not as high as expected from visual inspection. If application requires specific harmonics to be computed this methodology is highly recommended. Moreover, significant advantage is observed in terms of simulation run-time, as MFA finishes simulation in less than a third of switching model.

Mathematical foundation of MFA built in [1] [2] [3] has been expanded herein to incorporate variable frequency operation. An input LC filter to inverter has been added in the application in [3] which introduces the added complexity of convolving state signals (multiplication in time-domain), main impact of which are higher processing overhead and emergence of new dominant harmonics. The MFA methodology demonstrated here is limited to working on a common set of harmonics for the entire system whereas the above observation implies more optimization can be attained if different sub-systems can run on different sets of harmonics. *Future work would include the above improvements. Work is underway to operate the inverter-machine using a time-domain controller; and prospectively transform the whole system in the DQ plane for torque control.*

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