

# Generation of a squeezed state of an oscillator by stroboscopic back-action-evading measurement

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Continuous observation on an oscillator results in quantum back-action which limits the knowledge acquired by the measurement. A careful balance between the information obtained and the back-action disturbance leads to the standard quantum limit of precision. This limit can be surpassed by a measurement with strength modulated at twice the oscillator frequency, resulting in a squeezed state of the oscillator motion, as proposed decades ago [1–3]. Here, we report on the generation of a squeezed state of an oscillator by a stroboscopic back-action evading measurement. The oscillator is the spin of an atomic ensemble precessing in magnetic field. The oscillator prepared initially in nearly the ground state is stroboscopically coupled to an optical mode of a cavity. A measurement of the output light results in a  $2.2 \pm 0.3$  dB squeezed state of the oscillator. The demonstrated spin squeezed state of  $10^8$  atoms with the angular spin variance of  $8 \times 10^{-10}$  rad<sup>2</sup> is promising for magnetic field sensing.

The Heisenberg uncertainty sets the limit of how precisely two non-commuting variables, such as the canonical position and momentum with the commutation relation  $[\hat{X}_0, \hat{P}_0] = i$ , can be specified simultaneously; however, there are no physical limitations on determining the value of an individual variable. For an oscillator with frequency  $\Omega$  the position variable in the laboratory frame is  $\hat{X} = \hat{X}_0 \cos(\Omega t) + \hat{P}_0 \sin(\Omega t)$  where  $\hat{X}_0, \hat{P}_0$  are the rotating frame variables. Quantum states for which a canonical variable has reduced uncertainty with respect to the oscillator ground state, for example,  $\text{Var}(\hat{X}_0) < 1/2$ , are called squeezed states (SS). Squeezed states for matter oscillators have been first demonstrated for motion of a single ion [4] and later for magnetic spin oscillators by electron-nucleus entanglement [5, 6] and by spin-spin interaction [7, 8].

An intriguing approach towards generation of a SS in an oscillator is a stroboscopic quantum non demolition (QND) or back-action evading measurement proposed in [1–3]. If a meter is coupled to the oscillator with an interaction Hamiltonian  $\hat{H} \propto \hat{X}$  and if the measurement strength is maximized at times  $t = 0, \pi/\Omega, \dots, n\pi/\Omega$  the noise of the meter does not couple to  $\hat{X}_0$  and the read-out of this QND measurement yields a SS of the oscillator with a degree of squeezing defined as  $\xi^2 = 2\text{Var}(\hat{X}_0) < 1$ . Another version of this approach based on harmonic modulation of the measurement strength which works similarly to the stroboscopic measurement has been proposed in [9]. Previous attempts to implement this approach with a magnetic spin oscillator [10] and a mechanical oscillator [11] demonstrated reduction in the back-action noise but did not result in a SS due to the insuff-

ficient strength of the QND measurement compared to the decoherence caused by the environment. In a separate line of work back-action evasion has been demonstrated for a joint state of two spin oscillators [12, 13]. There a continuous QND measurement on both oscillators, one of which has an effective negative mass, has been shown to generate an entangled state of the two oscillators. Squeezed states of oscillating spin components belong to a broad field of spin squeezed states. Recent advances in spin squeezing by QND measurements of atomic state populations have been reported in [14–17]. Compared to these works, here we generate spin squeezing of an oscillating atomic spin coherence by a stroboscopic measurement.

A key requirement for achieving the SS is that the decoherence due to interaction with the environment is reduced to the extent that the oscillator maintains its quantum state for the time longer than a QND measurement takes. Every decoherence event is linked with loss of information about the measured variable and with the decay to the ground or thermally excited state of the oscillator depending on the decoherence mechanism. In this work this requirement is met by firstly enhancing the rate of the optical QND interaction by placing the spin ensemble in an optical cavity, and secondly by placing the spins in a designed spin-protecting environment.

The oscillator used in this study is a collective spin of an atomic ensemble precessing in a bias magnetic field (Fig. 1a). The collective spin components in the lab frame oscillate as  $\hat{J}_{z/y} = \pm \hat{J}_{z0/y0} \cos(\Omega t) / \sin(\Omega t) + \hat{J}_{y0/z0} \sin(\Omega t) / \cos(\Omega t)$ . For a macroscopic spin orientation  $J_x$  along the magnetic field, the energy spectrum

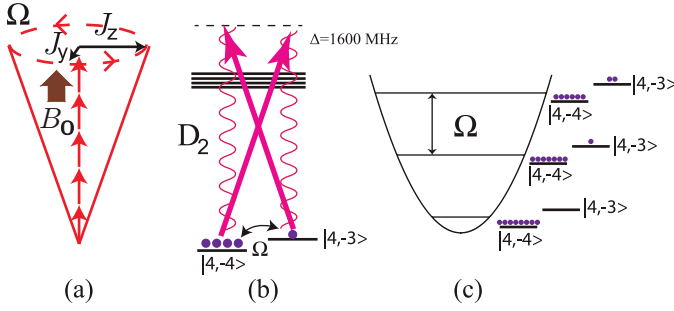


FIG. 1. **Magnetic oscillator.** **a**, The collective spin of an ensemble of atoms with macroscopic orientation along the magnetic field  $B_0$  precesses around  $B_0$  with a Larmor frequency  $\Omega$ . Normalized quantized values of the projections  $J_{y0,z0}$  in the rotating frame are the canonical variables for this oscillator. **b**, The energy diagram of an atomic constituent of the oscillator. In the ground state of the oscillator atoms are in the lower energy state  $|4, -4\rangle$ . Raman scattering of photons (wavy lines) driven by the linearly polarized input light orthogonal to the dc magnetic field (solid arrows) creates collective excitations in level  $|4, -3\rangle$  and corresponding quantum coherences  $|4, -3\rangle\langle 4, -4|$  responsible for  $\hat{J}_{y,z}$  and oscillating at  $\Omega$ . **c**, The  $n$ -th excited state of the oscillator corresponds to  $n$  collective spin flips generated as in (b). A squeezed state is a coherent superposition of such number states (comments in the text).

of the spin ensemble can be mapped onto that of a harmonic oscillator [6] with a resonance frequency set by the Larmor frequency of the spin precession (Fig. 1b,c). Mathematically this is expressed through the Holstein-Primakoff transformation [18]. Canonical position and momentum operators can be defined through the collective symmetric spin observables in the rotating frame  $[\hat{J}_{y0}, \hat{J}_{z0}] = iJ_x = iN_{\text{at}}F$  ( $\hbar = 1$ ) as:  $\hat{X}_0 = \hat{J}_{z0}/\sqrt{|\langle J_x \rangle|}$ , and  $\hat{P}_0 = \hat{J}_{y0}/\sqrt{|\langle J_x \rangle|}$  where it is assumed that the collective spin is well oriented, so that the population of the end state with the magnetic quantum number  $m_F = -F$  is close to the total number of spins  $N_{\text{at}}$  and hence the macroscopic quantity  $J_x$  is treated as a number rather than an operator. The ground state of the harmonic oscillator corresponds to all atoms being in the  $F = 4, m_F = -4$  state and the absence of the ensemble coherence  $|4, -3\rangle\langle 4, -4|$  (Fig. 1b). The mean number of excitations above the ground state can be evaluated as  $\bar{n} = \text{Var}(\hat{X}_0) + \text{Var}(\hat{P}_0) - 1$ . An excitation with the creation operator  $\hat{a}^\dagger = (\hat{X}_0 - i\hat{P}_0)/\sqrt{2}$  corresponds to a quantum of excitation, also called a polariton, distributed symmetrically among all the atoms of the oscillator. The ground state of the oscillator belongs to the class of Coherent Spin States (CSS) characterized by  $\text{Var}(\hat{J}_{y0}) = \text{Var}(\hat{J}_{z0}) = J_x/2 = N_{\text{at}}F/2$  [6].

The quantum state of the spin-oscillator can be created and probed through interaction with light field. In the limit of large probe detuning with respect to the atomic excited state hyperfine level (Fig. 1b), the light-spin in-

teraction can be approximated by the QND-type, Faraday Hamiltonian:  $H_{\text{int}} = 2(\kappa/\sqrt{N_{\text{ph}}})\hat{S}_z\hat{X}$ , where  $N_{\text{ph}}$  is the number of photons in the pulse of duration  $\tau$  and  $\hat{S}_z$  is the probe light Stokes operator in the circular basis normalized so that for a coherent pulse  $\text{Var}(S_z) = N_{\text{ph}}/(4\tau)$ . The coupling constant  $\kappa$  characterizes the QND interaction strength, which depends on the atom-light detuning, the excited-state linewidth and  $\kappa \propto \sqrt{N_{\text{at}}N_{\text{ph}}} \propto \sqrt{d_0\eta_\tau}$  [6], where  $d_0$  is the resonant optical depth of the atomic ensemble and  $\eta_\tau$  is proportional to the fractional number of decoherence events during the measurement time (see Supplementary Information).

For stroboscopic probing the relevant observable for the oscillator is the harmonic quadrature that evolves in phase with the modulation (chosen to be the cosine quadrature):  $\hat{x} = (1/TD) \int dt \hat{X}(t)\phi(t) \cos(\Omega t)$ , where  $T = 2\pi/\Omega$  is the oscillator period,  $D$  is the duty cycle of the probe,  $\phi(t)$  is a rectangular-pulse shaping function of unit amplitude, following the temporal evolution of the probe power, and the integration extends over one oscillator period. In the limit of zero duty cycle:  $\hat{x} = \hat{X}_0$ . The measurement record is the  $\cos\Omega t$  Fourier component of the photocurrent integrated over the pulse length:  $\hat{S}_{y,\tau} = \int_0^\tau dt \hat{S}_y(t)u(t) \cos(\Omega t)$ , where  $\hat{S}_y$  is the Stokes operator in the linear basis measured with polarization homodyning as shown in Fig. 2 and  $u$  is a mode function of unit energy over the pulse length that weights the measurement data according to the decoherence rate (see Methods). The evolution of variances due to the Hamiltonian interaction is described by (for  $u = 1$ ) (see Supplementary Information):

$$\text{Var}(\hat{S}_{y,\tau}) = \frac{\mathcal{B}N_{\text{ph}}}{8} \left[ 1 + \tilde{\kappa}^2 \text{Var}(\hat{x}_{\text{in}}) + \mathcal{C} \frac{\tilde{\kappa}^4}{3} \right], \quad (1)$$

$$\text{Var}(\hat{x}_{\text{out}}) = \text{Var}(\hat{x}_{\text{in}}) + \mathcal{C}\tilde{\kappa}^2, \quad (2)$$

where the subscripts (in), (out) indicate operators at the start and end of the interaction respectively,  $\mathcal{B} = 1 + \text{sinc}(\pi D)$ ,  $\tilde{\kappa} = \kappa\sqrt{\mathcal{B}}$  is the modified coupling constant, and  $\mathcal{C} \in [0, 1]$  quantifies the coupling of the probe noise to the observable. It can be shown (see Supplementary Information) that for a stroboscopic probe the back-action coupling constant is given by:

$$\mathcal{C} = \frac{1 - \text{sinc}(\pi D)}{1 + \text{sinc}(\pi D)}. \quad (3)$$

For a QND measurement  $\mathcal{C} = 0$ , while for a continuous probing  $\mathcal{C} = 1$ . In Eq. 1, the first term on the right-hand side specifies the imprecision due to the quantum noise of the meter (shot noise of light), whereas the second and third terms describe the oscillator input noise and back-action noise, respectively. In the following, the measurement noise associated with the oscillator in photon shot noise units will be denoted collectively with  $\text{Var}(\hat{x}_{\text{m}}) = \tilde{\kappa}^2 \text{Var}(\hat{x}_{\text{in}}) + \mathcal{C} \frac{\tilde{\kappa}^4}{3} + \eta_\tau$ , where  $\eta_\tau$  expresses

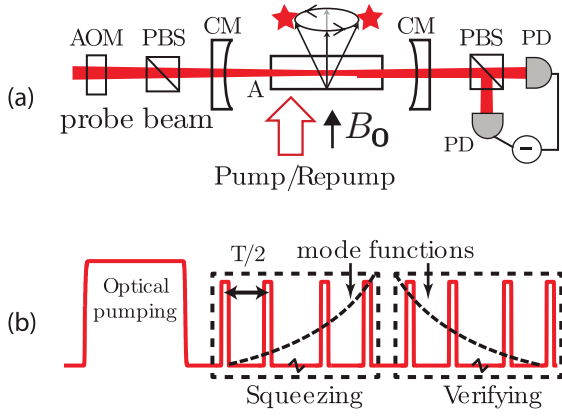


FIG. 2. **Outline of the experimental set-up.** **a**, The acousto-optic modulator (AOM) generates light pulses for stroboscopic measurement of the atomic spin oscillator placed in an optical cavity formed by mirrors (CM). Quantum measurement of the polarization state of light is performed with polarization beamsplitter (PBS) and two photodetectors (PD). The collective spin of the atomic ensemble contained in the micro-cell (A) oscillates in magnetic field  $B_0$ . The projection of the spin along the probe direction is subject to stroboscopic QND measurements every half period of oscillation as indicated with stars. **b**, Pulse sequence for generating and demonstrating conditional squeezing of an oscillator with stroboscopic QND measurement.

the uncertainty increase due to decoherence. Conditionally on a QND ( $\mathcal{C} = 0$ ) realization of  $\hat{S}_{y,\tau}$  the oscillator evolves to a quantum state with reduced position noise:  $\text{Var}(\hat{x}_{\text{out}}|\hat{S}_{y,\tau}) = \text{Var}(\hat{x}_{\text{out}}) - \text{Cov}^2(\hat{x}_{\text{out}}, \hat{S}_{y,\tau})/\text{Var}(\hat{S}_{y,\tau})$ , with Cov denoting the covariance. The quantum filtering then leads to conditional squeezing described by (see Supplementary Information):

$$\xi^2 = \frac{\text{Var}(\hat{x}_{\text{out}}|\hat{S}_{y,\tau})}{\langle \hat{x}^2 \rangle_0} = \frac{1}{1 + \tilde{\kappa}^2} + \eta_\tau \quad (4)$$

where  $\langle \hat{x}^2 \rangle_0$  is the imprecision in the ground state (zero point fluctuations). The decoherence term signifies the importance of the large optical depth  $d_0$  in achieving high degrees of squeezing, as  $\tilde{\kappa}^2 \propto d_0 \eta_\tau$ . We achieve an enhancement of the effective  $d_0$  by enclosing the spin oscillator in an optical resonator (Fig. 2). Optimal squeezing can be achieved with an impedance matched cavity where the output coupler transmission  $T_{\text{out}} \approx \mathcal{L}$ , where  $\mathcal{L}$  is the round-trip intensity loss (see Supplementary Information).

The atoms of the magnetic oscillator are initialized by optical pumping in a state close to the ground state. The ground state variance is calibrated against a measurement of the noise in the thermal state of the atomic ensemble obtained with unpolarized spins. The measured spin variance of unpolarized atoms  $\propto N_{\text{at}} F(F+1)/3$  can serve as a robust reference for the spin noise in the coherent end state  $\propto N_{\text{at}} F/2$  [5, 10], since the former is insensitive to classical fields and probe-induced noise. In

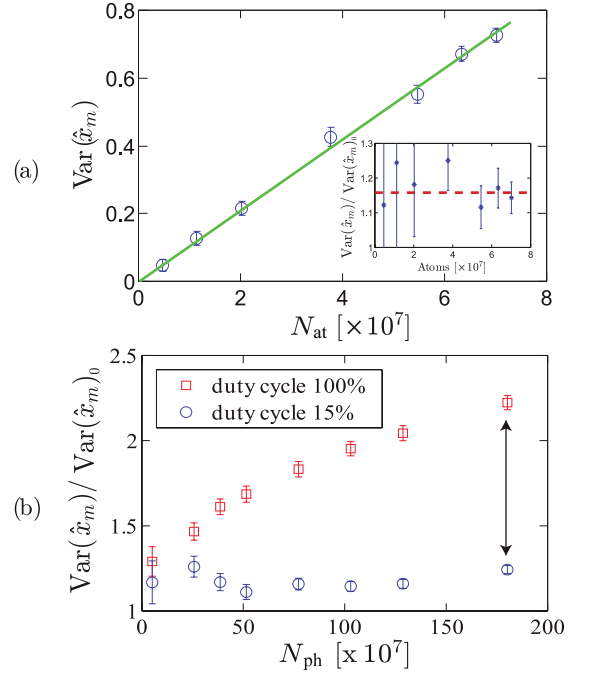


FIG. 3. **Noise characterization of the oscillator state prepared without any conditional evolution.** **a**, Measured noise in the oscillator state prepared by optical pumping. The noise is normalized to the probe light noise. Inset: Ratio of the measured oscillator variance to the expected measurement variance when the oscillator is in the ground state ( $\text{Var}(\hat{x}_m)_0$ ). The dashed line is the weighted average. The error bars represent 1 standard deviation, as estimated by  $\sim 2 \times 10^4$  independent repetitions. **b**, Demonstration of back-action noise suppression with stroboscopic quantum non-demolition measurement at twice the oscillator frequency. The oscillator noise has been normalized to the zero point fluctuations. Red squares - continuous measurement. Blue circles - stroboscopic measurement with 15% duty cycle. The arrow indicates the back-action suppression. Error bars correspond to 1 standard deviation.

Fig. 3a the oscillator noise variance in the state prepared by optical pumping is plotted as a function of the atomic density. The observed linear scaling indicates a quantum-limited performance and a QND character of the measurement. In the inset the ratio of the measured variance to the calibrated zero point imprecision is shown to be  $\sim 1.16$ . The increased measured variance in the initial oscillator state is due to the imperfect optical pumping, which leads to a finite oscillator temperature corresponding to thermal occupation  $\bar{n} \approx (8 \pm 1) \times 10^{-2}$ . The occupation probability distribution among the Zeeman levels can be found from the magneto-optical resonance signal [20] (see Fig. S3 in Supplementary Information). Assuming a spin-temperature distribution, it is found that after optical pumping  $\sim 98\%$  of the atoms are in the end state  $|F, m_F\rangle = |4, -4\rangle$ , with  $\sim 2\%$  occupation probability for the  $|F, m_F\rangle = |4, -3\rangle$  state, and negligible probabilities for the other states. This is consistent with the 16% in-

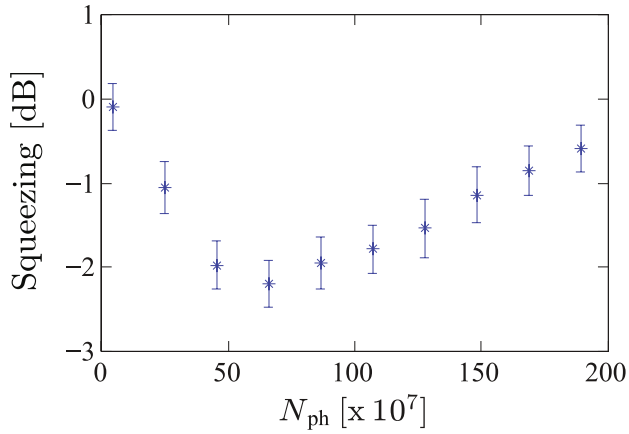


FIG. 4. **Conditional preparation of squeezed oscillator.** Demonstrated squeezing as a function of the number of photons in the first QND measurement pulse (see text for details). Error bars refer to 1 standard deviation of  $\sim 2 \times 10^4$  measurements and include uncertainties in the calibration of the ground state noise.

crease of the measured variance compared to the ground state noise.

It is instructive to compare decoherence and thermalisation properties of the magnetic oscillator and mechanical oscillators. For the former oscillator initialized in the ground state (fully polarized spin),  $\bar{n}(t) \propto f(F)e^{t/T_1} - 1$  in the range of  $t \leq T_1$  where  $T_1$  is the population life time ( $\sim 10$  msec) and  $F/2 \leq f(F) \leq (F+1)(2F+1)/2$  in the present case. The thermalisation time  $T_{th} = \frac{T_1 k_B T}{\hbar \Omega}$  is at least six orders of magnitude greater than  $T_1$ . For mechanical oscillators  $\bar{n}(t) \propto \bar{n}_{bath} e^{t/T_{th}} \approx \bar{n}_{bath}$  at  $t \approx T_{th}$  where the thermalisation time can reach at best a second in state-of-the-art experiments. This comparison illustrates the difficulty of quantum state preservation without cryogenic environment for mechanical oscillators [21] in comparison to magnetic spin oscillators.

The QND character of the measurement is demonstrated in Fig. 3b, where the measured variance of the oscillator is plotted as a function of the number of the probe photons in the pulse. For a continuous probe (100% duty cycle), the imprecision in units of zero point fluctuations increases with the number of photons in the pulse. In contrast, for a stroboscopic probe with a small duty cycle ( $\sim 15\%$ ) the noise remains nearly independent of the pulse strength over the measured range. The demonstrated reduction of probe back-action is more than 10 dB compared to a continuous probing of the oscillator.

To study the generation of squeezed oscillator states, two QND pulses are employed: the first provides information about the oscillator observable  $\hat{x}$ , and the second pulse evaluates the observable variance conditioned on the first measurement (see Fig. 2b). In Fig. 4 the reduction in conditional variance compared to the zero point imprecision is plotted as a function of the number

of photons in the first pulse, for a fixed photon number in the second pulse  $N_{ph,B} \approx 27 \times 10^7$ . The squeezing increases with the photon number in the pulse until the point where decoherence induced noise compensates for the reduction in uncertainty by measurement. The observed conditional variance is up to  $2.2 \pm 0.2$  dB below the ground state noise. Along with the dominant contribution of QND measurement to the degree of squeezing, a smaller contribution is due to the interaction nonlinear in the spin variables (see [22] and Supplementary Information for the effect of the second-rank tensor polarizability).

In comparison to the work where spin squeezed states were generated by QND measurements of atomic populations the present approach has two new features. Firstly, it directly creates a squeezed state of atomic coherences. Secondly, as any modulation technique, it is much less sensitive to classical fluctuations. As a result, we have been able to generate spin squeezing with two to three orders of magnitude more spins than in [14–17] and therefore obtain the angular spin variance of  $8 \times 10^{-10}$  for this non-classical state, more than 20 dB better than in the previous works using the criterion of [23]. One should note that squeezing of the populations as in [14–17] and direct squeezing of atomic coherences demonstrated here have different applications. For example, the former is relevant for clocks and the latter is relevant for sensing of ac magnetic fields or any other fields which couple to atomic spins.

Cavity-enhanced QND interaction utilizing a long-lived room-temperature spin oscillator can be further developed. Currently the cavity finesse is limited by the reflection losses at the cell windows. Straightforward improvements should allow to significantly increase this value, with the corresponding increase of squeezing (see Supplementary Information). The techniques developed in the paper will be useful for quantum metrology and sensing, as well as for generation of entanglement between disparate oscillators [24].

## Methods

The spin oscillator is realized in room-temperature, optically-pumped Cesium atoms, contained in a glass cell microchannel,  $300 \mu\text{m} \times 300 \mu\text{m}$  in cross section and 1 cm in length (see Fig. S1b in Supplementary Information). An alkene coating [19] deposited at the inner cell walls dramatically suppresses spin-relaxation due to the wall collisions. Atoms bounce off the walls and cross the optical mode cross section with the waist of  $55 \mu\text{m}$  approximately  $5 \times 10^3$  times before their quantum spin state decoheres in 10 msec due to wall collisions. The atom-atom collision rate at the low Cs pressure used here is negligible. As the typical light pulse duration of  $\approx 2$  msec is much greater than the atom transient

time of  $\approx 1.5 \mu\text{sec}$  and the oscillator period (typical  $\Omega \sim 380 \text{ kHz}$ ) the thermally moving atoms cross the optical mode many times in the same state and hence the detected optical mode couples to the symmetric spin mode (equivalently to the oscillator position  $\hat{X}$ ). We emphasize that the thermal motion of the atoms does not affect the oscillator temperature, which is determined by the spin distribution. The microcell is placed inside a standing wave optical cavity with a finesse  $\mathcal{F} \approx 17$ , the single-pass losses in the cell windows of 6.5% and the output coupler transmission of 80%, which is close to the optimal value  $T_{\text{out}} \approx \mathcal{L}$ . The cavity is kept on resonance with light using the Pound-Drever-Hall technique. The number of atoms in the  $F = 4$  hyperfine ground state coupled to the light field (Fig. 1b) has been adjusted within the  $\sim 10^7 - 10^8$  range by changing the cell temperature (typically  $\sim 26^\circ\text{C}$ ) and optical pumping for the maximal QND interaction strength. The frequency of the oscillator can be tuned with  $B_0$ .

An acousto-optic modulator is used to stroboscopically modulate the intensity of the probe beam at twice the Larmor frequency. The experiment was operated with  $\approx 15\%$  stroboscopic duty cycle, with probe wavelength blue detuned by 1.6 GHz with respect to the D2 transition (see Supplementary Information). The  $\hat{S}_{y,\tau}$  operator is measured by balanced polarimetry and lock-in detection. The data are weighted with an exponential mode function:  $u(t) \propto e^{\pm\gamma t}$ , where  $\gamma$  is the decoherence rate in the presence of the probe. The exponential falling mode function is used to assess the measured noise, except for the squeezing investigation where the first pulse measurement is defined with a rising mode (see Fig. 2b). To collect statistics for the variance estimation, each measurement is repeated  $\sim 2 \times 10^4$  times.

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### **Author contributions**

G.V. and H.S. contributed equally to this work. G.V., H.S., K.J., D.S. and B.C. performed the experiments and contributed to the analysis, M.B. and D.S. fabricated the microcell. G.V., H.S., K.J. and E.S.P. wrote the paper and all the authors provided feedback to the manuscript. E.S.P. supervised the research.

### **Competing Interests**

The authors declare that they have no competing financial interests.

### **Correspondence**

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