# Fresnel zone plate and ordinary lens antennas

Comparative study at microwave and terahertz frequencies

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Abstract-A number of lens antennas with single-dielectric phase-correcting Fresnel zone plate (FZP) lenses have been designed and studied numerically at microwave and terahertz frequencies, and contrasted to a similar in size and parameters ordinary refractive lens antenna. In a confined frequency band the terahertz grooved-dielectric FZP lenses/antennas with four or more phase-correction steps have gain, beamwidth, crosspolar isolation and input-mismatch comparable to those of the corresponding ordinary lens antenna. An examined 1.5-THz FZP lens antenna designed according to the here-proposed correction in FZP design equation can prevail in gain the ordinary lens antenna, but this finding is not valid for the examined 38-GHz (microwave) FZP lens antenna. Thus, the terahertz FZP lenses are thinner, lighter and effective options to the similar in diameter, focal length and building material ordinary lens for lens antennas construction.

Keywords-lens; terahertz lens; Fresnel zone plate lens; lens antennas;

## I. INTRODUCTION

The diffractive in nature Fresnel zone plate (FZP) lenses and the based on them lens or reflector antennas are popular in various quasioptical electronic systems [1]-[5]. Compared to the ordinary refractive lenses the FZP lenses are preferred whenever thinner, lighter and easier for production focusing devices are required. But despite of the fact that the FZP lenses and antennas are quite well examined theoretically their practical value is still underestimated. This is greatly due to the lack of precise comparative knowledge on similar in design and size diffractive and refractive lenses and lens antennas. The grooved-dielectric phase-correcting FZP lenses and antennas have been accurately studied by use of the bodyof-revolution (BOR)-FDTD method [5]. The focal length, diameter, number of zones, lens thickness, and number of phase corrections per full-wave zone on the focusing FZP lens and antenna ability are studied. In result, practical design graphs and good physical insight into focusing mechanism for the microwave/mm-wave are obtained. But in above cited article the influence of lens material and mismatch loss on antenna efficiency, and cross-polar properties are not studied.

With the present publication the authors pretend to partly fill up the gap regarding the realistic grooved-dielectric FZP lens antennas in two distinct frequency bands: microwave and terahertz. Groveed-dielectric FZP lens antennas with 2-, 4- and 8-step FZP lenses, and ordinary plane-hyperbolic lens having at each band the same feed-horn configurations, aperture and focal dimensions are examined, and contrasted by use of an accurate full-wave electromagnetic solver [6].

## II. STUDY OF FZP AND ORDINARY LENS ANTENNAS AT MICROWAVE AND TERAHERTZ FREQUENCIES

# A. Dielectric FZP and Ordinary Lens Designs

A refractive plane-hyperbolic dielectric lens (Fig. 1, on the left) has been chosen as a comparative ordinary lens [1]. In paralle, profile views of grooved-dielectric diffractive FZP lenses with stepwise phase-corrected aperture-field are illustrated: 2-step, 4-step, and 8-step FZP lenses.

The so called full-wave zone radius  $b_q$  is calculated using the next equation

$$b_q = \sqrt{2q\lambda_0 F + (q\lambda_0)^2}, \quad q = 1, 2, ..., Q$$
 (1)

where Q is the number of full-wave zones in the lens

Te full-wave zone in the FZP lens is divided in p = 2, 4, 8, ..., subzones. In each subzone the phase changes by an increment  $2\pi/p$  relative to the neighboring subzone.

The outer radius  $b_s$  of s th subzone is [1], [3].

$$b_s = \sqrt{\frac{2s\lambda_0 F}{p}} + \left(\frac{s\lambda_0}{p}\right)^2, \quad s = 1, 2, \dots, S$$
(2)

so that s = qp, and S = Qp.

The FZP lens phase correction is made by use of discrete phase shifters in the FZP subzones. Simple and most commonly used phase shifters are the circular grooves (corrugations) cut in a dielectric slab. In each grooved full-wave zone the corrugations make a stair with equal steps of height  $t_1$  each calculated by [1]



Figure 1. Dielectric lenses: (a) ordinary (plane-hyperbolic), 2-step, 4-step, and 8-step FZP lenses, and (b) FZP lens antenna with corrugated feed-horn

$$t_1 = \frac{\lambda_{01}}{p(n-1)} \tag{3}$$

The total thickness of the FZP lens  $t_p$  is found by

$$t_p = \frac{(p-1)\lambda_0}{p(n-1)} \tag{4}$$

The lens supporting base (plate) can be made as a lens extension slab by the same or distinct dielectric material. Like in the ordinary lenses, the thickness  $t_s$  of FZP supporting structure is chosen for a negligible influence on the FZP operation and good interface match between the lens and free-space, and thus, for improving the FZP efficiency.

The grooved-FZP dielectric lens makes a stepwise wave transformation by means of diffraction with a maximum allowed phase error in each wave zone equal to  $2\pi/p$ , where usually p = 2, 4, 8 or 16. For  $p \rightarrow \infty$  the diffractive Fresnel zone lens is converted into the zoned refractive lens.

The ordinary and FZP lenses have been designed for antenna operation at *microwave* frequency  $f_{mw} = 38$  GHz (or wavelength  $\lambda_{mw} = 7.89$  mm) and *terahertz* (THz) frequency  $f_{thz} = 1.5$  THz (or  $\lambda_{thz} = 0.48$  mm). Each microwave lens antenna has a focal length  $F_{mm} = 180$  mm. The lens/lens antenna aperture diameter for all lenses is the same,  $D_{mw} = 190.7$  mm. Therefore, these are relatively small in diameter and zone number lenses/lens antennas, with an aspect ratio  $F_{mw}/D_{mw} = 0.94$ .

All 38-GHz lenses are made of low-loss dielectric material (TPX polymer) with a relative permittivity  $\varepsilon_{r1} = 2.132$  (or refraction index  $n = \sqrt{\varepsilon_{r1}} = 1.46$ ) and a loss factor tan  $\delta_1 = 0.00043$  at 38 GHz [1]. The main physical data (depth, volume and weight) of microwave single-dielectric lenses, the ordinary and three FZP with 2, 4 and 8 phase correction steps in each wavelength zone, are compared in Table I.

The terahertz FZP and refraction lens focal length  $F_{thz}$  and diameter  $D_{thz}$  are calculated at the design frequency  $f_{thz}$  = 1.5 THz (wavelength  $\lambda_{thz}$  = 0.2 mm) by scaling down the corresponding 38-GHz dimensions  $F_{mm}$  and  $D_{mm}$ . The lens dimensions are not exactly scalable because they depend also on the refraction index that is slightly different at THz frequencies. All THz lenses are thought to have the same aperture diameter and focal length equal to  $F_{thz} = s_{sc}F_{mw} = 4.55$  mm and  $D_{thz} = s_{sc}D_{mw} = 4.83$  mm, respectively. The terahertz lenses are also supposed to be made of TPX polymer, which at 1.5 THz has a dielectric permittivity  $\varepsilon_{r2} = 2.09$  (n = 1.45), or almost equal to  $\varepsilon_{r1}$  at 38 GHz. On the contrary, at terahertz frequencies the TPX has much bigger absorbing loss dielectric  $(\tan \delta_2 = 0.0132), [1].$ 

TABLE I: Physical data for FZP and ordinary Lenses at 38 GHz

Lens	Depth (mm)	Volume (cm <sup>3</sup> )	Weight (kg)
Ordinary	45.22	638.6	0.53
8-step FZP	17.72	289.9	0.25
4-step FZP	15.58	259.3	0.22
2-step FZP	11.29	198.4	0.16

# B. Microwave and Terahertz Feed Horns: Design and Basic Characteristics

For obtaining an axially-symmetric and low-level crosspolar radiation field of the lens antenna a conical corrugated horn for  $f_{mw} = 38$  GHz is designed first and studied numerically in the microwave band from 30 to 50 GHz. Fig. 2(a) illustrates the design sketch of the horn, where the axial cut-plane view and front (aperture) view are shown. The basic horn dimensions are: outer horn diameter  $D_h = 24.89$  mm, horn aperture diameter  $D_a = 20.09$  mm, circular waveguide diameter  $D_w = 7.26$  mm, and horn length  $L_f = 18.30$  mm. The feed horn is also supposed to be golden with an electric conductivity of  $\sigma = 4.52 \text{ x} 10^7 \text{ S/m}.$ 

The 45-deg cut-plane co-polar (red solid line) and crosspolar (blue dash line) the radiation gain patterns are shown in Fig. 2(b), where the angle is read from the lens antenna axis. *Throughout in this text by antenna gain G the so called realized gain is understood obtained after deduction the material and mismatch loss from the aperture directive gain* (*directivity*). The input-match of the horn to the circular waveguide is also very good, with a mismatch loss of 0.05 dB from 30 to 33 GHz and less than 0.025 dB in the rest of the frequency band 30-50 GHz. In order to preserve at terahertz frequencies the feed horn radiation and input-match characteristics the computer simulation model of the microwave corrugated horn described above is made as a mini-scaled copy with the scale factor  $s_{sc}$ , and with the same

electric conductivity of  $\sigma = 4.52 \times 10^7$  S/m.



Figure 2. Conical corrugated horn: (a) axial plane-cut view and (b) co-polar (red solid line) and cross-polar (blue dashed line) radiation patterns

It is difficult and expensive to produce the tiny 1.5-THz conical corrugated terahertz horn counting on the use of very precise milling machines. Instead, the corrugated can be replaced by a similar in radiation and mismatch characteristics but much simpler in structure square-aperture diagonal horn that is available in sale for the frequencies up to about 2.0-2.5 THz.

# C. FZP and Ordinary Lens Antennas for Microwave and Terahertz Frequencies

Based on the microwave and terahertz lenses and feed horns described above, several FZP and ordinary lens antennas have been designed and investigated numerically. An axial cut-plane view of the 4-step FZP lens antenna is shown in Fig. 1(b).

The work described in the present paper is concentrated mainly on the 8-step FZP lens antenna, which is contrasted to the respective ordinary lens antenna and FZP lens antennas with smaller number of lens step corrections (p = 4 and p = 2). Our studies on 16-step FZP antenna have shown that its co-polar and cross-polar radiation patterns are slightly better than those corresponding to the 8-step FZP antenna and their maximum co-polar and cross-polar levels are almost the same. Because of the more complex and thicker structure of the 16-step FZP lens a step-correction number *p* bigger than 8 is not recommendable.

In Fig. 3 are shown the graphs of antenna gain vs. frequency in (a) 38-GHz band (30-50 GHz) and (b) 1.5-terahertz band (1.2-2.0 THz) for 2-step, 4-step and 8-step FZP lens antennas, and for an ordinary lens/lens antenna. From Fig. 3(a) is seen that in the microwave frequency band the TPX lenses do not suffer big material loss and the FZP gain is quite smaller compared to the ordinary lens antenna gain and this is especially valid in the case of 2-step FZP lens antenna.

The much bigger dielectric loss in the terahertz band causes significant focusing gain reduction in the bulky ordinary lens and the gain curves for 8-step and 4-step FZP lens antenas are reaching at some frequencies the gain curve of the ordinary lens antenna.

The gain vs frequency graphs in Fig. 3 illustrate the bandwidth character of the lenses involved: a filter-type narrowband for the FZP lenses and a broadband for the ordinary lens. In addition, with the FZP lens step-number p



Figure 3. Lens antennas gain versus frequency in (a) microwave band and (b) terahertz band for 2-step FZP (brown dash-dot lines), 4-step FZP (green dot lines), 8-step FZP (blue dash lines), and ordinary lens (red solid lines) antennas

and resultant depth increase an important anormal feature is observed: presence of two significantly displaced from the design frequency  $f_0$  gain maxima at  $f_{opt1}$  and  $f_{opt2}$  in the gain curves corresponding to 4-step (green dot lines) and 8-step (blue dash lines) FZP lens antennas, so that  $f_{opt2} > f_{opt1} > f_0$ .

More specifically, in Fig. 3(b) the ordinary lens antenna overpasses in gain the 8-step FZP lens antenna by about 1.7 dB at design frequency  $f_0 = 1.5$  THz while at the two optimum frequencies,  $f_{opt1} = 1.626$  THz and  $f_{opt2} = 1.750$  THz, the 8-step FZP lens antenna reaches in gain the ordinary lens antenna.

The terahertz 8-step FZP lens antenna with  $F/D \approx 1$  is much closer in radiation performance to the ordinary lens antenna compared to its microwave counterpart. This is concluded from the gain vs. frequency curves in Fig. 3(b), and is well verified by the gain radiation patterns in Fig. 4. It is found for instance that at the optimum frequency of 1.626 THz the FZP and ordinary lens antennas not only have equal maximum gains, but also very near in level and shape copolar and cross-polar radiation patterns (Fig. 4(a)) and (Fig. 4(b)).

### III. REMOVAL OF OPTIMUM FREQUENCY DISPLACEMENT

The FZP antenna gain displacement from the design frequency for the 8-step FZP microwave and terahertz lens antennas (Fig. 3) is about 8 % for the first and 18% for the second gain maximum. The gain frequency displacements are quite big and this indicates for an imperfection in the established design practice for the grooved-dielectric FZP dielectric lenses [1]-[2], [4]-[5]. From the original Fresnel zone equations (1) and (2) follows that the FZP focusing frequency can be reduced with the focal length increase. An empirical correction in equation (2) valid for the 2-step FZP lens is proposed initially by one of the present authors [3].

Here, in the process of numerical simulation and optimization of grooved-dielectric FZP lenses and antennas we more precisely found how much the focal length depends on the number of step corrections p or of the lens depth. In the case of 8-step FZP lens the conventional design focal



Figure 4. Gain co-polar (red solid lines) and cross-polar (blue dash lines) radiation patterns at 1.626 THz for (a) 8-step FZP lens antenna, and (b) ordinary lens antenna.

length F has to be increased by the lens thickness  $t_p$ , and the correct focal length becomes  $F' = F + t_g$ , where  $t_g$  is the 8-step FZP lens thickness calculated by (4) for p = 8. Thus, the equation (2) for the radius  $b_g$  of the 8-step FZP lens is corrected as follows

$$b_s = \sqrt{\frac{s\lambda_0 (F+t_8)}{4} + \left(\frac{s\lambda_0}{8}\right)^2}, \ s = 1, 2, ..., S$$
 (5)

where s = 8q and S = 8Q.

Fig. 5(a) proves fine the validity of newly-proposed equation (5) for the 8-step FZP lens. It is evident that the designed by use of (5) FZP lens antenna has a maximum gain near to the design frequency of 1.5 THz (cyan dash-dot line), and slightly overpasses the gain graph of the ordinary lens (red solid line). For comparison is shown also the antenna gain graph for 8-step FZP lens designed according to (4), and drawn in blue dash line. The corrected design equation (5) can be used for both terahertz and microwave bands, but only for 8-step FZP lens (or p = 8) with the specified aspect ratio. It is necessary to be checked further how the number of zones and lens aspect ratio influence on the above focal length correction. Now an appropriate optimization of (4) is searched also by the authors for the FZP lenses with other step-correction numbers like p = 4 and p = 16 and various aspect ratios and zone numbers for finding a more general design correction mechanism.

# IV. CONCLUSION

Several microwave and low-terahertz FZP lens antennas have been numerically studied and contrasted in two frequency bands: microwave and terahertz. It is found that for the classically-designed FZP with four or more phasecorrection levels the two lens antenna types, the FZP and ordinary, have comparable gain values, beamwidths, crosspolar isolations and mismatch qualities, but different design and maximum-gain frequencies. More precisely, at the microwave frequency of 38 GHz the 8-step microwave FZP antenna gives way to the ordinary lens antenna by 1.7 dB while at 1.5 THz this difference is only 0.8 dB. It is proved, however, that the 1.5-THz 8-step FZP lens/lens antenna designed according to the newly-proposed Fresnel zone design equation reaches in gain the ordinary lens/lens antenna at the original design frequency. Because the FZP lenses/lens antennas are narrowband compared to the ordinary ones all parameter similarities hold in a relatively smaller bandwidth, which, however, for the design frequency of 1.5 THz presents a big for many applications frequency band. The FZP lenses are very much smaller in thickness, volume and weight than the ordinary lenses and this leads to a creation of lighter lens antennas. Besides, the diffractive plane-step FZP lenses are easy to product and have better fabrication-error tolerance.



Figure 5. Effect of focal length correction: (a) antenna gain vs. frequency of corrected 8-step FZP (cyan dash-dot line), non-corrected 8-step FZP (blue dash line) and ordinary lens (red solid line) antennas, and (b) co-polar (red solid line) and cross-polar (blue dash line) radiation patterns of corrected 8-step FZP lens antenna at design frequency of 1.5 THz.

The application of microwave/terahertz FZP lenses and antennas includes fixed and mobile ground communication systems, rigid and collapsible space communication complexes, and imaging and security systems among many others. Another feasible application of FZP optics similar to the studied here is envisioned in a receiver for the Atacama Large Millimeter Array (ALMA) radio telescope in Chile, for operation in the low-frequency mm-wave band between 31 GHz and 45 GHz.

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