Intl. Trans. in Op. Res. xx (2018) 1–25

Supply Chain Financing with Advance Selling under Disruption

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Abstract

We study a financing problem in a supply chain consisting of one supplier and one buyer under supply disruption. The supplier could face a disruption at its end which could effectively reduce its yield in case of disruption, thereby resulting in supply yield uncertainty. The retailer can finance the supplier using advance selling that can help to mitigate the disruption. We model this problem as a Stackelberg game, where supplier as the leader announces the wholesale price and the retailer responds by deciding its optimal order quantity given stochastic demand and an exogenous fixed retail price. The supplier then commences production and a disruption can happen with a known probability. We assume that under disruption the quantity delivered is a fraction of the initial quantity ordered by the retailer. The retailer loses any unmet demand. We analyze three different scenarios of the Stackelberg game, namely: no advance selling with disruption, advance selling without disruption, and advance selling with disruption. Our results indicate that advance selling can be used to mitigate supply disruption and at the same time could lead to an increase in the overall supply chain profit.

Keywords: Supply chain finance; advance selling; supply disruption; supply chain risk management.

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1. Introduction

Supply chain finance (SCF) focuses on creating liquidity in the supply chain (SC) by means of various buyer-led or seller-led initiatives such as financial loans, trade credits, etc. The role of SCF is to ensure the availability of a working capital for supply chain partners that optimizes both the operational costs as well as the costs of financing. Large retailers, such as Target and Wal-Mart, often procure from small or medium-sized enterprises (SMEs) suppliers, who are often financially constrained and lack enough working capital to ensure a steady supply stream. In addition, these suppliers are generally located in developing countries where the bank loans are not easily available or when available could be very expensive. Even though the production and supply costs are generally low in the developing countries, the financial costs might not be and SCF has generated a lot of interest in successful partnerships between the retailers and the suppliers.

There are several different SC financing options available for a retailer who wishes to support her important but financially constrained supplier. These options include prepayment for the supplies and investing in the supplier. Prepayments to a supplier are typically *tactical* in nature and can take different forms such as reverse factoring, purchase order financing, advance selling, etc. Investments in a supplier is more *strategic* and is especially common when the supplier has a proprietary process or a specialized skill-set. The investment efforts can take many forms such as equity investments, joint-venture, subsidiary, etc. In this paper, we examine a particular SC financing strategy which is prepayment to the supplier by the retailer that could help to mitigate the SC disruption risk. We examine the impact of using the *advance-selling* (*cf.* Yu et al., 2014) financing option when the retailer prepays the supplier for the products before the production (and delivery) of the final product.

In recent years, major supply disruptions (*cf.* Oke and Gopalakrishnan, 2009; Gupta et al., 2015; He et al., 2016) have had negative effects on the ability of suppliers to satisfy the orders placed by retailers. This problem becomes more severe in the presence of a financially constrained supplier or a buyer (Blome and Schoenherr, 2011). Even though the impact of the loss of supply for retailers is well-known and can be severe, there has been little attention to this area of research which studies financially constrained suppliers who are prone to disruption. Some recent studies include Mizgier et al. (2015); Sahebjamnia et al. (2018); He et al. (2018); Zhang et al. (2018). Mizgier et al. (2015) tested whether risk of operational disruptions can be managed through a combination of process improvement and capital adequacy. They model capital amount allocation to the different risk event types using the loss distribution approach. Sahebjamnia et al. (2018) developed a multi-objective mixed-integer probabilistic programming model to assess the resilience of manufacturing in the face of multiple disruptions. The authors argued that the interaction between budget external resources and organizational resilience is critical for achieving the successful recovery strategy.

Our work is closely related to a study by Taleizadeh (2017). They developed a lot-sizing model for a retailer with advance selling and supply disruption with partial backordering. The supply disruption considered in the paper realizes when an entire batch of production is rejected once a defective product is discovered by a quality inspector at the production floor. The model studied in their paper considers a retailer-supplier supply chain where the supplier requires partial pre-payment for the order, and the retailer allows backordering of the deterministic demand. The wholesale and retail prices are considered to be exogenous and unaffected by the disruption. In our study, we relax some of these assumptions such as we assume the demand to be stochastic and allow for the wholesale price to be adjusted based on disruption. In addition, in our paper, we assume that the supply disruption affects the overall yield of the supplier that can deliver partial order quantity under disruption, and not modeled as accepting or rejecting a batch like Taleizadeh (2017). Specifically, we use the random yield modeling approach (Yano and Lee, 1995; Dada et al., 2007; Chen and Yang, 2014) in our study, i.e., in this model, the quantity received by a buyer is a random fraction of the quantity ordered from the disrupted supplier. Thus, a zero-percent yield in our model becomes a special case of rejecting a batch.

Another work closely related to our paper is Li et al. (2016), who studied a dyadic SC with a manufacturer and its supplier, where the supplier can be affected by disruption resulting in production disruption at the manufacturer. They investigate contracts where the supplier is penalized for the shortage and is provided with financial assistance to help maintain the supply stream. Their study indicates that integration of financial assistance and the non-delivery penalty is the best strategy for the manufacturer in most situations. In another study (without considering SC disruption), Xiao and Zhang (2018) studied a similar SC setting where the supplier considers offering a discounted price, before production starts, to the retailer to raise the necessary working capital. They investigated the supplier's optimal mix of financing strategy with advance selling by using a three-stage Stackelberg game between the SC members. They proposed an incentive scheme consisting of pre-ordering and bidirectional compensation contracts to stimulate the supplier to increase the production quantity as well as coordinate the supply chain. We also model the problem as a Stackelberg game and observe that due to advance selling, the order quantity increases when there is no disruption and also observe that the order quantity could increase in the case of disruption.

This paper is organized as follows. We present the literature review in §2. In §3, we discuss notation, the sequence of events, and details on model development. In §4, we formulate and analyze the models for the three scenarios, i.e., no advance selling with disruption in §4.1, advance selling without disruption in §4.2, and advance selling with disruption in §4.3. We obtain optimal solutions and analytical results in this section. Subsequently, in §5 we conduct numerical experiments to gain further insights on the impact of various model parameters on the optimal decisions and total SC profits to understand the impact of an advance selling arrangement. Finally, we conclude the paper and discuss some directions for future research in §6.

2. Literature Review

Our work contributes to two specific streams of SC literature - advance selling (and SC finance) and SC disruption management. We first discuss related works in the advance selling stream followed by key literature in the SC disruption literature.

Advance selling by a supplier works in the opposite direction to the commonly used trade credits by suppliers. The terms of a trade credit offered by a supplier entitle the retailer to pay the supplier at some later time for the purchase, whereas advance-selling requires payment prior to production/delivery of the product by the supplier. Therefore, a trade credit is more restrictive for SME suppliers who rely upon cash-flows for production and work with little to no working capital. Kouvelis and Zhao (2012) is one of the first studies in the area of SC financing which investigates early payment discount scheme as a framework to analyze the decisions for optimally structuring the trade credit contract (discounted wholesale price if paying early, financing rate if delaying payment) from the supplier's perspective. This study shows that a risk-neutral supplier should always finance the retailer at rates less than or equal to the risk-free rate. From the retailer's perspective, their study concludes that the retailer prefers supplier financing as compared to bank financing. In a later study, Kouvelis and Zhao (2015) studied the contract design problem for a one supplier-one buyer SC where both parties are financially constrained and in need of working capital for their operations subject to costs for defaulting on loans. They explored different SC contracts including buyback, revenue sharing, and quantity discount contracts. For a detailed review on the SC finance literature, we refer the readers to Wuttke et al. (2016); Zhao and Huchzermeier (2018a,b).

Lashgari et al. (2016) studied partial prepayment (*advance-selling*) and delayed payments in a SC. Specifically, they developed an EOQ model with downstream partial delayed payment and upstream partial prepayment with lost sales and backorder scenarios. Further, they proposed a solution algorithm and conduct numerical study including sensitivity analyses to obtain managerial insights. In another recent study in the area of *advance-selling* (preselling) contracts, Xiao and Zhang (2018) considered a SC with a manufacturer (supplier) and a retailer, where the manufacturer is financially constrained. In order to improve her cash-flows, the supplier advance sells the product to the retailer and receive cash flow to raise working capital for the production. The study proposed an advance selling advance-selling based incentive scheme that helps to coordinate the SC and also to increase the manufacturer's production quantity.

Xiao et al. (2017) considered a financially constrained SC using a Stackelberg game with the supplier as the leader and retailer as the follower. They analyzed a centralized SC obtaining corresponding co-ordination requirements and then examined if revenue-sharing, buyback, and all-unit quantity discount contracts can coordinate this SC. Their study indicates that the all-unit quantity discount contract does not coordinate and the revenue-sharing and buyback contracts can coordinate with sufficient working capital.

In certain situations, retailers and manufacturers can employ advance selling as a promotional tool to increase their sales by influencing customer demand. For instance, Ma et al. (2018) (and references therein) considered a market with a powerful manufacturer that can influence the spot market price of the raw materials and advance sells to its customers. They assume that a fraction of customers is risk-averse. Their study indicates that that the advance selling program should be offered in markets when risk aversion is low, or when it is high, and the manufacturer has high and low market power. By contrast, the advance selling program should not be offered when consumer risk aversion is high and the market power is medium. In addition, their study shows that the manufacturer benefits more from advance selling when consumers are not risk averse or are myopic.

Jin et al. (2018) investigated different kinds of financing strategies for a dyadic SC with financially constrained members using a Stackelberg game. They discussed three financing strategies: bank financing separately, bank financing with trade credit, and bank financing with the supplier's guarantee. They showed that collaborative financial strategies outperform competitive strategies, however, the competitive strategy can be better for the bank. Further, their results indicate that all SC members can perform better if the SC leader acts as a guarantor rather than as an intermediary creditor.

We now discuss the SC disruption literature and refer the readers to the following key studies that provide a detailed discussion: Gurnani et al. (2012); Heckmann et al. (2015); Ho et al. (2015); Wang et al. (2015); Snyder et al. (2016); Ivanov et al. (2017), and the references therein. Next, we discuss two studies which are most relevant to our paper that investigated SC financing under supply disruption.

He et al. (2018) studied the optimal ordering decision policy for a retailer whose supply is exposed to supply disruptions. In their study, they assume correlated demand and price uncertainty and optimize the inventory planning for the retailer in a two-stage supply chain. They use real-option pricing methodology to derive the profit function with the adoption of a dual sourcing strategy that helps with the disruption risk mitigation. They show that their optimization problem can be reduced to simply determining the fair value of the corresponding real options. Our study is different from theirs as we focus on the retailer's use of supplier financing to mitigate the supply disruption.

One of the many tools for coping with SC disruption risks is by allocating a budget for any future unexpected SC disruptions. In a recent study, Zhang et al. (2018) developed an optimization model to determine the optimal budget allocation plans in an auto parts manufacturing enterprise. In their study, the budget allocation coefficients (weights) of each response strategy need to be determined on the basis of failure and success probabilities of implementing each recovery strategy using an optimization model. Their model allows computing the budget allocation and expected losses against different total recovery budgets. The primary managerial implication of their study is that the risk prevention and risk mitigation with respect to disruption risks in supply chains should be attached importance (i.e., weights), especially for SMEs when the loss caused by the disruption risks is relatively severe.

According to the above literature review, supply chain financing with advance selling under disruption has not been studied and is an important problem to study. The value of advance selling to mitigate disruption has not been investigated in a supply chain setting and the model and the results presented in this study are applicable in real-world procurement situations. It is important for procurement managers in a retail setting to take note of the impact of the disruption and employ advance selling as a tool to build partnerships with their strategic suppliers to help strengthen their supply stream.

3. Model development

We consider a supply chain with a risk-neutral supplier and a risk-neutral retailer purchasing a single product. The supplier is financially constrained and also prone to supply disruption. The retailer purchases the product from the supplier at a wholesale price w and sells it in the market with demand D at an exogenous retail price $p < w$. We assume that the market demand D is stochastic with a c.d.f. $F(x)$ and p.d.f. $f(x)$. Also, let $\overline{F}(x) := 1 - F(x)$. We assume that the market demand D is uniformly distributed s.t. $F(x) \sim U[\mu - t, \mu + t]$ ($\mu \geq t$). The retailer places an order with the supplier before the market demand is realized. The supplier's per unit (manufacturing) cost is c if she has the working capital. However, we factor the financing cost for the working capital as she is financially constrained. This gives us the total per unit cost as $(1 + r)c$, where $r > 0$ is the loan rate received by the supplier from the retailer (or bank). This assumption is similar to Xiao and Zhang (2018), however, in contrast to their study, we assume that the retailer places only a single order with the supplier either by an advance selling purchase order or by a regular purchase order realized once the production is complete.

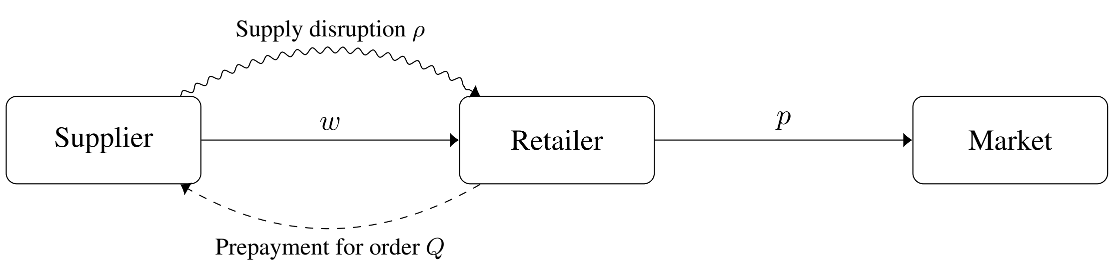


Fig. 1: Supply chain structure.

When there is no supply disruption, the supplier has enough working capital to deliver the order placed by the retailer. However, when the supply disruption occurs, based on the disruption level δ the supplier is able to produce only a portion of the desired order quantity. Specifically, we assume that if the disruption level is high then the order quantity delivered to the retailer is low. Under disruption, the supplier requires additional capital to recover the supply process. We assume that the capital required is directly proportional to the disruption level, i.e., the capital required increases in the disruption level δ. The disruption occurs with a probability ρ.

Following the literature, we assume that all information across the SC is common knowledge and the retailer and the supplier are both risk-neutral. Thus, both the retailer and supplier takes decisions in order to maximize their overall expected profits.

4. Supply Chain models

4.1 No Advance Selling with Supply Disruption

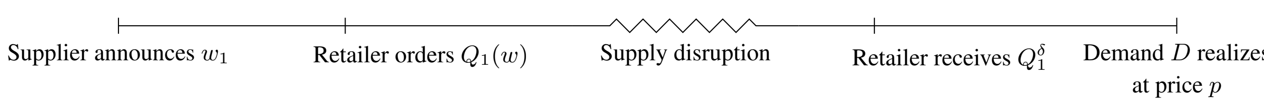


Fig. 2: Sequence of events when there is supply disruption in the SC without advance selling.

We first study the case without advance selling when the supply can be disrupted as shown in Figure 2. In addition, we assume that the supplier does not have any financial means to mitigate the disruption. Retailer decides the order quantity $Q_1(w_1)$ given the wholesale price w_1 and the market demand D, that maximizes her expected profit which is given by:

$$
\begin{aligned}
\mathrm{E}[\Pi_1^R(w_1, Q_1)] &= (1-\rho)\{p\mathrm{E}[\min(Q_1, D)] - wQ\} + \rho\{p\mathrm{E}[\min(Q_1^\delta, D) - w_1 Q_1^\delta\}, \\
&= (1-\rho)\{(p-w_1)Q_1 - pI(Q)\} + \rho\{(p-w_1)Q_1^\delta - pI(Q_1^\delta)\}, \\
&= (1-\rho)\{(p-w_1)Q_1 - pI(Q)\} + \rho\{(p-w_1)(1-\delta)Q_1 - pI((1-\delta)Q_1)\}, \quad (1)
\end{aligned}
$$

where $Q_1^\delta = (1-\delta)Q_1$, is the quantity delivered by the supplier under disruption and we know that

$\mathrm{E}[\min(Q,D)] = Q - \int_0^Q F(x)dx = Q - I(Q)dx$, s.t., $I(Q)$ is the left-over inventory. Using the first order condition (f.o.c.) w.r.t. Q_1[1], we obtain:

$$\begin{aligned}(1-\rho)(p-w_1) - p(1-\rho)F(Q_1) + \rho[(p-w_1)(1-\delta) - p(1-\delta)F(Q_1^\delta) &= 0,\\ (p-w_1)(1-\rho\delta) = p[(1-\rho)F(Q_1) + \rho(1-\delta)F(Q_1^\delta)]. \quad (2)\end{aligned}$$

Thus, the retailer's optimal order quantity $Q_1^*(w_1)$ for a given wholesale price w_1 is obtained by solving (2), which can be further rewritten in terms of the famous newsvendor solution as:

$$\frac{p-w_1}{p} = \frac{1-\rho}{1-\rho\delta}F(Q_1) + \frac{\rho(1-\delta)}{1-\rho\delta}F(Q_1^\delta),$$
$$\frac{p-w_1}{p} := \hat{Q}_1 = \frac{1-\rho}{1-\rho\delta}F(Q_1) + \frac{\rho(1-\delta)}{1-\rho\delta}F(Q_1^\delta), \quad (3)$$

where $\hat{Q}_1$ is the standard newsvendor quantity without supply disruption. It is straightforward to obtain the following proposition from (3), as $F(Q_1) \geq F(Q_1^\delta)$ because $Q_1 \geq Q_1^\delta$ and $\hat{Q}_1$ is a convex combination of Q_1 and Q_1^δ, therefore $Q_1 \geq \hat{Q}_1 \geq Q_1^\delta$.

Proposition 1. *The optimal quantity ordered by the retailer under supply disruption is always more than the quantity ordered without disruption, for a given wholesale price w_1, i.e. $Q_1^*(w_1) \geq \hat{Q}_1{}^*(w_1) \geq Q_1^{*\delta}(w_1)$.*

Proposition 1 states that the retailer always inflates its order when there is a supply disruption, however, the actual quantity received by the retailer under disruption will never be more than the quantity without the supply disruption. This insight is important as the order quantity is inflated only to the extend that even when the supply disruption occurs the quantity received is never more than the quantity without the supply disruption. Note that the above result does not depend on the type of demand distribution D. Next, we consider the supplier's decision to set the wholesale price w that maximizes her expected profit as follows:

$$\begin{aligned}\mathrm{E}[\Pi_1^S(w_1,Q_1)] &= (w_1-c)(1-\rho)Q_1 + (w_1-c)\rho Q_1^\delta = (w_1-c)(1-\rho)Q_1 + (w_1-c)\rho(1-\delta)Q_1,\\ &= (w_1-c)[1-\rho\delta]Q_1. \quad (4)\end{aligned}$$

[1]It is easy to see that the second order condition (s.o.c.) will be satisfied and we omit its presentation in the paper for brevity.

Using the f.o.c. w.r.t. w_1, we obtain: $Q_1 + (w_1 - c)\frac{\partial Q_1}{\partial w_1} = 0$. For $D \sim U[\mu - t, \mu + t]$ and with the knowledge of retailer's optimal order quantity $Q_1^*(w_1)$ we obtain the equilibrium wholesale price and the order quantity for the Stackelberg game. We now present the following result.

Proposition 2. *The equilibrium wholesale price and order quantity are given by* [2]

$$w_1^e = \frac{p(t+\mu)}{4t} + \frac{c}{2} \text{ and } Q_1^e = \frac{(p(t+\mu) - 2ct)(1-\delta\rho)}{2p(1-(2-\delta)\delta\rho)} \tag{5}$$

and the equilibrium profits of the two players are

$$\Pi_1^{S,e} = \frac{(p(t+\mu) - 2ct)^2(1-\delta\rho)^2}{8pt(1-(2-\delta)\delta\rho)} \text{ and } \Pi_1^{R,e} = \frac{(p(t+\mu) - 2ct)^2(1-\delta\rho)^2}{16p(1-(2-\delta)\delta\rho)}. \tag{6}$$

It can be seen that the supplier's profit is double that of the retailer. By taking first derivatives of the above expressions w.r.t. different model parameters in (5)-(6) we obtain insights on the sensitivity of optimal decisions and profits of the two players. It is interesting to note that w_1^e does not depend on disruption probability ρ and the impact of disruption δ. However, w_1^e increases in retail price p, raw material cost c, and expected market demand μ, and decreases in demand variability t. Q_1^e always increases in p, μ, and disruption probability ρ. It always decreases in c. In addition, we can also see that it is increasing in the demand spread t when $p > 2c$, decreasing when $p < 2c$, and does not change with t when $p = 2c$. We find that Q_1^e is increasing in δ for $\delta < \frac{1}{1+\sqrt{1-\rho}}$, and decreasing in δ when $\delta > \frac{1}{1+\sqrt{1-\rho}}$.

Sensitivity analysis of the profits of both the player's gives the following insights. The profits for both the players are increasing in retail price p and average demand μ. Their profits are decreasing in the unit manufacturing cost c, demand variability t, and the impact of disruption δ. Finally, on the sensitivity of the profits w.r.t. the disruption probability ρ, we find that the profits are decreasing in ρ for $\rho < \frac{1}{2-\delta}$ and increasing in ρ for $\rho > \frac{1}{2-\delta}$.

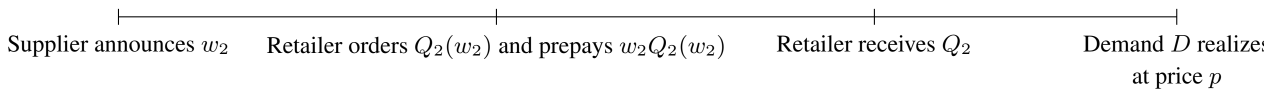


Fig. 3: Sequence of events when there is no supply disruption in the SC with advance selling.

4.2 Advance Selling without Supply Disruption

Now, we study the second case when the supplier advance sells the product to the retailer to obtain the required cash before the production and delivery of the product as presented above in Figure 3. The cash can be deposited by the supplier to obtain interest income at a rate r as part of revenues and by the supplier at a rate β to earn interest income. It is not straightforward to see that in lieu of the extra income if the supplier can offer an all-unit quantity discount to the retailer (*cf.* Xiao and Zhang, 2018). The retailer leads the SC as the Stackelberg and maximizes her profit as

$$\mathrm{E}[\Pi_2^R(w_2, Q_2)] = p\mathrm{E}[\min(Q_2, D)] - wQ_2(1+\beta), \tag{7}$$

and the supplier's expected profit is

$$\mathrm{E}[\Pi_2^S(w_2, Q_2)] \quad = \quad w_2Q_2(1+r) - cQ_2 = [w_2(1+r) - c]Q_2. \tag{8}$$

We use the F.O.C. conditions for (7)-(8), and specialize these profit expressions using $D \sim U[\mu - t, \mu + t]$, to obtain the equilibrium wholesale price and the order quantity without supply disruption as:

$$w_2^e \quad = \quad \frac{p(\mu+t)}{4t(\beta+1)} + \frac{c}{2(r+1)}, \text{ and } Q_2^e = \frac{1}{2}\left(\mu + t - \frac{2(\beta+1)ct}{p(r+1)}\right). \tag{9}$$

Therefore, an advance selling arrangement between the supplier and the retailers leads to an expected increase in the quantity ordered due to a discount in the wholesale price, as w_2^e (order quantity: Q_2^e) is decreasing (increasing) in the borrowing rate r for the supplier. This is a well-known SC phenomenon studied extensively in the advance selling literature (*e.g.;* Kouvelis and Zhao, 2012). Furthermore, the equilibrium wholesale price with advance selling is increasing in p, μ and c, and decreasing in β and t.

[2]To ensure feasible value of interior solution, we assume $p(t+\mu) > 2ct$.

As the opportunity cost for the retailer to prepay the supplier, i.e., β increases the supplier must provide a higher wholesale price discount to the retailer.

The retailer's equilibrium order quantity with advance selling Q_2^e increases with μ, p and r, and decreases in c and β. Also, when $p > 2c\left((\beta+1)/(r+1)\right)$ then Q_2^e increases in demand variability[3] t. From this discussion, we note that both w_2^e and Q_2^e are increasing (decreasing) in μ and p (β), and $w_2^e(1+r)$ and Q_2^e are both increasing in r. Thus, supplier's expected profit in equilibrium is increasing (decreasing) in μ, r, and p (β) in the case of advance selling which is summarized below in Proposition 1.

Corollary 1. *Supplier's expected profit with advance selling prepayment is increasing in μ, r, and p, and decreasing in β.*

4.3 Advance Selling with Supply Disruption

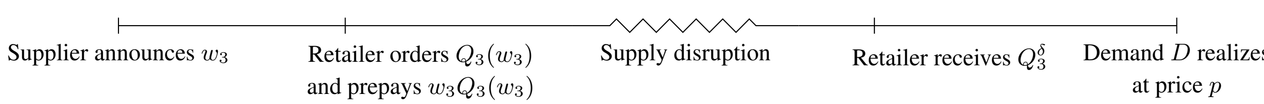


Fig. 4: Sequence of events when there is a supply disruption in the SC along with advance selling.

Finally, we consider an advance selling contract between the supplier and the retailer and supply disruption can happen with a probability equal to ρ. The sequence of events for this case is summarized in Figure 4 and is described as follows. The supplier first announces her wholesale price w_3, and the retailer in response decides her order quantity Q_3 in response to the supplier's decision while accounting for the possibility and impact of supply disruption. The retailer makes the payment, i.e., w_3Q_3 in advance before the production starts. We assume that due to internal discounting for both the players, any cash flow at the beginning of the time period (before production starts) is multiplied by appropriate factors

[3]The condition $p > 2c\left((\beta+1)/(r+1)\right)$ implies that the retail price is almost twice as high as compared to the supplier's raw material cost.

to account for time value of money. These factors are $(1 + r)$, and $(1 + \beta)$, for supplier and buyer, respectively, where $r, \beta \geq 0$. Thus the advance payment of w_3Q_3 is effectively worth $(1+r)w_3Q_3$, and $(1 + \beta)w_3Q_3$, for the supplier and the retailer, respectively. Similar to the model in previous sections, a disruption can happen with a probability ρ which effectively reduces the production size and eventual delivery to the retailer. The initial impact of this disruption is to effectively reduce the production size to $(1 - \delta)Q_3$. However, as part of the advance selling contract, the supplier has to invest some money into mitigating the effects of disruption. As part of the contract, this investment is proportional to the quantity ordered and is equal to ηQ_3, where η could be interpreted as the commitment in dollars for every unit of quantity ordered by the retailer. We assume that this investment by the supplier reduces the impact of disruption in the following way. We assume that the increase in final production post disruption is proportional to the money invested by the retailer. Let δ and Q_3^{δ} denote the impact of disruption and the effective production quantity before any investment by the supplier, and δ_1 and $Q_3^{\delta_1}$ denote the same after investment by the supplier. We assume the following relationships:

$$Q_3^{\delta_1} = Q_3^{\delta} + k\eta Q_3 = (1 - \delta)Q_3 + k\eta Q_3 = (1 - \delta_1)Q_3, \tag{10}$$

and therefore,

$$\delta_1 = \delta - k\eta, \tag{11}$$

where k is a constant. Since this investment of ηQ_3 is made at the beginning of time period before the production starts, we discount this by interest rate r when writing the overall profit for the supplier. The supplier then begins production and delivers the appropriate quantity to the retailer, i.e., Q_3 if no disruption happens, and $Q_3^{\delta_1}$ if indeed disruption happens. Finally, in the case of disruption, because the actual quantity $Q_3^{\delta_1}$ is less than the originally ordered and paid for quantity Q_3, the supplier refunds the difference, i.e., $w_3(Q_3 - Q_3^{\delta_1})$. This refund by the supplier is done at the end of the time period after the production ends and therefore this amount is not discounted. The market demand D is then realized and the retailer earns the sales revenue given the exogenous retail price p. We write the profits of supplier (Π_3^S), and retailer (Π_3^R) as follows.

$$\Pi_3^S = (1 - \rho)\big[w(1 + r) - c\big]Q_3 + \rho\big[(1 + r)wQ_3 - cQ_3^{\delta_1} - (1 + r)\eta Q_3 - w_3(Q_3 - Q_3^{\delta_1})\big]$$
$$\Pi_3^R = (1 - \rho)\big[p\mathrm{E}[\min(Q_3, D)] - (1 + \beta)w_3Q_3\big] + \rho\big[p\mathrm{E}[\min(Q_3^{\delta_1}, D)] - (1 + \beta)w_3Q_3 + w_3(Q_3 - Q_3^{\delta_1})\big].$$

Using (10), we can rewrite the profit functions as

$$\Pi_3^S = (1-\rho)\big[w_3(1+r)-c\big]Q_3 + \rho\big[w_3Q_3((1+r)(1+\eta)-\delta_1) - c(1-\delta_1)Q_3\big] \tag{12}$$

$$\Pi_3^R = (1-\rho)\big[p\mathrm{E}[min(Q_3,D)] - (1+\beta)w_3Q_3\big] + \rho\big[p\mathrm{E}[min((1-\delta_1)Q_3,D)] - w_3Q_3(1+\beta-\delta_1)\big]. \tag{13}$$

Using standard backward induction approach, we first look to obtain the retailer's order quantity given the wholesale price announced by the supplier. First-order-condition w.r.t. Q_3 in (13) gives

$$\frac{\partial \Pi_3^R}{\partial Q_3} = (1-\rho)\big[p - pF(Q_3) - (1+\beta)w_3\big] + \rho\big[p\frac{\partial Q_3^{\delta_1}}{\partial Q_3} - pF(Q_3^{\delta_1})\frac{\partial Q_3^{\delta_1}}{\partial Q_3} - (1+\beta)w_3 + w_3(1-\frac{\partial Q_3^{\delta_1}}{\partial Q_3})\big].$$

After a few steps of algebra, it can be shown that the retailer's optimal order quantity $Q_3^*(w_3)$ given a wholesale price is obtained by solving the following equation

$$p\big[(1-\rho)F(Q_3^*) + \rho(1-\delta_1)F((1-\delta_1)Q_3^*)\big] + \beta w = (p-w_3)(1-\rho\delta_1). \tag{14}$$

Proposition 3. $Q_3^{\delta_1} = (1-\delta_1)Q_3 \leq \hat{Q}_3$, *where* $\hat{Q}_3 = F^{-1}(\frac{p-w_3}{p})$ *is the standard optimal newsvendor quantity in the case of no disruption and no advance selling.*

Proof: From (14) it can be seen that since $\beta \geq 0$ and $Q_3^* \geq (1-\delta_1)Q_3^*$, the R.H.S. = $(p-w_3)(1-\rho\delta_1) \geq p\big[(1-\rho)F(Q_3^*) + \rho(1-\delta_1)F((1-\delta_1)Q_3^*)\big] \geq p\big[(1-\rho)F((1-\delta_1)Q_3^*) + \rho(1-\delta_1)F((1-\delta_1)Q_3^*)\big] = p(1-\rho\delta_1)F((1-\delta)Q_3^*)$. Thus, $(p-w_3) \geq pF((1-\delta_1)Q_3^*)$, which proves the above corollary.

We can obtain the retailer's optimal response from (14) and use it to rewrite the supplier's profit in (12) and then optimize it w.r.t. w_3. To obtain explicit results, we use a uniformly distributed demand of the form $D \sim Unif[\mu-t,\mu+t]$. We can now present the following result.

Proposition 4. *For a uniformly distributed demand function* $D \sim Unif[\mu-t,\mu+t]$, *supplier's optimal wholesale price and retailer's optimal initial order quantity are given by the following*

$$w_3^e = \frac{2(1+r)t\eta\rho(1+\beta-\delta_1\rho) + p(t+\mu)(1+r-\delta_1\rho)(1-\delta_1\rho) + 2ct(1+\beta-\delta_1\rho)(-1+\delta_1\rho)}{4t(1+r-\delta_1\rho)(1+\beta-\delta_1\rho)} \tag{15}$$

$$Q_3^e = \frac{p(t+\mu)(1+r-\delta_1\rho)(1-\delta_1\rho) - 2ct(1+\beta-\delta_1\rho)(1-\delta_1\rho) - 2(1+r)t\eta\rho(1+\beta-\delta_1\rho)}{2p(1+r-\delta_1\rho)(1-(2-\delta_1)\delta_1\rho)}. \tag{16}$$

The equilibrium profits of the supplier and the buyer, respectively are given as

$$\Pi_3^{S,e} = \frac{(2(1+r)t\eta\rho(1+\beta-\delta_1\rho) - p(t+\mu)(1+r-\delta_1\rho)(1-\delta_1\rho) + 2ct(1+\beta-\delta_1\rho)(1-\delta_1\rho))^2}{8pt(1+r-\delta_1\rho)(1+\beta-\delta_1\rho)(1+(-2+\delta_1)\delta_1\rho)} \tag{17}$$

$$\Pi_3^{R,e} = \frac{(2(1+r)t\eta\rho(1+\beta-\delta_1\rho) - p(t+\mu)(1+r-\delta_1\rho)(1-\delta_1\rho) + 2ct(1+\beta-\delta_1\rho)(1-\delta_1\rho))^2}{16pt(1+r-\delta_1\rho)^2(1+(-2+\delta_1)\delta_1\rho)} \tag{18}$$

Proof: Solving f.o.c. w.r.t. Q_3 in (14), we obtain retailer's optimal response as

$$Q_3^*(w_3) = \frac{p(t+\mu)(1-\delta_1\rho) - 2tw_3(1+\beta-\delta_1\rho)}{p(1-(2-\delta_1)\delta_1\rho)}. \tag{19}$$

We use (19) to rewrite the supplier's profit as follows

$$\Pi_3^S(w_3) = \frac{(w_3(1+r-\delta_1\rho) - c(1-\delta_1\rho) - \eta(1+r)\rho)(p(t+\mu)(1-\delta_1\rho) - 2tw_3(1+\beta-\delta_1\rho))}{p(1-(2-\delta_1)\delta_1\rho)} \tag{20}$$

We apply f.o.c. in (20) w.r.t. w_3 to obtain the optimal wholesale price in (15) and then use the optimal wholesale price in (15) to obtain the optimal quantity in (16). We then use (15) and (16) in (12)-(13) to obtain optimal profit functions in (17)-(18).

Corollary 2. $\dfrac{\Pi_3^{S,e}}{\Pi_3^{R,e}} = \dfrac{2(1+r-\delta_1\rho)}{1+\beta-\delta_1\rho}$.

This result shows that when the internal discounting factor is same for both the players, i.e., $r = \beta$, the supplier earns twice as much as the retailer. This is very much consistent with results in simple price based contracts given the fact that the supplier has the first-mover advantage in such a Stackelberg game. However, when $r \neq \beta$, it can also be shown that $\dfrac{\Pi_3^{S,e}}{\Pi_3^{R,e}}$ is increasing in ρ and δ_1 when $r > \beta$ and the opposite holds true when $r < \beta$. This shows that if the supplier has a higher discounting rate than the retailer, then the overall share of the supplier in total supply chain profits increases as the probability of disruption increases or the impact of disruption increases.

Corollary 3. *It can be shown that*

i. $\frac{\partial w_3^e}{\partial r} < 0$, *and* $\frac{\partial Q_3^e}{\partial r} > 0$

ii. $\frac{\partial w_3^e}{\partial \beta} < 0$, *and* $\frac{\partial Q_3^e}{\partial \beta} < 0$

iii. $\frac{\partial w_3^e}{\partial \mu} > 0$, *and* $\frac{\partial Q_3^e}{\partial \mu} > 0$

iv. $\frac{\partial w_3^e}{\partial t} < 0$

We performed numerical analysis to further understand the sensitivity of profits as well as both decision variables w.r.t. various model parameters. We present representative results and summarize our key findings in the next section.

5. Numerical Analysis

In order to further understand the impact of advance selling on mitigating the effects of supply disruption, we compare the wholesale price w_i^e, order quantity Q_i^e and quantity received $Q_i^{\delta,e}$, profits of the players $\Pi_i^{R,e}$ and $\Pi_i^{S,e}$, and the total supply chain profit Π_i^e for $i = \{1, 3\}$, as obtained in Sections 4.1 and 4.3. We also conduct sensitivity analysis of these important supply chain metrics w.r.t. the market demand volatility (t), disruption parameters (ρ, δ, η), and finance parameters (r, β). We conducted numerical experiments over a wide array of model parameters, and in this section discuss the wider insights that were consistent across our experiments. We present the results for a representative set of parameters. The following parameters are used in our baseline model: $c = 1,\ p = 5,\ t = 0.5,\ \mu = 1.0,\ \rho = 0.2,\ \delta = 0.2,\ k = 0.05,\ \eta = 1,\ r = 10\%, \beta = 5\%$.

Wholesale price comparisons

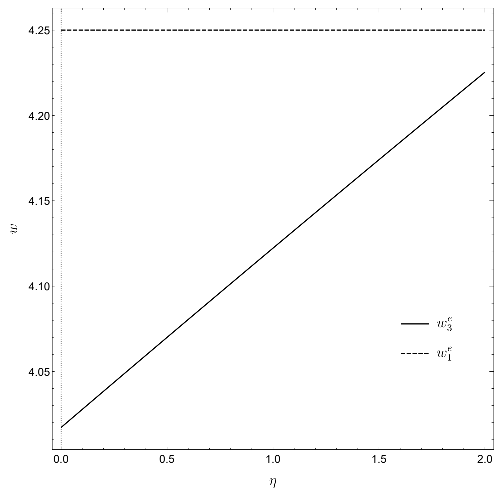
Figure 5 presents the wholesale prices charged by the supplier without (w_1^e) and with (w_3^e) disruption. From (5) we already know that w_1^e is increasing in μ, c and decreasing in t, and does not depend on disruption parameters – ρ, δ. However, w_3^e (15) depends on all the parameters and we observe the effect of change in parameters in Figure 5. w_3^e is increasing in η and ρ, and decreasing in δ, r, β and t. For the baseline model, we observe that in most cases $w_3^e < w_1^e$ which is the discount given by the supplier for advance selling, however, interestingly, for high (low) values of ρ (β) we observe $w_1^e < w_3^e$. Therefore, when the disruption probability is high or the interest rate for the supplier is low, the retailer does not give a wholesale price discount and instead charges a higher wholesale price.

Order and supply quantity comparisons

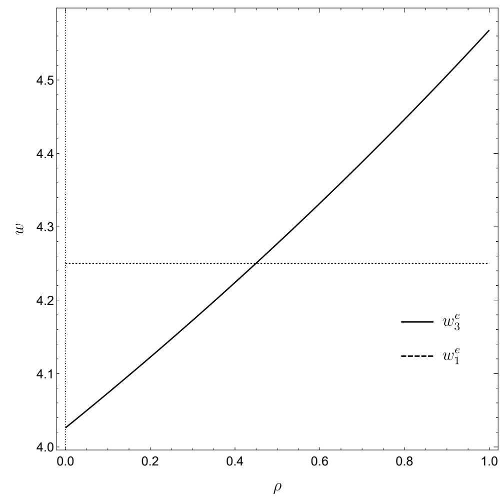
The retailer orders Q_i^e and the supplier supplies $Q_i^{\delta,e}$ to the retailer for $i \in \{1, 3\}$. We now investigate the sensitivity of these two equilibrium quantities w.r.t. to the parameters in Figure 6 for scenarios 1 and 3. We observe from the figure that Q_i^e increases in t, (and μ) and concave in δ (increases and then decreases). $Q_1^e > Q_3^e$ in most cases except for low η or ρ as noted in Figures 6(a)-6(b). Interestingly, Q_3^e is decreasing in ρ while Q_1^e is increasing in ρ. This phenomenon is driven by the yield recovery (see (11)) in case of advance selling and not by the wholesale prices as for low ρ as $w_1^e > w_3^e$ from Figure 5(b). Further, Q_3^e is decreasing in η, β and increasing in r, however, Q_1^e is not affected by η, r, and β. The sensitivity of $Q_1^{\delta,e}$ is identical to Q_1^e in all the cases except for δ as seen in Figure 6(e) as $Q_1^{\delta,e}$ is decreasing in δ. Similarly, $Q_3^{\delta,e}$ and Q_3^e also have identical sensitivity results except for δ, η which is again driven by (11). Finally, we compare $Q_1^{\delta,e}$ and $Q_3^{\delta,e}$. We note that $Q_1^{\delta,e} < Q_3^{\delta,e}$ in most cases, i.e., advance selling does help in supply restoration as the retailer receives a larger quantity to sell when advance selling, except, when ρ is high or β is very high. Therefore, when the disruption probability is low and the retailer has the ability to advance sell with a reasonable interest rate advance selling can help with mitigating supply disruption.

Profit comparisons

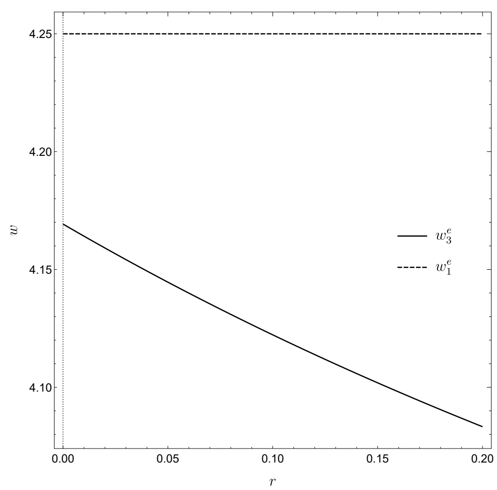
Finally, we compare the profits for the supplier and the retailer ($\Pi_i^{S,e}, \Pi_i^{R,e}$, respectively) as well as the overall SC profit (Π_i^e) and analyze their sensitivity w.r.t. the parameters. Supplier's profit without advance selling $\Pi_1^{S,e}$ decreases in δ and t and appears to be mildly convex in ρ. However, in the presence of advance selling the profit $\Pi_3^{S,e}$ decreases in η, ρ, β, δ and t, and increases in r. Consequently, for low values of η, ρ, β, we have $\Pi_3^{S,e} > \Pi_1^{S,e}$ and for higher values of r and δ, we have $\Pi_3^{S,e} < \Pi_1^{S,e}$, as observed in Figure 6. Similar to supplier's profit, the retailer's profit without advance selling $\Pi_1^{R,e}$ also decreases in δ and t and appears to be mildly convex in ρ. $\Pi_3^{R,e}$ decreases in η, ρ, β, δ and t, and increases in r, similar to $\Pi_3^{S,e}$. It is interesting to note that the retailer is only better off by advance selling, i.e.,



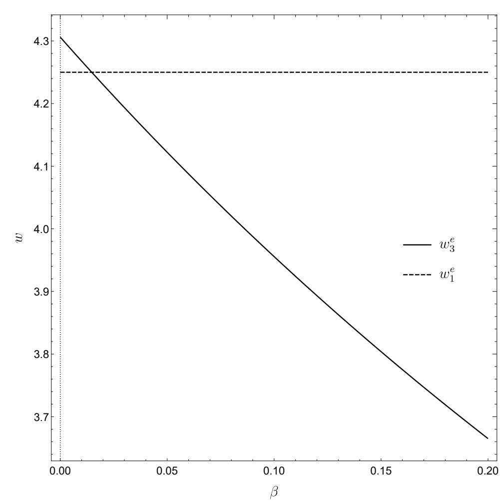
(a) w vs. η



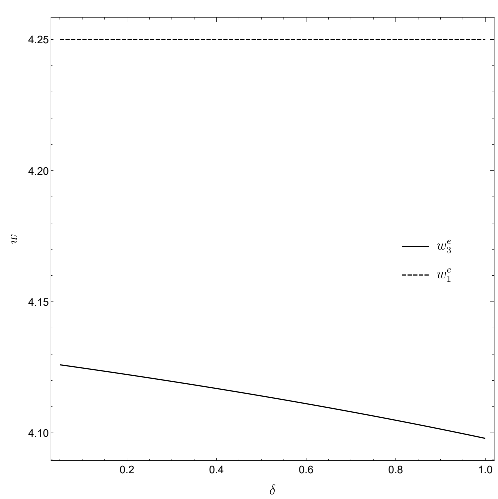
(b) w vs. ρ



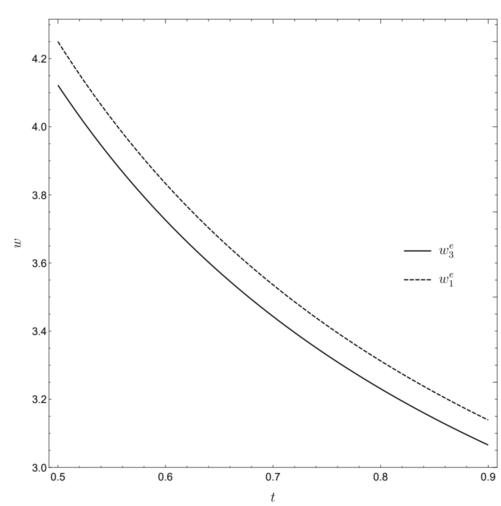
(c) w vs. r



(d) w vs. β

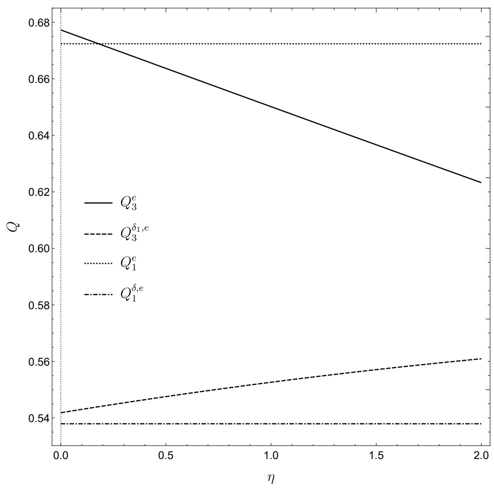


(e) w vs. δ

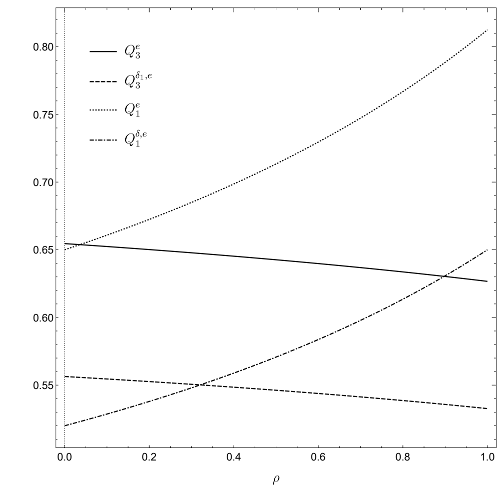


(f) w vs. t

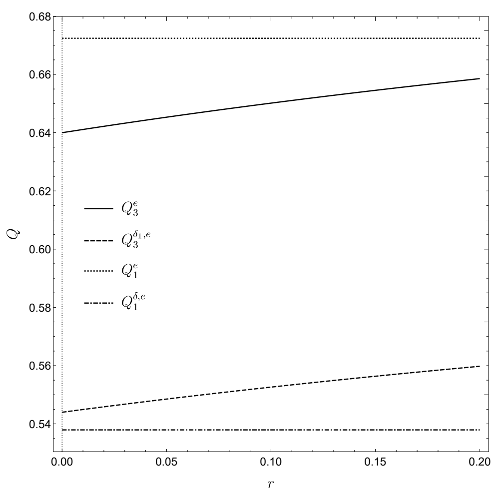
Fig. 5: Sensitivity of wholesale prices.



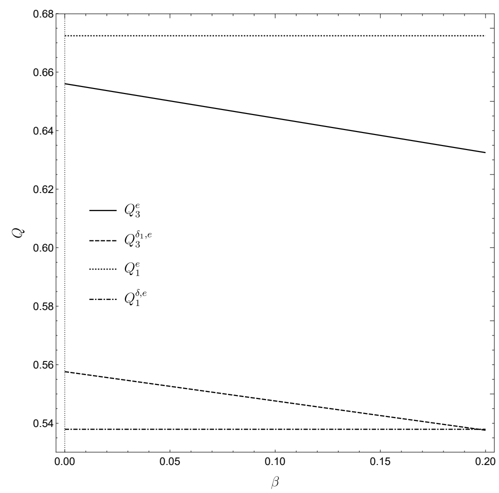
(a) Q vs. η



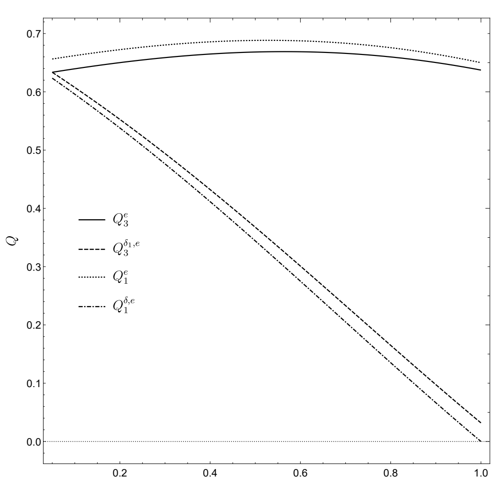
(b) Q vs. ρ



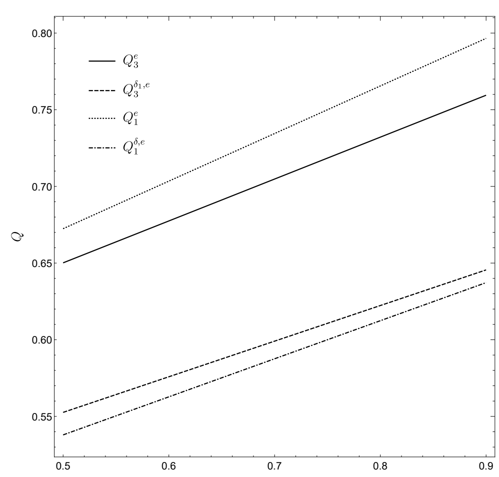
(c) Q vs. r



(d) Q vs. β



(e) Q vs. δ

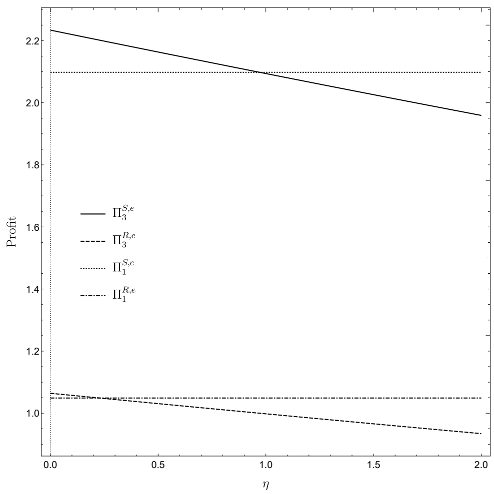


(f) Q vs. t

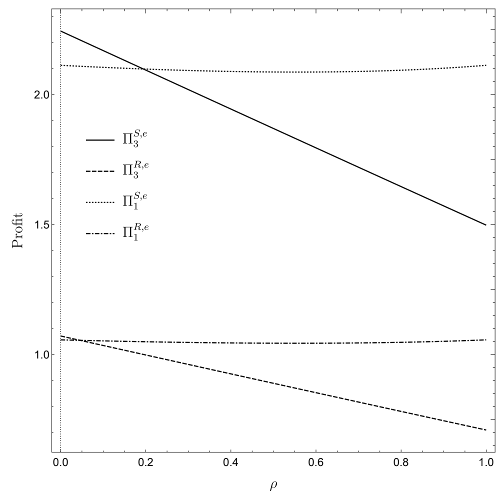
Fig. 6: Sensitivity of quantity Q.

$\Pi_3^{R,e} > \Pi_1^{R,e}$, for low values of η and ρ as observed in Figures 7(a)-7(b).

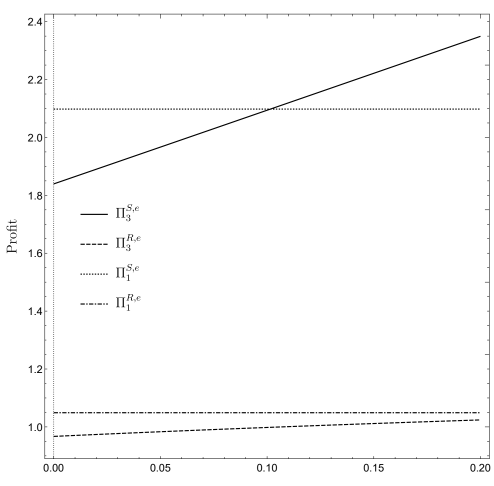
The overall SC profit is higher for low values of η and ρ where both the retailer and the supplier are better off (Pareto optimal) as compared to the profits without advance selling. However, for large values of interest rate r, even though the overall SC profit is higher but the retailer is worse off in the presence of advance selling earning a lower profit. In conclusion, advance selling is a useful strategy for not only mitigating supply disruption but can also help increases the SC profit, for both retailer and supplier, when the cost of recovering eta and the disruption probability ρ are not too high.



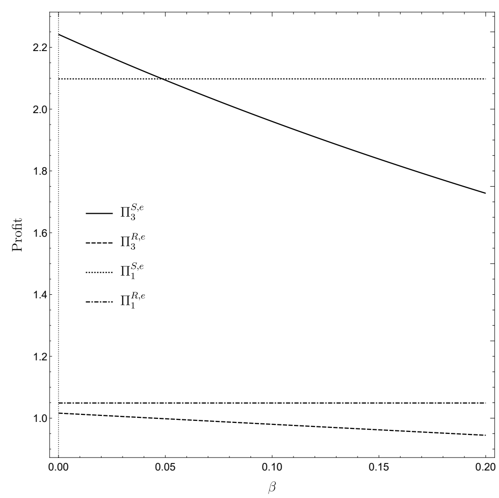
(a) Profit vs. η



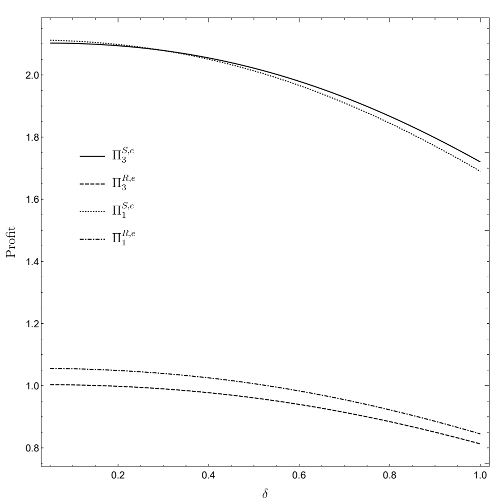
(b) Profit vs. ρ



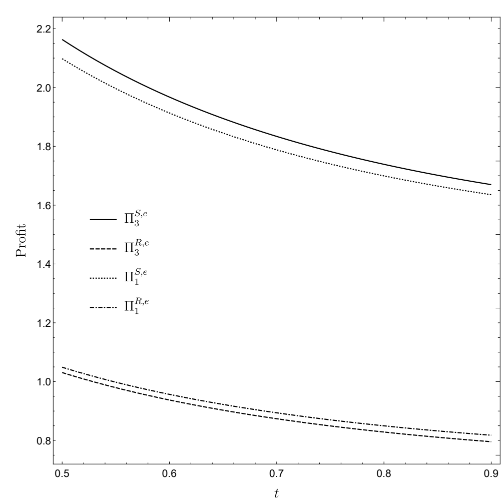
(c) Profit vs. r



(d) Profit vs. β

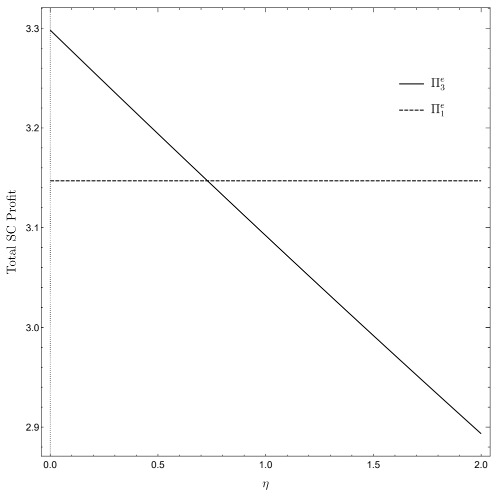


(e) Profit vs. δ

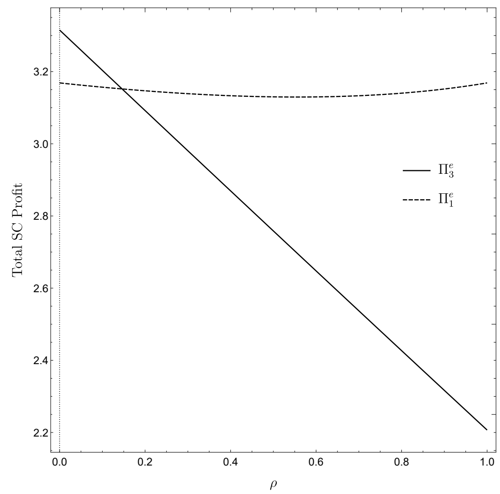


(f) Profit vs. t

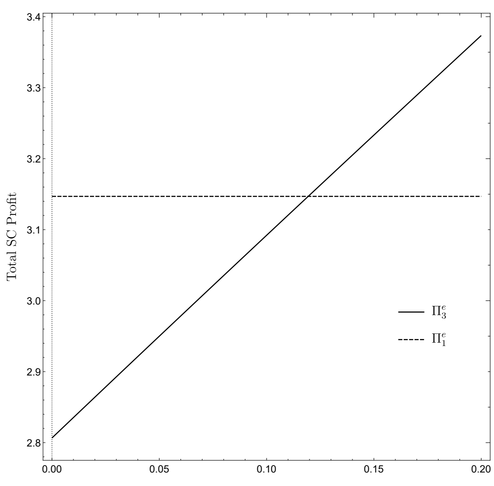
Fig. 7: Sensitivity of supplier and retailer profit's.



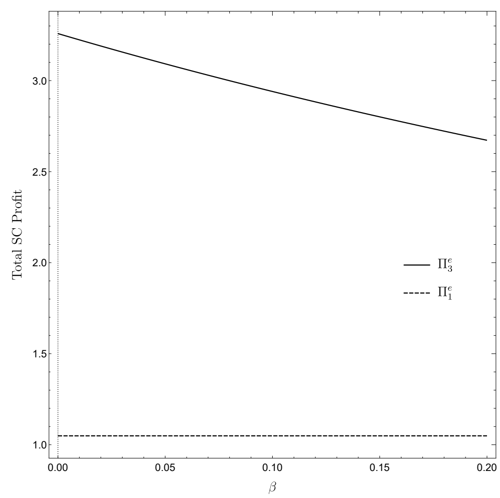
(a) SC Profit vs. η



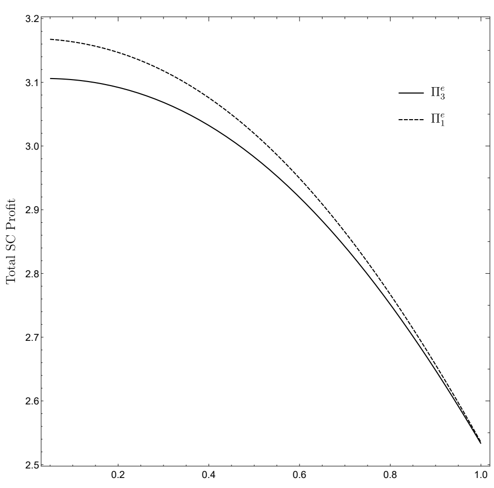
(b) SC Profit vs. ρ



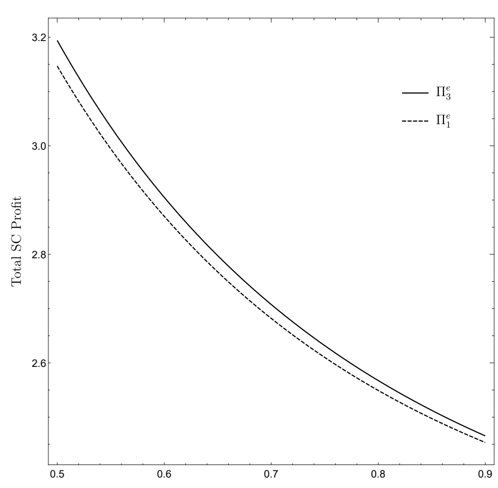
(c) SC Profit vs. r



(d) SC Profit vs. β



(e) SC Profit vs. δ



(f) SC Profit vs. t

Fig. 8: Sensitivity of wholesale prices.

6. Conclusions

This paper studies a dyadic SC where a retailer helps its financially constrained supplier by advance selling to help mitigate its supply disruption. We develop and present the analytical models as Stackelberg games with the supplier as the SC leader who decides the wholesale price followed by the retailer placing an order with or without advance selling in the absence or presence of supply disruption. Accordingly, we discussed three scenarios in the paper and investigate the overall efficacy of advance selling strategy to help the supplier to maintain its supply stream. We discuss useful managerial insights obtained through analytical means as well as some numerical studies by comparing the order quantities, wholesale prices, and profits of the supply chain members under these different scenarios.

The larger managerial insights that emerge from our analysis are as follows. We find that an advance selling arrangement can mitigate the impact of disruption and increase the supplier's yield. This 'improvement' in the supplier's yield is higher when the supplier commits more to invest towards mitigation, and when the supplier's internal rate of discounting is higher (i.e. when it gets more value by an advance payment). This improvement, however, is lower when an advance payment gets financially more costly for the retailer. We also find that advance selling is more likely to improve yield when the likelihood of disruption is not too high. An advance selling contract can also improve the total supply chain profits. We find that with advance selling the total supply chain profit is likely to improve when the supplier's commitment to invest, and when the probability of disruption are not too high. We find that a higher internal discounting at the supplier's end improves the supply-chains overall profits under advance selling.

Our model could be extended in several different directions for future research in this area. In our paper, we have considered a form of an advance selling contract where the supplier commits to a predetermined level of investment to mitigate the supply disruption. One could potentially consider different types of commitments from the supplier in return for an advance payment, such as for e.g., a fixed investment, investment as a fraction of total payment, quantity commitments, etc. In addition, we have considered an exogenous retail price given market constraints. Our model could also be extended to consider the retail price as a decision variable as well.

References

Blome, C., Schoenherr, T., 2011. Supply chain risk management in financial crises—a multiple case-study approach. *International Journal of Production Economics* 134, 1, 43–57.

Chen, K., Yang, L., 2014. Random yield and coordination mechanisms of a supply chain with emergency backup sourcing. *International Journal of Production Research* 52, 16, 4747–4767.

Dada, M., Petruzzi, N.C., Schwarz, L.B., 2007. A newsvendor's procurement problem when suppliers are unreliable. *Manufacturing & Service Operations Management* 9, 1, 9–32.

Gupta, V., He, B., Sethi, S.P., 2015. Contingent sourcing under supply disruption and competition. *International Journal of Production Research* 53, 10, 3006–3027.

Gurnani, H., Mehrotra, A., Ray, S., 2012. *Supply chain disruptions: Theory and practice of managing risk*. Springer.

He, B., Huang, H., Yuan, K., 2016. Managing supply disruption through procurement strategy and price competition. *International Journal of Production Research* 54, 7, 1980–1999.

He, J., Alavifard, F., Ivanov, D., Jahani, H., 2018. A real-option approach to mitigate disruption risk in the supply chain. *Omega*

Heckmann, I., Comes, T., Nickel, S., 2015. A critical review on supply chain risk–definition, measure and modeling. *Omega* 52, 119–132.

Ho, W., Zheng, T., Yildiz, H., Talluri, S., 2015. Supply chain risk management: a literature review. *International Journal of Production Research* 53, 16, 5031–5069.

Ivanov, D., Dolgui, A., Sokolov, B., Ivanova, M., 2017. Literature review on disruption recovery in the supply chain. *International Journal of Production Research* 55, 20, 6158–6174.

Jin, W., Zhang, Q., Luo, J., 2018. Non-collaborative and collaborative financing in a bilateral supply chain with capital constraints. *Omega*

Kouvelis, P., Zhao, W., 2012. Financing the newsvendor: supplier vs. bank, and the structure of optimal trade credit contracts. *Operations Research* 60, 3, 566–580.

Kouvelis, P., Zhao, W., 2015. Supply chain contract design under financial constraints and bankruptcy costs. *Management Science* 62, 8, 2341–2357.

Lashgari, M., Taleizadeh, A.A., Ahmadi, A., 2016. Partial up-stream advanced payment and partial down-stream delayed payment in a three-level supply chain. *Annals of Operations Research* 238, 1-2, 329–354.

Li, Y., Zhen, X., Qi, X., Cai, G.G., 2016. Penalty and financial assistance in a supply chain with supply disruption. *Omega* 61, 167–181.

Ma, S., Li, G., Sethi, S.P., Zhao, X., 2018. Advance selling in the presence of market power and risk-averse consumers. *Decision Sciences*

Mizgier, K.J., Hora, M., Wagner, S.M., Jüttner, M.P., 2015. Managing operational disruptions through capital adequacy and process improvement. *European Journal of Operational Research* 245, 1, 320–332.

Oke, A., Gopalakrishnan, M., 2009. Managing disruptions in supply chains: A case study of a retail supply chain. *International Journal of Production Economics* 118, 1, 168–174.

Sahebjamnia, N., Torabi, S.A., Mansouri, S.A., 2018. Building organizational resilience in the face of multiple disruptions. *International Journal of Production Economics* 197, 63–83.

Snyder, L.V., Atan, Z., Peng, P., Rong, Y., Schmitt, A.J., Sinsoysal, B., 2016. Or/ms models for supply chain disruptions: A review. *IIE Transactions* 48, 2, 89–109.

Taleizadeh, A.A., 2017. Lot-sizing model with advance payment pricing and disruption in supply under planned partial backordering. *International Transactions in Operational Research* 24, 4, 783–800.

Wang, Y., Wallace, S.W., Shen, B., Choi, T.M., 2015. Service supply chain management: A review of operational models. *European Journal of Operational Research* 247, 3, 685–698.

Wuttke, D.A., Blome, C., Heese, H.S., Protopappa-Sieke, M., 2016. Supply chain finance: Optimal introduction and adoption decisions. *International Journal of Production Economics* 178, 72–81.

Xiao, S., Sethi, S.P., Liu, M., Ma, S., 2017. Coordinating contracts for a financially constrained supply chain. *Omega* 72, 71–86.

Xiao, Y., Zhang, J., 2018. Preselling to a retailer with cash flow shortage on the manufacturer. *Omega* 80, 43–57.

Yano, C.A., Lee, H.L., 1995. Lot sizing with random yields: A review. *Operations Research* 43, 2, 311–334.

Yu, M., Ahn, H.S., Kapuscinski, R., 2014. Rationing capacity in advance selling to signal quality. *Management Science* 61, 3, 560–577.

Zhang, Y., Zhao, C., Pang, B., 2018. Budget allocation in coping with supply chain disruption risks. *International Journal of Production Research* 56, 12, 4152–4167.

Zhao, L., Huchzermeier, A., 2018a. Research overview of operations-finance interface. In *Supply Chain Finance*. Springer, pp. 143–183.

Zhao, L., Huchzermeier, A., 2018b. *Supply Chain Finance: Integrating Operations and Finance in Global Supply Chains*. Springer.