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Optimal licensing contract: the implications of preference function*

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Abstract: We show the implications of preference function on the optimal licensing contract. As the market expansion effect gets stronger, it increases the possibility of a royalty only contract, thus reducing the co-existence of positive fixed-fee and output royalty.

Key Words: Licensing; Fixed-fee; Royalty

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1. Introduction

The seminal papers by Kamien and Tauman (1984 and 1986) show that "outside innovators"[1] prefer fixed-fee licensing and auction than royalty licensing, regardless of the industry size and/or the magnitude of innovation. Hence, the wide prevalence of output royalty in the licensing contracts (see, e.g., Rostoker, 1984) remained a puzzle, and created significant interest in analysing the implications of technology licensing.[2]

In an earlier work, Rockett (1990) considers a duopoly market with an inside innovator and homogeneous products and shows that the equilibrium licensing contract consists of a positive output royalty only if there is no imitation. In a duopoly market with homogeneous products, Wang (1998) shows that a licenser prefers royalty licensing than fixed-fee licensing if the licenser is an inside innovator. Although these papers provide new insights, they cannot explain an important fact, i.e., the existence of positive fixed-fee and output royalty in the licensing contracts, in the absence of imitation, which may be the outcome of a strong patent system.[3]

Sen and Tauman (2007), Mukherjee and Balasubramanian (2001) and Mukherjee (2014) show the implications of number of firms, product differentiation, and decreasing returns to scale, in explaining the existence of positive fixed-fee and output royalty in the licensing contracts. Sen and Tauman (2007) show that the result of Rockett (1990) holds if the

[1] Outside (inside) innovator refers to the situation were the innovator is not (is) a product-market competitor of the licensees.
[2] In the case of an outside innovator, Gallini and Wright (1990) show that a technology licensing contract can consists of fixed-fee and output royalty if quality of the licensed technology is private information.
[3] For example, Rostoker (1984) shows that royalty and fixed-fee was used for 46% of time among the firms surveyed.

number of licensees is not more than two; however, if the number of licensees is at least three, the equilibrium contract can involve fixed-fee and output royalty. In a duopoly market with an inside innovator, fixed-fee and output royalty can occur in equilibrium if the firms produce differentiated products (Mukherjee and Balasubramanian, 2001). Mukherjee (2014) shows that fixed-fee and output royalty can occur in the presence of decreasing returns to scale technologies.

We focus on a different aspect in this paper. We show how the consumer's preference function affects the licensing contracts.

We consider a duopoly market with horizontally differentiated products to show how the market expansion effect influences the licensing contract. Considering two popular demand functions due to Shubik and Levitan (1980) (where the market size is independent of the degree of product differentiation) and Bowley (1924) (where the market size is significantly affected by the degree of product differentiation) as two extreme cases, we show that as the market expansion effect gets stronger, the range of product differentiation over which the equilibrium licensing contract consists of output royalty only increases. Hence, the consumer's preference function affects the possibility of having positive fixed-fee and output royalty in the licensing contracts.

The remainder of the paper is organised as follows. Section 2 describes the model and shows the results. Section 3 concludes.

2. The model and the results

Assume that there are two firms, firm 1 and firm 2, competing in a product market like Cournot duopolists with horizontally differentiated products. Assume that the technology of firm 1 is better than the technology of firm 2. The marginal cost corresponding to the technology of firm

1 is c_1, which we normalise to 0 for simplicity, and the marginal cost corresponding to the technology of firm 2 is $c > 0$. This cost difference creates the possibility of technology licensing, which is the focus of this paper. Our results do not depend on the simplifying assumption of $c_1 = 0$.

The inverse market demand function for the *i*th good, is $P_i = 1 - [1 + s(1-g)]q_i - gq_j$, $i = 1, 2,\ i \neq j$, where P_i is the price of the *i*th good, q_i and q_j are the outputs and $g \in [0,1]$ is the degree of product differentiation. This demand function is generated from the utility function $U(q_1, q_2) = (q_1 + q_2) - [1 + s(1-g)]\frac{1}{2}(q_1^2 + q_2^2) - gq_1q_2$. If $g = 0$, the goods are isolated and if $g = 1$, they are perfect substitutes. The parameter $s \in [0, 1]$ measures the degree of market expansion, where $s = 1$ corresponds to no market expansion effect, as in Shubik and Levitan (1980), and $s = 0$ generates a preference function due to Bowley (1924), which shows that the market size significantly increases with higher product differentiation. It is worth noting that product differentiation is important for our analysis. Without product differentiation, i.e., if $g = 1$, the market expansion effect, captured by *s*, has no effect, since the demand functions are independent of *s* for $g = 1$.

If we aggregate the demand functions, we get $(q_1 + q_2) = [1 + g + s(1-g)]^{-1} 2(1 - \overline{P})$, where $\overline{P} = \frac{P_1 + P_2}{2}$ is the average price. As *s* reduces, the total demand increases, implying that the market size increases. If $s = 1$, we get $(q_1 + q_2) = (1 - \overline{P})$, suggesting that the total demand is independent of *g*, as in Shubik and Levitan (1980). If $s = 0$, we get $(q_1 + q_2) = [1 + g]^{-1} 2(1 - \overline{P})$, suggesting that a lower *g* increases the total demand, i.e., the market size increases with higher product differentiation, as in Bowley (1924).

We consider the following game. At stage 1, firm 1 decides whether to license its technology to firm 2. In the case of licensing, firm 1 gives a take-it-or-leave-it licensing contract with a non-negative up-front fixed-fee (*F*) and a non-negative per-unit output royalty (*r*).[4] At stage 2, Firm 2 accepts the licensing contract if it is not worse off by accepting it than rejecting it. At stage 3, conditional on the licensing decision, the firms compete like Cournot duopolists and the profits are realised. We solve the game through backward induction.

2.1. No licensing

First, consider the game under no licensing, which creates reservation payoffs of the firms under licensing.

If there is no licensing, firms 1 and 2 determine their outputs to maximise $\underset{q_1}{Max}[1-(1+\mathrm{s}(1-\mathrm{g}))q_1-\mathrm{g}\mathrm{q}_2]q_1$ and $\underset{q_2}{Max}[1-(1+\mathrm{s}(1-\mathrm{g}))q_2-gq_1-c]q_2$ respectively.

Standard calculation shows that the equilibrium outputs and profits of firms 1 and 2 are respectively $q_1^0=\frac{2[1+s(1-g)]-\mathrm{g}(1-\mathrm{c})}{4[1+s(1-g)]^2-g^2}$, $q_2^0=\frac{2(1-c)[1+s(1-g)]-\mathrm{g}}{4[1+s(1-g)]^2-g^2}$,

$$\pi_1^0=\frac{[1+s(1-g)][2(1+s(1-g))-g\left(1-c\right)]^2}{\left(4(1+s(1-g))^2-g^2\right)^2} \text{ and } \pi_2^0=\frac{[1+s(1-g)][2\left(1-c\right)(1+s(1-g))-\mathrm{g}]^2}{\left(4(1+s(1-g))^2-g^2\right)^2}.$$

We consider that $c<\frac{1}{2}$, which ensures that both firms always produce positive outputs.

2.2. Licensing

[4] It is usual to consider non-negative fixed-fee and royalty in the licensing contract (see, e.g., Rockett, 1990). This may be due to anti-trust regulation to prevent collusion among the firms.

Now consider the game under licensing, which allows both firms to use the technology of firm 1. Under licensing, firms 1 and 2 determine their outputs to maximise $\underset{q_1}{Max}[1-(1+\text{s}(1-\text{g}))q_1-\text{g}\text{q}_2]q_1+rq_2+F$ and $\underset{q_2}{Max}[1-(1+\text{s}(1-\text{g}))q_2-\text{g}\text{q}_1]q_2-rq_2-F$ respectively.

We get the equilibrium outputs and profits as $q_1^l=\frac{2[1+s(1-g)]-g\left(1-r\right)}{4[1+s(1-g)]^2-g^2}$,

$q_2^l=\frac{2\left(1-r\right)[1+s(1-g)]-g}{4[1+s(1-g)]^2-g^2}$,

$\pi_1^l=\frac{[1+s(1-g)]\left(2(1+s(1-g))-g\left(1-r\right)\right)^2}{[4(1+s(1-g))^2-g^2]^2}+\frac{r[2(1+s(1-g))\left(1-r\right)-g}{4(1+s(1-g))^2-g^2}+F$ and

$\pi_2^l=\frac{[1+s(1-g)][2(1+s(1-g))\left(1-r\right)-g]^2}{[4(1+s(1-g))^2-g^2]^2}-F$.

Since firm 1 gives a take-it-or-leave-it contract, it charges the fixed-fee in a way so that the net profit of firm 2 is the same under licensing and no licensing. Hence, the equilibrium fixed-fee is determined by

$$F^*=\frac{[1+s(1-g)][2(1+s(1-g))\left(1-r\right)-g]^2}{[4(1+s(1-g))^2-g^2]^2}-\pi_2^0. \quad (1)$$

The equilibrium royalty is determined by maximizing the following expression:

$$\frac{[1+s(1-g)]\left(2(1+s(1-g))-g\left(1-r\right)\right)^2}{[4(1+s(1-g))^2-g^2]^2}+\frac{r[2(1+s(1-g))\left(1-r\right)-g}{4(1+s(1-g))^2-g^2}+F^* \quad (2)$$

where F^* is given in (1), and the equilibrium fixed-fee and equilibrium royalty are non-negative.

The royalty rate that maximises (2) is given by

$$r^{opt} = \frac{g[2(1+s(1-g))-g]^2}{[1+s(1-g)][8(1+s(1-g))^2-6g^2]} > 0 . \qquad (3)$$

We get from (3) that $r^{opt}(g=1) = \frac{1}{2} > c$, $r^{opt}(g=0) = 0 < c$, r^{opt} is continuous in $g \in [0,1]$ and

$$\frac{\partial r^{opt}}{\partial g} = \frac{(1+s)\left(-2+g+2(-1+g)s\right)\begin{pmatrix} -8(1+s)^3 + 12g(1+s)^2(1+2s) \\ -6(1+s)(g+2gs)^2 + g^3\left(-3+2s\left(3+6s+4s^2\right)\right)\end{pmatrix}}{2\left(-1+(-1+g)s\right)^2\left(4-3g^2-8(-1+g)s+4(-1+g)^2 s^2\right)^2} > 0 .$$

Hence, there exists a critical value of *g*, say, $g^*(s)$, such that $r^{opt} > c$ for $g > g^*(s)$, $r^{opt} = c$ at $g = g^*(s)$ and $r^{opt} < c$ for $g < g^*(s)$.

Since the fixed-fee is non-negative, the equilibrium royalty rate must not exceed *c*, implying that the equilibrium royalty rates are $r^* = c$ for $g \geq g^*(s)$ and $r^* = r^{opt}$ for $g < g^*(s)$. Hence, the equilibrium licensing contract is given by $\{F^* = 0, r^* = c\}$ for $g \geq g^*(s)$, and by $\{F^*(r^* = r^{opt}) > 0, r^* = r^{opt} < c\}$ for $g < g^*(s)$.

The following proposition summarises the above discussion.

Proposition 1: *(i) If* $g \geq g^*(s)$, *the equilibrium licensing contract consists of royalty only and the royalty rate is equal to c (i.e.,* $F^* = 0$ *and* $r^* = c$*).*

(ii) If $g < g^*(s)$, *the equilibrium licensing contract consists of a positive output royalty equal to* r^{opt} *and a positive fixed-fee corresponding to the royalty rate* r^{opt} *(i.e.,* $F^*(r^* = r^{opt}) > 0$ *and* $r^* = r^{opt} < c$*).*

The reason for the above result is as follows. The reason for charging a positive output royalty is to soften competition after technology licensing. However, it distorts the output choice of the licensee, which reduces the fixed-fee. As the products get differentiated, competition between the firms reduces, which reduces the need for softening competition through output royalty. Hence, if the products get differentiated, the royalty rate reduces. If the products are sufficiently differentiated, the royalty rate is less than *c*, which allows firm 1 to charge a positive fixed-fee. However, if the products are close substitutes, the competition softening motive becomes important, and firm 1 charges no fixed-fee and the equilibrium royalty is equal to *c*.

We find that

$$\frac{\partial r^{opt}}{\partial s} = \frac{2g\left(-2+g+2(-1+g)s\right)\begin{pmatrix} 8-20g+18g^2-3g^3-3g^4+6\left(2-3g+g^2\right)^2 s \\ +12(-2+g)(-1+g)^3 s^2+8(-1+g)^4 s^3 \end{pmatrix}}{\left(6g^2\left(-1+(-1+g)s\right)+8\left(1+s-gs\right)^3\right)^2} < 0,$$

implying that as *s* reduces (i.e., the market expansion effect gets stronger), r^{opt} increases and therefore, $g^*(s)$ falls since $\frac{\partial r^{opt}}{\partial g} > 0$. This implies that a lower *s* increases the range of *g* over which the equilibrium royalty rate is equal to *c*.

Hence, the following proposition is immediate.

Proposition 2: *If s falls, i.e., the market expansion effect gets stronger, the range of g over which the equilibrium royalty rate is equal to c increases, implying that the possibility of an equilibrium licensing contract with output royalty only increases.*

The reason for the above result is as follows. As the market expansion effect gets stronger for a given degree of product differentiation, i.e., as *s* decreases, it increases the market size and therefore, firm 1's incentive for softening competition through output royalty. Hence, as the market expansion effect gets stronger, i.e., *s* decreases, the equilibrium royalty rate increases and thus, increases the range of product differentiation over which firm 1 charges only royalty under licensing.

As an example, Figure 1 shows r^{opt} for $s = 0$ and $s = 1$.

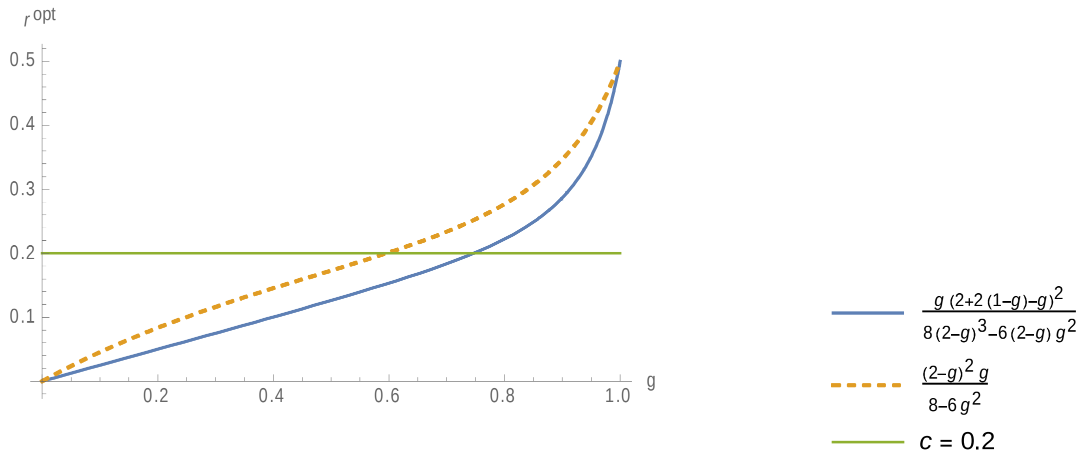

Figure 1

The solid positively sloped curve shows $r^{opt}(s=1) = \frac{g\left(2+2(1-g)-g\right)^2}{8(2-g)^3 - 6(2-g)g^2}$ and the dashed positively sloped curve shows $r^{opt}(s=0) = \frac{(2-g)^2 g}{8-6g^2}$. The horizontal line is $c = .2$. Hence, the equilibrium royalty rate is *c* for those values of *g* for which the curves are above the horizontal line. However, the equilibrium royalty rates are given by the curves for those values of *g* for which the curves are below the horizontal line. It is then immediate that the range of *g* for which the equilibrium royalty is *c* is higher under $r^{opt}(s=0)$ compared to $r^{opt}(s=1)$. The above diagram shows that there are values of g for which the equilibrium licensing contract consists of royalty only for $s = 0$ while the licensing contract consists of positive fixed-fee and output royalty for $s = 1$, implying that the consumer's preference function, which affects the market expansion effect (i.e., s), affects the possibility of having positive fixed-fee and output royalty in the licensing contracts.

3. Conclusion

The extant literature shows that the possibility of imitation, number of firms, product differentiation and decreasing returns to scale are the reasons for the co-existence of fixed-fee and royalty in the licensing contracts.

We show in this paper how the consumer's preference function, affecting the market size, influences the licensing contracts. As the market expansion effect gets stronger, the range of product differentiation over which the equilibrium licensing contract consists of output royalty only increases. Hence, the consumer's preference function affects the possibility of having positive fixed-fee and royalty in the licensing contracts.

References

Gallini, N.T. and B.D. Wright, 1990, 'Technology transfer under asymmetric information', Rand Journal of Economics, *Rand Journal of Economics*, 21: 147-160.

Kamien, M.I. and Y. Tauman, 1984, 'The private value of a patent: A game theoretic analysis', *Journal of Economics*, 4 (Supplement): 93-118.

Kamien, M.I. and Y. Tauman, 1986, 'Fees versus royalties and the private value of a patent', *Quarterly Journal of Economics*, 101: 471-491.

Mukherjee, A. and N. Balasubramanian, 2001, 'Technology transfer in a horizontally differentiated product market', *Research in Economics*, 55: 257-274.

Mukherjee, A., 2014, 'Licensing under convex costs', *Journal of Economics*, 111: 288-299.

Rockett, K., 1990, 'The quality of licensed technology', *International Journal of Industrial Organization*, 8: 559-574.

Rostoker, M.D., 1984, 'A survey of corporate licensing', *IDEA: Journal of Law and Technology*, 24: 59-92.

Sen, D. and Y. Tauman, 2007, 'General licensing scheme for a cost-reducing innovation', *Games and Economic Behavior*, 59: 163-186.

Wang, X.H., 1998, ‘Fee versus royalty licensing in a Cournot duopoly model’, *Economics Letters*, 60: 55-62.