A model of firm heterogeneity in factor intensities and international trade[*]

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Abstract

Empirical evidence suggests that exporters – besides being more productive – are significantly more skilled labor intensive than non–exporters. In a setting which captures both these features, we show that the firm selection induced by trade liberalization works along two dimensions. First, export growth increases competition for skilled labor. This leads to the exit of some of the skilled labor intensive firms, while benefitting unskilled labor intensive ones. Second, within the group of firms with the same factor intensities, the reallocation of factors is towards the exporters. We show that the increased competition for skilled labor dampens the positive effect of trade liberalization on sector–wide TFP and real income.

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1   Introduction

In this paper we study the impact of trade liberalization in a model where firms can choose their factor intensities in production. As this choice affects their export–status, we analyze a selection mechanism that complements the one highlighted by Melitz (2003) which works instead through heterogeneity in total factor productivity (TFP). By doing so, we develop a theoretical framework that can account for the large heterogeneity in factor intensities that has been identified in the empirical literature.[1] Importantly, our model helps also rationalizing the recent evidence suggesting that the effects of trade liberalization on sector–wide TFP might only be moderate (Lawless and Whelan 2008; Chen, Imbs, and Scott 2009).

We cast our discussion in a general equilibrium setting with one monopolistically competitive sector in each country. Each firm produces a unique variety of a differentiated final good using skilled and unskilled labor. Upon market entry, firms choose the factor share parameter characterizing their CES production function and, afterwards, are randomly assigned a TFP level. Importantly, firms find it optimal to adopt different factor intensities to limit competition in factor markets. Our analysis starts by characterizing the autarkic equilibrium. Next, we study the trade equilibrium arising in a symmetric N–country world. In a setting with fixed export costs, and in which skilled labor intensive firms are more likely to serve the foreign market, we show that the firm selection induced by trade liberalization works along two dimensions.

First, more intense competition in factor markets induced by the additional production required to serve the export markets increases the relative price of skilled labor. This has a negative effect on those firms that use this factor intensively, and a positive one on unskilled labor intensive firms and this effect becomes stronger, the larger is the difference in factor intensities between the two types of firms. As a result, some of the skilled labor intensive firms might be forced to cease production. Second, within each of the two types of firms with the same factor input choice, we observe a selection against the non–exporters, as in Melitz (2003). While the latter process increases sector–wide TFP, the first one has a priori an ambiguous effect. Still, under some mild assumptions, we show that the larger is the difference in factor intensities between firms, the smaller is the increase in sector–wide TFP induced by trade liberalization. Thus, factor market competition dampens the positive effect of trade on sector–wide TFP and on the change in real income.[2]

Our paper contributes to the literature on trade with firm heterogeneity, which has been pioneered by Bernard, Eaton, Kortum, and Jensen (2003) and Melitz (2003). Bernard et al. (2007) extend the Melitz (2003) setup by considering two factors of production and, additionally, two monopolistically competitive sectors with different capital–labor ratios in production. As a result, they are able to provide important insights into the inter–industry and intra–industry factor reallocations induced by trade liberalization. At the same time, in their model firms are homogeneous with

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[1]See Bernard and Jensen (1995) for the United States, Alvarez and López (2005) for Chile, Munch and Skaksen (2008) for Denmark, Wagner (2010) and Klein, Moser, and Urban (2013) for Germany and Martins and Opromolla (2011) for Portugal among many others.

[2]For recent alternative explanations on the moderate TFP impact of trade liberalization see Atkeson and Burstein (2010) or Raff and Wagner (2010).

respect to factor intensities within each sector and a firm's export status only depends on its TFP. Thus, they do not analyze how firm heterogeneity in factor intensities interacts with globalization. In Yeaple (2005), firms choose instead their technology upon market entry. Labor is the only factor of production, but workers differ in their skills and for each technology, a higher skill level is assumed to lead to higher revenues per worker. Similarly, a more advanced technology also leads to higher revenues for any given skill level of the employee. Because of this monotone relationship, trade liberalization generates the same type of firm selection as in Melitz (2003): the relative mass of exporters increases, whereas the relative mass of non–exporters decreases. In our setup, on the other hand, firms produce with standard CES technologies with two inputs, and for this reason we do not have a monotone relationship between factor intensities and profits. While the paper by Yeaple (2005) provides important insights on how trade liberalization affects workers' skill–premia, it does not consider firm heterogeneity in factor intensities and thus it cannot explain those stylized facts about trade liberalization, which refer to factor market competition.

The papers that come closest to ours are Emami Namini (2014), Crozet and Trionfetti (2011) and Furusawa and Sato (2008). All of these contributions develop models of trade in which firms within the same sector differ in factor intensities. Emami Namini (2014) considers a setting in which the factor intensity parameter is randomly assigned to firms and studies the impact of trade liberalization on welfare and growth. Because of the randomness of the technology assignment, the relative mass of firms with different factor intensities is given exogenously, whereas the study of the effect of trade liberalization on firm selection is the focus of this paper. Crozet and Trionfetti (2011) also consider a model with random factor intensities and study how a firm's technology and a country's relative factor endowment interact to determine a firm's sales volume. Furusawa and Sato (2008) assume instead a random TFP parameter like in Melitz (2003) and a technology in which a continuum of intermediate inputs, which differ in their factor intensities, is used to produce a final good. Their focus is on the effects of trade liberalization on the adoption of a new technology for the intermediate good and like in Crozet and Trionfetti (2011), Furusawa and Sato (2008) do not consider the effect of the heterogeneity in factor intensities on firm selection.[3]

The remainder of the paper is organized as follows. Section 2 lies out our model, whereas in section 3 we characterize the autarkic equilibrium. In section 4 we solve for the open economy equilibrium in a symmetric N–country setting, and in section 5 we study how trade liberalization impacts sector–wide TFP and real income. Section 6 concludes the paper.

2   Model setup

Home's economy is populated by a continuum of households of unit mass and has a single monopolistically competitive industry. We start by describing the demand side, and proceed then to consider production, focusing on the technologies available to the firms and on market entry.

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[3]Our analysis is also related to the vast body of literature that has studied the link between globalization and wage inequality. For a recent survey, see Goldberg and Pavcnik (2007).

The preferences of the representative household are given by a CES utility function of the type

$$U = \left[\int_{v \in \Upsilon} q(v)^{\frac{\xi-1}{\xi}} dv \right]^{\frac{\xi}{\xi-1}}, \tag{1}$$

where $\xi > 1$ is the elasticity of substitution between different varieties, and Υ is the set of available varieties $q(v)$, indexed by v. Each household is endowed with fixed amounts of skilled and unskilled labor, respectively denoted by S and L. The country's aggregate factor income is given by:

$$Y = w_S \overline{S} + w_L \overline{L},$$

where w_S and w_L are respectively the returns to skilled and unskilled labor and $\overline{S}$ and $\overline{L}$ the aggregate factor supplies.[4] The aggregate demand for each individual variety is given by:

$$q(v) = Y P^{\xi-1} p(v)^{-\xi}, \tag{2}$$

where $P = \left[\int_{v \in \Upsilon} p(v)^{1-\xi} dv \right]^{\frac{1}{1-\xi}}$ is the price index, which is dual to the utility function, and $p(v)$ the price of variety $q(v)$.

Turning to the supply side of the economy, there is a continuum of identical potential entrants, each of which can produce a different variety of the same good, combining skilled and unskilled labor according to a CES technology. Firms start by choosing the parameter $\phi_i \in \{\phi_L, \phi_S\}$, with $\phi_S > \phi_L$, determining the factor intensities in production. Next, to actually enter the market, we follow Melitz (2003) and assume that firms pay a sunk market entry fee f^E, which allows them to draw a TFP parameter A from a common and exogenously given Pareto distribution with support $[1, \infty)$ and cumulative density $G(A) = 1 - A^{-k}$, $k > \xi - 1$.[5] Since the random TFP parameter reflects a firm's uncertainty about, e.g., how well workers perform, it is reasonable to assume that a firm learns its TFP after it has chosen its skilled labor share parameter. A firm's ϕ and TFP parameter remain fixed thereafter, but a firm faces a constant and exogenous death probability θ, $0 < \theta < 1$, forcing it to exit the market.[6] The production function of a firm with skilled labor share

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[4]Note that we are assuming each household, which we index with β, $\beta \in B$, to supply S and L units of the inputs. Aggregate factor supplies are thus given by $\int_{\beta \in B} S d\beta = \overline{S}$ and $\int_{\beta \in B} L d\beta = \overline{L}$.

[5]We assume that both skilled and unskilled labor intensive firms draw their TFP parameter from the same distribution to separately capture the effects of heterogeneity in factor intensities and TFP. In a recent paper Harrigan and Resheff (2011) consider instead a setting in which skill intensity is strongly positively correlated with TFP, and trade liberalization induces a firm selection process that is very similar to that identified in Melitz (2003). We will show later that the assumption $k > \xi - 1$ for the shape parameter k is necessary for the equilibrium to exist. Axtell (2001) and Luttmer (2007), amongst others, have shown that a Pareto distribution describes appropriately the distribution of TFP across firms in manufacturing.

[6]As in Melitz (2003), we will focus only on steady state equilibria. Moreover we assume that households do not discount the future and that there are no savings opportunities in the economy. The constant death probability implies a constant firm turnover and a constant amount of sunk entry costs in each instant of time in the steady state.

parameter ϕ_i is given by:

$$q_i(A) = A \left[\phi_i^{1-\alpha}(S_i\Omega_S)^\alpha + (1-\phi_i)^{1-\alpha}(L_i\Omega_L)^\alpha \right]^{\frac{1}{\alpha}}, \quad \alpha < 1, \tag{3}$$

where $q_i(A)$ is the firm's output, S_i and L_i are the inputs of skilled and unskilled labor of firm i, and Ω_S and Ω_L are factor specific productivity parameters. As a result, $S_i\Omega_S$ and $L_i\Omega_L$ denote the effective units of factor inputs, and we assume $\Omega_S > 1$ and $\Omega_L = 1$ to capture the idea that one unit of skilled labor is more productive than one unit of unskilled labor.[7] The elasticity of substitution between inputs is given by $\sigma = \frac{1}{1-\alpha} > 0$.[8] Based on the empirical literature, we will assume $\xi > \sigma$ in the remainder of the analysis, i.e. that varieties are closer substitutes in consumption than factors in production.[9]

The marginal cost $c_i(A)$ of a firm with factor share parameter ϕ_i is given by:

$$c_i(A) = \frac{1}{A} \left[\phi_i \left(\frac{w_S}{\Omega_S} \right)^{1-\sigma} + (1-\phi_i)w_L^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad \sigma > 0.$$

Clearly, if $\frac{w_S}{\Omega_S} \neq w_L$, different values of ϕ lead to different marginal costs, and if $\frac{w_S}{\Omega_S} < w_L$, $c_S(A) < c_L(A)$, and viceversa. Production also requires a fixed cost which takes the following form:

$$F_i^P = Ac_i(A)f_i^P = \left[\phi_i \left(\frac{w_S}{\Omega_S} \right)^{1-\sigma} + (1-\phi_i)w_L^{1-\sigma} \right]^{\frac{1}{1-\sigma}} f_i^P, \quad i = S, L.$$

Thus, as in Melitz (2003), we assume that TFP does not influence the fixed production cost F_i^P. The structure we have chosen for the fixed cost is common in the literature and implies that the latter is expressed in terms of output that must be produced, but which ultimately cannot be sold (see Yeaple 2005). We assume that $f_i^P > f_j^P$ if $\phi_i > \phi_j$, i.e. that the fixed input requirement is higher the more skilled labor intensive is the technology. This captures for instance the idea that more skill intensive firms tend to spend more in $R\&D$ investment (see also Long, Raff, and Stähler 2011). To simplify the algebra, we assume that the sunk market entry fee f^E is also expressed in terms of a firm's output, i.e. the sunk market entry cost is given by $F_i^E = Ac_i(A)f^E$.

Finally, a firm's profits are given by: $\pi_i(A) = \frac{Yp_i(A)^{1-\xi}}{P^{1-\xi}\xi} - Ac_i(A)f_i^P$. Profit maximization leads to the following pricing rule: $p_i(A) = \frac{\xi}{\xi-1}c_i(A)$.

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[7]We thank one of the referees for suggesting this normalization.

[8]Note that in the representative consumer's utility function (equation 1) each variety receives an identical weight, regardless of its factor intensities in production. While we could assume that, e.g., varieties with a higher skilled labor intensity get a larger weight in utility (e.g., Haruyama and Zhao 2008), this would not add to our analysis of the factor market effect of trade liberalization, while complicating the algebra.

[9]Typical estimates for ξ report values around 4 (e.g., Broda and Weinstein 2006), whereas point estimates for σ average around 1 (Antras 2004).

3   Autarkic equilibrium

In this section, we solve for the autarkic equilibrium in the Home country, which is characterized by the following set of equations:

i) production (equation 3) equals demand (equation 2) for each variety at the price $p_i(A)$, $i = S, L$;

ii) two zero cutoff profit conditions for the supply to the domestic market (one for the skilled, one for the unskilled labor intensive technology);

iii) two free entry conditions (one for the skilled, one for the unskilled labor intensive technology);

iv) two factor market clearing conditions.

Choosing unskilled labor as the numéraire ($w_L = 1$), these equations can be solved for the autarkic equilibrium (subscript a) values of: the average TFP parameters $\widetilde{A}_{a,i}$ $i = L, S$, the relative price of skilled labor $w_{a,S}$, the mass $\eta_{a,i}$ of each type of firm $i = L, S$, and the output of each variety $q_i(A)$.

We start by determining the minimum productivity level $A^*_{a,i}$, such that a firm of type i actually starts production after market entry. This is done by setting $\pi_i(A) = 0$, which results in the following zero cutoff profit condition:

$$Y P^{\xi-1} p_i \left(A^*_{a,i}\right)^{-\xi} = A^*_{a,i}(\xi - 1) f_i^P, \quad i = S, L. \tag{4}$$

Assuming an infinite time horizon for potential entrants, free entry drives the ex–ante expected profits from market entry to zero, which implies:

$$\left[1 - G(A^*_{a,i})\right] \left[\sum_{t=0}^{\infty}(1-\theta)^t \int_{A^*_{a,i}}^{\infty} \pi_i(A)\mu_{a,i}(A)dA\right] = F_i^E, \quad \text{where} \quad \mu_{a,i}(A) = \frac{g(A)}{1 - G(A^*_{a,i})}. \tag{5}$$

The first term in squared brackets on the left hand side represents the probability that a firm of type i starts producing after entry. The second term in squared brackets represents the expected lifetime profits, given that market entry has been successful. The term $(1-\theta)^t$ accounts for the risk of death in each period, and t is a time index. The term on the right hand side represents instead the sunk entry cost. We now combine the zero cutoff profit condition with the free entry condition to characterize the threshold TFP parameter in the autarkic equilibrium:[10]

Lemma 1 *The threshold TFP parameter in the autarkic equilibrium is given by:*

$$A^*_{a,i} = \left(\frac{f_i^P}{f^E \theta} \frac{\xi - 1}{k + 1 - \xi}\right)^{\frac{1}{k}}, \quad i = S, L.$$

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[10]Note that we do not include any time index in the following equations since we focus only on steady state equilibria in which all sector–wide variables are constant.

Proof. See appendix A. ∎

We can introduce now the *price of skilled labor* (PS) equation, which results from taking the ratio of the two zero cutoff profit conditions (see equation 4) and determines the relative price of skilled labor, given that both types of firms are active:

$$w_{a,S} = \Omega_S \left[\frac{\Psi_a \left(1 - \phi_L\right) - \left(1 - \phi_S\right)}{\phi_S - \Psi_a \phi_L} \right]^{\frac{1}{1-\sigma}}, \quad \text{where} \quad \Psi_a \equiv \left(\frac{f_S^P}{f_L^P}\right)^{\frac{\sigma-1}{\xi}} \left(\frac{A_{a,S}^*}{A_{a,L}^*}\right)^{\frac{(1-\sigma)(1-\xi)}{-\xi}}. \tag{6}$$

Substituting $A_{a,S}^*$ and $A_{a,L}^*$ from lemma 1 into equation 6, we can solve for $w_{a,S}$.[11]

In equilibrium factor markets clear. Applying Shephard's Lemma to the marginal cost functions leads to:

$$\overline{L} = \sum_{i=L,S} \eta_i \int_{A_{a,i}^*}^{\infty} a_{Li}(A) \left[q_i(A) + A\widehat{f}_{a,i}\right] \mu_{a,i}(A) dA \tag{7}$$

$$\overline{S} = \sum_{i=L,S} \eta_i \int_{A_{a,i}^*}^{\infty} a_{Si}(A) \left[q_i(A) + A\widehat{f}_{a,i}\right] \mu_{a,i}(A) dA, \tag{8}$$

where $a_{Li}(A) \equiv A^{\sigma-1} \left(1 - \phi_i\right) c_i(A)^{\sigma}$ and $a_{Si}(A) \equiv A^{\sigma-1} \phi_i w_S^{-\sigma} \Omega_S^{\sigma-1} c_i(A)^{\sigma}$ are, respectively, the per unit skilled and unskilled labor requirements for variety i, and $\widehat{f}_{a,i} \equiv \frac{f^E \theta}{1 - G(A_{a,i}^*)} + f_i^P$, i.e. $\eta_i \widehat{f}_{a,i}$ denotes total fixed input requirements of firms of type i in general equilibrium.[12]

Using the free entry condition and substituting $q_i(A)$ from equation 2 into equations 7 and 8 and taking their ratio, we obtain the *relative factor market clearing* (FMC) equation:

$$\frac{\overline{L}}{\overline{S}} w_{a,S}^{-\sigma} \Omega_S^{\sigma-1} = \frac{\left(1 - \phi_S\right) + \left(1 - \phi_L\right) \frac{\widetilde{A}_{a,L}^{\xi-1}}{\widetilde{A}_{a,S}^{\xi-1}} \frac{\eta_L}{\eta_S} \left[\frac{\phi_L \left(\frac{w_{a,S}}{\Omega_S}\right)^{1-\sigma} + 1 - \phi_L}{\phi_S \left(\frac{w_{a,S}}{\Omega_S}\right)^{1-\sigma} + 1 - \phi_S}\right]^{\frac{\sigma-\xi}{1-\sigma}}}{\phi_S + \phi_L \frac{\widetilde{A}_{a,L}^{\xi-1}}{\widetilde{A}_{a,S}^{\xi-1}} \frac{\eta_L}{\eta_S} \left[\frac{\phi_L \left(\frac{w_{a,S}}{\Omega_S}\right)^{1-\sigma} + 1 - \phi_L}{\phi_S \left(\frac{w_{a,S}}{\Omega_S}\right)^{1-\sigma} + 1 - \phi_S}\right]^{\frac{\sigma-\xi}{1-\sigma}}}, \tag{9}$$

where $\widetilde{A}_{a,i} \equiv \left[\int_{A_{a,i}^*}^{\infty} A^{\xi-1} \mu_{a,i}(A) dA\right]^{\frac{1}{\xi-1}}$ is the average TFP parameter of all active firms of type i in the autarkic equilibrium. Since $\frac{\widetilde{A}_{a,L}^{\xi-1}}{\widetilde{A}_{a,S}^{\xi-1}}$ and $w_{a,S}$ are already known, we can solve equation 9 for $\frac{\eta_{a,L}}{\eta_{a,S}}$. Using either of the two zero cutoff profit conditions, we can then determine $\eta_{a,S}$ and $\eta_{a,L}$. Once $\widetilde{A}_{a,S}$, $\widetilde{A}_{a,L}$, $w_{a,S}$, $\eta_{a,S}$ and $\eta_{a,L}$ are known, we can solve for $q_i(A)$, $i = S, L$ and establish the following:

Proposition 1 *If* $\frac{1 - \phi_S}{1 - \phi_L} < \left(\frac{f_S^P}{f_L^P}\right)^{\frac{(\sigma-1)(k+1-\xi)}{\xi k}} < \frac{\phi_S}{\phi_L}$ *there exists a unique autarkic equilibrium with both*

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[11]For $w_{a,S}$ to be defined, the term in squared brackets on the right hand side of equation 6 must be positive. See proposition 1 for the exact parametric restrictions required.

[12]Note that, if $\widehat{\eta}_i$ firms of type i have entered the market, $\eta_i \equiv [1 - G(A_{a,i}^*)]\widehat{\eta}_i$ actually become active. Furthermore, since in the steady state a share θ of active firms is replaced by new firms in each instant of time, the total sunk market entry requirements for firms of type i are given by $\eta_i \widehat{f}_i$.
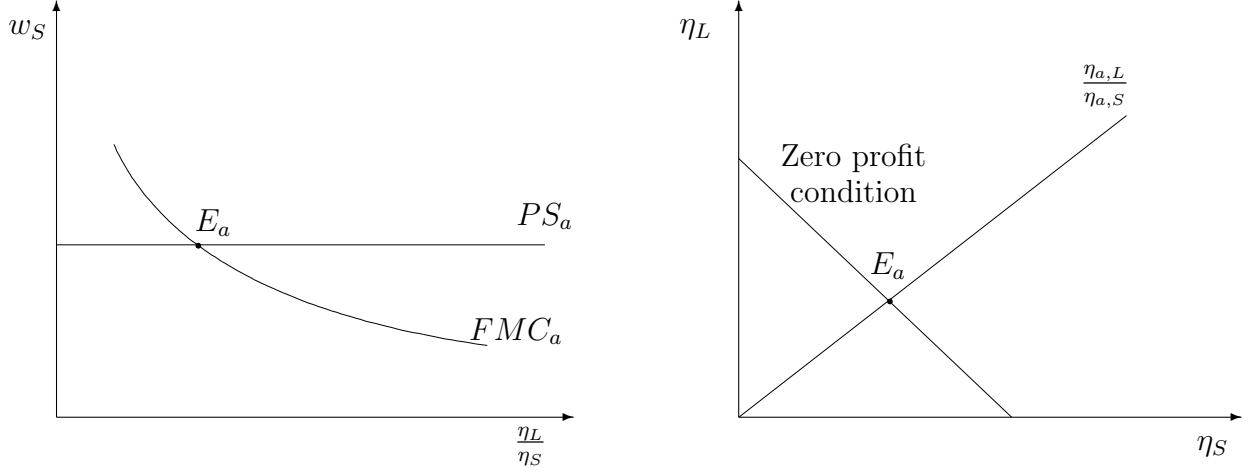
Figure 1: Autarkic equilibrium

skilled and unskilled labor intensive firms active in the market. Otherwise, a unique equilibrium exists with only skilled or only unskilled labor intensive firms active.

Proof. See Appendix B. ■

In the remainder of the analysis we will assume that ϕ_i and f_i^P, $i = S, L$, are such that both skilled and unskilled labor intensive firms are active in equilibrium.[13] Substituting $A_{a,i}^*$ (see lemma 1) into equation 6 then yields $\frac{w_{a,S}}{\Omega_S} < 1$. As a result, $c_S(A) < c_L(A)$ for any given A, i.e. the marginal cost of a skilled labor intensive firm must be lower than the marginal cost of an unskilled labor intensive firm. Intuitively, entrants will choose the skilled labor intensive technology with higher fixed costs only if they are compensated by a lower marginal cost.

In the left panel of Figure 1, we depict the PS and FMC curve. Their intersection establishes the relative price of skilled labor w_S and the relative mass of unskilled labor intensive firms $\frac{\eta_L}{\eta_S}$ in the autarkic equilibrium. Once the relative mass of unskilled labor intensive firms $\frac{\eta_{a,L}}{\eta_{a,S}}$ is known (see the upward sloping line in the right panel), we can obtain the absolute number of firms by using one of the two zero cutoff profit conditions.

4 Open economy equilibrium

In this section, we extend our analysis to a setting with N symmetric countries to study the effect of a move from autarky to an open economy equilibrium in the presence of variable and fixed export costs. The subscript "op" denotes variables in the open economy equilibrium, and our analysis focuses on a representative country.

The new equilibrium is characterized by the same equations that describe the autarkic equilibrium (see section 3), with the addition of two zero cutoff profit conditions for the supply to the

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[13]The analysis with a single type of firm would be comparable to the one in Melitz (2003).

foreign market (one for the skilled, one for the unskilled labor intensive technology). Using these conditions we can determine: the average TFP parameters $\widetilde{A}_{op,i}$ for the two types of firms, the relative price of skilled labor $w_{op,S}$, the mass $\eta_{op,i}$ and the share of exporters $s_{X,i}$ among the two types of firms and, finally, the aggregate production of each variety.

Let $\tau \geq 1$ be an iceberg transportation cost common to all varieties. Utility maximization abroad results in foreign demand for a domestic variety given by: $q_{X,i}(A) = YP^{\xi-1}p_i(A)^{-\xi}\tau^{1-\xi}$, where $p_i(A)$ is the producer price. Thus, the aggregate output of an exporting firm is given by:

$$\left[1 + (N-1)\tau^{1-\xi}\right]q_i(A) = \left[1 + (N-1)\tau^{1-\xi}\right]YP^{\xi-1}p_i(A)^{-\xi}, \quad N \geq 2. \tag{10}$$

In order to export, we assume that a firm must set up a distribution network, which leads to a fixed export cost given by $F_{X,i} = Ac_i(A)f_X$, i.e. TFP also does not influence the fixed export cost.

In the open economy equilibrium, we have to consider two threshold parameters for TFP for each type of firm. The first one, denoted by $A^*_{op,i}$, identifies the marginal firm supplying the domestic market, and it is the solution to the zero cutoff profit condition described in equation 4. The second one is denoted by $A^*_{X,i}$ and characterizes the minimum productivity level that enables a firm to serve the $N-1$ foreign markets profitably. This threshold is determined from the following zero cutoff profit condition:

$$YP^{\xi-1}p_i(A^*_{X,i})^{-\xi}\tau^{1-\xi} = q_{X,i}(A^*_{X,i}) = A^*_{X,i}(\xi-1)f_X. \tag{11}$$

Equation 11 implies that $A^*_{X,L} > A^*_{X,S}$, i.e. unskilled labor intensive firms need a higher TFP level to export, as compared to skilled labor intensive firms. Intuitively, a higher TFP is needed to compensate for the otherwise higher marginal cost of unskilled labor intensive firms.

Finally, dividing equations 4 and 11 by each other and solving for $A^*_{X,i}$ yields:

$$A^*_{X,i} = A^*_{op,i}\tau\left(\frac{f^P_i}{f_X}\right)^{\frac{1}{1-\xi}}, \quad i = S, L. \tag{12}$$

Following Melitz (2003), we will assume that $\tau^{\xi-1}f_X \geq f^P_i$. As a result, $A^*_{X,i} \geq A^*_{op,i}$, i.e. not all firms necessarily export in the open economy equilibrium.

The free entry condition has to be modified to account for the additional ex–ante expected export profits, and can be written as:

$$\sum_{t=0}^{\infty}(1-\theta)^t\left[\int_{A^*_{op,i}}^{\infty}\pi_i(A)\mu_{op,i}(A)dA + (N-1)s_{X,i}\int_{A^*_{X,i}}^{\infty}\pi_{X,i}(A)\mu_{X,i}(A)dA\right] = \frac{F^E_i}{1-G(A^*_{op,i})}, \tag{13}$$

where $\mu_{op,i}(A) = \frac{g(A)}{1-G(A^*_{op,i})}$, $s_{X,i} = \frac{1-G(A^*_{X,i})}{1-G(A^*_{op,i})}$ and $\mu_{X,i}(A) = \frac{g(A)}{1-G(A^*_{X,i})}$. The term $1 - G(A^*_{X,i})$ denotes the probability that a firm of type i exports after market entry, and $\int_{A^*_{X,i}}^{\infty}\pi_{X,i}(A)\mu_{X,i}(A)dA$ is the average export profits of exporting firms. The following result characterizes the threshold TFP parameter for each type of firm in the open economy equilibrium and the impact of trade

liberalization on it:

Lemma 2 *The open economy threshold TFP parameter is* $A^*_{op,i} = \left[\dfrac{(f_i^P)^{\frac{k+1-\xi}{1-\xi}} + \frac{N-1}{\tau^k} f_X^{\frac{k+1-\xi}{1-\xi}}}{(f_i^P)^{\frac{k}{1-\xi}} f^E \theta} \dfrac{\xi-1}{k+1-\xi} \right]^{\frac{1}{k}}$,
$i = S, L$. *Trade liberalization increases* A^*_i, *and the increase is larger, the less restricted is trade* (*i.e. the smaller are* τ *and* f_X).[14] *Furthermore, trade liberalization increases* $\frac{A^*_S}{A^*_L}$.

Proof. See appendix C. ∎

To understand the intuition behind lemma 2, note that trade liberalization increases ex–ante expected profits from market entry and thus triggers additional entry of both skilled and unskilled labor intensive firms. Competition becomes stronger, which implies that only the more productive firms of each type will survive. Since the share of exporters among skilled labor intensive firms is larger, i.e. $s_{X,S} > s_{X,L}$, new entry of skilled labor intensive firms exceeds new entry of unskilled labor intensive firms. Thus, the average productivity increase among skilled labor intensive firms is larger than that among unskilled labor intensive firms.

The relative price of skilled labor in the open economy equilibrium can be derived by taking the ratio of the zero cutoff profit conditions for the supply to the export market (equation 11), considering equation 12 and then solving for $w_{op,S}$:

$$w_{op,S} = \Omega_S \left[\frac{\Psi_{op}(1 - \phi_L) - (1 - \phi_S)}{\phi_S - \Psi_{op}\phi_L} \right]^{\frac{1}{1-\sigma}}, \text{ with } \Psi_{op} = \left(\frac{f_S^P}{f_L^P} \right)^{\frac{\sigma-1}{\xi}} \left(\frac{A^*_{op,S}}{A^*_{op,L}} \right)^{\frac{(1-\sigma)(1-\xi)}{-\xi}}. \quad (14)$$

Equation 14 shows that $w_{op,S} > w_{a,S}$, i.e. trade liberalization shifts the PS–curve upward.[15]

Since firms can now also export, the factor market clearing conditions become:

$$\overline{L} = \sum_{i=L,S} \eta_i \left\{ \int_{A^*_{op,i}}^{\infty} a_{Li}(A) \left[q_i(A) + A\widehat{f}_{op,i} \right] \mu_{op,i}(A)dA + \int_{A^*_{X,i}}^{\infty} a_{Li}(A)[q_{X,i}(A) + Af_X] s_{X,i}(N-1)\mu_{X,i}(A)dA \right\} \quad (15)$$

$$\overline{S} = \sum_{i=L,S} \eta_i \left\{ \int_{A^*_{op,i}}^{\infty} a_{Si}(A) \left[q_i(A) + A\widehat{f}_{op,i} \right] \mu_{op,i}(A)dA + \int_{A^*_{X,i}}^{\infty} a_{Si}(A)[q_{X,i}(A) + Af_X] s_{X,i}(N-1)\mu_{X,i}(A)dA \right\}, \quad (16)$$

where $\widehat{f}_{op,i} \equiv \frac{f^E \theta}{1 - G(A^*_{op,i})} + f_i^P$. Substituting domestic and foreign demands into equations 15 and 16

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[14]The same result holds if the number N of trading partners increases.

[15]Remember that from lemma 2 we have $\frac{A^*_{op,S}}{A^*_{op,L}} > \frac{A^*_{a,S}}{A^*_{a,L}}$. Thus, if $\sigma > 1$, $\Psi_{op} < \Psi_a$ and $\frac{\partial w_S}{\partial \Psi} < 0$, and if $\sigma < 1$, $\Psi_{op} > \Psi_a$ and $\frac{\partial w_S}{\partial \Psi} > 0$. Finally, for $w_{op,S}$ to be defined, the term in squared brackets on the right hand side of equation 14 must be positive. Thus, if $\sigma > 1$ we must have $\frac{\phi_S}{\phi_L} > \Psi_{op}$ and if $\sigma < 1$ we must have $\Psi_{op} > \frac{1-\phi_S}{1-\phi_L}$ (see appendix B for the same argument concerning $w_{a,S}$). Since trade liberalization increases $\frac{A^*_S}{A^*_L}$, we have $\Psi_{op} < \Psi_a$ if $\sigma > 1$ and $\Psi_{op} > \Psi_a$ if $\sigma < 1$. Thus, the same parametric restrictions which are necessary for $w_{a,S}$ to be defined (see proposition 1), also imply that $w_{op,S}$ is defined.

and taking their ratio results in:

$$
\frac{\overline{L}}{\overline{S}}w_{op,S}^{-\sigma}\Omega_S^{\sigma-1} \;=\; \frac{(1-\phi_S)+(1-\phi_L)\dfrac{\widetilde{A}_{op,L}^{\xi-1}}{\widetilde{A}_{op,S}^{\xi-1}}\dfrac{\Delta_L}{\Delta_S}\dfrac{\eta_L}{\eta_S}\left[\dfrac{\phi_L\left(\frac{w_{op,S}}{\Omega_S}\right)^{1-\sigma}+1-\phi_L}{\phi_S\left(\frac{w_{op,S}}{\Omega_S}\right)^{1-\sigma}+1-\phi_S}\right]^{\frac{\sigma-\xi}{1-\sigma}}}{\phi_S+\phi_L\dfrac{\widetilde{A}_{op,L}^{\xi-1}}{\widetilde{A}_{op,S}^{\xi-1}}\dfrac{\Delta_L}{\Delta_S}\dfrac{\eta_L}{\eta_S}\left[\dfrac{\phi_L\left(\frac{w_{op,S}}{\Omega_S}\right)^{1-\sigma}+1-\phi_L}{\phi_S\left(\frac{w_{op,S}}{\Omega_S}\right)^{1-\sigma}+1-\phi_S}\right]^{\frac{\sigma-\xi}{1-\sigma}}}, \tag{17}
$$

where $\Delta_i \equiv 1 + \frac{(N-1)s_{X,i}}{\tau^{\xi-1}}\frac{\widetilde{A}_{X,i}^{\xi-1}}{\widetilde{A}_{op,i}^{\xi-1}}$. $\widetilde{A}_{X,i}\equiv\left[\int_{A_{X,i}^*}^{\infty}A^{\xi-1}\mu_{X,i}(A)dA\right]^{\frac{1}{\xi-1}}$ and $\widetilde{A}_{op,i}\equiv\left[\int_{A_{op,i}^*}^{\infty}A^{\xi-1}\mu_{op,i}(A)dA\right]^{\frac{1}{\xi-1}}$ are respectively the average TFP parameter of exporting and all active firms of type i in the open economy equilibrium. Comparing the right hand sides of equations 9 and 17, note that trade liberalization decreases $\widetilde{A}_L^{\xi-1}$ relative to $\widetilde{A}_S^{\xi-1}$ and $\frac{\Delta_L}{\Delta_S}<1$.[16] Thus, for a *given* w_S, $\frac{\eta_L}{\eta_S}$ must increase after trade liberalization for factor markets to clear. This implies that trade liberalization shifts the FMC–curve to the right. Summarizing our results so far we obtain:

Lemma 3 *Compared to autarky, a multilateral trade liberalization has the following consequences:*

i) exporting firms increase their production;

ii) the relative price of skilled labor w_S increases since the share of exporters among skilled labor intensive firms is larger than among unskilled labor intensive firms ($s_{X,S}>s_{X,L}$);

iii) the increase in w_S ceteris paribus decreases (increases) the ex–ante expected profits from choosing the skilled (unskilled) labor intensive technology.

Proof. See appendix D. ∎

Our analysis so far suggests that the effect of trade liberalization on firm selection is in general ambiguous, i.e. we do not know whether $\frac{\eta_L}{\eta_S}$ increases or decreases. The additional availability of foreign varieties adversely affects both skilled and unskilled labor intensive firms. At the same time, the increased profit opportunities abroad affect the average skilled labor intensive firm more positively than the average unskilled labor intensive firm. Finally, the increased competition in factor markets, which is reflected by the rightward shift of the FMC–curve, affects skilled labor intensive firms negatively and unskilled labor intensive firms positively.

The net effect of trade liberalization on the two types of firms crucially depends on the *factor intensity gap*, i.e. on the difference in the skilled labor share parameters $\phi_S-\phi_L$, which determines (i) the extent to which w_S increases with trade liberalization and (ii) the extent to which firms are affected by the increase in w_S. Its role is characterized in the following:

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[16]Due to our distributional assumption for A, we have $\widetilde{A}_i = A_i^*\left(\frac{k}{k+1-\xi}\right)^{\frac{1}{\xi-1}}$. Thus, as $\frac{A_S^*}{A_L^*}$ increases with trade liberalization (see lemma 2), $\frac{\widetilde{A}_S}{\widetilde{A}_L}$ increases as well. $\frac{\Delta_L}{\Delta_S}$ is smaller than 1 since $s_{X,L}\frac{\widetilde{A}_{X,L}^{\xi-1}}{\widetilde{A}_{op,L}^{\xi-1}}<s_{X,S}\frac{\widetilde{A}_{X,S}^{\xi-1}}{\widetilde{A}_{op,S}^{\xi-1}}$, which can be transformed to $\left(\frac{A_{X,L}^*}{A_{op,L}^*}\right)^{\xi-1-k}<\left(\frac{A_{X,S}^*}{A_{op,S}^*}\right)^{\xi-1-k}$, due to our distributional assumption for A. Since $\frac{A_{X,i}^*}{A_{op,i}^*}=\tau\left(\frac{f_i^P}{f_X}\right)^{\frac{1}{1-\xi}}$, the latter condition can be transformed to $\left(f_L^P\right)^{\frac{\xi-1-k}{1-\xi}}<\left(f_S^P\right)^{\frac{\xi-1-k}{1-\xi}}$, which holds since $f_L^P<f_S^P$ and $\frac{\xi-1-k}{1-\xi}>0$.

$$\frac{\eta_{op,L}}{\eta_{op,S}} - \frac{\eta_{a,L}}{\eta_{a,S}}$$

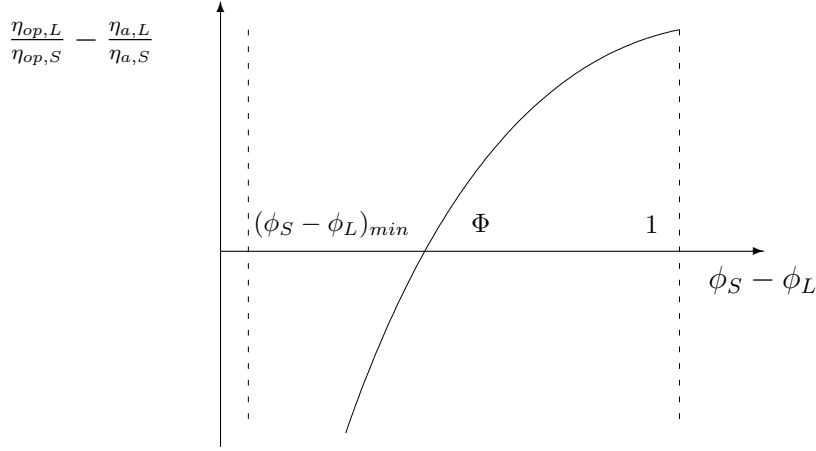$(\phi_S - \phi_L)_{min}$ Φ 1

$\phi_S - \phi_L$

Figure 2: The role of the factor intensity gap

Proposition 2 *There exists a threshold value for the factor intensity gap, denoted by Φ, such that if $\phi_S - \phi_L > (<) \Phi$ trade liberalization increases (decreases) the relative mass of unskilled labor intensive firms $\frac{\eta_L}{\eta_S}$. Furthermore, the larger is $\phi_S - \phi_L$, the more detrimental (beneficial) is trade liberalization for skilled (unskilled) labor intensive firms.*

Proof. See appendix E. ■

Figure 2 illustrates the result. $\frac{\eta_{op,L}}{\eta_{op,S}}$ stands for the relative mass of unskilled labor intensive firms in the open economy equilibrium, while $\frac{\eta_{a,L}}{\eta_{a,S}}$ stands for the relative mass of unskilled labor intensive firms in the autarkic equilibrium. The minimum factor intensity gap, which is denoted by $(\phi_S - \phi_L)_{min}$, is defined as that difference $\phi_S - \phi_L$, which leads to $\eta_{a,S} = 0$.[17]

The intuition behind proposition 2 is as follows. First, the increase in the relative price of skilled labor w_S due to trade liberalization is larger, the larger is the difference $\phi_S - \phi_L$. Second, for a given increase in w_S, the losses (gains) for the skilled (unskilled) labor intensive firms are larger, the larger is $\phi_S - \phi_L$. Thus, if the factor intensity gap is sufficiently large, unskilled (skilled) labor intensive firms will gain (lose) from trade liberalization and will enter (exit) the market.

Figure 3 illustrates the effect of trade liberalization on the mass of firms active in equilibrium. The left panel shows that, starting from the autarkic equilibrium E_a, trade liberalization shifts the PS–curve upward. This results from the increase in A_S^* relative to A_L^* (see lemma 2) due to trade liberalization, which requires an increase in the relative price of skilled labor w_S for the zero cutoff profit conditions to hold again.[18] Trade liberalization also increases competition in factor markets, which shifts the FMC–curve rightward. In fact, if the relative demand for skilled labor increases, $\frac{\eta_L}{\eta_S}$ has to increase for any given w_S to re–establish factor market clearing. The open economy

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[17]See appendix E for a formal proof that the relationship between $\frac{\eta_{op,L}}{\eta_{op,S}} - \frac{\eta_{a,L}}{\eta_{a,S}}$ and $\phi_S - \phi_L$ is monotonically increasing.

[18]Note that an increase in w_S increases $\frac{p_S(A)}{p_L(A)}$, which shifts demand from skilled to unskilled labor intensive firms.
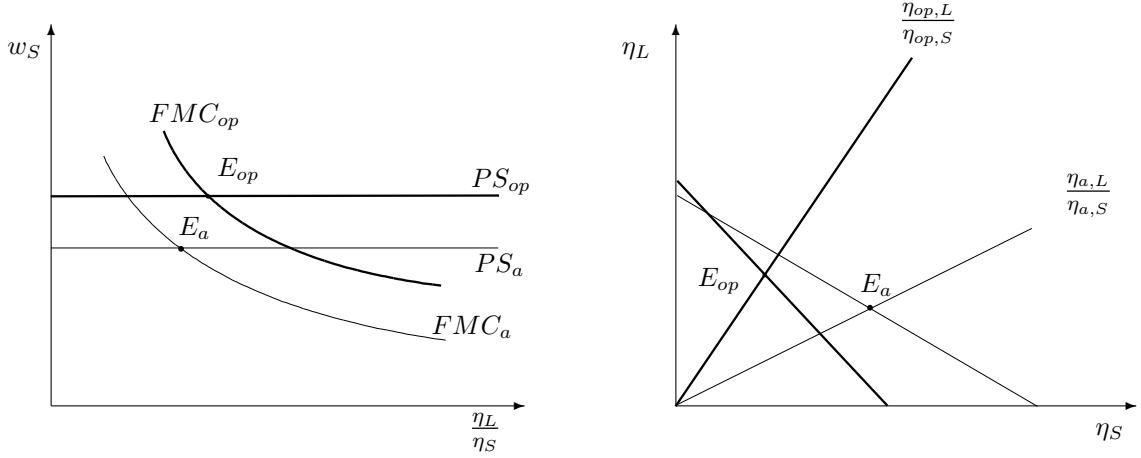
Figure 3: Trade liberalization

equilibrium is illustrated by point E_{op}. Note that we have drawn the curves for a "large" factor intensity gap, such that $\frac{\eta_L}{\eta_S}$ increases with trade liberalization.

The right panel of the same figure illustrates also the role played by the increased availability of foreign varieties. Starting from the autarkic equilibrium E_a, holding factor prices constant, increased availability of foreign varieties and new profit opportunities abroad make the line illustrating the zero cutoff profit condition for skilled labor intensive firms shift inward and become steeper. Allowing factor prices to adjust (w_S increases) flattens the curve and makes it shift inward.[19] The new equilibrium point is indicated by E_{op}. In general, the mass of skilled labor intensive firms η_S decreases, whereas η_L can increase or decrease.

Finally, note that in our model the increased factor market competition, reflected by the shift of the FMC–curve, does not induce a skill intensive exporting firm to become a non–exporter and stay active.[20] This is because the resulting increase in w_S negatively impacts both profits from serving the domestic market and profits from exporting. Thus, an increase in w_S does not induce the marginal skill intensive exporter to become a non–exporter.[21] Instead, it induces fewer (more) firms to choose the skill (unskill) intensive technology upon market entry.

5   Average TFP and real income

We turn now to consider how trade liberalization affects productivity and welfare. We start by focusing on average productivity. In particular, we measure productivity at the factory gate, i.e.

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[19]Still, the zero cutoff profit condition in the open economy equilibrium is steeper than the one in the autarkic equilibrium. Appendix F formally derives the shift of the zero cutoff profit condition.

[20]This result follows, of course, from our focus on steady states. It is well–known that, in the short run, firms also enter and exit the export market without necessarily dying, as pointed out by Schröder and Sørensen (2012).

[21]Remember that $A_{op,i}^*$ (lemma 2) and $A_{i,X}^*$ (equation 12) are independent of w_S.

we use an output weighted measure defined as follows:

$$\widetilde{A} = \widetilde{A}_S \Xi_S + \widetilde{A}_L \Xi_L, \tag{18}$$

where Ξ_S and Ξ_L are the share of GDP respectively produced by skilled and unskilled labor intensive firms and $\widetilde{A}_i$ is the average TFP parameter of firms of type i, $i = S, L$.[22] The effect of trade liberalization on average productivity is characterized in the following:

Proposition 3 *Trade liberalization increases sector–wide average productivity, and this increase is larger, the less restricted trade becomes. This increase is smaller, the larger is the factor intensity gap $\phi_S - \phi_L$.*

Proof. See appendix G. ∎

The intuition for this result is as follows: on the one hand, the less restricted trade becomes, i.e. if τ and f_X decrease, the larger is the increase in A_i^* (see lemma 2). On the other hand, the share of exporters among the skill intensive firms is larger than among the unskill intensive ones. Thus, trade liberalization not only increases $\widetilde{A}_S$, $\widetilde{A}_L$ and $\widetilde{A}_S$ relative to $\widetilde{A}_L$, but it ceteris paribus also increases Ξ_S relative to Ξ_L. However, as shown by proposition 2, the factor intensity gap determines whether $\frac{\eta_L}{\eta_S}$ increases or decreases with trade liberalization. If $\phi_S - \phi_L$ is large, the increased factor market competition (rightward shift of the FMC-curve in Figure 3) dominates the impact of increased profit opportunities abroad (upward shift of the PS-curve), so that η_L increases relative to η_S. If this is the case, the relative frequency of those firms, which experience a larger increase in their average TFP, decreases. This dampens the positive effect of trade liberalization on sector–wide TFP.

The theoretical analyses that have built upon Melitz's (2003) model have emphasized the positive TFP effect of trade liberalization. At the same time, recent empirical evidence (Lawless and Whelan 2008; Chen, Imbs, and Scott 2009) points out that these effects might be only moderate. Our analysis suggests that, in the presence of substantial heterogeneity in factor intensities, the increase in factor market competition actually dampens the increase in average TFP brought about by trade liberalization, by forcing some of the skilled labor intensive firms out of the market.[23] Looking at factor markets is thus crucial to gain a more nuanced understanding of the firm selection process and of its consequences.

We turn next to study the effect of trade on real income, which is done in the following:

Proposition 4 *Trade liberalization increases real income, and this increase is larger, the less restricted trade becomes. The increase in real income is smaller, the larger is the factor intensity gap $\phi_S - \phi_L$.*

Proof. See appendix H. ∎

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[22]See appendix G for the derivation of Ξ_S and Ξ_L used for the computation of $\widetilde{A}$.

[23]For empirical evidence on this point see Emami Namini, Facchini, and López (2013).

To understand this result, note that trade liberalization increases the mass of available varieties, which decreases the aggregate price index and thus insures that real income increases. As trade becomes less restricted, i.e. as τ and f_X decrease, the TFP gains from trade liberalization become larger (see lemma 2), which implies that the increase in real income becomes larger as well. At the same time, the increased factor market competition due to trade liberalization hurts the skill intensive firms, which are those that supply at a lower price and are more likely to export. Thus, a larger factor intensity gap implies a smaller rise in real income since it makes the increase in factor market competition more detrimental for skill intensive firms.

6   Conclusions

A large empirical literature has shown that exporting and non–exporting firms differ not only in their TFP, but also in the mix of inputs used in production, even within narrowly defined sectors. In this paper, we have developed a new theoretical framework to analyze how these two sources of heterogeneity affect the firm selection process brought about by trade liberalization.

In a setting in which exporters are more productive than non–exporters and in which skill intensive firms are more likely to export, we have shown that the firm selection induced by trade liberalization works along two dimensions. First, more intense competition in factor markets induced by the additional production needed to export increases the relative price of skilled labor, negatively affecting those firms that use this input intensively, while positively affecting unskilled labor intensive firms. This effect becomes stronger, the larger is the difference in factor intensities between the two types of firms. As a result, some of the skill intensive firms might be forced to cease production and exit. Second, within each type of firms, we observe selection against the non–exporters, as in Melitz (2003). While the second process increases sector–wide TFP, the first one has a priori an ambiguous effect. Still, under some mild assumptions, we have established that the larger is the difference in the skill intensity between firms, the smaller is the increase in sector–wide TFP induced by a trade expansion. In other words, factor market competition dampens the positive effect of trade on sector–wide TFP and on the increase in real income. Our analysis thus suggests that to fully understand the welfare implications of trade liberalization, competition in factor markets should be taken into account.

Our research can be extended to tackle several additional important questions. First, we could consider a model with multiple sectors with differences in factor intensities both within and across industries and Heckscher–Ohlin trade. In this context we could study whether reallocations within sectors can dominate reallocations between sectors, so that even the unskilled labor abundant country might experience an increase in country–wide skilled labor intensity following a trade liberalization. Secondly, it would be interesting to carry out a quantitative exploration of the effects of factor market competition on the gains from trade within our model. While both these questions are important, they are left for future research.

Appendix

A   Proof of lemma 1

Substituting the expression for $\pi_i(A)$ into equation 5, using the marginal cost function $c_i(A)$ and the formula for an infinite geometric series, the free entry condition can be rewritten as follows:

$$\frac{q_i(\widetilde{A}_{a,i})}{(\xi - 1)\widetilde{A}_{a,i}} = \frac{f^E \theta}{1 - G\left(A^*_{a,i}\right)} + f^P_i, \quad \text{where} \quad \widetilde{A}_{a,i} = \left[\int_{A^*_{a,i}}^{\infty} A^{\xi-1} \mu_{a,i}(A) dA\right]^{\frac{1}{\xi-1}}. \tag{19}$$

Since $\frac{q_i\left(A^*_{a,i}\right)}{q_i(\widetilde{A}_{a,i})} = \left(\frac{\widetilde{A}_{a,i}}{A^*_{a,i}}\right)^{-\xi}$, equation 4 implies that $q_i(\widetilde{A}_{a,i}) = \left(\frac{\widetilde{A}_{a,i}}{A^*_{a,i}}\right)^{\xi} A^*_{a,i}(\xi - 1)f^P_i$. Substituting $q_i(\widetilde{A}_{a,i})$ into equation 19 and recalling that A follows a Pareto distribution, we can determine $A^*_{a,i}$. Note that the assumption $k > \xi - 1$ is necessary for $A^*_{a,i}$ to be defined.

B   Proof of proposition 1

w_S is defined for all possible values of σ only if $\dfrac{(1-\phi_L)\left(\frac{f^P_S}{f^P_L}\right)^{\frac{\sigma-1}{\xi}}\left(\frac{A^*_{a,S}}{A^*_{a,L}}\right)^{\frac{(1-\xi)(\sigma-1)}{\xi}} - (1-\phi_S)}{\phi_S - \phi_L\left(\frac{f^P_S}{f^P_L}\right)^{\frac{1-\sigma}{-\xi}}\left(\frac{A^*_{a,S}}{A^*_{a,L}}\right)^{\frac{(1-\xi)(1-\sigma)}{-\xi}}} > 0$ (see equation 6). Since $\frac{\phi_S}{\phi_L} > \frac{1-\phi_S}{1-\phi_L}$, the numerator and the denominator have the same sign only if they are both positive. Using the solution for $A^*_{a,i}$ from lemma 1, this is true if the following conditions hold: (*i*) $\frac{1-\phi_S}{1-\phi_L} < \left(\frac{f^P_S}{f^P_L}\right)^{\frac{(\sigma-1)(k+1-\xi)}{\xi k}}$; (*ii*) $\left(\frac{f^P_S}{f^P_L}\right)^{\frac{(\sigma-1)(k+1-\xi)}{k\xi}} < \frac{\phi_S}{\phi_L}$. Condition (*i*) holds if $\sigma > 1$, while condition (*ii*) holds if $\sigma < 1$.

To establish existence, substitute $w_{a,S}$ from equation 6 into the right hand side of equation 9 and solve for $\frac{\eta_{a,L}}{\eta_{a,S}}$ to obtain:

$$\frac{\eta_{a,L}}{\eta_{a,S}} = \frac{\frac{1-\phi_S}{w_{a,S}^{-\sigma}\Omega_S^{\sigma-1}\phi_S} - \frac{\overline{L}}{\overline{S}}}{\frac{\overline{L}}{\overline{S}} - \frac{1-\phi_L}{w_{a,S}^{-\sigma}\Omega_S^{\sigma-1}\phi_L}} \frac{\phi_S}{\phi_L} \frac{\Psi_a^{\frac{\sigma-\xi}{1-\sigma}}}{\frac{\widetilde{A}_{a,L}^{\xi-1}}{\widetilde{A}_{a,S}^{\xi-1}}} > 0. \tag{20}$$

To understand why $\frac{\eta_{a,L}}{\eta_{a,S}} > 0$, note that $\frac{1-\phi_S}{w_{a,S}^{-\sigma}\Omega_S^{\sigma-1}\phi_S}$ is the relative unskilled labor demand by skilled labor intensive firms, while $\frac{1-\phi_L}{w_{a,S}^{-\sigma}\Omega_S^{\sigma-1}\phi_L}$ is the relative unskilled labor demand by unskilled labor intensive firms (remember the derivation of the factor input coefficients in equations 7 and 8). While the former is smaller than $\frac{\overline{L}}{\overline{S}}$, the latter is larger than $\frac{\overline{L}}{\overline{S}}$.

To establish uniqueness, we totally differentiate equation 9 with respect to w_S and $\frac{\eta_L}{\eta_S}$ and solve for $\frac{dw_S}{d(\frac{\eta_L}{\eta_S})}$ to obtain: $\frac{dw_S}{d(\frac{\eta_L}{\eta_S})} = \frac{-(\phi_S-\phi_L)\widetilde{A}_{a,S}^{\sigma-1}c_S(\widetilde{A}_{a,S})^{\sigma-\xi}\widetilde{A}_{a,L}^{\sigma-1}c_L(\widetilde{A}_{a,L})^{\sigma-\xi}w_S^{\sigma}\Omega_S^{1-\sigma}\eta_S^2}{\sigma w_S^{-1}\frac{\overline{L}}{\overline{S}}\chi + (\phi_S-\phi_L)^2\widetilde{A}_{a,L}^{2(\sigma-1)}c_L(\widetilde{A}_{a,L})^{2\sigma-\xi-1}\eta_L\widetilde{A}_{a,S}^{2(\sigma-1)}c_S(\widetilde{A}_{a,S})^{2\sigma-\xi-1}\eta_S(\xi-\sigma)}$, with $\chi \equiv \left[\sum_{i=S,L}\phi_i\widetilde{A}_{a,i}^{\sigma-1}c_i(\widetilde{A}_{a,i})^{\sigma-\xi}\eta_i\right]^2$. $\frac{dw_S}{d(\frac{\eta_L}{\eta_S})} < 0$ since $\phi_S - \phi_L > 0$ and $\xi - \sigma > 0$ by assumption. Since the PS equation shows that $w_{a,S}$ in equilibrium is independent from $\frac{\eta_L}{\eta_S}$, while the FMC equation implies a negative relationship between w_S and $\frac{\eta_L}{\eta_S}$, it follows that the autarkic equilibrium is unique.

If $\frac{1-\phi_S}{1-\phi_L} < \left(\frac{f^P_S}{f^P_L}\right)^{\frac{(\sigma-1)(k+1-\xi)}{\xi k}} < \frac{\phi_S}{\phi_L}$ does not hold, there exists no $w_{a,S}$ that satisfies the zero

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cutoff profit condition (equation 4) for the skilled and the unskilled labor intensive technology simultaneously, i.e. only one technology is used in equilibrium in this case.

C   Proof of lemma 2

Substituting the expressions for $\pi_i(A)$ and $\pi_{X,i}(A)$ into equation 13, using the marginal cost function $c_i(A)$ and the formula for an infinite geometric series, equation 13 can be rewritten as follows:

$$\frac{q_i(\widetilde{A}_{op,i})}{(\xi-1)\widetilde{A}_{op,i}} + (N-1)s_{X,i}\left[\frac{q_i(\widetilde{A}_{X,i})\tau^{1-\xi}}{(\xi-1)\widetilde{A}_{X,i}} - f_X\right] = \frac{f^E\theta}{1-G\left(A_{a,i}^*\right)} + f_i^P, \tag{21}$$

where $\widetilde{A}_{op,i} = \left[\int_{A_{op,i}^*}^{\infty} A^{\xi-1}\mu_{op,i}(A)dA\right]^{\frac{1}{\xi-1}}$, $\widetilde{A}_{X,i} = \left[\int_{A_{X,i}^*}^{\infty} A^{\xi-1}\mu_{X,i}(A)dA\right]^{\frac{1}{\xi-1}}$ and $s_{X,i} = \frac{1-G(A_{X,i}^*)}{1-G(A_{op,i}^*)}$. Since $\frac{q_i\left(A_{op,i}^*\right)}{q_i\left(\widetilde{A}_{op,i}\right)} = \left(\frac{\widetilde{A}_{op,i}}{A_{op,i}^*}\right)^{-\xi}$ and $\frac{q_i\left(A_{X,i}^*\right)}{q_i\left(\widetilde{A}_{X,i}\right)} = \left(\frac{\widetilde{A}_{X,i}}{A_{X,i}^*}\right)^{-\xi}$, the zero cutoff profit conditions (equations 4 and 11) can be transformed to: $q_i(\widetilde{A}_{a,i}) = \left(\frac{\widetilde{A}_{a,i}}{A_{a,i}^*}\right)^{\xi} A_{a,i}^*(\xi-1)f_i^P$ and $q_i(\widetilde{A}_{X,i})\tau^{1-\xi} = \left(\frac{\widetilde{A}_{X,i}}{A_{X,i}^*}\right)^{\xi} A_{X,i}^*(\xi-1)f_X$. Substituting these terms for $q_i(\widetilde{A}_{a,i})$ and $q_i(\widetilde{A}_{X,i})$, as well as $A_{X,i}^*$ (equation 12) into equation 21 and recalling that A follows a Pareto distribution, equation 21 can be solved for $A_{op,i}^*$. The solution for $A_{op,i}^*$ shows the following: (i) $A_{op,i}^* > A_{a,i}^*$ since $k+1-\xi > 0$; (ii) $A_{op,i}^*$ increases if τ and f_X become smaller, or if N becomes larger. Finally, if we define $\Gamma \equiv \frac{N-1}{\tau^k f_X^{\frac{k+1-\xi}{\xi-1}}}$ we can derive the following partial derivative:

$$\frac{\partial\left(A_{op,S}^*/A_{op,L}^*\right)}{\partial\Gamma} = \left(\frac{A_{op,S}^*}{A_{op,L}^*}\right)^{1-k} \frac{(f_L^P)^{\frac{k+1-\xi}{1-\xi}} - (f_S^P)^{\frac{k+1-\xi}{1-\xi}}}{k\left[(f_L^P)^{\frac{k+1-\xi}{1-\xi}} + \Gamma\right]^2} \frac{(f_L^P)^{\frac{k}{1-\xi}}}{(f_S^P)^{\frac{k}{1-\xi}}} > 0$$

since $f_S^P > f_L^P$ and $\frac{k+1-\xi}{1-\xi} < 0$. Since Γ becomes larger if τ or f_X become smaller or if N becomes larger, the ratio $\frac{A_{op,S}^*}{A_{op,L}^*}$ increases with trade liberalization.

D   Proof of lemma 3

Part (i) follows from equation 10, while part (ii) follows from equation 14. To prove part (iii), note that the ex–ante expected per period profits $\pi_S^{exp}(\widetilde{A}_{op,S})$ from choosing a skilled labor intensive technology are given by:

$$\pi_S^{exp}(\widetilde{A}_{op,S}) = \left[1 - G(A_{op,S}^*)\right]\left[\int_{A_{op,S}^*}^{\infty}\pi_S(A)\mu_{op,S}(A)dA + (N-1)s_{X,S}\int_{A_{X,S}^*}^{\infty}\pi_{X,S}(A)\mu_{X,S}(A)dA\right] - F_S^E\theta. \tag{22}$$

Substituting the terms for $\pi_S(A)$ and $\pi_{X,S}(A)$ into equation 22 leads to:

$$\pi_S^{exp}(\widetilde{A}_{op,S}) = \left[1 - G(A_{op,S}^*)\right]\left[\frac{Yp_S(\widetilde{A}_{op,S})^{1-\xi}}{P^{\xi-1}\xi}\Delta_S - \widetilde{A}_{op,S}c_S(\widetilde{A}_{op,S})f_S'\right], \tag{23}$$

with $\Delta_S \equiv 1 + \frac{N-1}{\tau^{\xi-1}} s_{X,S} \frac{\widetilde{A}_{X,S}^{\xi-1}}{\widetilde{A}_{op,S}^{\xi-1}}$ and $f_S' \equiv \frac{f^E \theta}{1-G(A_{op,S}^*)} + f_S^P + s_{X,S} f_X$. Considering that the aggregate price index is given by $P = \left[\sum_{i=S,L} \eta_{op,i} p_i(\widetilde{A}_{op,i})^{1-\xi} \Delta_i \right]^{\frac{1}{1-\xi}}$, we can now determine the partial derivative $\frac{\partial \pi_S^{exp}(\widetilde{A}_{op,S})}{\partial w_S}$ and consider afterwards that $f_S' = \frac{Y p_S(\widetilde{A}_{op,S})}{P^{1-\xi} \widetilde{A}_{op,S}(\xi-1)} \Delta_S$ in the initial equilibrium (see equation 13) to obtain:

$$\frac{\partial \pi_S^{exp}(\widetilde{A}_{op,S})}{\partial w_S} = \frac{\left(\frac{\overline{S}}{\overline{L}} - \frac{\phi_S w_S^{-\sigma}}{1-\phi_S} \Omega_S^{\sigma-1} \right) \frac{(1-\phi_S)\overline{L}}{P^{\xi-1}} - \frac{Y(\phi_S-\phi_L) w_S^{-\sigma}}{\xi^{\xi-1}(\xi-1)^{-\xi}} \eta_L \Delta_L c_L(\widetilde{A}_L)^{\sigma-\xi} \widetilde{A}_L^{\sigma-1} \Omega_S^{\sigma-1}}{\widetilde{A}_S^{1-\sigma} c_S(\widetilde{A}_S)^{\xi-\sigma} P^{2(1-\xi)} (\xi-1)^{1-\xi} \xi^\xi \Delta_S^{-1}}. \tag{24}$$

Since $\frac{\phi_S}{1-\phi_S} w_S^{-\sigma} \Omega_S^{\sigma-1}$ denotes relative skilled labor demand by the skilled labor intensive firms we can conclude that $\frac{\overline{S}}{\overline{L}} - \frac{\phi_S}{1-\phi_S} w_S^{-\sigma} \Omega_S^{\sigma-1} < 0$. Furthermore, since $\phi_S - \phi_L > 0$, we get $\frac{\partial \pi_S^{exp}(\widetilde{A}_{op,S})}{\partial w_S} < 0$. It can be shown along the same lines that $\pi_L^{exp}(\widetilde{A}_L)$ increases with w_S.

E Proof of proposition 2

The proof proceeds in four steps. *First*, the upward shift of the *PS*–curve becomes smaller, the larger is ϕ_S for any given level of ϕ_L. This shift is reflected by $\frac{w_{op,S}}{w_{a,S}}$, and it is easy to show that:

$$\frac{\partial \left(\frac{w_{op,S}}{w_{a,S}} \right)}{\partial \phi_S} = \frac{(\phi_S - \Psi_{op}\phi_L)(\phi_S - \Psi_a\phi_L) \left[\phi_L w_{op,S}^{1-\sigma} w_{a,S}^{1-\sigma} + 1 - \phi_L \right]}{\left(\frac{w_{a,S}}{w_{op,S}} \right)^\sigma \frac{1-\sigma}{\Psi_a - \Psi_{op}} \{ [\Psi_a(1-\phi_L) - (1-\phi_S)][\phi_S - \Psi_{op}\phi_L] \}^2} < 0$$

since $\Psi_a > \Psi_{op}$ if $\sigma > 1$, $\Psi_a < \Psi_{op}$ if $\sigma < 1$ and $\phi_S - \Psi\phi_L > 0$ (see proposition 1).

Second, the rightward shift of the *FMC*–curve with trade liberalization does not depend on the factor intensity gap. Solving equations 9 and 17 for $\frac{\eta_{a,L}}{\eta_{a,S}}$ and $\frac{\eta_{op,L}}{\eta_{op,S}}$, and taking their ratio results in:

$$\frac{\eta_{op,L}/\eta_{op,S}}{\eta_{a,L}/\eta_{a,S}} = \frac{\frac{(1-\phi_S) - \frac{\overline{L}}{\overline{S}} w_{op,S}^{-\sigma} \Omega_S^{\sigma-1} \phi_S}{\frac{\overline{L}}{\overline{S}} w_{op,S}^{-\sigma} \Omega_S^{\sigma-1} \phi_L - (1-\phi_L)} \left[\frac{\phi_S \left(\frac{w_{op,S}}{\Omega_S} \right)^{1-\sigma} + 1 - \phi_S}{\phi_L \left(\frac{w_{op,S}}{\Omega_S} \right)^{1-\sigma} + 1 - \phi_L} \right]^{\frac{\sigma-\xi}{1-\sigma}} \left(\frac{\widetilde{A}_{op,L}}{\widetilde{A}_{op,S}} \right)^{1-\xi} \frac{\Delta_S}{\Delta_L}}{\frac{1-\phi_S - \frac{\overline{L}}{\overline{S}} w_{a,S}^{-\sigma} \Omega_S^{\sigma-1} \phi_S}{\frac{\overline{L}}{\overline{S}} w_{a,S}^{-\sigma} \Omega_S^{\sigma-1} \phi_L - (1-\phi_L)} \left[\frac{\phi_S \left(\frac{w_{a,S}}{\Omega_S} \right)^{1-\sigma} + 1 - \phi_S}{\phi_L \left(\frac{w_{a,S}}{\Omega_S} \right)^{1-\sigma} + 1 - \phi_L} \right]^{\frac{\sigma-\xi}{1-\sigma}} \left(\frac{\widetilde{A}_{a,L}}{\widetilde{A}_{a,S}} \right)^{1-\xi}}.$$

Thus, if we use the solutions for $\widetilde{A}_{a,i}$ and $\widetilde{A}_{op,i}$, we can express $\frac{\eta_{op,L}/\eta_{op,S}}{\eta_{a,L}/\eta_{a,S}}$ for a *constant* level of $\overline{w}_S = w_{op,S} = w_{a,S}$:

$$\frac{\eta_{op,L}/\eta_{op,S}}{\eta_{a,L}/\eta_{a,S}} \Bigg|_{w_{op,S}=w_{a,S}} = \left[\frac{1 + \frac{N-1}{\tau^k} \left(\frac{f_S^P}{f_X} \right)^{\frac{k+1-\xi}{\xi-1}}}{1 + \frac{N-1}{\tau^k} \left(\frac{f_L^P}{f_X} \right)^{\frac{k+1-\xi}{\xi-1}}} \right]^{\frac{k+\xi-1}{k}} > 1 \tag{25}$$

since $k - \xi + 1 > 0$ and $f_S^P > f_L^P$. As a result, trade liberalization shifts the *FMC*–curve rightward, and the magnitude of this shift does not depend on the factor intensity gap.

Third, $\frac{\eta_L}{\eta_S}$ decreases with trade liberalization if the factor intensity gap is at its minimum level. To derive the minimum factor intensity gap, note that $\frac{\eta_{a,S}}{\eta_{a,L}}$ is given by:

$$\frac{\eta_{a,S}}{\eta_{a,L}} = \frac{\frac{\overline{L}}{\overline{S}}w_{a,S}^{-\sigma}\Omega_S^{\sigma-1}\phi_L - (1-\phi_L)}{1-\phi_S - \frac{\overline{L}}{\overline{S}}w_{a,S}^{-\sigma}\Omega_S^{\sigma-1}\phi_S}\left(\frac{\widetilde{A}_{a,L}}{\widetilde{A}_{a,S}}\right)^{\xi-1}\left[\frac{\phi_S\left(\frac{w_{a,S}}{\Omega_S}\right)^{1-\sigma}+1-\phi_S}{\phi_L\left(\frac{w_{a,S}}{\Omega_S}\right)^{1-\sigma}+1-\phi_L}\right]^{\frac{\xi-\sigma}{1-\sigma}}. \tag{26}$$

Thus, $\frac{\eta_{a,S}}{\eta_{a,L}} = 0$ if $\frac{\overline{L}}{\overline{S}}w_{a,S}^{-\sigma}\Omega_S^{\sigma-1}\phi_L - (1-\phi_L) = 0$. Remember that $w_{a,S}$ (see equation 6) is a function of ϕ_S and ϕ_L. Since $\frac{\partial w_{a,S}}{\partial \phi_S} = \frac{w_{a,S}^\sigma \Omega^{1-\sigma}}{(\phi_S - \Psi_a \phi_L)^2}\frac{1-\Psi_a}{1-\sigma} > 0$ (note that $\Psi_a > 1$ if $\sigma > 1$ and $\Psi_a < 1$ if $\sigma < 1$) for each given level of ϕ_L, there exists a unique ϕ_S that leads to $\frac{\eta_{a,S}}{\eta_{a,L}} = 0$. Since the term $1-\phi_S - \frac{\overline{L}}{\overline{S}}w_{a,S}^{-\sigma}\Omega_S^{\sigma-1}\phi_S$ on the right hand side of equation 26 is negative and since w_S increases with trade liberalization (see lemma 3), we can conclude that $\frac{\eta_S}{\eta_L}$ becomes strictly positive with trade liberalization if the factor intensity gap is such that $\frac{\eta_{a,S}}{\eta_{a,L}} = 0$.

Finally, if the factor intensity gap is at its maximum, i.e. if $\phi_S = 1$ and $\phi_L = 0$, we get $\frac{\eta_{a,S}}{\eta_{a,L}} = \frac{\overline{S}f_L^P}{\overline{L}f_S^P}\Omega_S$ and $\frac{\eta_{op,S}}{\eta_{op,L}} = \frac{\overline{S}f_L^P}{\overline{L}f_S^P}\Omega_S\frac{\Delta_L}{\Delta_S}$. Since $\frac{\Delta_L}{\Delta_S} = \frac{1+\frac{N-1}{\tau^k}\left(\frac{f_L^P}{f_X}\right)^{\frac{k-\xi+1}{\xi-1}}}{1+\frac{N-1}{\tau^k}\left(\frac{f_S^P}{f_X}\right)^{\frac{k-\xi+1}{\xi-1}}} < 1$ (remember that $k+1-\xi > 0$

and $f_S^P > f_L^P$), $\frac{\eta_S}{\eta_L}$ decreases with trade liberalization if $\phi_S = 1$ and $\phi_L = 0$.

F   The zero cutoff profit condition in the right panel of figure 3

Let op_1 denote and op_2 denote respectively the open economy equilibrium *before* any adjustment of relative factor prices and *after* it. The axis intercepts of the zero cutoff profit condition of the skilled labor intensive firms (see equation 4) in the right panel of figure 3 are given by:

$$\eta_{a,S} = \left[\frac{Y}{p_S(A_S^*)A_S^*}\right]_a\frac{k+1-\xi}{k(\xi-1)f_S^P} \quad \text{and} \quad \eta_{a,L} = \left[\frac{Yp_L(A_L^*)^{\xi-1}(A_L^*)^{\xi-1}}{p_S(A_S^*)^\xi(A_S^*)^\xi}\right]_a\frac{k+1-\xi}{k(\xi-1)\left(\frac{f_S^P}{f_L^P}\right)^{\frac{1}{k}}f_S^P}.$$

Since $P = \left[\sum_{i=S,L}\eta_i p_i(\widetilde{A}_i)^{1-\xi}\Delta_i\right]^{\frac{1}{1-\xi}}$ in the open economy equilibrium, the axis intercepts after trade liberalization and before any adjustment of relative factor prices are given by:

$$\eta_{op_1,S} = \left[\frac{Yp_S(A_S^*)^{-1}}{A_S^*\Delta_S}\right]_{op_1}\frac{k+1-\xi}{k(\xi-1)f_S^P} \quad \text{and} \quad \eta_{op_1,L} = \left[\frac{Yp_L(A_L^*)^{\xi-1}(A_L^*)^{\xi-1}}{p_S(A_S^*)^\xi(A_S^*)^\xi\Delta_L}\right]_{op_1}\frac{(k+1-\xi)\left(\frac{f_L^P}{f_S^P}\right)^{\frac{1}{k}}}{k(\xi-1)f_S^P}.$$

Thus, $\frac{\eta_{op_1,i}}{\eta_{a,i}} = \frac{1}{\Delta_i}$, $i = S, L$ and $\frac{\eta_{op_1,S}}{\eta_{a,S}} < \frac{\eta_{op_1,L}}{\eta_{a,L}}$ since $\Delta_S > \Delta_L$. This implies that the zero cutoff profit condition becomes steeper and shifts inward (note that $\Delta_i > 1$).

In order to determine how the increase in w_S affects the η_S–axis intercept, we consider the following partial derivative: $\frac{\partial\left[\frac{Y}{p_S(A_S^*)A_S^*}\right]}{\partial w_S} = \left(\frac{\overline{S}}{\overline{L}} - \frac{\phi_S w_S^{-\sigma}\Omega_S^{\sigma-1}}{1-\phi_S}\right)\frac{(1-\phi_S)\overline{L}\xi(A_S^*)^\sigma c_S(A_S^*)^\sigma}{p_S(A_S^*)^2(A_S^*)^2(\xi-1)} < 0$. Thus, we obtain $\left[\frac{Y}{p_S(A_S^*)A_S^*}\right]_{op_2} < \left[\frac{Y}{p_S(A_S^*)A_S^*}\right]_{op_1}$, which implies $\eta_{op_2,S} < \eta_{op_1,S}$.

To determine how the increase in w_S affects the η_L–axis intercept, *first* note that the increase in w_S makes the zero cutoff profit condition ceteris paribus flatter since its slope is given by $\frac{d\eta_L}{d\eta_S} =$

$-\frac{\Delta_S}{\Delta_L}\left[\frac{p_S(\widetilde{A}_S)}{p_L(\widetilde{A}_L)}\right]^{1-\xi}$ and $\frac{\partial\left[p_S(\widetilde{A}_S)/p_L(\widetilde{A}_L)\right]}{\partial w_S} > 0$. *Second,* taking the ratio of the zero cutoff profit conditions

of the two types of firms leads to: $\left[\frac{p_L(A_L^*)}{p_S(A_S^*)}\right]^{\xi} = \frac{A_S^* f_S^P}{A_L^* f_L^P}$. Thus, the η_L–axis intercepts become:

$$\eta_{a,L} = \left[\frac{Y\frac{(f_S^P)^{\xi}}{(f_L^P)^{\xi}}}{p_L(A_L^*)A_L^*}\right]_a \frac{(k+1-\xi)\left(\frac{f_L^P}{f_S^P}\right)^{\frac{1}{k}}}{k(\xi-1)f_S^P} \quad \text{and} \quad \eta_{op2,L} = \left[\frac{Y\frac{(f_S^P)^{\xi}}{(f_L^P)^{\xi}}}{p_L(A_L^*)A_L^*}\right]_{op2} \frac{(k+1-\xi)\left(\frac{f_L^P}{f_S^P}\right)^{\frac{1}{k}}}{k(\xi-1)f_S^P}.$$

It follows immediately that $\eta_{a,L} < \eta_{op2,L}$ since $\frac{\partial\left[\frac{Y}{p_L(A_L^*)A_L^*}\right]}{\partial w_S} > 0$. *Third,* since the zero cutoff profit condition becomes flatter as w_S increases and since $\eta_{op2,S} < \eta_{op1,S}$, we have that $\eta_{op2,L} < \eta_{op1,L}$.

G   Proof of proposition 3

Let $\Xi_j = \frac{\int_{A_j^*}^{\infty} q_j(A)p_j(A)\eta_j\mu(A)dA + \int_{A_{X,j}^*}^{\infty} q_{X,j}(A)p_j(A)s_{X,j}\eta_j\mu_{X,j}(A)dA}{\sum_{i=L,S}\left[\int_{A_i^*}^{\infty} q_i(A)p_i(A)\eta_i\mu(A)dA + \int_{A_{X,i}^*}^{\infty} q_{X,i}(A)p_i(A)s_{X,i}\eta_i\mu_X(A)dA\right]}$ be the share of GDP produced by

firms using intensively factor j. Substituting the demand functions, the pricing condition and the equilibrium value of w_S into Ξ_j we obtain:

$$\Xi_j = \frac{\mathfrak{F}_j^{\frac{\xi-1+k\xi}{k\xi}}\left(f_j^P\right)^{\frac{k}{\xi-1}}}{\mathfrak{F}_L^{\frac{\xi-1+k\xi}{k\xi}}\frac{\eta_L}{\eta_j}\left(f_L^P\right)^{\frac{k}{\xi-1}} + \mathfrak{F}_S^{\frac{\xi-1+k\xi}{k\xi}}\frac{\eta_S}{\eta_j}\left(f_S^P\right)^{\frac{k}{\xi-1}}},$$

where $\mathfrak{F}_j = \left(f_j^P\right)^{\frac{k+1-\xi}{1-\xi}} + \Gamma$, $j = S, L$, and $\Gamma \equiv \frac{N-1}{\tau^k f_X^{\frac{k+1-\xi}{\xi-1}}}$. Trade liberalization impacts $\mathfrak{F}_j$ as well as

$\frac{\eta_S}{\eta_L}$, and if the factor intensity gap is at its maximum, the impact on $\frac{\eta_S}{\eta_L}$ is most detrimental. Thus, to prove proposition 3, we first analyze how trade liberalization impacts Ξ_L if $\phi_S = 1$ and $\phi_L = 0$. Substituting $\frac{\eta_S}{\eta_L}\big|_{\phi_S=1,\phi_L=0}$ into the term for Ξ_L, we can determine the following partial derivative:

$$\frac{\partial\Xi_L\big|_{\phi_S=1,\phi_L=0}}{\partial\Gamma} = \frac{\left[(f_S^P)^{\frac{k+1-\xi}{1-\xi}} - (f_L^P)^{\frac{k+1-\xi}{1-\xi}}\right]\frac{\xi-1+2k\xi}{k\xi}\frac{\overline{L}}{\overline{S}}\left[\frac{(f_S^P)^{\frac{k}{\xi-1}}}{(f_L^P)^{\frac{k-1+\xi}{1-\xi}}}\right]^2 \mathfrak{F}_S^{\frac{\xi-1+k\xi}{k\xi}}\mathfrak{F}_L^{\frac{\xi-1+k\xi}{k\xi}}}{\left[\mathfrak{F}_L^{\frac{\xi-1+2k\xi}{k\xi}}(f_L^P)^{\frac{2k}{\xi-1}} + \mathfrak{F}_S^{\frac{\xi-1+2k\xi}{k\xi}}(f_S^P)^{\frac{2k}{\xi-1}}\frac{\overline{L}}{\overline{S}}\Omega_S^{\xi-1}\right]^2 \Omega_S^{1-\xi}} < 0$$

since $k + 1 - \xi > 0$ and $f_S^P > f_L^P$. Thus, even when the firm selection is most in favor of unskilled labor intensive firms, the weighting factor Ξ_L decreases with trade liberalization, and the opposite is true for Ξ_S since $\Xi_S = 1 - \Xi_L$. Thus, since $\widetilde{A}_S > \widetilde{A}_L$ and since $\frac{\partial\widetilde{A}_S}{\partial\Gamma} > \frac{\partial\widetilde{A}_L}{\partial\Gamma}$ (see lemma 2), $\widetilde{A} = \widetilde{A}_L\Xi_L + \widetilde{A}_S\Xi_S$ increases with trade liberalization, even when the firm selection is most detrimental for the skilled labor intensive firms. A fortiori the result is true if the factor intensity gap becomes smaller, and the increase in $\widetilde{A}$ becomes larger. Finally, as trade becomes less restricted, both $\widetilde{A}_L$ and $\widetilde{A}_S$ increase, which increases $\widetilde{A}$.

H   Proof of proposition 4

Real income is given by: $\frac{Y}{P} = \frac{w_S\overline{S}+\overline{L}}{[\int_{\nu\in\Upsilon} p(\nu)^{1-\xi}d\nu]^{\frac{1}{1-\xi}}}$. Note first that any change in w_S across trade equilibria does not impact real income. This follows immediately from calculating $\frac{\partial\left(\frac{Y}{P}\right)}{\partial w_S}$ and substituting the equilibrium values for $\frac{\eta_S}{\eta_L}$ (see appendix E) and w_S into the resulting term. As a consequence, we will set w_S equal to a constant level $\overline{w}_S$ in the following. Solving equations 7 and 8 for $\eta_{a,L}$ and $\eta_{a,S}$ and equations 15 and 16 for $\eta_{op,L}$ and $\eta_{op,S}$ and using the term for P yields:

$$\frac{\left(\frac{Y}{P}\right)_{op}}{\left(\frac{Y}{P}\right)_{a}} = \left[\frac{(1-\phi_L)\frac{k}{k+1-\xi}f_L^P(\widetilde{A}_{op,L})^{1-\xi}+\Theta f_S^P(1-\phi_S)\frac{k}{k+1-\xi}(\widetilde{A}_{op,S})^{1-\xi}}{(1-\phi_L)\frac{k}{k+1-\xi}f_L^P(\widetilde{A}_{a,L})^{1-\xi}+\Theta f_S^P(1-\phi_S)\frac{k}{k+1-\xi}(\widetilde{A}_{a,S})^{1-\xi}}\right]^{\frac{1}{1-\xi}}, \qquad (27)$$

where $\Theta \equiv \frac{\frac{\overline{L}}{\overline{S}}\overline{w}_S^{-\sigma}\Omega_S^{\sigma-1}\phi_L-(1-\phi_L)\left[\phi_L\left(\frac{\overline{w}_S}{\Omega_S}\right)^{1-\sigma}+1-\phi_L\right]^{\frac{\xi}{\sigma-1}}}{1-\phi_S-\frac{\overline{L}}{\overline{S}}\overline{w}_S^{-\sigma}\Omega_S^{\sigma-1}\phi_S\left[\phi_S\left(\frac{\overline{w}_S}{\Omega_S}\right)^{1-\sigma}+1-\phi_S\right]^{\frac{\xi}{\sigma-1}}}$. Thus, as trade becomes less restricted, $\widetilde{A}_{op,L}$ and $\widetilde{A}_{op,S}$ increase (see lemma 2), and so does real income. Furthermore,

$$\frac{\partial\left[\frac{\left(\frac{Y}{P}\right)_{op}}{\left(\frac{Y}{P}\right)_{a}}\right]}{\partial\phi_S} = \frac{\left[\frac{\left(\frac{Y}{P}\right)_{op}}{\left(\frac{Y}{P}\right)_{a}}\right]^{\xi}}{(1-\xi)\widetilde{A}_{a,S}^{\xi-1}\widetilde{A}_{a,L}^{\xi-1}}\frac{\left(\frac{\widetilde{A}_{op,L}^{1-\xi}}{\widetilde{A}_{a,L}^{1-\xi}}-\frac{\widetilde{A}_{op,S}^{1-\xi}}{\widetilde{A}_{a,S}^{1-\xi}}\right)f_S^P f_L^P(1-\phi_L)\left[\Theta-\frac{\partial\Theta}{\partial\phi_S}(1-\phi_S)\right]}{\left[(1-\phi_L)f_L^P\widetilde{A}_{a,L}^{1-\xi}+\Theta f_S^P(1-\phi_S)\widetilde{A}_{a,S}^{1-\xi}\right]^2},$$

with $\frac{\partial\Theta}{\partial\phi_S} = \Theta\left\{\frac{1+\frac{\overline{L}}{\overline{S}}\overline{w}_S^{-\sigma}\Omega_S^{\sigma-1}}{\left(\frac{1-\phi_S}{\phi_S}-\frac{\overline{L}}{\overline{S}}\overline{w}_S^{-\sigma}\Omega_S^{\sigma-1}\right)\phi_S}+\frac{\frac{\xi}{1-\sigma}\left[\left(\frac{\overline{w}_S}{\Omega_S}\right)^{1-\sigma}-1\right]}{\left(\frac{\overline{w}_S}{\Omega_S}\right)^{1-\sigma}\phi_S+1-\phi_S}\right\} < 0$. Note that $\frac{\widetilde{A}_{op,L}^{1-\xi}}{\widetilde{A}_{a,L}^{1-\xi}}-\frac{\widetilde{A}_{op,S}^{1-\xi}}{\widetilde{A}_{a,S}^{1-\xi}} > 0$, which follows from lemma 2, and $1-\xi < 0$. Thus, $\frac{\partial\left[\frac{\left(\frac{Y}{P}\right)_{op}}{\left(\frac{Y}{P}\right)_{a}}\right]}{\partial\phi_S} < 0$.

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