Rapid Communications

Multipartite Einstein-Podolsky-Rosen steering sharing with separable states

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The distribution of quantum correlations among remote users is a key procedure underlying many quantum information technologies. Einstein-Podolsky-Rosen steering, which is one kind of such a correlation stronger than entanglement, has been identified as a resource for secure quantum networks. We show that this resource can be established between two and even more distant parties by transmission of a system being separable from all the parties. For two-user scenarios, we design a protocol allowing one to distribute one-way Gaussian steering which can be used subsequently for one-sided device-independent (1sDI) quantum key distribution. Further, we extend the protocol to three-user scenarios to distribute richer steerability properties including one-to-multimode steering and collective steering which can be used for 1sDI quantum secret sharing. All the proposed protocols can be implemented with squeezed states, beam splitters, and displacements, and thus they can be readily realized experimentally. Our findings reveal that not only entanglement but even steering can be distributed via communication of a separable system. Viewed from a different perspective, the present protocols also demonstrate that one can switch multipartite states between different steerability classes by operations on parts of the states.

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Einstein-Podolsky-Rosen (EPR) steering was put forward by Schrödinger [1] to describe the "spooky action-at-a-distance" debated in the original EPR paradox [2,3], which allows one observer to adjust ("steer") the state of another remote observer by local measurements. This special type of quantum correlation offers insights into directional nonlocality [4–14] and differs conceptually from inseparable correlations, also known as entanglement [15,16]. The fact that steering enables verification of shared entanglement even when one party's measurements are untrusted [17–19] makes it an essential resource for a number of applications, such as one-sided device-independent (1sDI) quantum key distribution (QKD) [20–23] and quantum secret sharing (QSS) [24–26], secure quantum teleportation [27–29], and subchannel discrimination [30,31].

However, in general it is harder to establish EPR steering than entanglement, as the former requires a stronger interaction and tolerates less noise than the latter [6,32]. In a practical quantum network, not all users might have the ability to produce steering, and distributing it directly might expose the transmitted quantum states to the unwanted attacks of an eavesdropper. A more efficient scenario would be to have a quantum *cloud server* which can generate quantum states and perform appropriate operations for different tasks, and then establish desired correlations between the users, mediated by transmission of ancillary systems with as little as possible quantum resources. Somehow counterintuitively, it has been shown theoretically [33–35] and experimentally [36–38] that entanglement (inseparability) can be distributed between two parties via a separable ancilla. However, the resources needed to distribute more stringent forms of correlations such as EPR steering and Bell nonlocality remained unexplored.

In this Rapid Communication, we show that EPR steering, which is strictly stronger than entanglement and possesses intrinsical asymmetry, can be faithfully distributed between two and even more distant parties with minimal resources in continuous-variable (CV) Gaussian systems. By preparing locally initial quantum states, performing suitably tailored local correlated displacements on them, and transmitting a *separable* ancilla mode across multiuser networks (see Fig. 1), we show how to establish a plethora of useful steering properties, such as one-way Gaussian steering in two-user scenarios, two-way steering and collective steering which can be used for CV QSS [39,40] in three-user scenarios, and so on. Furthermore, we prove that the distributed steerability among distant users can be maximized by optimal displacements. We further present a modified scheme with a relaxed condition that the ancilla mode used for distribution is nonsteerable instead of separable from the users, yielding a broader range of parameters for which the protocol succeeds. These results

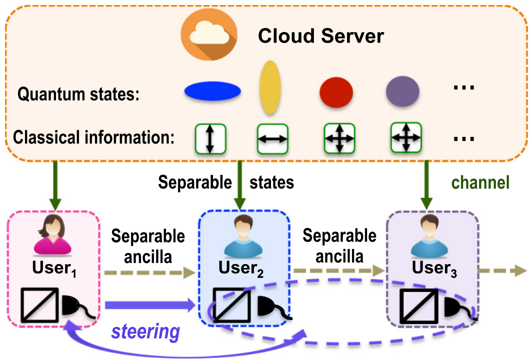

FIG. 1. Scheme to distribute Gaussian EPR steering among different users via a separable ancilla: The quantum cloud server locally produces the quantum states and analyzes the classical information of the displacements required by the task, then sends the separable quantum states to the users. By transmitting a separable ancilla, the users can successfully share EPR steering with desired properties.

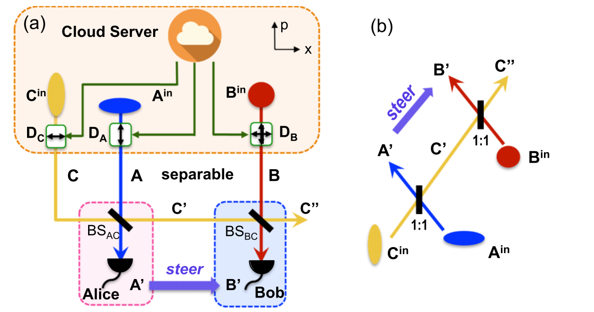

FIG. 2. (a) Sketch of the one-way Gaussian steering distribution protocol. See text for details. The best steerability is recovering the steerability in the tripartite entangled state created by the optical network, as illustrated in (b).

shed light on the rich steering sharing structure in multipartite CV systems and yield experimentally feasible recipes for the scalable realization of 1sDI communication tasks across multiuser quantum networks.

Gaussian steerability. For any $(n_A + m_B)$-mode Gaussian state, we put the amplitude (position) and phase (momentum) quadratures of each mode into a column vector $\hat{\xi} := (\hat{x}_1^A, \hat{p}_1^A, \dots, \hat{x}_{n_A}^A, \hat{p}_{n_A}^A, \hat{x}_1^B, \hat{p}_1^B, \dots, \hat{x}_{m_B}^B, \hat{p}_{m_B}^B)^\top$, satisfying the canonical commutation rules $[\hat{x}_j, \hat{p}_k] = 2i\delta_{jk}$. The properties of the state can be fully specified by its covariance matrix (CM) γ_{AB} with elements $(\gamma_{AB})_{ij} = \langle \hat{\xi}_i \hat{\xi}_j + \hat{\xi}_j \hat{\xi}_i \rangle /2 - \langle \hat{\xi}_i \rangle \langle \hat{\xi}_j \rangle$, which reads as $\gamma_{AB} = \begin{pmatrix} A & C \\ C^\top & B \end{pmatrix}$. Here, the submatrices A and B are the CMs corresponding to the reduced states of each subsystem, respectively. The steerability from Alice to Bob via Gaussian measurements can be quantified by [7]

$$\mathcal{G}^{A\to B}(\gamma_{AB}) := \max\left\{0, -\sum\nolimits_{j:\bar{\nu}_j^{AB\backslash A}<1} \ln\left(\bar{\nu}_j^{AB\backslash A}\right)\right\}, \quad (1)$$

where $\bar{\nu}_j^{AB\backslash A}$ $(j = 1, \dots, m_B)$ are the symplectic eigenvalues of the Schur complement of A defined as $\bar{\gamma}_{AB\backslash A} = B - C^\top A^{-1} C$. The quantity $\mathcal{G}^{A\to B}$ is a monotone under Gaussian local operations and classical communication [41] and vanishes when Alice cannot steer Bob by Gaussian measurements [7,42]. This quantifier has been experimentally measured in Gaussian cluster states by reconstructing the CM [13].

For the sake of experimental feasibility, one can also confirm the presence of steering when the EPR variance product $E_{B|A} := \Delta_{\inf,A}\hat{x}_B \Delta_{\inf,A}\hat{p}_B < 1$. Here, $\Delta_{\inf,A}\hat{x}_B = \Delta(\hat{x}_B - g_x\hat{x}_A)$ is the minimum inferred variance of Bob's position outcome given Alice's result with optimal gain factor $g_x = \langle \hat{x}_B, \hat{x}_A \rangle / (\Delta \hat{x}_A)^2$ [6], where $\langle x, y\rangle := \langle xy \rangle - \langle x \rangle \langle y \rangle$, and $\Delta_{\inf,A}\hat{p}_B$ is defined similarly. The criterion $E_{B|A} < 1$ is necessary and sufficient to test steering by Gaussian measurements. The quantity $E_{B|A}$, which can be efficiently measured by homodyne detection [24], in fact directly quantifies the Gaussian steerability, as $\mathcal{G}^{A\to B} = \max\{0, -\ln E_{B|A}\}$ for all two-mode CMs γ_{AB} in standard form (i.e., with diagonal A, B, C) [7,26].

In the following, we first show that one-way steering can be distributed from Alice to Bob by a separable ancilla. We improve the protocol developed for entanglement [35] by optimizing the displacements to distribute the highest steerability. As depicted in Fig. 2(a), the initial modes A, B, and C sent from the cloud server are in a fully separable state, and the ancillary mode C' is separable from the modes held by Alice and Bob, ensuring the security of the process of establishing Gaussian steering. We prove that the highest steerability which can be distributed by a separable ancilla is determined by the utilized optical network composed only of input squeezed states and beam splitters, as depicted in Fig. 2(b).

Protocol. The protocol consists of three steps. In step 1, the cloud server initially produces modes A^{in} and C^{in} in momentum- and position-squeezed vacuum states, respectively, while mode B^{in} is in a vacuum state. All three modes are in a product state described by the CM $\gamma_{ABC}^{\text{in}} = \text{diag}(e^{2t}, e^{-2t}, 1, 1, e^{-2t}, e^{2t})$, where t is the squeezing parameter. The three modes are then appropriately displaced by local correlated displacements $\hat{p}_{A^{\text{in}}} \to \hat{p}_{A^{\text{in}}} - D_A p_d$, $\hat{x}_{C^{\text{in}}} \to \hat{x}_{C^{\text{in}}} + D_C x_d$, $\hat{x}_{B^{\text{in}}} \to \hat{x}_{B^{\text{in}}} + D_B x_d$, $\hat{p}_{B^{\text{in}}} \to \hat{p}_{B^{\text{in}}} + D_B p_d$. Here, x_d and p_d obey a zero mean Gaussian distribution with the same variance and D_A, D_B, D_C are the strengths of the displacements, to be specified in the second step. The resulting state is fully separable with CM $\gamma_{ABC} = \gamma_{ABC}^{\text{in}} + \tilde{D}$, where $\tilde{D}$ denotes a positive noise matrix created by the displacements.

In step 2, Alice superimposes modes A and C on a balanced beam splitter BS_{AC}, thereby creating a three-mode state with CM $\gamma_{A'BC'} = U_{AC}(\gamma_{ABC}^{\text{in}} + \tilde{D})U_{AC}^\top = U_{AC}\gamma_{ABC}^{\text{in}}U_{AC}^\top + xP$, $x \geqslant 0$. Here, the symplectic matrix U_{AC} describes the beam splitter, and $U_{AC}\gamma_{ABC}^{\text{in}}U_{AC}^\top$ describes a product state of vacuum mode B^{in} and a two-mode squeezed state obtained by mixing the undisplaced squeezed modes A^{in} and C^{in} at the beam splitter [see Fig. 2(b)]. However, the entanglement between the output modes A' and C' can be smeared by adding a sufficiently large noise term xP, where P is a suitable positive matrix and x regulates the strength of noise. Using the method developed in Ref. [44], the matrix P can be constructed as $P = q_1 q_1^\top + q_2 q_2^\top$ from the 6×1 vectors

$q_1 = (0, -1, 0, d_B, 0, -1)^\top$ and $q_2 = (1, 0, d_B, 0, -1, 0)^\top$ to negate the entanglement between A' and C', where the parameter d_B can be optimized to reach the highest steerability from Alice to Bob in the final state. The CM of the state after step 2 becomes

$$\gamma_{A'BC'} = \begin{pmatrix} m\mathbb{1} & d_B x\sigma_z & n\sigma_z \\ d_B x\sigma_z & (1+d_B^2 x)\mathbb{1} & -d_B x\mathbb{1} \\ n\sigma_z & -d_B x\mathbb{1} & m\mathbb{1} \end{pmatrix}, \qquad (2)$$

where $m = \cosh 2t + x$ and $n = \sinh 2t - x$. Hence, one can determine the correlation matrix of displacements $\tilde{D}$ prior to the beam splitter BS_{AC} as $\tilde{D} = xU_{AC}^\top PU_{AC}$. This corresponds to displacement strengths $D_A = D_C = \sqrt{2}$, $D_B = d_B$, and displacement variances $\langle(\Delta x_d)^2\rangle = \langle(\Delta p_d)^2\rangle = x$. The free parameters d_B and x need to be suitably adjusted for the protocol to work, as done in the next step.

In step 3, Bob mixes mode C' sent by Alice with his mode B on the balanced beam splitter BS_{BC}, which yields the final CM $\gamma_{A'B'C''} = U_{BC}\gamma_{A'BC'}U_{BC}^\top$ in the form

$$\gamma_{A'B'C''} = \begin{pmatrix} m\mathbb{1} & \frac{d_B x+n}{\sqrt{2}}\sigma_z & \frac{d_B x-n}{\sqrt{2}}\sigma_z \\ \frac{d_B x+n}{\sqrt{2}}\sigma_z & \frac{1+m+d_B x(d_B-2)}{2}\mathbb{1} & \frac{1+d_B^2 x-m}{2}\mathbb{1} \\ \frac{d_B x-n}{\sqrt{2}}\sigma_z & \frac{1+d_B^2 x-m}{2}\mathbb{1} & \frac{1+m+d_B x(d_B+2)}{2}\mathbb{1} \end{pmatrix}. \qquad (3)$$

Now, by quantifying the amount of the distributed Gaussian steering from mode A' to mode B' via Eq. (1), we get

$$\mathcal{G}^{A'\to B'} = \ln[2\cosh 2t/(\cosh 2t + 1)], \qquad (4)$$

with optimal displacement $d_B^{\text{opt}} = \tanh 2t + 1$. Note that $\mathcal{G}^{A'\to B'} > 0$ for any $t > 0$. Interestingly, the right-hand side of Eq. (4) equals the amount of steerability from A' to B' in the scheme without any displacements, shown in Fig. 2(b) [45]. This means that the optimal displacements ensure separability of the transmitted ancilla from the other modes, while not reducing the maximum steering that can be distributed. Experimentally, one can verify the Gaussian steerablity via the minimum EPR variance product $E_{B'|A'} = (\cosh 2t + 1)/(2\cosh 2t)$ by homodyne detection. One finds $E_{B'|A'} < 1$ for any $t > 0$.

Discussion. From Eq. (4), we find that the Gaussian steerability from Alice to Bob with displacement d_B^{opt} can be distributed for any $t > 0$; however, we need also check the separability of the states in steps 2 and 3 to assure that the transmitted ancilla stays separable from the rest at all stages.

After step 2, the shared state transforms from a fully separable state to a two-mode biseparable state following the classification of Ref. [44]. Making use of the positive partial transpose (PPT) criterion [46] one finds that the state with CM (2) is entangled across $A' - (BC')$ splitting for any $x > 0$ and $t > 0$, but it is separable with respect to $C' - (A'B)$ splitting when $x \geqslant x_{C'-(A'B)} = 2\cosh^2 2t \sinh t/(\cosh t + \cosh 3t + \sinh t)$ [see the blue solid curve in Fig. 3(a)], and furthermore, it is separable with respect to $B - (A'C')$ splitting for any $x > 0$ and $t > 0$. Since steering is strictly stronger than entanglement, the CM (2) also represents a state that is nonsteerable

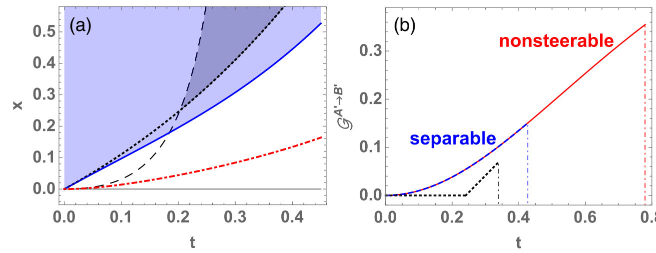

FIG. 3. (a) The blue solid curve represents the threshold of $x_{C'-(A'B)}$, above which the state is separable with respect to $C' - (A'B)$ splitting. Since $\mathcal{G}^{A'\to B'} > 0$ for any $t > 0$ with optimal d_B^{opt}, the parameters above this curve can be used to distribute Gaussian steering by transmitting a separable mode C'. With a nonoptimal $d_B = 2$, a much narrower range of parameters within the dark blue (dark gray) area between $x_{A'\to B'}$ (black dashed) and x_{sep} (black dotted) can be used to distribute steering from A' to B'. The red dashed-dotted curve shows a relaxed threshold for absence of steering instead of entanglement across $C' - (A'B)$ splitting in step 2, above which one can use a nonsteerable mode C' to distribute steering. (b) The distributed steerability $\mathcal{G}^{A'\to B'}$ for $x = 0.5$ in the protocol with optimal d_B^{opt} by a separable (blue dashed) or nonsteerable (red solid) ancilla, and in the case of nonoptimal $d_B = 2$ (black dotted).

with respect to $C' - (A'B)$ splitting if $x \geqslant x_{C'-(A'B)}$ and $B - (A'C')$ splitting for any $x > 0$ and $t > 0$. However, the steerability $\mathcal{G}^{A'\to(BC')} > 0$ for all $x > 0$ and $t > 0$, which is essential for the performance of the steering distribution from Alice to Bob in the final state. Without the help of the transmitted mode C', the second beam splitter alone cannot create steering. In the blue (light gray) area above the blue solid curve $x \geqslant x_{C'-(A'B)}$, the state after step 3 described by the CM (3) remains separable with respect to $C'' - (A'B')$ splitting. Therefore, in this area, the ancilla mode is separable from the rest at all stages, nevertheless, for the Gaussian steerablity of the final state one gets $\mathcal{G}^{A'\to B'} > 0$ for any $t > 0$, which means that Gaussian steering is successfully distributed from Alice to Bob. If we relax the condition that the ancilla is separable to that it is nonsteerable from the rest (i.e., it may be entangled), then the distribution task can be accomplished in an even larger region of parameters, as shown in the area above the red dashed-dotted curve in Fig. 3(a).

Figure 3(b) shows the amount of one-way Gaussian steerability that can be distributed via sending a separable ancilla (blue dashed) and a nonsteerable ancilla (red solid), respectively. One can find that both of them distribute equal steerability from Alice to Bob, but work at two different ranges of initial squeezing parameter t for a fixed value $x = 0.5$. By sending a nonsteerable ancilla, the initial squeezing level is requested to satisfy $0 < t < 0.78$ to guarantee that the transmitted mode C' is nonsteerable from $(A'B)$ at all stages, while by sending a separable ancilla, the initial squeezing level is requested to satisfy a more stringent inequality $0 < t < 0.43$.

Comparing previous results with a nonoptimal displacement, say, $d_B = 2$ [35], one finds that the distribution of steering via a separable ancilla can only work in the range of $x_{\text{sep}} = (e^{2t} - 1)/2 \leqslant x < x_{A'\to B'} = (1 - e^{2t})^2/(4 - 2e^{2t})$, depicted by the dark blue (dark gray) area in Fig. 3(a), which

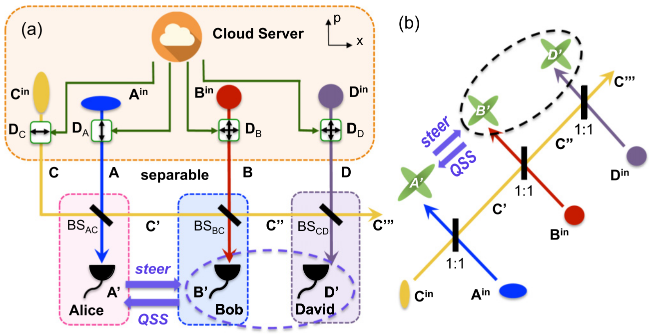

FIG. 4. (a) Scheme for distribution of tripartite Gaussian steering via a separable state. (b) Optical network with the same amount of steering as the optimal scheme with displacements.

is much narrower than the area corresponding to the optimal $d_B^{\rm opt}$. In addition, the distributed steerability [black dotted in Fig. 3(b)] is also lower than that given by the optimal protocol (blue dashed curve). For a fixed value of $x = 0.5$, the protocol with $d_B = 2$ can only work for squeezings obeying $0.241 < t < 0.346$, thus requiring a nontrivial threshold as opposed to the condition $t > 0$ for optimal $d_B^{\rm opt}$.

We have discussed the distribution of Gaussian steering from Alice to Bob with a separable or nonsteerable ancilla. Can the distributed state simultaneously display also Gaussian steering from Bob to Alice in the setup given above? The answer is no. According to the CM (3), mode B' and mode C'' are completely symmetric in the final state. Due to the monogamy relation of Gaussian steering with two observables $\hat{x}$ and $\hat{p}$ [47], neither of them can steer mode A', so that only one-way Gaussian steering from Alice to Bob is distributed using the above setup. If Bob wants to steer Alice, he can send the request to the cloud server, and the server can simply switch the initial quantum states and displacements for Alice and Bob.

Multiuser distribution. The scheme can be extended to the multiuser case as shown in Fig. 4(a). Bob continues to send the separable mode C'' to David who mixes it with his mode D on a balanced beam splitter BS_{CD}. It not only distributes tripartite steering from Alice to Bob and David, but also creates a collective steering in the opposite direction.

To accomplish steering distribution in the direction $A' \to B'D'$, apart from the condition $x \geqslant x_{C'-(A'B)}$ that assures the mode C' to be separable from modes $(A'B)$ after step 2, we also need to find further constraints on x and t guaranteeing that the ancilla mode C'' is separable from subsystem $(A'B'D)$ when $x \geqslant x_{C''-(A'B'D)}$ after step 3. Besides, we also need to suitably adjust d_D in the displacement vectors $q_1 = (0, -1, 0, d_B^{\rm opt}, 0, -1, 0, d_D)^\top$ and $q_2 = (1, 0, d_B^{\rm opt}, 0, -1, 0, d_D, 0)^\top$ to distribute steerability as large as possible. The CM of the resulting four-mode state is detailed in the Supplemental Material [45]. One can prove that with optimal displacement $d_D^{\rm opt} = \sqrt{2} d_B^{\rm opt}$, the highest distributed steerability reads $\mathcal{G}^{A'\to B'D'} = \ln[4\cosh 2t/(3 + \cosh 2t)]$, $\mathcal{G}^{A'\to D'} = \ln[4\cosh 2t/(1 + 3\cosh 2t)]$, $\mathcal{G}^{A'\to B'} = \ln[2\cosh 2t/(1 + \cosh 2t)]$, and it can be achieved for any $t > 0$. Since $x_{C'-(A'B)} > x_{C''-(A'B'D)}$ reported in Ref. [45], in the blue area in Fig. 3(a) one can perfectly restore the steering of $A' \to B'D'$, $A' \to B'$, and $A' \to D'$ generated by the displacement-free optical network shown in Fig. 4(b) [45].

For the opposite direction $B'D' \to A'$, keeping $d_B^{\rm opt}$ we can distribute the maximum steerability $\mathcal{G}^{B'D'\to A'} = \ln[(1 + 3\cosh 2t)/(3 + \cosh 2t)]$ with $d_D^{\rm opt} = (2 + 2\coth t + \tanh t - \tanh 2t)/\sqrt{2}$, which is recovering the steering created in Fig. 4(b). Note that the protocol works only when $t \geqslant 0.943$ and $x \geqslant \max\{x_{C'-(A'B)}, x_{C''-(A'B'D)}\} = x_{C''-(A'B'D)}$ [45] to assure the state to be separable with respect to $C' - (A'B)$ splitting, as well as $C'' - (A'B'D)$ splitting. For smaller t, we need to optimize d_B and d_D simultaneously. We find that when $0.28 \leqslant t < 0.943$, the distributed steering $\mathcal{G}^{B'D'\to A'}$ can be still maximized by choosing some numerically optimized displacements d_B and d_D, while when $t < 0.28$, it is impossible to achieve the same amount of steerability as in the scheme in Fig. 4(b) [45]. In this case, $\mathcal{G}^{B'\to A'} = \mathcal{G}^{D'\to A'} = 0$, which means that neither Bob nor David can individually steer Alice, but they can do that only if they collaborate.

This makes the state a perfect resource for 1sDI QSS, where Alice does not trust Bob and David's devices. Assume Alice acts as the dealer who sends a secret encoded in her state, while Bob and David are players aiming at decoding the message together. To provide security against eavesdropping, a guaranteed secret key rate for the QSS protocol is given by $K \geqslant \ln[2/(eE_{A'|B'D'})] = \mathcal{G}^{B'D'\to A'} - \ln(e/2)$ [25,26]. A state whose correlations fulfill $K > 0$ is certified as a useful resource. Referring to the studied scheme, the condition translates into $\cosh 2t > (3e - 2)/(6 - e)$, which means that a squeezing level of 5.4 dB ($t > 0.62$), is required to ensure a nonzero key rate. This is well within the current experimental feasibility, since up to 10 dB of squeezing has been demonstrated [48,49].

Conclusions. We proposed a protocol for the distribution of Gaussian EPR steering across multiuser networks with a separable ancilla. Rich steering properties, such as one-way, one-to-multimode, and collective Gaussian steering, can be distributed via local operations on parts of an initially fully separable state and communication of a separable part of the state. In particular, we derive analytical thresholds on the displacements as a function of the squeezing degree of the initial states such that the protocol succeeds, and prove that the largest steerability that can be distributed recovers that of the multimode states created by the same optical network without correlated displacements. The proposed protocols can be implemented by performing suitable local correlated displacements on the input states of linear optical networks usually used to generate multipartite CV entangled states and hence they are feasible with current technology. Realization of the protocols in the laboratory is particularly attractive in view of the fact that previous experiments on Gaussian entanglement distribution by separable states [37,38,50] were unable to distribute any steering. While our study focused on CV systems and Gaussian measurements, future interesting work can be devoted to steering distribution protocols for qubits as well, building on the seminal study of entanglement distribution with separable states [33]. It would also be interesting to examine our result by applying other feasible criteria, such as the steering inequality with tolerance for measurement-setting errors [51], for non-Gaussian systems.

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