Analysis of Low-Frequency-Oscillations in Single-Phase AC Systems by LTP Theory

Valerio Salis, Alessandro Costabeber, Pericle Zanchetta
Power Electronics, Machines and Control Group
University of Nottingham
University Park, NG7 2RD
Nottingham, United Kingdom
Email: eexvs2@nottingham.ac.uk

Stephen Cox
School of Mathematical Sciences
University of Nottingham
University Park, NG7 2RD
Nottingham, United Kingdom

ABSTRACT

The paper presents the analysis of a single-phase active rectifier, with DC constant power load, affected by Low-Frequency Oscillation (LFO) behaviour. Conventional LTI analysis tools might fail to reveal the LFO phenomenon, whereas the LTP (Linear Time Periodic) approach, here exploited with analytical derivation and detailed simulations, provides superior results.

INTRODUCTION

In recent years, system instabilities due to low-frequency-oscillation (LFO) problems have occurred in different single-phase traction power supply system. In [1] a report of the last cases of LFO is provided and the main cause of these phenomena is found to be in the harmonic interaction between the line-side converters of the vehicle-train and the single-phase AC grid. By the analysis provided in this paper, based on the linearised models of the vehicle-grid system, it is found that the LFO behaviour arise when the closed-loop system shows two dominant poles that are close to the imaginary axis. Because of the non-linearities of the system, like saturations for the maximum value of the reference signals in the control algorithm, these unstable modes do not diverge, but they appear as a stable and undesired LFO. In the same way as in [1], in [2] an impedance model valid below $10-15$Hz is used to assess stability in presence of LFOs, making the proposed method useful to detect only LFO instabilities within this frequency range. However, instabilities in power systems might occur at any frequency, hence a general method able to predict if a system is stable or not, regardless the frequency range of oscillations, is required. One promising and used method is Harmonic Linearisation (HL), [3], [4], although it might not reveal LFO, as stated in [3]. Another established method is Dynamic Phasor approach (DP) [5], however it relies on the approximation that the system is dominated by the first-harmonic component, which is not true in the system under analysis in this paper, that has a not negligible second-harmonic component represented by the ripple in the DC-link voltage.

Thus, in this paper, a general method for the stability analysis of power systems, based on the LTP theory developed by Wereley and Hall [6], is exploited to analyse a single-phase active rectifier that exhibits a LFO behaviour. This method detects the instability regardless the frequency range of the oscillation. The analysis is based on the average-model of the system (defined either in continuous or discrete-time), which is a Non-Linear Time Periodic (NLTP) model. Then, linearisation is performed around the steady-state solution of the system and stability is assessed by evaluating the eigenvalue loci plot of the linearised model. A comparison with the standard LTI analysis is also provided, showing that LTP approach provides more accurate and precise results, as also reported in [7], [8].

FULL-BRIDGE RECTIFIER MODEL

The single-phase active rectifier is connected on the AC side to an equivalent grid represented by a sinusoidal voltage source, $v_g(t) = V_g \sin(\omega_g t)$, $\omega_g = 2\pi f_g = 2\pi/T_g$, in series with an $R_g L_g$ impedance, while on the DC side it is connected to a capacitor, C_{dc}, in parallel with a constant power load, which is represented as an equivalent current source generator,

TABLE I: system parameters

V_{grid} = $115\sqrt{2}$ V	f_{grid} = 50 Hz	f_{pwm} = 10 kHz	V^*_{dc} = 300 V
L_g = 2.78 mH	R_g = 0.4 Ω	C_{dc} = 400 μF	P^* = 1 kW
k_{pi} = 16.28	k_{ii} = 3984	$\omega_0 = 2\omega_g$	Q = 20
k^A_{pv} = 9.9e-5	k^A_{iv} = 0.035	k^B_{pv} = 7.9e-6	k^B_{iv} = 0.035
$T_s = 1/f_s$ = 5e-5	M = 0.08	i_g^{max} = 13.5 A	

controlled by the voltage across the DC link, $v_{dc}(t)$, thus $i_p(t) = P^*/v_{dc}(t)$. Unity-power-factor mode of operation is considered, with direct measurement of the grid voltage typical of relatively low power applications. A notch filter is used in order to attenuate the ripple of the voltage, $v_{dc}(t)$, at twice the grid frequency $(2f_g)$ and saturation is applied to the output produced by the voltage PI controller, in order to guarantee that the grid current, $i_g(t)$, will not exceed the maximum peak value i_g^{max}. The unit digital computational delay is considered, making the example system similar to a real-case converter. A double-update mode is considered, thus the control algorithm is executed at twice the switching frequency, $f_s = 2f_{pwm}$.

In general, stability analysis can be performed in two equivalent ways:

- continuous-time domain: the control algorithm is represented in the Laplace domain and features like the unit computational delay must be taken into account by an approximated model like the Padé's one, in order to obtain an accurate model; however current and voltage dynamics of AC and DC subsystems are naturally described in Laplace;
- discrete-time domain: the control algorithm is naturally represented in the z-domain, as well as digital computation delay, but AC and DC subsystems must be discretized.

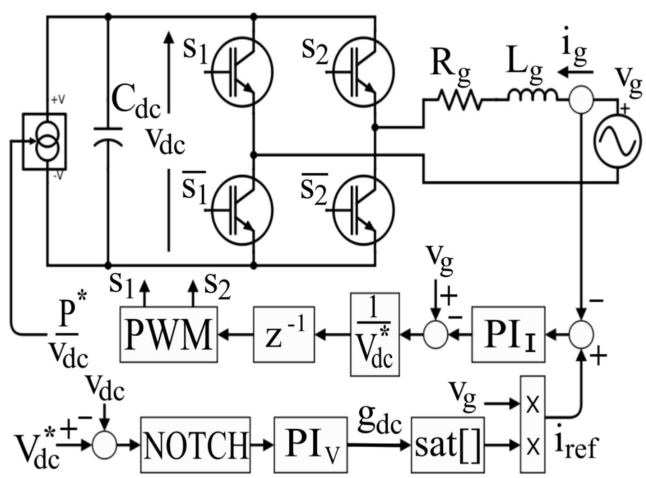

Fig. 1: full-bridge rectifier - switching model

In this work the analysis is carried out in the discrete-time domain. Thus, the average model on which stability analysis is based includes the same control algorithm that would be implemented in the DSP (with computational delay) and the AC and DC subsystems are discretized using a zero-order-hold (ZOH) transformation.

The design of the controllers has been made in continuous-time domain, just because it makes it easier to define the controller gains in order to meet design specifications such us crossover frequency, f_c, and phase margin, PM, of the closed loop system. The inner current control is implemented using the linearised open-loop transfer function $TOL_I(s) = (sL_g + R_g)^{-1}$, with a closed-loop crossover frequency $f_c = 1$kHz and phase margin $PM = 70°$:

$$\mathrm{PI}_I(z) = \text{Tustin}\left[k_{pi} + \frac{k_{ii}}{s}\right] = F_1 + \frac{F_0}{z + E_0} \tag{1}$$

whereas for the outer voltage control $TOL_V(s) = V_g^2/(2C_{dc}V^*_{dc}s)$ is used. It is known, in general, that reducing the phase margin of the closed-loop system implies lower stability margins; thus, two cases are considered: case (A), with $f_c = 10$Hz and $PM = 10°$, which will provide a stable system, and case (B), with $f_c = 10$Hz and $PM = 0.8°$, which will provide a system affected by an LFO behaviour:

$$\mathrm{PI}_V(z) = \text{ZOH}\left[k_{pv}^{A,B} + \frac{k_{iv}^{A,B}}{s}\right] = D_1 + \frac{D_0}{z + C_0} \tag{2}$$

The notch filter is discretized as:

$$\mathrm{N}(z) = \text{Tustin}\left[\frac{s^2+\omega_0^2}{s^2+\omega_0/Qs+\omega_0^2}\right] = B_2 + \frac{B_1 z + B_0}{z^2 + A_1 z + A_0} \tag{3}$$

The AC and DC subsystems are discretized applying the ZOH transformation, and the overall system is described by the following model:

$$\begin{aligned}
x_1(k+1) &= x_2(k)\ , \quad x_2(k+1) = -A_0x_1(k) - A_1x_2(k) + V_{dc}^* - v_{dc}(k)\\
u_v(k) &= B_0x_1(k) + B_1x_2(k) + B_2(V_{dc}^* - v_{dc}(k))\\
x_3(k+1) &= -C_0x_3(k) + u_v(k)\ , \quad g_{dc}(k) = D_0x_3(k) + D_1u_v(k)\\
i_{ref}(k) &= sat_M[g_{dc}(k)]v_g(k)\ , \quad x_4(k+1) = -E_0x_4(k) + i_{ref}(k) - i_g(k)\\
v_{co}(k) &= F_0x(k) + F_1(i_{ref}(k) - i_g(k))\ , \quad d(k+1) = (v_g(k) - v_{co}(k))/V_{dc}^*\\
i_g(k+1) &= i_g(k) + T_s/L_g[v_g(k) - R_gi_g(k) - d(k)v_{dc}(k)]\\
v_{dc}(k+1) &= v_{dc}(k) + T_s/C_{dc}[d(k)i_g(k) - P^*/v_{dc}(k)]
\end{aligned} \tag{4}$$

which, with proper substitutions can be represented as a Discrete Non-Linear Time Periodic (DNLTP) state-space model of the form $x(k+1) = f(x(k)) + g(x(k))v_g(k)$, where all the state-space variables are L-periodic, with $L = T_g/T_s$. $sat_M(q)$ is the saturation function such that $sat_M(q) = q$ for $|q| < M$ and $sat_M(q) = sign(q)M$ for $|q| > M$.

Remark - For real system application, the analysed active rectifier is not well designed, since the phase margin of the voltage PI is badly selected (usually it is chosen to be around 70°), and also a PR controller is usually preferred. However, this is not an issue for the objective of this work, which is to prove that the stability method based on the LTP theory can be successfully exploited to detect LFO behaviours, even when LTI traditional approaches fail. To demonstrate this, the system is intentionally brought to operate in a LFO operation mode by moving the eigenvalues of the closed-loop system close to the imaginary axis (hence with small phase margin), as in [7], and both LTP and LTI approaches are exploited to assess the stability.

ANALYTICAL RESULTS

Based on the DNLTP model (4), first the steady-state solution is calculated, either numerically or applying Harmonic Balance. Then, linearisation around the calculated L-periodic solution is performed and the Discrete Linear Time Periodic (DLTP) model of the system is derived:

$$x(k+1) = A(k)x(k) \tag{5}$$

with the matrix $A(k)$ being L-periodic as well. As reported in [9], [10], the system described in (5) can be equivalently represented by a time-invariant model of the form:

$$q(k+1) = \bar{A}q(k) \tag{6}$$

with $q(k)$ being a sampled version of $x(k)$, such that $q(k) = x(kL)$, and the matrix $\bar{A}$ being constant. The most relevant feature for our purposes is the fact that stability can be equivalently assessed based on either (5) or (6). Since $\bar{A}$ can be easily calculated by:

$$\bar{A} = A(k+L-1)\cdot A(k+L-2)\cdots A(k+1)\cdot A(k) \tag{7}$$

stability analysis is performed based on the eigenvalue loci of $\bar{A}$. If all the eigenvalues of (7) lie inside the unit-circle, the system is stable, otherwise it is unstable. Results from the LTP analysis are shown in Fig.2(a)-(c) for the cases (A) and (B), respectively (refer to the voltage-control design). It can be seen that case (B) leads to an unstable system, which is in agreement with the simulation results. In Fig.2(b)-(d) the same cases are analysed based on the LTI closed-loop system $G_{CL} = G_{OL}/(1+G_{OL})$, with $G_{OL} = N(z)PI_V(z)z^{-1}TOL_V(z)$ and $TOL_V(z) = \mathrm{ZOH}[TOL_V(s)]$. As it can be seen, in both cases the system results to be stable, which is in disagreement for case (B) with the results from simulations. It is worth noting that two of the LTI eigenvalues in case (B), Fig.2-(d), are very close to the unit circle, which means that the error committed by using the LTI analysis is not very big. However, as also stated in [7], the difference between LTP and LTI analysis becomes important when fast voltage control loops are implemented, leading in general to the conclusion that LTP approach is preferable for the analysis of systems with high performance.

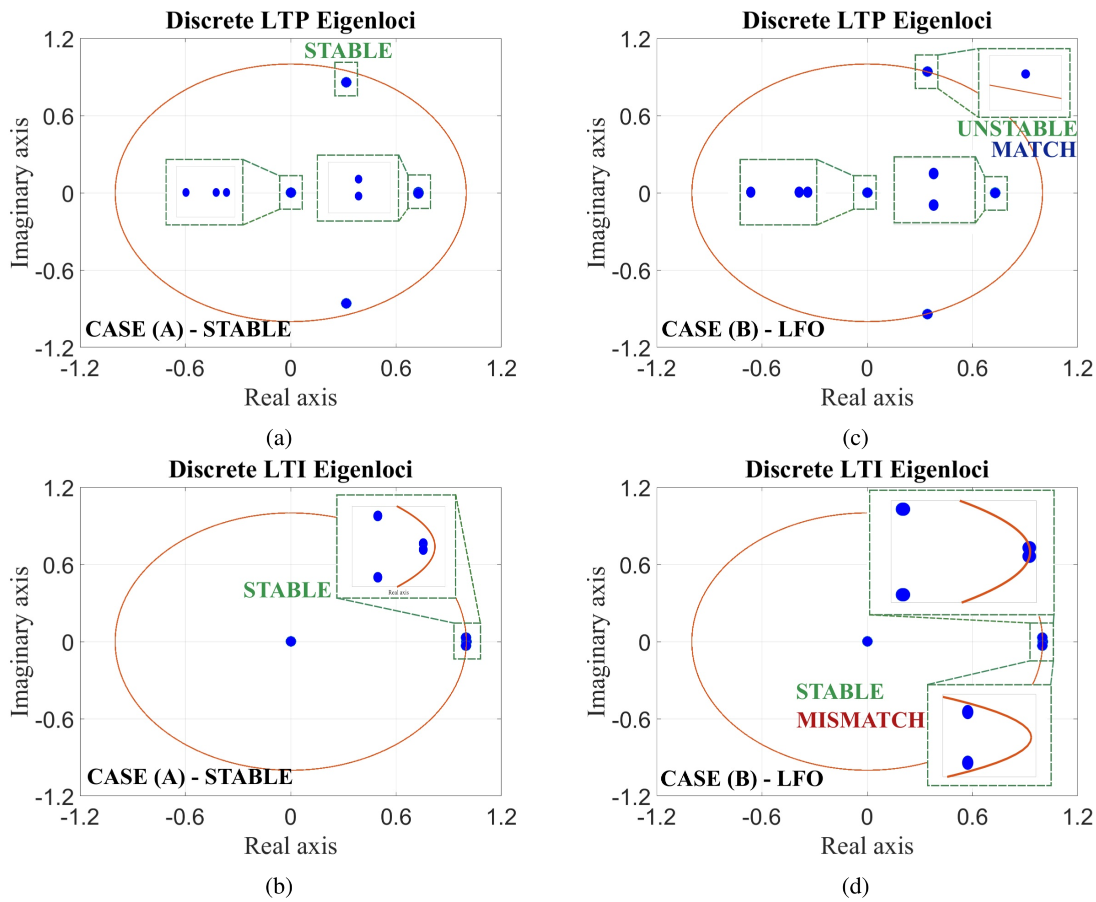

Fig. 2: (a), (b) stable system with $PM = 10°$, (c), (d) unstable system with $PM = 0.8°$

SIMULATION RESULTS

Simulations have been performed in Matlab-Simulink and PLECS toolbox, based on the switching-model reported in Fig.1 and control algorithm implemented as a script in C-language. The two cases (A) and (B) have been simulated and in Fig.3 the steady-state behaviour of the relevant quantities is reported. It can be seen that for the stable case (A), Fig.3 (a)-(b), the current $i_g(t)$ follows the reference one, $i_{ref}(t)$, and the voltage $v_{dc}(t)$ follows the reference one, V_{dc}^*, with the normal ripple at twice the grid frequency, $2f_g$, with a peak ripple of 12.8V. For the unstable case (B), Fig.3 (c)-(d), both current $i_g(t)$ and voltage $v_{dc}(t)$ show a LFO behaviour. Such LFO instability does not diverge, but it remains stable in time, due to the saturation block that limits the maximum peak-value of the reference current $i_{ref}(t)$; whereas without the saturation block, voltage and currents will diverge until the control system trips. From the Fast Fourier Transform (FFT) analysis, it has been found that the frequency of this oscillation is 8.3Hz, with peak-amplitude 7.4V for the DC-link voltage and 0.55A for the grid current. These results agree with the analytical ones obtained using the LTP approach. Moreover, the current $i_g(t)$ follows well the reference one $i_{ref}(t)$, confirming that the LFO arise due to the intentional poor design of the voltage loop. A further decrease of the phase margin used for the design of the voltage controller, such that $PM < 0.8°$, will result in a system with a stronger constant LFO behaviour, i.e. with higher voltage and current peak-amplitudes at the unstable low frequency. These simulations are not reported in this abstract for the sake of brevity.

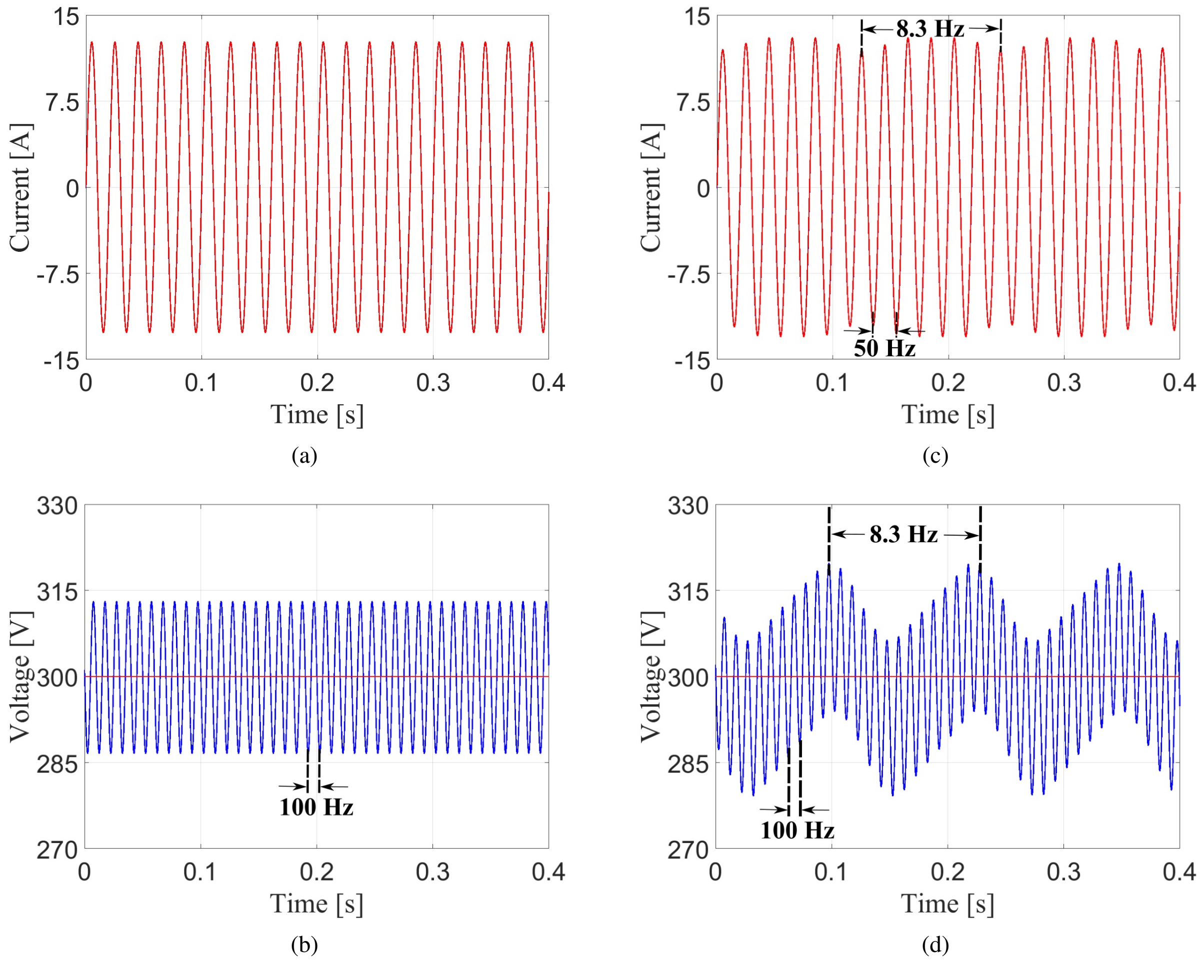

Fig. 3: currents: blue - $i_g(t)$, red - $I_{ref}(t)$; voltages: blue - $v_{dc}(t)$, red - V_{dc}^*; (a), (b) case (A) - stable system, (c), (d), case (B) - unstable system affected by LFO behaviour

CONCLUSIONS

In this paper a general method is exploited, based on LTP theory, to perform stability analysis of complex non-linear power systems. A case study of a single-phase active rectifier with constant power load has been used to show the practical application of the method. The system has been intentionally brought to operate in a steady-state operation mode affected by LFO. It has been shown that the LTP approach, compared to the LTI one, provides more accurate results. Analytical and simulation results are provided to validate the method.

References

[1] H. Wang, W. Mingli, and J. Sun, "Analysis of low-frequency oscillation in electric railways based on small-signal modeling of vehicle-grid system in dq frame," *IEEE Transactions on Power Electronics*, vol. 30, pp. 5318–5330, Sept 2015.

[2] J. Suarez, P. Ladoux, N. Roux, H. Caron, and E. Guillame, "Measurement of locomotive input admittance to analyse low frequency instability on ac rail networks," in *2014 International Symposium on Power Electronics, Electrical Drives, Automation and Motion*, pp. 790–795, June 2014.

[3] J. Sun, "Small-signal methods for ac distributed power systems; a review," *Power Electronics, IEEE Transactions on*, vol. 24, pp. 2545–2554, Nov 2009.

[4] S. Shah and L. Parsa, "On impedance modeling of single-phase voltage source converters," in *2016 IEEE Energy Conversion Congress and Exposition (ECCE)*, pp. 1–8, Sept 2016.

[5] S. Lissandron, L. D. Santa, P. Mattavelli, and B. Wen, "Experimental validation for impedance-based small-signal stability analysis of single-phase interconnected power systems with grid-feeding inverters," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 4, pp. 103–115, March 2016.

[6] N. M. Wereley and S. R. Hall, "Frequency response of linear time periodic systems," in *Decision and Control, 1990., Proceedings of the 29th IEEE Conference on*, pp. 3650–3655 vol.6, Dec 1990.

[7] R. Z. Scapini, L. V. Bellinaso, and L. Michels, “Stability analysis of pfc ac-dc full-bridge converters with reduced dc-link capacitance,” *IEEE Transactions on Power Electronics*, vol. PP, no. 99, pp. 1–1, 2017.

[8] J. Kwon, X. Wang, F. Blaabjerg, C. L. Bak, V. S. Sularea, and C. Busca, “Harmonic interaction analysis in grid-connected converter using harmonic state space (hss) modeling,” *IEEE Transactions on Power Electronics*, vol. PP, no. 99, pp. 1–1, 2016.

[9] R. Meyer and C. Burrus, “A unified analysis of multirate and periodically time-varying digital filters,” *IEEE Transactions on Circuits and Systems*, vol. 22, pp. 162–168, Mar 1975.

[10] S. Bittanti and P. Colaneri, “Invariant representations of discrete-time periodic systems,” *Automatica*, vol. 36, no. 12, pp. 1777 – 1793, 2000.

[11] M. Cespedes and J. Sun, “Renewable energy systems instability involving grid-parallel inverters,” in *2009 Twenty-Fourth Annual IEEE Applied Power Electronics Conference and Exposition*, pp. 1971–1977, Feb 2009.

[12] J. Kwon, X. Wang, F. Blaabjerg, and C. L. Bak, “Comparison of lti and ltp models for stability analysis of grid converters,” in *2016 IEEE 17th Workshop on Control and Modeling for Power Electronics (COMPEL)*, pp. 1–8, June 2016.