Thermality from a Rindler quench

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Abstract

Ultracold fermionic atoms in an optical lattice, with a sudden position-dependent change (a quench) in the effective dispersion relation, have been proposed by Rodríguez-Laguna *et al.* as an analogue spacetime test of the Unruh effect. We provide new support for this analogue by analysing a massless scalar field on a $(1 + 1)$-dimensional continuum spacetime with a similar quench: an early time Minkowski region is joined at a constant time surface, representing the quench, to a late time static region in which left and right asymptotically Rindler domains are connected by a smooth negative curvature bridge. We show that the quench is energetically mild, and late time static observers, modelled as a derivative-coupling Unruh-DeWitt detector, see thermality, in a temperature that equals the Unruh temperature for observers in the asymptotic Rindler domains. The Unruh effect hence prevails, despite the energy injected into the field by the quench and despite the absence of a late time Killing horizon. These results strengthen the motivation to realise the experimental proposal.

1 Introduction

In relativistic quantum field theory, an observer's measurements of a quantum field depend on the observer's motion. A celebrated example is the Unruh effect [1, 2, 3], in which a linearly uniformly accelerated observer in Minkowski spacetime reacts to a field in its Minkowski vacuum by excitations and de-excitations characteristic of a thermal state, in the Unruh temperature $a\hbar/(2\pi c k_B)$, where a is the observer's proper acceleration (for textbooks and reviews, see [4, 5, 6]). An experimental confirmation of the Unruh effect has remained elusive, due to the required magnitude of acceleration (for a discussion of the magnitudes, and a proposal to enhance the effect through the Berry

phase, see [7]). A related effect exists for nonlinear uniform accelerations [8], including circular motion [9, 10, 11, 12], and the circular motion version is related to the spin depolarisation of particle beams in accelerator storage rings [13, 14, 15, 16], originally predicted by different methods [17, 18] and observed [19], but also here establishing a direct connection between the observation and the circular motion Unruh effect has remained elusive [16]. The prospects to observe versions of the Unruh effect with high-power laser systems are discussed in [20, 21, 22, 23]. An experimental confirmation of the Unruh effect would be significant since the mathematics underpinning the effect is closely related to the mathematics in Hawking's prediction of black hole radiation [24] and to the mathematics of the early universe quantum effects that may be responsible for the origin of structure in the present-day Universe [25, 26].

It is by now well recognised that classical and quantum field theory phenomena in relativistic spacetimes can be simulated in laboratory systems described by mathematically similar effective field theories, classical or quantum [27]. Recent experimental work includes the observation of a classical mode conversion that underlies the Hawking effect in the quantum theory [28], the observation of classical superradiance [29], the observation of quantum phenomena characteristic of an expanding cosmology [30], and observations interpreted as analogue Hawking radiation [31, 32]. Laboratory analogues of the Unruh effect have been proposed in a Bose-Einstein condensate [33] and in ultra-cold fermionic atoms in an optical lattice [34, 35], and related proposals are discussed in [36, 37]. A laboratory analogue of the Gibbons-Hawking effect, a curved spacetime counterpart of the Unruh effect, has been proposed in [38, 39].

The purpose of this paper is to provide new evidence that the optical lattice proposal of [34, 35] has the requisite properties to simulate the Unruh effect, despite having energetic and causal properties that differ from those in the usual setting of the Unruh effect.

The system analysed in [34, 35] consists of fermionic atoms held in an optical lattice, with a dispersion relation that can be adjusted to depend on both space and time. The detailed experimental implementation is described in [34, 35]. Mathematically, the system is a spatially discretised fermionic field in an effective $(2 + 1)$-dimensional spacetime whose spatial sections are flat but the time-time component of the metric, determining the effective dispersion relation, may depend on both space and time. To simulate the Unruh effect, the spacetime metric is engineered to undergo a sudden change, a quench, from the $(2 + 1)$-dimensional Minkowski metric to a metric given by the $(1 + 1)$-dimensional part

$$ds^2 = -\chi^2 d\eta^2 + d\chi^2 \tag{1.1}$$

plus one flat spatial dimension. (We set from now on $c = \hbar = k_B = 1$.) It is shown in [34, 35] by a combination of analytic and numerical methods that the field's behaviour at constant χ, sufficiently far from $\chi = 0$ in terms of the lattice scale, has thermal characteristics, in a position-dependent temperature that approximates $1/(2\pi|\chi|)$. This thermality is interpreted as an analogue of the Unruh effect, on the grounds that the regions $\chi > 0$ and $\chi < 0$ of (1.1) each cover one Rindler wedge of Minkowski spacetime, the worldlines of constant $\chi \neq 0$ have proper acceleration $1/|\chi|$, and the usual Unruh effect states that an observer in a Rindler wedge at constant χ experiences the Minkowski

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vacuum as thermal at the Unruh temperature $1/(2\pi|\chi|)$ [1, 2, 3, 4, 5, 6].

The optical lattice quench and the usual Unruh effect setup have however three qualitative differences, each of which could potentially limit the ability of the lattice to simulate the Unruh effect. First, in the usual setup the field is prepared in its Minkowski vacuum, implying in particular that the field's stress-energy tensor has a vanishing expectation value (see [40, 41] for a recent reinforcement of this point). By contrast, a sudden quench can be expected to inject energy into the field, potentially lots of it, and the spatial inhomogeneity of the quench suggests that the post-quench energy may not remain static: does the post-quench stress-energy tensor remain small, in some controllable sense? Second, the degeneracy of (1.1) at $\chi = 0$ gets regularised in [34, 35] in terms of the spatial lattice. How sensitive are the Unruh effect results to this regularisation, which does not feature in the usual setup? Third, the lattice-regularised metric does not have a counterpart of the Minkowski spacetime future and past quadrants that join the two Rindler wedges in the usual setup. Yet the arrangement of the four quadrants is essential when the thermality in the Unruh effect is described in terms of the entanglement between two opposing Rindler wedges [1, 2, 3, 4, 5, 6], and new phenomena emerge when this arrangement is modified [42]. Does the quench still create spacelike entanglement, similar to that in the usual setup?

In this paper we address the first two of these questions in a simplified, analytically solvable quench model that shares the potentially troublesome features of the optical lattice. We consider a massless scalar field in the $(1+1)$-dimensional continuum spacetime in which the singularity of (1.1) at $\chi = 0$ is regularised by modifying the $\eta\eta$ component to remain negative everywhere, including at $\chi = 0$. We find that the post-quench renormalised stress-energy tensor is well defined and nonvanishing, and the 'total energy', defined as the integral of the energy density over the constant η surface, is finite. When the regulator is small, in a sense that we describe, the stress-energy tensor is small everywhere except in a narrow region near $\chi = 0$. In this sense, the regularised quench is energetically mild. We then probe the thermality of the post-quench region by an Unruh-DeWitt detector on a worldline of constant χ. The detector's late time response is Planckian, in a χ-dependent temperature, and for nonzero χ this temperature approaches the Unruh temperature when the regulator is taken to zero; further, when the regulator is small, the response is Planckian in the usual Unruh temperature also at times shortly after the quench. In this sense, the Unruh-type thermality in the post-quench region is relatively insensitive to the regulator.

In short, we find that the Unruh effect prevails, despite the energy injected into the field by the quench and despite the absence of a late time Killing horizon. While our model is simplified, these results do strengthen the motivation to realise the experimental proposal of [34, 35].

Given our results, particularly our closed expression for the post-quench Wightman function, post-quench spacelike entanglement could be investigated by the harvesting techniques of [42, 43, 44, 45, 46]. We shall comment on the prospects and challenges of such harvesting in Section 7.

The plan of the paper is as follows: We begin by introducing in Section 2 a regularisation of the double Rindler metric (1.1). Section 3 presents the regularised quench, and quantises the scalar field individually in the pre-quench and post-quench regions. The Wightman function in the post-quench region, with the field prepared in the Minkowski

vacuum of the pre-quench region, is evaluated in Section 4. The stress-energy tensor is analysed in Section 5 and the response of an Unruh-DeWitt detector in Section 6. Section 7 presents a summary and a brief discussion.

Spacetime points are denoted by sans serif letters. Overline denotes complex conjugate and dagger Hermitian conjugate. $\text{sgn}(x)$ denotes the signum function, equal to 1 for $x > 0$, -1 for $x < 0$, and 0 for $x = 0$.

2 Regularised double Rindler

We consider the spacetime

$$ds^2 = -\bigl(b^2 + \chi^2\bigr)d\eta^2 + d\chi^2 \ , \tag{2.1}$$

where $-\infty < \eta < \infty$, $-\infty < \chi < \infty$, and b is a positive constant. A conformally flat form, obtained by the coordinate transformation $\chi = b \sinh y$, is

$$ds^2 = b^2 \cosh^2 y \left(-d\eta^2 + dy^2\right) \ , \tag{2.2}$$

where $-\infty < \eta < \infty$ and $-\infty < y < \infty$.

The spacetime is curved, with the Ricci scalar

$$R = -\frac{2b^2}{\bigl(b^2 + \chi^2\bigr)^2} = -\frac{2}{b^2 \cosh^4 y} \ . \tag{2.3}$$

It is static, with the timelike Killing vector ∂_η. The integral curves of ∂_η are timelike worldlines of constant χ, and their proper acceleration is

$$a_\chi = \frac{|\chi|}{b^2 + \chi^2} \ . \tag{2.4}$$

It can be verified that the spacetime is geodesically complete.

Comparison of (1.1) and (2.1) shows that the spacetime consists of asymptotically Rindler regions at $\chi \gg b$ and $\chi \ll -b$, joined by a negative curvature bridge whose effective length is of the order of b. In the limit $b \to 0$, (2.1) reduces to (1.1), which has exact Rindler wedges at $\chi > 0$ and $\chi < 0$ and a degeneracy at $\chi \to 0_\pm$, where the Rindler horizons would be. We may think of b as a regulator of the degeneracy of (1.1) at the Rindler horizon. Note that the regularised spacetime does not have a Killing horizon since ∂_η is everywhere timelike.

The behaviour of $g_{\eta\eta}$ near $\chi = 0$ in (2.1) is reminiscent of the near-throat region of the $(2+1)$-dimensional and $(3+1)$-dimensional static wormhole spacetimes discussed in [47, 48, 49], where a regulator analogous to our b was introduced to remove a black hole Killing horizon. Interpreting (2.1) as a wormhole spacetime would however be stretching the reminiscence because (2.1) has no transverse dimensions whose size would attain a minimum at $\chi = 0$.

3 Quench

3.1 Quench spacetime

Our quench spacetime consists of the $\eta > 0$ half of the regularised double Rindler spacetime (2.1) joined to the $t < 0$ half of Minkowski spacetime, given by

$$ds^2 = -dt^2 + dx^2 \ , \tag{3.1}$$

so that $x = \chi$ at $t = 0 = \eta$. The metric has a discontinuous time-time component at the quench but the other components are continuous.

We wish to quantise a massless minimally coupled scalar field ϕ on the quench spacetime. The field equation is $\Box \phi = 0$. We discuss the pre-quench region and the post-quench region first separately and then connect the two.

3.2 Quantum scalar field: pre-quench region

In the pre-quench region, $t < 0$, we employ a standard Fock quantisation adapted to the Killing vector ∂_t. A standard basis of mode functions that are positive frequency with respect to ∂_t is

$$u_{\omega,\epsilon} = \frac{1}{\sqrt{4\pi\omega}} \exp[-i\omega(\eta - \epsilon x)] \ , \tag{3.2}$$

where $\omega > 0$ and $\epsilon \in \{1, -1\}$. The mode functions with $\epsilon = 1$ are right-movers and the mode functions with $\epsilon = -1$ are left-movers. Adopting the conventions of [4], the Klein-Gordon (indefinite) inner product on a constant t hypersurface reads

$$(\phi, \psi) = -i \int_{-\infty}^{\infty} \left(\phi \partial_t \overline{\psi} - \overline{\psi} \partial_t \phi \right) dx \ , \tag{3.3}$$

and the mode functions are normalised in this inner product to

$$(u_{\omega,\epsilon}, u_{\omega',\epsilon'}) = -(\overline{u}_{\omega,\epsilon}, \overline{u}_{\omega',\epsilon'}) = \delta_{\epsilon\epsilon'}\delta(\omega - \omega') \ , \tag{3.4}$$

with the mixed inner products vanishing.

The quantised scalar field is expanded as

$$\phi = \sum_{\epsilon} \int_0^{\infty} \left(a_{\omega,\epsilon} u_{\omega,\epsilon} + a_{\omega,\epsilon}^{\dagger} \overline{u}_{\omega,\epsilon} \right) d\omega \ , \tag{3.5}$$

where $\left[a_{\omega,\epsilon}, a_{\omega',\epsilon'}^{\dagger} \right] = \delta_{\epsilon\epsilon'}\delta(\omega - \omega')$ and the other commutators vanish. The Fock space is built in the usual way on the Minkowski vacuum $|0_M\rangle$ that is normalised and satisfies $a_{\omega,\epsilon}|0_M\rangle = 0$.

3.3 Quantum scalar field: post-quench region

In the post-quench region, $\eta > 0$, we employ a standard Fock quantisation adapted to the Killing vector ∂_η. A standard basis of mode functions that are positive frequency

with respect to ∂_η is

$$
\begin{aligned}
U_{\Omega,\epsilon} &= \frac{1}{\sqrt{4\pi\Omega}} \exp[-i\Omega(\eta - \epsilon y)] \\
&= \frac{1}{\sqrt{4\pi\Omega}} \exp\big[-i\Omega\big(\eta - \epsilon\operatorname{arsinh}(\chi/b)\big)\big] \; ,
\end{aligned}
\tag{3.6}
$$

where $\Omega > 0$ and $\epsilon \in \{1, -1\}$. The mode functions with $\epsilon = 1$ are again right-movers and the mode functions with $\epsilon = -1$ are left-movers. The Klein-Gordon inner product on a constant η hypersurface reads

$$
(\phi, \psi) = -i \int_{-\infty}^{\infty} \big(\phi\partial_\eta\overline{\psi} - \overline{\psi}\partial_\eta\phi\big) \frac{d\chi}{\sqrt{b^2 + \chi^2}} \; ,
\tag{3.7}
$$

and the mode functions are normalised in this inner product to

$$
\big(U_{\Omega,\epsilon}, U_{\Omega',\epsilon'}\big) = -\big(\overline{U}_{\Omega,\epsilon}, \overline{U}_{\Omega',\epsilon'}\big) = \delta_{\epsilon\epsilon'}\delta(\Omega - \Omega') \; ,
\tag{3.8}
$$

with the mixed inner products vanishing.

The quantised scalar field is expanded as

$$
\phi = \sum_{\epsilon} \int_0^{\infty} \big(A_{\Omega,\epsilon} U_{\Omega,\epsilon} + A_{\Omega,\epsilon}^\dagger \overline{U}_{\Omega,\epsilon}\big) \, d\Omega \; ,
\tag{3.9}
$$

where $[A_{\Omega,\epsilon}, A_{\Omega',\epsilon'}^\dagger] = \delta_{\epsilon\epsilon'}\delta(\Omega - \Omega')$ and the other commutators vanish. A Fock space can be built in the usual way on the normalised state $|0_D\rangle$ that satisfies $A_{\Omega,\epsilon}|0_D\rangle = 0$, and we may regard $|0_D\rangle$ as a regularised double-sided Rindler vacuum. In what follows we shall however not be interested in $|0_D\rangle$ but instead in the post-quench state to which the pre-quench Minkowski vacuum evolves.

4 Post-quench Wightman function

We fix the state of the field to be the pre-quench Minkowski vacuum $|0_M\rangle$. To analyse the effects of the quench, we need to evaluate the Wightman function,

$$
W(\mathsf{x}, \mathsf{x}') = \langle 0_M | \phi(\mathsf{x})\phi(\mathsf{x}') | 0_M \rangle \; ,
\tag{4.1}
$$

when both x and x' are in the post-quench region.

We first match the pre-quench and post-quench mode functions at the quench, using the Bogoliubov transformation formalism in the conventions of [4]. Since the left-movers and right-movers decouple, we shall now drop the subscript ϵ, understand the transformation formulas to hold separately each value of ϵ, and add the left-mover and right-mover contributions to the Wightman function at the end.

With this notation, we write the Bogoliubov transformation of the modes at $t = 0 =$

η as

$$U_\Omega = \int_0^\infty \left(\alpha_{\Omega\omega} u_\omega + \beta_{\Omega\omega} \overline{u}_\omega \right) d\omega \ , \tag{4.2}$$

where $\alpha_{\Omega\omega}$ and $\beta_{\Omega\omega}$ are the Bogoliubov coefficients. The coefficients are given by [4]

$$\alpha_{\Omega\omega} = \left(U_\Omega, u_\omega \right) \ , \tag{4.3a}$$

$$\beta_{\Omega\omega} = -\left(U_\Omega, \overline{u}_\omega \right) \ , \tag{4.3b}$$

where the Klein-Gordon inner products are taken at the quench hypersurface $t = 0 = \eta$. Note that in these inner products the time derivative on u_ω is as in (3.3) and the time derivative on U_Ω is as in (3.7).

The inner products in (4.3) can be evaluated using 3.471.10 in [50], with the result

$$\alpha_{\Omega\omega} = \frac{1}{\pi} \sqrt{\frac{\Omega}{\omega}} \, e^{\pi\Omega/2} K_{i\Omega}(b\omega) \ , \tag{4.4a}$$

$$\beta_{\Omega\omega} = -\frac{1}{\pi} \sqrt{\frac{\Omega}{\omega}} \, e^{-\pi\Omega/2} K_{i\Omega}(b\omega) \ , \tag{4.4b}$$

where K is the modified Bessel function of the second kind. As a consistency check, it can be verified that the coefficients (4.4) satisfy the Bogoliubov identities [4], using the Bessel function identity

$$\int_0^\infty \frac{dx}{x} K_{i\Omega}(x) K_{i\Omega'}(x) = \frac{\pi^2}{2\Omega \sinh(\pi\Omega)} \, \delta(\Omega - \Omega') \ . \tag{4.5}$$

(4.5) can be justified informally by observing that $K_{i\Omega}(e^z)$ are the (improper) eigenfunctions of the essentially self-adjoint differential operator $-\partial_z^2 + e^{2z}$, which implies orthogonality, and considering the small argument behaviour of $K_{i\Omega}(x)$ [51], which determines the normalisation constant. A rigorous discussion of (4.5) is given in Section 4.15 of [52].

Next, we recall [4] that

$$A_\Omega = \int_0^\infty \left(\alpha_{\Omega\omega} a_\omega - \beta_{\Omega\omega} a_\omega^\dagger \right) d\omega \ , \tag{4.6}$$

where we have used the reality of the Bogoliubov coefficients. Proceeding for the moment informally, we substitute (3.9) in (4.1), use (4.6) and its Hermitian conjugate, interchange the integrals, and use the identity (4.5). Adding finally the left-mover and right-mover contributions, we arrive at

$$W(\mathsf{x}, \mathsf{x}') = W_0(\eta - y, \eta' - y') + W_0(\eta + y, \eta' + y') \ , \tag{4.7}$$

where

$$W_0(z, z') = \int_0^\infty \frac{d\Omega}{8\pi\Omega \sinh(\pi\Omega)} \left(e^{\pi\Omega/2} e^{-i\Omega z} + e^{-\pi\Omega/2} e^{i\Omega z} \right) \left(e^{\pi\Omega/2} e^{i\Omega z'} + e^{-\pi\Omega/2} e^{-i\Omega z'} \right) \ . \tag{4.8}$$

The expression (4.8) for $W_0(z, z')$ is ill defined because of the small Ω behaviour of the integrand. This was to be expected because of the well known infrared ambiguity of the Wightman function of a massless scalar field in two dimensions [53]. To extract a meaningful expression for $W_0(z, z')$, we differentiate both sides of (4.8) with respect to z and take the derivative on the right hand side to operate under the integral. The resulting integral has the distributional interpretation

$$\partial_z W_0(z, z') = -\frac{i}{4}\delta(z - z') - \frac{1}{8\pi}P\coth\left(\frac{z - z'}{2}\right) - \frac{1}{8\pi}\tanh\left(\frac{z + z'}{2}\right) , \qquad (4.9)$$

where P stands for the Cauchy principal value and we have used 3.981.1 and 3.981.8 in [50]. We now integrate (4.9) with respect to z, fixing the integration constant (which a priori could depend on z') by requiring $W_0(z, z') = \overline{W_0(z', z)}$, which a Wightman function must satisfy. We find

$$W_0(z, z') = -\frac{i}{8}\operatorname{sgn}(z - z') - \frac{1}{4\pi}\ln\left[\sinh\left(\frac{|z - z'|}{2}\right)\right] - \frac{1}{4\pi}\ln\left[\cosh\left(\frac{z + z'}{2}\right)\right] , \quad (4.10)$$

up to an additive purely numerical real-valued constant, which we have dropped from (4.10) as it will not affect what follows.

To summarise, we have arrived at the post-quench Wightman function $W(\mathsf{x}, \mathsf{x}')$ given by (4.7) with (4.10). The infrared divergence was removed by a procedure that can be interpreted as dropping an infinite additive constant. As a consistency check, we note that our $W(\mathsf{x}, \mathsf{x}')$ has the correct small separation asymptotic form [53].

We record here that the asymptotic late time form of $W(\mathsf{x}, \mathsf{x}')$ is

$$\begin{aligned}
W_{\text{late}}(\mathsf{x}, \mathsf{x}') = &-\frac{i}{8}\operatorname{sgn}(\eta - \eta' + y - y') - \frac{i}{8}\operatorname{sgn}(\eta - \eta' - y + y') \\
&-\frac{1}{4\pi}\ln\left[\sinh\left(\frac{|\eta - \eta' + y - y'|}{2}\right)\sinh\left(\frac{|\eta - \eta' - y + y'|}{2}\right)\right] - \frac{\eta + \eta'}{2\pi} .
\end{aligned}$$
$$(4.11)$$

We shall return to the implications of (4.11) in Section 7.

5 Post-quench stress-energy

We now evaluate the post-quench renormalised stress-energy tensor.

We use Hadamard renormalisation, adapting the Feynman Green's function formalism of [53] to the Wightman function. This gives

$$T_{ab}(\mathsf{x}) = \lim_{\mathsf{x}' \to \mathsf{x}}\left(g_b{}^{b'}\partial_a\partial_{b'} - \tfrac{1}{2}g_{ab}g^{cd'}\partial_c\partial_{d'}\right)\left(W(\mathsf{x}, \mathsf{x}') - W_{\text{sing}}(\mathsf{x}, \mathsf{x}')\right) + \frac{1}{48\pi}R(\mathsf{x})g_{ab} , \quad (5.1)$$

where the purely geometric subtraction term is

$$W_{\text{sing}}(\mathsf{x}, \mathsf{x}') = -\frac{1}{4\pi}\ln|\sigma(\mathsf{x}, \mathsf{x}')| - \frac{i}{8}\operatorname{sgn}(\eta - \eta' + y - y') - \frac{i}{8}\operatorname{sgn}(\eta - \eta' - y + y') , \quad (5.2)$$

and $\sigma(\mathsf{x}, \mathsf{x}')$ is half of the geodesic distance squared between x and x', with the convention

that $\sigma(\mathsf{x}, \mathsf{x}') > 0$ when the geodesic is spacelike and $\sigma(\mathsf{x}, \mathsf{x}') < 0$ when the geodesic is timelike. For a metric of the form $ds^2 = F(y)\big(-d\eta^2 + dy^2\big)$, a small separation expansion gives

$$
\sigma(\mathsf{x}, \mathsf{x}') = \tfrac{1}{2}\big((y - y')^2 - (\eta - \eta')^2\big) F(\tilde{y})
$$
$$
\times \left[1 + \frac{F''(\tilde{y})}{24 F(\tilde{y})}(y - y')^2 - \frac{1}{48}\left(\frac{F'(\tilde{y})}{F(\tilde{y})}\right)^2 \big((y - y')^2 - (\eta - \eta')^2\big) + (\text{cubic})\right] ,
$$
$$(5.3)$$

where $\tilde{y} := (y + y')/2$. Using (5.3) with $F(y) = b^2 \cosh^2 y$, the Wightman function given by (4.7) and (4.10), and the Ricci scalar (2.3), we find

$$
T_{\eta\eta} = \frac{1}{8\pi \cosh^2 y} - \frac{1}{16\pi}\left(\frac{1}{\cosh^2(\eta - y)} + \frac{1}{\cosh^2(\eta + y)}\right) , \qquad (5.4\text{a})
$$

$$
T_{yy} = \frac{1}{24\pi \cosh^2 y} - \frac{1}{16\pi}\left(\frac{1}{\cosh^2(\eta - y)} + \frac{1}{\cosh^2(\eta + y)}\right) , \qquad (5.4\text{b})
$$

$$
T_{\eta y} = \frac{1}{16\pi}\left(\frac{1}{\cosh^2(\eta - y)} - \frac{1}{\cosh^2(\eta + y)}\right) . \qquad (5.4\text{c})
$$

T_{ab} is hence well defined and finite everywhere in the post-quench region. As a consistency check, it can be verified that T_{ab} is conserved, $\nabla_a T^{ab} = 0$, and it has the correct trace anomaly, $T^a{}_a = R/(24\pi)$ [53].

From the expressions in (5.4) we may make the following three observations.

First, at the quench, $\eta \to 0_+$, we have $T_{yy} \to -1/(12\pi \cosh^2 y)$, while the other components vanish. The quench creates initially a negative pressure but no energy density.

Second, in the evolution after the quench, $T_{\eta\eta}$ and T_{yy} each consist of a positive static contribution, peaked around $y = 0$, and negative pulses travelling to the left and right at the speed of light, peaked around $y = \pm\eta$. In the late time limit at fixed y, the pulses have passed, and we have $T_{\eta\eta} \to 1/(8\pi \cosh^2 y)$, $T_{yy} \to 1/(24\pi \cosh^2 y)$ and $T_{\eta y} \to 0$. For fixed y, the late time energy density and pressure are hence static and positive.

Third, in view of the analogue system of [34, 35], an energetic quantity of interest is the 'total energy' at constant η, defined as the integral of the energy density $-T^\eta{}_\eta$ over the spatial volume,

$$
E_\eta := -\int_{-\infty}^{\infty} T^\eta{}_\eta \, b \cosh y \, dy
$$
$$
= \frac{\tanh^2(\eta/2)}{16\pi b} . \qquad (5.5)
$$

E_η is finite for all η, and it increases monotonically from 0 to $1/(16\pi b)$ as η increases from 0 to infinity. The initial negative pressure hence evolves at late times into a finite and static positive total energy.

If we view the parameter b as a regulator that is small compared with length scales

of interest, it is useful to express T_{ab} in the coordinates (η, χ) of (2.1), with the result

$$T_{\eta\eta} = \frac{b^2}{8\pi} \left(\frac{1}{\chi^2 + b^2} - \frac{\chi^2 \cosh(2\eta) + b^2 \cosh^2\eta}{\left(\chi^2 + b^2 \cosh^2\eta\right)^2} \right) , \tag{5.6a}$$

$$T_{\chi\chi} = \frac{b^2}{8\pi\left(\chi^2 + b^2\right)} \left(\frac{1}{3\left(\chi^2 + b^2\right)} - \frac{\chi^2 \cosh(2\eta) + b^2 \cosh^2\eta}{\left(\chi^2 + b^2 \cosh^2\eta\right)^2} \right) , \tag{5.6b}$$

$$T_{\eta\chi} = \frac{b^2}{8\pi} \frac{\chi \sinh(2\eta)}{\left(\chi^2 + b^2 \cosh^2\eta\right)^2} . \tag{5.6c}$$

In these coordinates, the pointwise limit of T_{ab} as $b \to 0$ vanishes for $\chi \neq 0$ but diverges for $\chi = 0$. For b small but finite, T_{ab} is large only within the narrow region $|\chi| \lesssim b \cosh \eta$, and in particular it is this narrow region that contributes to the total energy E_η (5.5) the piece that diverges as $b \to 0$.

In summary, the regularised quench produces a well-defined stress-energy tensor everywhere to the future of the quench. When the regulator is small, the stress-energy tensor is small everywhere except in a narrow wedge about $\chi = 0$.

6   Post-quench thermality

To examine thermality in the post-quench region, we probe the field with a pointlike Unruh-DeWitt detector [3, 54], specifically with a variant that is coupled linearly to the field's proper time derivative rather than the field itself, since this makes the detector less sensitive to the infrared ambiguity in the Wightman function (for selected references see [55, 56, 57, 58, 59, 60]). We follow the notation of [60], to which we refer for the details.

We take the detector to follow a worldline of constant χ, that is, an orbit of the Killing vector ∂_η. Let τ be the proper time on this worldline, with the additive constant chosen so that $\tau = 0$ at the quench. The detector's response is determined by the pull-back of the Wightman function on this worldline, given by

$$W_\chi(\tau, \tau') = -\frac{i}{4} \operatorname{sgn}(\tau - \tau') - \frac{1}{2\pi} \ln\left[\sinh\left(\frac{|\tau - \tau'|}{2\sqrt{\chi^2 + b^2}} \right) \right]$$
$$- \frac{1}{4\pi} \ln\left[\cosh\left(\frac{\tau + \tau'}{\sqrt{\chi^2 + b^2}} \right) + 1 + \frac{2\chi^2}{b^2} \right] , \tag{6.1}$$

where we have dropped an additive numerical constant. Comparing (6.1) to Section 3.3 in [60] shows that if the last term in (6.1) can be neglected, and the detector operates so long that switch-on and switch-off effects are negligible, the transition rate, evaluated to first order in perturbation theory and dropping an overall multiplicative constant, takes the Planckian form

$$\dot{\mathcal{F}}(E) = \frac{E}{e^{E/T_\chi} - 1} , \tag{6.2}$$

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where E is the detector's energy gap and

$$T_\chi = \frac{1}{2\pi\sqrt{b^2 + \chi^2}} \ . \tag{6.3}$$

When (6.2) holds, $\dot{\mathcal{F}}$ is hence thermal in temperature T_χ, in the sense of the detailed balance condition,

$$\dot{\mathcal{F}}(-E) = e^{E/T_\chi}\dot{\mathcal{F}}(E) \ . \tag{6.4}$$

We note that when $|\chi| \to \infty$ with fixed b, T_χ is asymptotically equal to $a_\chi/(2\pi)$, where a_χ is the trajectory's proper acceleration (2.4); conversely, for fixed $\chi \neq 0$, taking the regulator b to zero makes both T_χ and $a_\chi/(2\pi)$ tend to $1/(2\pi|\chi|)$, which is the Unruh temperature on the Rindler trajectory of constant $\chi \neq 0$ in the unregularised Rindler metric (1.1). When (6.2) holds, the regularised quench hence makes the detector respond identically to the Unruh effect, at scales that are large compared with the regulator b.

Now, when does (6.2) hold? That is, when does the last term in (6.1) make a negligible contribution to $\partial_\tau \partial_{\tau'} W_\chi(\tau, \tau')$? For any fixed χ and b, it is clear from (6.1) that one regime where this happens is the late time limit. However, if we view b as a regulator that is small compared with length scales of interest, the situation to consider is to fix $\chi \neq 0$ and take $b \ll |\chi|$. The late time limit in which (6.2) holds is then at proper times much larger than $|\chi|\ln(2|\chi|/b)$. But (6.1) shows that (6.2) then holds also at early post-quench proper times, much smaller than $|\chi|\ln(2|\chi|/b)$. This might have been expected from the stress-energy analysis of Section 5, since $|\chi|\ln(2|\chi|/b)$ is the proper time at which the detector crosses a travelling peak in $T_{\eta\eta}$.

We conclude that when $\chi \neq 0$ and $b \ll |\chi|$, the detector's transition rate is approximately Planckian at approximately the usual Unruh temperature $1/(2\pi|\chi|)$ at proper times much larger and much smaller than $|\chi|\ln(2|\chi|/b)$. The sense of the approximations can be made precise using (6.1) and the transition rate formalism of [60]. Inclusion of finite time switch-on and switch-off effects would be analytically more involved (cf. [61]), but straightforward to implement numerically.

7 Summary and discussion

We have provided new support for the proposal of [34, 35] to simulate the Unruh effect experimentally with ultracold fermionic atoms in an optical lattice. We first identified three qualitative differences between the optical lattice system and the usual Unruh effect setup, in their energetic and causal properties, and in the fact that the lattice provides a horizon regulator that has no counterpart in the usual Unruh effect. These differences could cast doubt on the ability of the lattice to simulate the Unruh effect. We then presented a simplified continuum field theory model that shares the potentially troublesome features of the optical lattice, and showed that in this model the energetic and causal properties can be brought under analytic control, and the Unruh effect prevails. While our simplifications included going from effective spacetime dimension $(2 + 1)$ to effective spacetime dimension $(1 + 1)$, and replacing a discrete fermion field by a continuum scalar field, our analytic results are compatible with the analytic and

numerical conclusions obtained in [34, 35].

In summary, our results strengthen the motivation to realise the experimental proposal of [34, 35].

A key technical property that made our analysis feasible was that the Wightman function could be written down in closed form, and we used this Wightman function to evaluate the stress-energy tensor and to establish the thermal response of a static Unruh-DeWitt detector. Given the Wightman function, it would be possible to study also the spatial entanglement in the field, harvesting the entanglement by a pair of Unruh-DeWitt detectors [42, 43, 44, 45, 46], and to compare with the entanglement that is present in Minkowski vacuum for Rindler observers in opposing Rindler wedges [1, 2, 3, 4, 5, 6]. Because of the late time growth in the Wightman function, shown in (4.11), a pair of Unruh-DeWitt detectors coupled linearly to the field would be problematic. A pair of Unruh-DeWitt detectors coupled linearly to the proper time derivative of the field, used in Section 6, would avoid this problem, but the short distance properties of the twice differentiated Wightman function then require the detectors to be smeared in time and space [46], increasing the parameter space of the harvesting protocol, and suggesting the need for a numerical approach. We leave this question to future work.

Acknowledgments

Note added in proof: A metric obtained from (2.1) by continuing b^2 to negative values has been considered in [62, 63] as a consequence of an upper bound on proper acceleration. I thank Maurizio Gasperini for bringing this work to my attention.

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