Higher order clockwork gravity

Florian Niedermann,* Antonio Padilla,† and Paul M. Saffin‡
School of Physics and Astronomy, University of Nottingham, Nottingham NG7 2RD, United Kingdom

(Received 11 May 2018; published 12 November 2018)

We present a higher order generalization of the clockwork mechanism starting from an underlying nonlinear multigravity theory with a single scale and nearest neighbor ghost-free interactions. Without introducing any hierarchies in the underlying potential, this admits a family of Minkowski vacua around which massless graviton fluctuations couple to matter exponentially more weakly than the heavy modes. Although multidiffeomorphisms are broken to the diagonal subgroup in our theory, an asymmetric distribution of conformal factors in the background vacua translates this diagonal symmetry into an asymmetric shift of the graviton gears. In particular we present a TeV scale multigravity model with $\mathcal{O}(10)$ sites that contains a massless mode whose coupling to matter is Planckian, and a tower of massive modes starting at a TeV mass range and with TeV strength couplings. This suggests a possible application to the hierarchy problem as well as a candidate for dark matter.

DOI: 10.1103/PhysRevD.98.104014

The exponentially large hierarchy between the electroweak scale and the Planck scale suggests that new physics could be very close to the scale of current collider experiments (see e.g., [1]). Generically, the Higgs mass is quadratically unstable against radiative corrections coming from any physics in this large ultraviolet window. If we are to retain the notion of naturalness [2], any new theory must incorporate a mechanism to ensure cancellation between loops, as in supersymmetry [3]. Alternatively, the exponential hierarchy could merely be an illusion, with the fundamental scale lying much closer to the electroweak scale, thanks, say, to large [4] or warped extra dimensions [5], or the weakness of the string coupling [6]. Yet another possibility is that the observed vacuum expectation value of the Higgs is just one of a much larger landscape: when these values can be scanned by the theory in some way, we can invoke anthropic considerations [7,8], or employ some sort of cosmological relaxation procedure [9] to pick out the observed value. This list of proposals for addressing or rephrasing the hierarchy problem is far from exhaustive and as yet no experimental evidence in favor of any particular model has been forthcoming (see e.g., [10,11]).

With this in mind it is important to continue to explore new ideas. Recently, the *clockwork mechanism* [12,13] was proposed in order to generate a hierarchy between the fundamental scale in the theory and the effective coupling of the zero mode to external sources at low energies. It was originally applied to axions with a view to explaining the super-Planckian decay constants required by cosmological relaxation models [9]. The idea is to have a modest number of fields, or *gears*, π_i, whose masses mix with some characteristic strength $q > 1$. The structure of the mass terms are governed by an asymmetrically distributed unbroken subgroup of $U(1)^N$ in the fundamental theory. This is nonlinearly realized by the "pion" fields as gear shifts, $\pi_i \to \pi_i + c/q^i$, that are equal up to a rescaling with increasing powers of $1/q$. The result is a zero mode whose overlap with each of the gears also scales asymmetrically, $a_0 \propto \pi_0 + \pi_1/q + \cdots \pi_N/q^{N-1}$. By coupling external sources to one end of the clockwork we are able to engineer very little overlap with the zero mode thanks to the high power of $1/q$. At low energies, this leads to an exponentially large hierarchy of scales from a theory with a single mass scale, and order one parameters. See also [14] for similar ideas applied to dark energy.

The clockwork mechanism was later generalized to a much wider class of fields in [15], in particular to linearized gravity, where it was used to explain the hierarchy between the electroweak scale and the Planck scale. These generalizations were criticized in [16] who argued, amongst other things, that one could not apply the clockwork mechanism to non-Abelian theories, including gravity. The claim rested on the assumption that there is no site (i.e., gear number) dependence in the couplings, as one might expect from a fundamental theory free of large parameters, and made use of elegant group theoretic arguments that forbid an asymmetric distribution in the

*florian.niedermann@nottingham.ac.uk
†antonio.padilla@nottingham.ac.uk
‡paul.saffin@nottingham.ac.uk

structure of the unbroken subgroup. At the level of the low energy effective field theory, such site *independence* might be viewed as a model dependent statement, making a concrete assumption about the underlying UV theory [17]. If we allow site *dependence* in the couplings, we can again obtain interesting phenomenology but one might worry about the origin of this hierarchy at a fundamental level even though we only ever couple to one external site. The clockwork idea has seen a number of interesting applications, especially in the context of dimensionally deconstructed setups (see e.g., [18–22]).

This is something of a linguistic debate about what is and is not a meaningful clockwork but one that teaches us some valuable lessons [16,17]. It is certainly true that the *standard* clockwork cannot be obtained from a discrete theory with a single scale [16]. Indeed, as we will see by investigating the corresponding nonlinear ghost-free multigravity setup [23], to obtain the classic clockwork mass matrix of [13,15] in the linearized theory one must introduce hierarchies at the level of the underlying nonlinear theory. As in [17], we could simply introduce a dilaton to account for these underlying hierarchies. Here we take a very different approach, generalizing the clockwork philosophy to the four dimensional arrays that govern the metric interactions in the discrete multigravity framework. By focusing on a sparsely populated array with nearest neighbor interactions only, we show how the desired asymmetric decomposition of the zero mode can be obtained from an underlying theory with a single scale and no large parameters. This yields a low energy effective theory of a massless graviton with exponentially suppressed couplings. More specifically, if we were to take $M \sim$ TeV to be unique across all sites, we can generate a low energy effective theory of gravity with Planckian coupling with $\mathcal{O}(10)$ sites and no exponentially large parameters.

It remains the case (as it must) that the interactions break the symmetry down to the symmetric diagonal subgroup of diffeomorphisms [16,23,24]. Nevertheless, as we will explain below, we can still obtain an asymmetric distribution in how this acts on the canonically normalized metric perturbations if that distribution is also present in the background value. It turns out that the form of the zero mode, the massless graviton, is completely fixed by the structure of the underlying vacua, and that this can be rendered asymmetric with only very mild assumptions on the underlying nonlinear theory.

Our starting point is the general action for a ghost-free multigravity theory described by [23,25]

$$S = S_{\rm K} + S_{\rm V}, \tag{1}$$

where the "kinetic" part for the N metric fields $(g_i)_{\mu\nu}$ is

$$S_{\rm K} = \sum_{i=0}^{N-1} \frac{M_i^2}{2} \int {\rm d}^4x \sqrt{-g_i} R[g_i]. \tag{2}$$

Here we include possible site dependence in the spectrum of Planck scales, although we emphasize that we have in mind that each M_i is of order a unique underlying scale, M. It is convenient to express the potential in terms of vielbeins, $(E_i)^a_\mu$, such that [23]

$$S_{\rm V} = -\sum_{i,j,k,l} \int T_{ijkl}\epsilon_{abcd}(E_i)^a \wedge (E_j)^b \wedge (E_k)^c \wedge (E_l)^d, \tag{3}$$

where $(E_i)^a = (E_i)^a_\mu {\rm d}x^\mu$ and $(g_i)_{\mu\nu} = \eta_{ab}(E_i)^a_\mu (E_i)^b_\nu$. Here and in the following, the sums run from 0 to $N-1$ (unless otherwise stated). The interaction matrix T_{ijkl} is required to be totally symmetric and is assumed to depend on the unique underlying scale $T_{ijkl} \sim M^4$. The potential part breaks N copies of the diffeomorphism group acting at each site, down to the diagonal subgroup. Working in the vielbein formalism, N copies of local Lorentz invariance are also broken down to their diagonal subgroup by the potential.

The equivalence between the vielbein and an explicit metric formulation is not automatic. Indeed, if we go beyond pairwise interactions and/or allow "cycles" of interactions between sites, e.g., $1 \to 2 \to 3 \to 1$, the equivalence is broken because the field equations no longer imply a symmetric vielbein condition [23]. Such structures, in either vielbein or explicit metric formulations, generically lead to ghosts [26–28] (see [29,30] for recent constructions that evade this rule). For nearest neighbor interactions only, as we consider here, we have a chain of pairwise interactions linking each of the sites $0 \to 1 \to \cdots N-2 \to N-1$, rather than a cycle, and this means the vielbein formulation is equivalent to a metric one and the theory is ghost free.

The theory admits N Minkowski vacua, $(\bar{g}_i)_{\mu\nu} = c_i^2 \eta_{\mu\nu}$, provided the constants c_i fulfill

$$\sum_{j,k,l} c_j c_k c_l T_{ijkl} = 0. \tag{4}$$

Note that the c_i cannot all be gauged to unity because there is only one diagonal copy of diffeomorphisms left intact by the potential. In this sense their values are physical up to an overall normalization. There does exist a pseudosymmetry that allows us to conformally rescale each metric $(g_i)_{\mu\nu} \to \lambda_i^{-2}(g_i)_{\mu\nu}$ at the expense of rescaling the couplings $M_i \to \lambda_i M_i$, $T_{ijkl} \to \lambda_j\lambda_j\lambda_k\lambda_l T_{ijkl}$. Since we want to work in a frame in which all scales in the action correspond to the unique underlying scale M, without any large parameters, this pseudosymmetry is essentially fixed and cannot be used to remove the c_i in our background solution.

The overall normalization of the c_i is fixed by the matter Lagrangian. As we will explain below, matter is only allowed to couple to one particular site and we normalize all of the conformal factors relative to this site. This ensures that any mass scales appearing in the matter Lagrangian correspond to the physical masses for the canonically normalized matter fields propagating on the background geometry.

We now consider fluctuations about our vacua

$$(g_i)_{\mu\nu} = c_i^2\eta_{\mu\nu} + \frac{c_i}{M_i}(h_i)_{\mu\nu}, \tag{5}$$

where the normalization ensures a canonical form of the Fierz-Pauli kinetic term [31]. Thanks to the symmetric vielbein condition,[1] $\eta_{ab}(E_i)^a_{[\mu}(E_j)^b_{\nu]} = 0$, we can use the one diagonal copy of local Lorentz invariance so that the vielbein fluctuations are symmetric and correspond to

$$\delta^\mu_a(\delta E_i)^a_\nu = \frac{1}{2M_i}(h_i)^\mu_\nu, \tag{6}$$

where Lorentz indices are raised and lowered with $\eta_{\mu\nu}$. The second variation of the potential then becomes

$$\delta_2 S_V = \int d^4x \sum_{i,j}\mathcal{M}_{ij}[(h_i)^\mu_\mu(h_j)^\nu_\nu - (h_i)^\mu_\nu(h_j)^\nu_\mu], \tag{7}$$

where the mass matrix is given by

$$\mathcal{M}_{ij} = \frac{3}{M_iM_j}\sum_{k,l}c_kc_lT_{ijkl}. \tag{8}$$

Let us now choose a frame for which $M_i = M$ for all i. We immediately see the presence of the zero mode $(a_0)_{\mu\nu} \propto \sum_j c_j(h_j)_{\mu\nu}$ from Eq. (4). To obtain the desired asymmetric distribution, we only really require that $c_j/c_{j+1} = \mathcal{O}(1) > 1$. However, for simplicity let us suppose that the T_{ijkl} are such that the $c_j = cq^{-j}$ exactly, for some overall normalization constant, c, as an example of the desired asymmetric distribution in the overlap between the zero mode and the graviton gears. This is a consequence of the diagonal subgroup of diffeomorphisms applied to fluctuations on an asymmetric distribution of background vacua. To see this we note that the diagonal diffs act on the metric fluctuations as $\delta(g_i)_{\mu\nu} \to \delta(g_i)_{\mu\nu} + 2c_i^2\partial_{(\mu}\xi_{\nu)}$, where $\xi_\mu = \eta_{\mu\nu}\xi^\nu$ is site independent. In terms of the graviton gears this reads as $(h_i)_{\mu\nu} \to (h_i)_{\mu\nu} + 2Mc_i\partial_{(\mu}\xi_{\nu)}$, which is analogous to the asymmetric gear shifts familiar to the original clockwork proposal [13]. It follows that the form of the zero mode is entirely dictated by the conformal factors in the background vacua and the unbroken diagonal subgroup of diffeomorphisms. If those conformal factors exhibit the desired asymmetry then the zero mode has a classic clockwork distribution. In [16] this possibility was not considered as it was assumed that $c_i = 1$ for all i. Of course, one might expect that an asymmetric and hierarchical distribution in the c_i is not possible without introducing dangerously large hierarchies in the T_{ijkl} although, as we will now show, this is not the case.

To proceed, we recall that we are assuming nearest neighbor interactions only, consistent with the ghost-free assumption. This implies that the interaction matrix $T_{ijkl} = \tau_{(ijkl)}$ where

$$\tau_{ijkl} = A_{ij}\delta_{jk}\delta_{kl} + B_{ik}\delta_{ij}\delta_{kl}. \tag{9}$$

The first term above forces three identical indices while the second forces two pairs of identical indices. Both matrices A_{ij} and B_{ij} are of tridiagonal form and can be expressed as

$$A_{ij} = \lambda^A_i\delta_{ij} + \mu^A_i\delta_{i,j-1}\theta_{i,N-1}\theta_{j,0} + \nu^A_j\delta_{i-1,j}\theta_{i,0}\theta_{j,N-1}, \tag{10}$$

where $\theta_{ij} = 1-\delta_{ij} = \begin{cases} 0 & i=j \\ 1 & i\neq j\end{cases}$ and a similar expression given for B_{ij} in terms of λ^B_i, μ^B_i, ν^B_i. Basically, the diagonal components are given by $\lambda^{A,B}_i$, the upper diagonal by $\mu^{A,B}_i$ and the lower diagonal by $\nu^{A,B}_i$. In terms of A_{ij} and B_{ij}, we have a mass matrix (8) proportional to

$$\sum_{k,l}c_kc_lT_{ijkl} = \frac{1}{4}(A_{ij}c_j^2 + A_{ji}c_i^2) + \frac{1}{2}\delta_{ij}c_i\sum_k A_{ki}c_k + \frac{2}{3}B_{(ij)}c_ic_j + \frac{1}{3}\delta_{ij}\sum_k B_{(ik)}c_k^2 \tag{11}$$

and a vanishing vacuum condition (4) given by

$$\sum_{j,k,l}c_jc_kc_lT_{ijkl} = \frac{1}{4}\sum_j(A_{ij}c_j^3 + 3A_{ji}c_jc_i^2) + \sum_j B_{(ij)}c_ic_j^2. \tag{12}$$

Note that the antisymmetric part of B_{ij} drops out which means we could identify μ^B_i with ν^B_i. In any event, assuming vacua with $c_i = cq^{-i}$, the vanishing of (11) yields a very weak condition of the form

$$\begin{aligned}\lambda^A_i + \lambda^B_i = &-\frac{1}{4}\left(\mu^A_i\frac{\theta_{i,N-1}}{q^3} + 3\mu^A_{i-1}q\theta_{i,0}\right)\\ &-\frac{1}{4}\left(\nu^A_{i-1}\theta_{i,0}q^3 + 3\nu^A_i\frac{\theta_{i,N-1}}{q}\right)\\ &-\frac{1}{2}(\mu^B_i+\nu^B_i)\frac{\theta_{i,N-1}}{q^2} - \frac{1}{2}(\mu^B_{i-1}+\nu^B_{i-1})\theta_{i,0}q^2.\end{aligned} \tag{13}$$

[1]The symmetric vielbein condition follows from the field equations whenever there are pairwise interactions only and no cycles, as is the case here [23].

This implies[2] that in order to obtain the desired asymmetric distribution in background conformal factors, we only need to tolerate hierarchies in T_{ijkl} at order q^2. Furthermore, we note that we have focused on a special case for which $c_j = cq^{-j}$. Detuning this choice of the λ, μ, ν by order 1 (in units of M) would simply induce a relative correction of order 1 to the c_j, which will generically preserve the desired hierarchy in the conformal factors. Perturbing the theory to include couplings that lie off the tridiagonal will weaken the efficiency of the resulting "clockwork." However, recall that such a deformation would generically introduce new, ghostlike degrees of freedom that are not expected to be radiatively generated below the cutoff. We shall elaborate on this later. In any event, the mass matrix for our chosen parametrization is given by

$$\mathcal{M}_{ij} = \frac{3c^2 q^{-2i}}{M^2}\left[-\delta_{ij}\left(\frac{\theta_{i,N-1}}{q}Z_i^+ + q\theta_{i,0}Z_i^-\right) + \delta_{i,j-1}\theta_{i,N-1}Z_i^+ + \delta_{i-1,j}\theta_{i,0}Z_i^-\right], \tag{14}$$

where

$$Z_i^+ = \frac{1}{4}\left(\frac{\mu_i^A}{q^2} + \nu_i^A\right) + \frac{1}{3q}(\mu_i^B + \nu_i^B) \tag{15}$$

and $Z_i^- = q^2 Z_{i-1}^+$. We now see the difficulty in generating the classic clockwork mass matrix of [13,15] in the absence of exponentially large hierarchies in the underlying theory. In [13,15], the mass matrix for the field fluctuations depends on a single overall scale. This is not the case in (14) unless the μ_i, ν_i are chosen to absorb the exponential prefactor of q^{-2i}. Such a choice would amount to choosing a hierarchy of scales in T_{ijkl}. This result could have been anticipated from the no-go claims of [16]. Of course, the presence/absence of hierarchies in T_{ijkl} is only a meaningful statement up to possible conformal rescalings of the metric. However, we recall that we have chosen to work in a conformal frame in which all the Planck scales $M_i = M$, so there is no ambiguity in what we are saying here.

Given that the mass matrix for gravitons is an emergent object, not independent of the background, we would argue that there is actually no compelling reason for us to require it to depend on a single scale, as in [13]. Instead we choose to impose the single scale requirement at the level of the fundamental theory, the T_{ijkl}, and ask whether or not the spectrum of fluctuations about consistent vacua gives rise to an emergent hierarchy, with a zero mode that is exponentially more weakly coupled to external states than the heavy modes. This is certainly possible with the setup described in this paper. Our clockwork is really a higher order one governed by the four-point vielbein interactions. Although there are no large parameters in the potential, it admits an exponential distribution of conformal factors in the corresponding vacua. This in turn yields a graviton zero mode with a classic asymmetric clockwork decomposition in graviton gears.

Let us now study the phenomenology of the mass eigenstates for the graviton fluctuations that emerge from our single scale theory. To simplify the analysis, let us assume that the μ_i, ν_i are site independent, in other words, $\mu_i^A = \mu^A$ etc. The mass matrix now takes the simple form

$$\mathcal{M}_{ij} = \frac{F(q)}{M^2}c^2 q^{-2i}\left[\delta_{ij}\left(\frac{\theta_{i,N-1}}{q^2} + q^2\theta_{i,0}\right) - \delta_{i,j-1}\frac{\theta_{i,N-1}}{q} - \delta_{i-1,j}q\theta_{i,0}\right], \tag{16}$$

where

$$F(q) = -\frac{3}{4}\left(\frac{\mu^A}{q} + \nu^A q\right) - \mu^B - \nu^B. \tag{17}$$

As anticipated earlier, we also assume that matter is minimally coupled to a single site, given by $i = i_*$:

$$S_m = \int d^4x\sqrt{-g_{i_*}}\mathcal{L}_m[g_{i_*}]. \tag{18}$$

Note that coupling the same matter to multiple sites will generically yield a ghost [32–34], although in some special cases its mass may exceed the scale of strong coupling [28,33,35–37] (see, also, [29] for novel constructions that remain ghost free at higher energies). In any event, our conservative choice is a consistent one and yields an effective interaction between the canonically normalized i_*th graviton gear and the energy momentum tensor of the form

$$\delta S_m = \int d^4x \frac{1}{2M}(h_{i_*})_{\mu\nu}T^{\mu\nu}, \tag{19}$$

where we have used the fact that $c_{i_*} = 1$ and $M_i = M$. The condition on c_{i_*} follows from the fact that we have fixed the overall normalization of the c_i's relative to the site to which matter couples. Since $c_i = cq^{-i}$, this fixes the overall conformal normalization factor to be $c = q^{i_*}$.

The mass matrix (18) can be diagonalized by a rotation in field space (suppressing indices), $h_i = \sum_j O_{ij}a_j$. The orthogonal matrix, O_{ij}, has its columns given by the unit mass eigenstates. In particular, the zeroth column is given by the unit zero mode so that $O_{i0} = \mathcal{N}q^{-i}$, where $\mathcal{N} = (\sum_{k=0}^{N-1} q^{-2k})^{-1/2} = (\frac{1-q^{-2N}}{1-q^{-2}})^{-1/2}$. Numerical investigations suggest that the jth massive eigenstate generically has

[2] This is obvious in the B sector. In the A sector we can see it by assuming $\mu_i^A \sim q, \nu_i^A \sim 1/q$.

$O_{i,j>0} \approx 1$, for some i. The corresponding massive eigenvalues are given by

$$m_{j>0}^2 \sim \frac{F(q) q^{2(1+i_*-j)}}{M^2}, \tag{20}$$

where we have again used the fact that $c = q^{i_*}$. These results can be obtained analytically in the large q limit, when the mass matrix approximates as $\mathcal{M}_{ij} \approx \frac{F(q)}{M^2} q^{2(1+i_*-i)} \delta_{ij} \theta_{i,0}$.

In terms of the mass eigenstates, the coupling to matter reads

$$\delta S_m = \int \mathrm{d}^4 x \left(g_0 (a_0)_{\mu\nu} + \sum_{j=1}^{N-1} g_j (a_j)_{\mu\nu} \right) T^{\mu\nu}, \tag{21}$$

where the zero mode coupling is

$$g_0 = \frac{\mathcal{N}}{2Mq^{i_*}} = \frac{\left(\frac{1-q^{-2N}}{1-q^{-2}}\right)^{-1/2}}{2Mq^{i_*}}. \tag{22}$$

If we couple matter to the end of the clockwork, at site $i_* = N-1$, then for $q > 1$ the zero mode coupling is at an exponentially higher scale than the fundamental scale, $M_0^{\mathrm{eff}} \sim M q^{(N-1)}$. Taking $M \sim \mathrm{TeV}$ and $q = 4$, we can achieve a Planck scale effective coupling $M_0^{\mathrm{eff}} \sim M_{\mathrm{Pl}}$ with $N = 26$ sites. Recall that the level of hierarchy in T_{ijkl} need not exceed $q^2 \sim 16$ in this case.

Turning to the heavy modes, these couple to matter with strength $g_j = \frac{O_{i_*,j}}{2M}$, which is given by the fundamental scale, M. Taking $\mu^{A,B}$, $\nu^{A,B} \sim M^4$, consistent with our single scale theory, there is a mass gap of order $M^2 q^2$ to the spectrum of heavy modes. These are then distributed exponentially, with the heaviest mode having a mass, $M^2 q^{2(N-1)}$. Choosing our parameters as in the previous paragraph, this yields lightest and next to lightest heavy modes whose masses lie beyond the TeV scale, with TeV strength and weaker coupling to the energy-momentum tensor. In principle, this spectrum could include an interesting dark matter candidate (see [38–41] for some work on spin two dark matter).

Before presenting this as a robust solution to the hierarchy problem, we need to ask whether or not the structure of the potential is radiatively stable. For example, do loop corrections generate large next-to-nearest neighbor interactions that could weaken the efficiency of our higher order clockwork? For the case of matter loops the answer is obviously negative since we took matter to only couple to a single site. For graviton loops, the question is more subtle and the only possible statement we can make is to ask what happens far below the cutoff (TeV) when we treat this as an effective field theory (if indeed that is a reasonable thing to do). We anticipate that gauge invariance will prevent zero mode loops from generating any new potential interactions. Heavy mode loops could be more dangerous although one might just assume that they decouple at low energies since the masses start at a TeV scale. Of course, it is possible that decoupling is subtle, at least if the interactions between the light and heavy modes also diverge as we send the masses to infinity. A thorough investigation of this is obviously going to be very involved, as with any calculation involving graviton loops. Indeed, its scope extends beyond the context of this paper to a more general question regarding the radiative stability of ghost-free multigravity theories. This is because additional beyond nearest neighbor interactions introduce a trivertex and/or a cycle in our potential, which would resurrect the Boulware-Deser ghost [28,35]. This represents new degrees of freedom and in analogy with higher order curvature corrections generated in a perturbative approach to quantum general relativity, we might expect them to have mass scales at or above the cutoff of the theory (see [42,43] for a similar statement in a massive gravity and bigravity context). A more detailed analysis will be very involved but is clearly a priority for future work.

Another important feature of our model is the absence of an underlying dilaton, in contrast to the original proposals presented in [15,17]. From a four-dimensional perspective, this allows us to have a fully nonlinear multigravity clockwork governed by a single (TeV) scale, representing a completely new approach to the electroweak hierarchy problem. The flip side of this particular structure is that it could prove to be an obstacle in obtaining it from a dimensional deconstruction of a five dimensional model. Of course, the ultimate goal would be to realize this setup as a string theory compactification.

To summarize, we have shown that a consistent single scale multigravity model can yield a clockwork graviton spectrum where the massless graviton couples to matter exponentially more weakly than the heavy modes. This is achieved through a higher order generalization of the standard clockwork mechanism involving nearest neighbor interactions in the ghost-free nonlinear theory. Although multidiffeomorphisms are broken to the diagonal subgroup by these interactions, this translates into an asymmetric shift of the graviton gears thanks to an asymmetric distribution of conformal factors in the background vacua. This has led us to a TeV scale multigravity model with $\mathcal{O}(10)$ sites that contains a massless mode whose coupling to matter is Planckian, and a tower of massive modes starting at a TeV mass range and with TeV strength matter couplings. However, before presenting this as a complete resolution of the naturalness question, we emphasize the need to compute radiative corrections including those mediated by graviton loops in an effective description below the cutoff. This will be a priority in future investigations.

ACKNOWLEDGMENTS

This work was supported by a STFC Consolidated Grant. A. P. is also funded by a Leverhulme Trust Research Project Grant. We would like to thank E. del Nobile, M. Sloth and S. Y. Zhou for useful discussions.

[1] P. Nath *et al.*, The hunt for new physics at the Large Hadron Collider, Nucl. Phys. B, Proc. Suppl. **200–202**, 185 (2010).
[2] G. F. Giudice, Naturally speaking: The naturalness criterion and physics at the LHC, *Perspectives on LHC Physics* (World Scientific, Singapore, 2008), pp. 155–178.
[3] J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton University, Princeton, NJ, 1992), p. 259.
[4] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, The hierarchy problem and new dimensions at a millimeter, Phys. Lett. B **429**, 263 (1998).
[5] L. Randall and R. Sundrum, A Large Mass Hierarchy from a Small Extra Dimension, Phys. Rev. Lett. **83**, 3370 (1999).
[6] I. Antoniadis, S. Dimopoulos, and A. Giveon, Little string theory at a TeV, J. High Energy Phys. 05 (2001) 055.
[7] V. Agrawal, S. M. Barr, J. F. Donoghue, and D. Seckel, The anthropic principle and the mass scale of the standard model, Phys. Rev. D **57**, 5480 (1998).
[8] A. Arvanitaki, S. Dimopoulos, V. Gorbenko, J. Huang, and K. Tilburg, A small weak scale from a small cosmological constant, J. High Energy Phys. 05 (2017) 071.
[9] P. W. Graham, D. E. Kaplan, and S. Rajendran, Cosmological Relaxation of the Electroweak Scale, Phys. Rev. Lett. **115**, 221801 (2015).
[10] A. de Gouvea, D. Hernandez, and T. M. P. Tait, Criteria for natural hierarchies, Phys. Rev. D **89**, 115005 (2014).
[11] C. Csáki and P. Tanedo, Beyond the Standard Model, *2013 European School of High-Energy Physics, Paradfurdo, Hungary*, (CERN, Geneva, 2015), pp. 169–268.
[12] K. Choi and S. H. Im, Realizing the relaxion from multiple axions and its UV completion with high scale supersymmetry, J. High Energy Phys. 01 (2016) 149.
[13] D. E. Kaplan and R. Rattazzi, Large field excursions and approximate discrete symmetries from a clockwork axion, Phys. Rev. D **93**, 085007 (2016).
[14] K. Enqvist, S. Hannestad, and M. S. Sloth, Seesaw Mechanism for Scalar Fields as Possible Basis for Dark Energy, Phys. Rev. Lett. **99**, 031301 (2007).
[15] G. F. Giudice and M. McCullough, A clockwork theory, J. High Energy Phys. 02 (2017) 036.
[16] N. Craig, I. Garcia Garcia, and D. Sutherland, Disassembling the clockwork mechanism, J. High Energy Phys. 10 (2017) 018.
[17] G. F. Giudice and M. McCullough, Comment on Disassembling the Clockwork Mechanism, arXiv:1705.10162.
[18] G. F. Giudice, Y. Kats, M. McCullough, R. Torre, and A. Urbano, Clockwork/linear dilaton: Structure and phenomenology, J. High Energy Phys. 06 (2018) 009.
[19] D. Teresi, Clockwork without supersymmetry, Phys. Lett. B **783**, 1 (2018).
[20] A. Ahmed and B. M. Dillon, Clockwork Goldstone bosons, Phys. Rev. D **96**, 115031 (2017).
[21] T. Hambye, D. Teresi, and M. H. G. Tytgat, A clockwork WIMP, J. High Energy Phys. 07 (2017) 047.
[22] A. Kehagias and A. Riotto, Clockwork inflation, Phys. Lett. B **767**, 73 (2017).
[23] K. Hinterbichler and R. A. Rosen, Interacting spin-2 fields, J. High Energy Phys. 07 (2012) 047.
[24] N. Arkani-Hamed, H. Georgi, and M. D. Schwartz, Effective field theory for massive gravitons and gravity in theory space, Ann. Phys. **305**, 96 (2003).
[25] N. Khosravi, N. Rahmanpour, H. R. Sepangi, and S. Shahidi, Multimetric gravity via massive gravity, Phys. Rev. D **85**, 024049 (2012).
[26] K. Nomura and J. Soda, When is multimetric gravity ghost-free?, Phys. Rev. D **86**, 084052 (2012).
[27] J. H. C. Scargill, J. Noller, and P. G. Ferreira, Cycles of interactions in multigravity theories, J. High Energy Phys. 12 (2014) 160.
[28] C. de Rham and A. J. Tolley, Vielbein to the rescue? Breaking the symmetric vielbein condition in massive gravity and multigravity, Phys. Rev. D **92**, 024024 (2015).
[29] M. Lben and A. Schmidt-May, Ghost-free completion of an effective matter coupling in bimetric theory, Fortschr. Phys. **66**, 1800031 (2018).
[30] S. F. Hassan and A. Schmidt-May, Interactions of multiple spin-2 fields beyond pairwise couplings, arXiv: 1804.09723.
[31] M. Fierz and W. Pauli, On relativistic wave equations for particles of arbitrary spin in an electromagnetic field, Proc. R. Soc. A **173**, 211 (1939).
[32] Y. Yamashita, A. De Felice, and T. Tanaka, Appearance of Boulware-Deser ghost in bigravity with doubly coupled matter, Int. J. Mod. Phys. D **23**, 1443003 (2014).
[33] C. de Rham, L. Heisenberg, and R. H. Ribeiro, On couplings to matter in massive (bi)gravity, Classical Quantum Gravity **32**, 035022 (2015).
[34] Q. G. Huang, R. H. Ribeiro, Y. H. Xing, K. C. Zhang, and S. Y. Zhou, On the uniqueness of the nonminimal matter coupling in massive gravity and bigravity, Phys. Lett. B **748**, 356 (2015).
[35] J. Noller and S. Melville, The coupling to matter in massive, bi- and multigravity, J. Cosmol. Astropart. Phys. 01 (2015) 003.
[36] C. de Rham, L. Heisenberg, and R. H. Ribeiro, Ghosts and matter couplings in massive gravity, bigravity and multigravity, Phys. Rev. D **90**, 124042 (2014).
[37] V. O. Soloviev, Bigravity in tetrad Hamiltonian formalism and matter couplings, arXiv:1410.0048.
[38] E. Babichev, L. Marzola, M. Raidal, A. Schmidt-May, F. Urban, H. Veerme, and M. von Strauss, Heavy spin-2 dark matter, J. Cosmol. Astropart. Phys. 09 (2016) 016.
[39] L. Marzola, M. Raidal, and F. R. Urban, Oscillating spin-2 dark matter, Phys. Rev. D **97**, 024010 (2018).
[40] X. Chu and C. Garcia-Cely, Self-interacting spin-2 dark matter, Phys. Rev. D **96**, 103519 (2017).
[41] N. L. Gonzlez Albornoz, A. Schmidt-May, and M. von Strauss, Dark matter scenarios with multiple spin-2 fields, J. Cosmol. Astropart. Phys. 01 (2018) 014.
[42] C. de Rham, L. Heisenberg, and R. H. Ribeiro, Quantum corrections in massive gravity, Phys. Rev. D **88**, 084058 (2013).
[43] L. Heisenberg, Quantum corrections in massive bigravity and new effective composite metrics, Classical Quantum Gravity **32**, 105011 (2015).