Developing mathematical patterning in ECE classrooms: participatory research with teachers of 3-5 year olds

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Abstract

Research shows that attention to pattern and structure is fundamental to mathematical learning and attainment yet early mathematics curricula in England underplay the importance of patterning. In a critical realist notion of powerful knowledge, pattern teaching has the potential to empower children to notice patterns, mathematise their everyday experiences and engage in mathematical sense-making. This study investigated how to harness this potential. It reports on participatory research with ten teachers of three to five year old children in England as they developed pattern teaching in their classrooms. Findings indicate that teacher knowledge, pedagogic interactions and pattern-rich environments (all underpinned by an appropriate developmental progression and extended to form a setting-wide shared approach) support the development of patterning praxis in early childhood classrooms. These offer potential priorities for ECE teachers in developing their patterning praxis in order to support children's mathematical learning.

Keywords: mathematics; patterning; pattern; teachers

Introduction

Mathematics is the science of patterns (Steen, 1990). Pattern is central to mathematical learning and 'fundamental to human knowing and sense-making' (Björklund & Pramling 2014, 89). It is essentially about regularity and predictability (Orton 2005; Wijns et al. 2019a). Through helping children to mathematise their everyday

experiences (Gravemeijer & Terwel 2000) and to generalise from these (McGarvey 2012), pattern brings predictability, meaning and connectedness which supports their mathematical well-being and understanding. It is more than just an area of mathematics, it is a way to think and be mathematical; seeking regularity, order, structure and sense. As Clements and Sarama (2021, 280) argue, pattern is 'a process, a domain of study, and a habit of mind'. In this way, the young child is mathematical as both a learner and a maker of patterns (Hardy 1967)

A pattern has both regularity and predictability (Wijns et al. 2019a). It is some overarching order or organisation which governs how elements continue or change; a rule which allows us to recursively predict what would stay the same and what could change if it were extended or developed. This rule enables us to perceive an overall structure (Wijns et al. 2019a, McGarvey 2012). Patterns exist within numerical, spatial and logical relationships (Mulligan & Mitchelmore 2009). They can occur within single objects or multiple objects as well as in intangible items or ideas (Papic et al. 2011). In England, where this research was conducted, successive curriculum documents have included expectations that young children learn to recognise and recreate patterns but it has not been a major focus (Gripton 2022).

Although the importance of pattern in mathematics has long been recognised, its position as a fundamental concept of early childhood mathematics (Perry & Dockett 2002) has been further strengthened with evidence from research of a relationship between patterning abilities and mathematical abilities overall (Elia et al. 2021, Wijns et al. 2019a). The challenge then is how to realise this in the Early Childhood Education (ECE) classroom. This study addresses this challenge reporting on the experiences of

ten teachers[1] of three to five year olds as they sought to develop patterning in their classrooms.

The increased recognition of the importance of patterning in ECE

Pattern has traditionally formed part of the early childhood mathematics education offered to young children. Teachers typically encourage children to notice patterns in their environment and create repeating patterns (such as blue, red, blue, red) using toys, beads or printing, for example. ECE teachers have become increasingly aware of the importance of this early patterning in recent years (Sarama et al. 2008). This is in part fuelled by as an increased focus on early algebraic thinking in mathematics education, which has a well-established connection to pattern (Kaput 2008, Mason 1996, Warren 2005), with some national curricula now incorporating algebra/pattern strands from ECE onwards[2].

From recent research studies we now know that children who spontaneously focus on patterns are better at mathematics (Wijns et al. 2020) and that patterning abilities predict later general mathematics achievement (Burgoyne et al. 2019, Fyfe et al. 2019, Lüken 2012, Nguyen et al. 2016, Rittle-Johnson et al. 2017, Schmerold et al. 2017, Warren & Miller 2013, Wijns et al. 2019b). There is a clear relationship between patterning and numerical ability (Wijns et al. 2021) with pattern found to be a significant longitudinal predictor of arithmetic (Burgoyne et al. 2019) and calculation

[1] For this paper, teachers is used as an inclusive term for all ECE pedagogues that support the learning and development of young children

[2] For example in South Africa (patterns), Australia (patterns and algebra), Scotland (patterns and relationships), the United States (structure) and Chinese Taipai (algebra)

(Fyfe et al. 2017; Mackay & DeSmedt 2019 for growing patterns). The case for pattern as an underutilised route to supporting early mathematical learning has become a compelling one.

The power of patterning in young children's mathematical learning

The central role of pattern to mathematical learning means that it can be conceptualised as powerful knowledge. From a critical realist standpoint, powerful knowledge is core disciplinary knowledge learned within the social and cultural structures of the child's context which is powerful for the child (Alderson, 2020). Pattern meets this criterion as both a central and unique contributor to children's mathematical understanding (Wijns et al. 2019b, Zippert et al. 2019) as well as a mathematical habit of mind (Cuoco, Goldenberg & Mark 1996), supporting them to 'mathematise' their individual socially and culturally structured realities (Gravemeijer & Terwel 2000). It reduces reliance on adults as children make connections and sense-make for themselves. By enhancing children's agency, an important dimension of critical realist powerful knowledge (Alderson 2020), patterning is powerful and potentially emancipatory for the child.

Given its power, it is important for all children to have access to pattern learning, particularly those at greatest risk of mathematical underachievement. Superficial attention to pattern might lead to children using inefficient and inflexible strategies as they grow older, making mathematics tasks and mathematics learning more difficult (Mulligan 2011). In studies, pattern instruction has been shown to be particularly effective for lower attaining children (Lüken & Kampmann 2018) and children from low-income households (Rittle-Johnson et al. 2017), indicating its potential contribution for addressing societal inequalities through ECE. To realise this potential, ECE teachers need to know how to develop patterning in their classrooms.

Teaching patterning in ECE

Whilst patterning varies considerably between young children (Mulligan & Mitchelmore 2009; Papic, Mulligan & Mitchelmore 2011), it can be taught (Lüken & Kampmann 2018; Mulligan & Mitchelmore 2009; Papic & Mulligan 2007; Papic, Mulligan & Mitchelmore 2011). Early childhood is the ideal time for this teaching where pattern learning can undergo substantial growth (Ginsburg 2017, Wijns 2019a). Patterning development can be particularly strong for three to five year olds (Lüken 2018, Rittle-Johnson et al. 2013), perhaps accounting for the slightly larger effect sizes for 4-5 year olds than 5-6 year olds of pattern learning on mathematical learning (Wijns et al. 2019b for general mathematics; Wijns et al. 2021 for early number). From three to five years children are able to increase the number of patterning strategies they use and develop more advanced patterning strategies (Lüken & Sauzet 2020). Children can move from pattern recognition to copying and then to fixing and extending patterns, alongside developing the complexity of the patterns they do this with (Sarama & Clements 2009). Given the potential for pattern learning when children are young, pattern practice in early childhood classrooms is crucial in providing all children with access to this powerful knowledge.

ECE teachers need to make explicit provisions for teaching patterning rather than expecting this to arise from general mathematics provision, as it is likely that this approach will be less effective (Strauss et al., 2020). Indications from intervention trials are that pattern teaching is more effective at supporting children's performance on general mathematical tasks than general mathematics intervention (Kidd et al. 2013, 2014; Pasnak et al. 2015). Whilst high fidelity discrete, time-limited intervention is useful to identify the impact of pattern-focussed teaching, application to everyday

practice does not demand an intervention approach but does urge teachers to prioritise pattern within their mathematics practice.

Teacher knowledge of how to teach patterning

Generally, teachers do not feel well-supported to teach patterning. There is relatively little high-quality pre- or in-service professional development in early mathematics for teachers and what there is rarely focusses on pattern (Verschaffel, Torbeyns & De Smedt 2017). Priorities for professional learning tend to follow practice which is typically dominated by numeracy-related activities at the expense of pattern (Björklund & Barendregt 2016; Ginsburg, Lee & Boyd 2008). Teachers have reported that they need more support to translate pattern curriculum content into practice (Wilkie 2012).

Unsurprisingly given the lack of attention to pattern in professional development, teachers understand pattern differently and not all teachers connect it to structure (Houssart 2000, Tirosh et al. 2019). Functional competence with common patterns, for example being able to draw an AB repeating pattern, is insufficient understanding to teach patterning effectively. ECE teachers need a more profound understanding including knowledge of a wide range of pattern types, strategies and activities (McGarvey 2012, Wijns et al. 2019a). Teachers need to know how patterns can be extended and be able to identify repeated units, including where a unit is incomplete (Tirosh et al. 2019). Similarly, they need to know that attention to different characteristics might lead individuals to form different criteria and see different rules. This is important pedagogically so that teachers seek to understand how the child sees the pattern.

The teacher role in fostering children's engagement with and attention to patterning is a crucial one (Fox 2005). Their pedagogic knowledge for patterning is essential to enable them to identify and develop the rich spontaneous patterning that

occurs whilst children are following their own interests through play in ECE (Waters 2004) and to encourage more complex patterning strategies such as within pattern transference to an alternative media or context (Collins & Laski 2015). Development of patterning practice is therefore a key concern and the focus of this paper.

The research

To investigate the development of patterning practice and harness the power of pattern for young children's mathematical learning, the research investigated three questions:

a. What practices or approaches can be used to teach pattern to young children in ECE classrooms?
b. What might assist ECE teachers in supporting young children's early pattern development?
c. How can the development of patterning practice in ECE classrooms be facilitated?

A co-learning approach (Wagner 1997) is appropriate to developing mathematics teaching given the particular complexities involved which Jaworski (2001) lists as: educators' theories, the specialist nature of mathematics, issues of determining 'effective' mathematics teaching and the sensitivities involved in recognising the need for improvement. These issues are particularly pronounced for an area such as pattern which has been significantly underrepresented in early childhood mathematics education in England over time (Gripton 2022). Whilst co-learning can lead to a 'smoothing' effect where ideas are shared and reinforced between group members, it has the democratic, participatory benefits of a praxeological approach with practitioners working together at the heart of the research process (Pascal & Bertram 2012). With this in mind, a co-learning methodology shaped the research design where teachers were co-researchers in the study. These were a self-selecting group who responded to an

open call across five locations in the East Midlands region of England to engage in pattern teaching development research with 3-5 year old children.

As co-researchers, the teachers were involved in participant selection and recruitment, data collection and data analysis (but not literature review, research design or final report writing). The teachers provided written and on-going verbal consent and had control over what data they collected. Whilst this reduced consistency in data collection, teacher control enhanced their power and status within the research (within the ethical framing of the study), utilised their knowledge of their classes and supported the authenticity of the data collected ensuring that the research was close-to-practice. Their eclectic and pragmatic approach was essentially a form of qualitative bricolage[3] where they collected what was 'to hand' to them as pedagogues, enabling them to capture the development process and evidence of multiple perspectives in their ECE classrooms. To mediate issues of inconsistency, each school was treated as a separate case within a collective case study approach. Thus the data was collated and analysed within cases to retain the integrity of each (Stake 1995) in an attempt to prevent differences in data quantity from disproportionality impacting results.

[3] Bricolage in the everyday sense of an improvised 'do-it-yourself' approach rather than a critical approach with a complex amalgam of methodologies requiring extensive knowledge of research methods (Denzin & Lincoln 2018).

The ten teachers were supported by methods training including on research ethics and all followed the project's ethics protocol (using the projects' information sheets and consent forms, for example). The sample of children (n=58) was drawn from the teachers' ten schools (school characteristics in Table 1.). Each teacher developed pattern teaching in their ECE classroom over a one year period. During this time, they used a structured action planning process to identify and evaluate priorities for their setting with the overall goal of developing pattern teaching. In line with the co-learning approach, the teachers met together four times within the year where there were opportunities to learn from each other.

Table 1. School characteristics

Schools							ECE classrooms		
School name (pseudonyms)	School type	Location (pseudonyms)	Children from low income circumstances*	Children for whom English is an additional language*	School size (no. of pupils)	Children with identified special educational needs or disabilities*	Teacher co-researcher (pseudonyms)	Age range in ECE classroom	ECE provision type (part-time, full-time)
Sacred Heart School	3-11	City A	10%	45%	200-400	15%	Claire	3-5 years	PT/FT
New Road School	3-11	City A	40%	25%	>400	15%	Asha	4-5 years	FT
Parkside School	3-11	City B	35%	55%	>400	20%	Paula	4-5 years	FT
River Mead School	3-11	County A	50%	25%	200-400	15%	Georgie	3-4 years	PT/FT
Field Mill School	3-7	County B	30%	5%	>400	10%	Cara	3-5 years	PT/FT
Mackley School	3-11	County A	35%	10%	>400	20%	Vicky	3-4 years	PT/FT
Netherglade School	3-11	County A	20%	5%	200-400	10%	Sara	3-4 years	PT/FT
Central School	2-4	City A	75%	60%	<200	20%	Ana	2-4 years	PT/FT
Redgate School	3-11	County A	25%	25%	>400	20%	Ruth	4-5 years	FT
Northside School	4-7	County C	10%	0%	<200	5%	Helen	4-5 years	FT

*rounded to nearest 5% to avoid identification of schools from publicly available data

Each teacher identified a sample of focus children to collect evidence for (child and parental consent obtained) and each completed written reflections on their professional learning throughout the process. The teachers, as experts in their own practice, collected what they deemed to be relevant data for the children and these constituted photographs, assessment records and written observations. Whilst this led to

further, significant variation in the data collected, it did provide authentic accounts of the teachers' practice in a way that each individual was comfortable with and found manageable. The teachers also collated relevant ethnographic documentary evidence of their practice throughout the year: their written reflections, photographs of the physical environment, teaching materials, planning records and resources used with colleagues or families.

At the end of the year, the teachers each engaged in thematic analysis of the data from their school. This data-driven approach required the teacher co-researchers to generate their own themes arising from their interpretation of the data. Whilst this is clearly subjective, there was a real benefit to the first analysis being conducted by someone with such deep knowledge of the data and context. With strategies being more important than solutions in children's patterning (Lüken & Sauzet 2020), teacher knowledge of children's processes was invaluable to the research. The teachers worked separately to review their data and look for patterns. They moved between the data and themes to clarify categories, writing as an integral part of the analysis to each create an individual case study report to accurately reflect the entire data set for their school (Braun & Clarke 2006). Facilitated by the researcher, the teachers then worked collaboratively to analyse all of the evidence from the ten schools by identifying common themes (again clarifying codes and categorisation), collapsing similar or duplicate themes as needed to arrive at five main themes present in all ten ECE classrooms. To do this, they collated data across the schools and worked in teams to write a collective report on each of the five main themes.

The data analysis process is shown in Table 2. There were two stages of separate thematic analysis for each school (steps 1 and 3) and two of collective thematic analysis (steps 2 and 4) through which the findings emerged. Researcher triangulation

was achieved through a separate analysis of the data by the author (steps 3 and 4) which confirmed the same five main findings and in addition identified some further commonalities between the data, although these were limited to a subset of schools and therefore deemed secondary findings.

Table 2. The data analysis process

1. Individual thematic analysis of data for each setting (individual case study reports)	Individual settings	Teacher co-researchers
2. Collective thematic analysis of all cases (collective report on findings for all settings)	All settings	Teacher co-researchers
3. Individual thematic analysis of data for each setting using categories from 2 (researcher triangulation)	Individual settings	Researcher
4. Collective thematic analysis of all cases (researcher triangulation)	All settings	Researcher

Discussion of findings

The ten teachers in this study took different approaches to developing patterning in their ECE classrooms. Differences were due to existing approaches to mathematics teaching, class types/sizes and setting ethoi. Teachers had ownership of and agency within the bespoke approaches to embed patterning within their setting. Findings were consistent across the ten schools, supported by the co-learning methodology, but the relative strength and importance of the findings varied between schools. The teachers identified five components as important for the development of patterning in their ECE classrooms. These are:

(1) teacher knowledge of patterning

(2) pattern-focussed pedagogic interactions (in direct teaching and children's play)

(3) a pattern rich learning environment

(4) an appropriate developmental progression for pattern

(5) a shared approach to pattern teaching

There was consistent evidence across the schools that children enjoyed patterning, applied pattern learning across and beyond the mathematics curriculum, were attentive to pattern and structure, encountered pattern more frequently within their everyday experiences, had increased freedom and agency as mathematical thinkers and were perceived differently as mathematicians by teachers and themselves. This evidence is discussed for each of the findings.

Finding 1. Teacher knowledge of patterning

The ten teachers agreed that teacher knowledge was crucial for patterning praxis. There was, however, substantial variety in the relative importance they attributed to this compared to the other four findings, perhaps dependent on individual prior knowledge. As one might expect with a co-learning approach, definitions of pattern were stable across the teacher case studies which contrasts with the findings of both Tirosh et al. (2019) and Waters (2004) where greater variation was apparent. All of the ten teachers identified defining pattern as important in developing pattern teaching. This helped to avoid issues such as those identified by McGarvey (2012) around teacher differences in understanding of what constitutes a pattern where some of the teachers in that study did not include single-item patterns, for example.

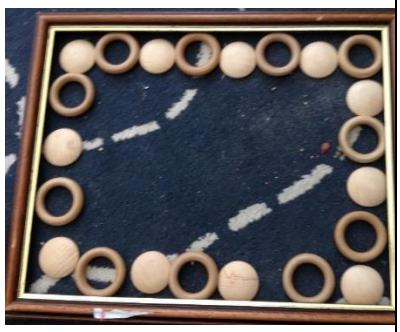
The teachers identified knowledge of pattern types as important for teachers and identified three broad types:

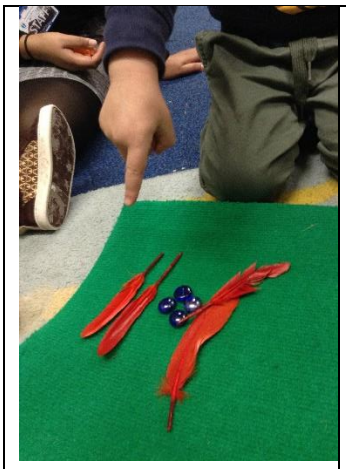
- repeating (linear, cyclical and hopscotch)
- spatial (including reflective and rotational symmetry as well as subitising spatial arrangements)

- growing (including staircase and other number patterns)

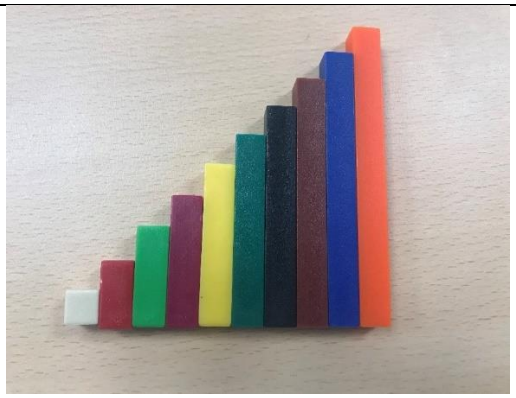
In their collective report, these are the categories and order used by the teacher co-researchers, echoing Papic & Mulligan (2007)'s spatial structure, repeating and growing pattern types which are relatively consistent classifications for patterns, even if terminology sometimes differs (Wijns et al. 2019a). Figure 1. contains examples of children's patterns, identified by the teachers. They show a mixture of patterns created during free play or independently within teacher guided activity.

Figure 1. Pattern types

<table>
<tr><td colspan="3">Repeating patterns</td></tr>
<tr><td>linear
  </td><td>'hopscotch' (number of items creates a repeating pattern)
  </td><td>cyclical
  </td></tr>
<tr><td colspan="3">Spatial patterns</td></tr>
<tr><td>arrangement</td><td>radiating</td><td>symmetry</td></tr>
</table>



Growing patterns

As ‘pattern aware teachers’ (Waters 2004) or pattern sensitive teachers, patterning development in ECE classrooms was supported by teacher knowledge of children’s typical development in patterning (related to finding 4) including key indicators of relational patterning such as the unit of repeat (Threlfall 2005) and pattern transference where the same pattern structure is created using different items or media (Collins & Laski 2015). Teacher knowledge of progression and patterning pedagogies informed their interactions (planned for and ‘in the moment’) and provision. These interactions are discussed in finding 2.

Finding 2. Adult modelling and pattern-focussed pedagogic interactions

The teachers reported that the adult role is crucial to pattern development. This was in both direct teaching and teacher-child interactions during children's play. Individual teachers adopted and adapted existing structures for pattern-focussed direct teaching sessions. Central and Sacred Heart schools used their own four part structures for discrete pattern sessions, for example, and Redgate school embedded a patterning activity into the beginning of every mathematics-focussed session. In five of the classrooms, images were routinely used as a stimuli to begin pattern sessions, encouraging pattern recognition as a first step, which aligns with Clements & Sarama's developmental progression (2021).

During play, teachers modelled patterning strategies. These sometimes demonstrated immediate benefits for the children, such as when Georgie (teacher at River Mead) playfully made an error in her ABC linear repeating pattern when playing alongside four-year-old Sidra, who spotted the issue and corrected it for her. Longer term benefits were also evident where three-year-old Lai, for example, noticed the AB structure in her teacher's repeating pattern. Lai drew upon her teacher's modelling of pattern descriptions over time to use only the items in the 'unit of repeat' (the smallest unit in the pattern, Orton 2005) to describe it as 'red and green' and to use the word 'repeat' when she stated, "the pattern goes red and green around in a circle. The red and green repeats over and over."

Finding 3. Pattern rich learning environment

Provision of resources and spaces for patterning enabled significant change in the quantity, quality and variety of patterning that children chose to engage in. The data shows that resources provided were hugely varied with a deep range of characteristics

(including orientation, colour, shape, size, type). The richness of the resources prompted decisions around which characteristics were being included/ignored for the pattern, something that pattern-making is dependent on (Björklund & Pramling 2014). As Paula, the teacher at Parkside School, explained the children had to determine which attributes were 'part of their rule'. Her emphasis upon the child's ownership of the rule suggests her perception that children were experiencing increased child agency in patterning.

All schools provided specific patterning areas in their provision (see Figure 2) and some chose to introduce new items over time to allow familiarity to build (e.g. Netherglade) or encourage different types of patterning (e.g. circular base boards to encourage cyclic repeating patterns, Parkside). It was important for children to experience freedom in their access to and use of resources for patterning. This feeling of entitlement is evident in the children asking for an outdoor pattern area at Northside School where only an indoor one had been provided (the trays of resources in Figure 2). Pattern areas included spaces for patterning (such as mats and trays) as well as stimuli (such as photographs, patterned borders and children's drawings). One of the 'pattern stations' at Central Nursery School included a light box, introduced following the children's 'fascination with pattern sniffing and pattern making' using light, according to their teacher Ana.

Figure 2. Examples of areas of provision for patterning

Class routines were also used by the teachers and children for patterning with children spotting patterns in their personal care routines (such as hand washing and changing clothes) as well as setting routines (such as the sequence of the day, registration times and queuing or turn-taking). Here, attention to pattern soothed children, bringing coherence to their day (Björklund & Pramling 2014).

The emotional, behavioural and attitudinal aspects of pattern learning were emphasised by the teachers as crucial contributors to creating a pattern-rich learning environment and potentially the longest term benefits. Enhanced mathematical behaviours, confidence and agency are evident with examples from each school. The starkest changes were in the mathematical self-concepts of the children judged as lower attaining or 'not ready' for more challenging mathematics. Examples of these are provided in the next section.

Finding 4. An appropriate developmental progression for pattern

In line with Frye et al.'s systematic review of research evidence in early mathematics (2013), using a developmental progression to underpin teaching of pattern was important for developing patterning in all the ECE classrooms studied. These developmental progressions provided a sequence of patterning skills and concepts that children typically develop. Knowledge of these enabled specific assessment of individual children as well as guiding planning and pedagogic interactions. More importantly, they provided a tool to support teachers to perceive children differently. Paula at Parkside, for example, found that the children in her classroom were at a much earlier stage in their understanding of linear repeating patterns than she had previously realised. Helen, at Northside Infants, had the opposite experience with five year old Deanne where Helen's thinking shifted from perceiving Deanne as a child struggling with mathematics to one of a competent mathematician.

> "As well as being a huge confidence boost to her, this also gave me a different way of looking at how I teach her … she has advanced into the 'Expected level' group. Never underestimate your children!" (extract from teacher reflection)

Understanding developmental stages in patterning supports teachers to recognise earlier stages of development and adapt their practice accordingly. Not all teachers in the study found existing developmental progressions (such as Clements and Sarama 2021) appropriate for the children in their class, although these were helpful to all. The progressions used were a mixture of sourced (3), adapted (3) and created (4) documents. Teachers who adapted or created pattern progressions explained that they did so because:

- they needed these to include greater detail for the earlier stages of pattern development
- existing progressions focussed too much on specific pattern types and not on those that their children were engaging with
- existing progressions were too prescriptive or too detailed to work with
- the progression sequence did not match the developmental progress of most of the children in their setting

Two schools created quite original progressions. Central Nursery School created a detailed structure which each child was assessed against, whilst Mackley created a six step child-facing schematic (including symbols) which was displayed around the classroom environment for parents, practitioners and children to draw upon. The use and refinement of pattern progressions developed across the year. The teachers collective reporting of theme 4 revealed that all children engaged in pattern spotting and that this was important in both the earlier and later stages of pattern development. The teachers suggest that this should been a constant alongside other more stage-related aspects of pattern development. They also felt that each stage needed to include a 'maker' and 'expert' level statement to emphasise both earlier and later stages of development due to experience with specific pattern types or aspects of pattern learning.

It was important to allow time this time for experience to build and deeper understanding to develop, preventing practice from moving on too quickly. In their written reflections, Ana and Claire (Central and Sacred Heart) emphasised that pattern learning needs to be 'slow' and 'deep', requiring a developmental progression which represents this to support practice.

Finding 5. A shared approach to pattern teaching

Despite taking quite different paths to achieving it, establishing a shared approach was key to developing patterning in all of the schools. This shared approach was developed democratically with practitioners working together and learning from each other. Findings 1 to 4 were therefore important for all practitioners in each ECE classroom and not just the teacher co-researchers. All schools included specific professional development events for their staff teams. At Mackley School, for example, the team spent time making and exploring patterns for themselves, attempting to perceive patterning through the eyes of the children in their setting.

Three schools (Central, Mackley and Redgate) developed their shared approach further and overtly included children's families in this. Ruth, the teacher at Redgate explained, 'home learning is key and parents need to be aware of patterning and how it develops' (extract from case study report). In these three settings, families were included in specific sessions to learn about pattern in early mathematics and provided with information or resources to support patterning in the home. The children's faces and body language in the photographs of these sessions demonstrate the joy of sharing patterning with a loved one.

Secondary findings. A holistic approach, support for children's well-being and precise patterning vocabulary

A holistic approach to pattern teaching was evident in four of the ten schools in this study. Whilst a range of pedagogies emerged across the ECE classrooms (for example 'pattern of the week' and pattern stations within the environment), the practice in these settings was more blended, with mathematical patterning consistently embedded in music, art, dance, reading, physical and social development activities. In these schools, this seemed more than opportunistic cross-curricular links and was deliberate pedagogy to teach mathematical patterning in a holistic way.

Pattern learning within routines formed part of a pattern-rich learning environment in all ECE classrooms (finding 3) but Sacred Heart and New Road showed deliberate use of pattern-learning opportunities to support transition of new children into the setting, supporting them to understand and feel secure in an environment which was new to them.

Within their patterning praxis, two teachers (one with 3-4 year olds and one with 4-5 year olds) used specific patterning vocabulary for pattern types and within repeating pattern teaching including 'unit of repeat' and letters to generalise repeating pattern structures (e.g. an AB pattern or an AABC pattern). Whilst other teachers used this type of language in their documentation and with colleagues, these were the only two to encourage children to use it. They reported that it supported application of learning to new situations and confidence in communicating their pattern thinking to others.

Conclusion

The teacher researchers in this study were able to develop patterning in their ECE classrooms, through a setting-wide shared approach and supported by teacher knowledge, pedagogic interactions, a pattern-rich environment and an underpinning

developmental progression for pattern. This required them to take a broad view of early childhood mathematics, being open to adjusting and developing their understanding of children's learning and the practice that supports this, something that Mulligan & Mitchelmore (2018) argue is necessary in order to embrace the emerging research evidence. The teachers in this study offer the five components that they found were important for the development of patterning in their ECE classrooms to others as potential starting points for how to harness the power of pattern in their settings. These might also be useful to those leading professional development of teachers to help shape or select professional learning opportunities for early childhood teachers in mathematics.

Moving forwards, nine of the teachers plan to share their patterning praxis with teachers of 5-7 year olds in their schools. This might develop teaching of the 'unit of repeat' (Threlfall 2005) which happens around 6 years (Clements and Sarama 2021), but sometimes younger non-verbally (Papic, Mulligan & Mitchelmore 2011). For older children, growing patterns may feature more prominently (fewer examples of this pattern type in this study), particularly in relation to the number system and multiplication patterns (Mulligan & Mitchelmore 2018). Further research is needed in the development of patterning praxis with older children, including in the development of number concepts, as well as how to sustain patterning development in settings for 3 to 5 year olds over time. In general, more research is needed into developing teacher knowledge of patterning in early childhood education including for pre-service teachers in order to harness the power of pattern for more young children.

Finally, there were examples in this study of pattern learning being joyful, authentic and even emancipatory for the child. This suggests that pattern has the potential to support early mathematical learning in a way that embraces child agency

and well-being (particularly for children at greatest risk of later underachievement in mathematics). The teacher from Central Nursery School explained, 'Pattern is harmony. It makes you feel reassured and secure'. Pattern has the potential to unify mathematics through drawing disparate areas of mathematics content together and to connect the child with the mathematics, as knowledge that is powerful for the child (Alderson's powerful knowledge, 2020). This study indicates that patterning praxis can be authentically developed by ECE teachers to support children's mathematical learning.

References

Alderson, P. (2020). Powerful knowledge and the curriculum: Contradictions and dichotomies. *British Educational Research Journal*, 46(1), 26-43.

Björklund, C. & Barendregt, W. (2016). Teachers' pedagogical mathematical awareness in Swedish early childhood education. *Scandinavian Journal of Educational Research*, 60(3), 359-377.

Björklund, C. & Pramling, N. (2014). Pattern discernment and pseudo-conceptual development in early childhood mathematics education. *International Journal of Early Years Education,* 22(1), 89-104.

Braun V. & Clarke V. (2006). Using thematic analysis in psychology. *Qualitative Research Psychology*, 3(2), 77–101.

Burgoyne, K., Malone, S., Lervag, A. & Hulme, C. (2019). Pattern understanding is a predictor of early reading and arithmetic skills. *Early Childhood Research Quarterly*, 49, 69-80.

Clements, D.H. & Sarama, J. (2021). *Learning and teaching early math: The learning trajectories approach*. Oxon: Routledge.

Collins, M.A. & Laski, E.V. (2015). Preschoolers' strategies for solving visual pattern tasks. *Early Childhood Research Quarterly*, 32, 204-214.

Cuoco, A., Goldenberg, E.P & Mark, J. (1996). Habits of mind: An organizing principle for mathematics curricula. *The Journal of Mathematical Behavior*, 15(4), 375-402.

Denzin, N.K. & Lincoln, Y.S. (2018). *The SAGE handbook of qualitative research.* 5th ed. London: Sage.

Fox, J. (2005). Child-initiated mathematical patterning in the pre-compulsory years. *International Group for the Psychology of Mathematics Education*, 2, 313-320.

Frye, D., Baroody, A. J., Burchinal, M., Carver, S. M., Jordan, N. C. & McDowell, J. (2013). *Teaching math to young children: A practice guide NCEE 2014-4005*. Washington DC: National Center for Education Evaluation and Regional Assistance (NCEE).

Fyfe, E.R., Evans, J.L., Matz, L.E., Hunt, K.M. & Alibali, M.W. (2017). Relations between patterning skill and differing aspects of early mathematics knowledge. *Cognitive Development*, 44, 1-11.

Fyfe, E. R., Rittle-Johnson, B., & Farran, D. C. (2019). Predicting success on high-stakes math tests from preschool math measures among children from low-income homes. *Journal of Educational Psychology*, 111(3), 402–413.

Ginsburg, H.P. (2017). *What young children know and need to learn about pattern and algebraic thinking.* DREME TE. Retrieved from http://prek-math-te.stanford.edu/system/files/media/document/2021/What-Children-Know-and-Need-to-Know-About-Pattern-and-Algebra-Expanded-Version.pdf

Ginsburg, H.P., Lee, J.S. & Boyd, J.S. (2008). Mathematics education for young children: What it is and how to promote it. *Social Policy Report*, 22(1), 1-24.

Gravemeijer, K. & Terwel, J. (2000). Hans Freudenthal: A mathematician on didactics and curriculum theory. *Journal of Curriculum Studies*, 32(6), 777-796.

Gripton, C. (2022). Pattern in early years mathematics curriculum: A 25 year review of the status, positioning and conception of pattern in England. *Research in Mathematics Education*, 1-21.

Hardy, G.H. (1967). *A mathematician's apology*. Cambridge: Cambridge University Press.

Elia, I., Baccaglini.-Frank, A., Levenson, E., Matsuo, N. & Feza, N. (2021, July 11-18). *Early childhood mathematics education* [ICME Research Survey]. The 14th International Congress on Mathematical Education (ICME 14), Shanghai, China.

Houssart, J. (2000). Perceptions of mathematical pattern amongst primary teachers. *Educational Studies*, 26(4), 489-502.

Jaworski, B. (2001). Developing mathematics teaching: Teachers, teacher educators, and researchers as co-learners. In F-L. Lin & T.J. Cooney (Eds.), *Making sense of mathematics teacher education,* 295-320. Dordrecht: Springer Netherlands.

Kaput, J.J. (2008). What is algebra? What is algebraic reasoning? In J.J. Kaput, D.W. Carraher & M.L. Blanton (Eds.), *Algebra in the early grades*, 5-18. Oxon: Routledge.

Kidd, J.K., Carlson, A.G., Gadzichowski, K.M., Boyer, C.E., Gallington, D.A. & Pasnak, R. (2013). Effects of patterning instruction on the academic achievement of 1st-grade children. *Journal of Research in Childhood Education*, 27(2), 224-238.

Kidd, J.K., Pasnak, R., Gadzichowski, K.M., Gallington, D.A., McKnight, P., Boyer, C. E. & Carlson, A. (2014). Instructing first-grade children on patterning improves reading and mathematics. *Early Education & Development*, 25(1), 134-151.

Lüken, M.M. (2012). Young Children's Structure Sense. *Journal für Mathematik-Didaktik*, 33(2), 263-285.

Lüken, M.M. (2018). Repeating pattern competencies in three- to five-year old kindergartners: A closer look at strategies. In I. Elia, J. Mulligan, A. Anderson, A. Baccaglini-Frank, & C. Benz (Eds.), *Contemporary Research and Perspectives on Early Childhood Mathematics Education*, ICME-13 Monographs, 35-53. Cham: Springer.

Lüken, M.M & Kampmann, R. (2018). The Influence of Fostering Children's Patterning Abilities on Their Arithmetic Skills in Grade 1. In M. J. Elia I., Anderson A., Baccaglini-Frank A., Benz C. (Ed.), *Contemporary Research and Perspectives on Early Childhood Mathematics Education.* ICME-13 Monographs, 55-66. Cham: Springer.

Lüken, M.M. & Sauzet, O. (2020). Patterning strategies in early childhood: a mixed methods study examining 3- to 5-year-old children's patterning competencies. *Mathematical Thinking and Learning*, 23(1), 28-48.

MacKay, K.J. & De Smedt, B. (2019). Patterning counts: Individual differences in children's calculation are uniquely predicted by sequence patterning. *Journal of Experimental Child Psychology*, 177, 152-165.

Mason, J. (1996). Expressing generality and roots of algebra. In N. Bednarz, C. Kieran, & L. Lee. *Approaches to algebra*, 65-86. Dordrecht: Springer.

McGarvey, L.M. (2012). What is a pattern? Criteria used by teachers and young children. *Mathematical Thinking and Learning*, 14(4), 310-337.

Mulligan, J. (2011). Towards understanding the origins of children's difficulties in mathematics learning. *Australian Journal of Learning Difficulties*, 16(1), 19-39.

Mulligan, J. & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal*, 21(2), 33-49.

Mulligan, J. & Mitchelmore, M. (2018). Promoting early mathematical structural development through an integrated assessment and pedagogical program. In I. Elia, J. Mulligan, A. Anderson, A. Baccaglini-Frank & C. Benz (Eds.) *Contemporary research and perspectives on early childhood mathematics education* 17-33. Cham: Springer.

Nguyen, T., Watts, T. W., Duncan, G. J., Clements, D. H., Sarama, J. S., Wolfe, C. & Spitler, M. E. (2016). Which preschool mathematics competencies are most predictive of fifth grade achievement? Early Childhood Research Quarterly, 36, 550-560.

Orton, A. (2005). *Pattern in the teaching and learning of mathematics*. London: Continuum.

Papic, M. & Mulligan, J. (2007). The growth of early mathematical patterning: An intervention study. Proceedings of the 30th annual conference of the mathematics education research group of Australia (MERGA)

Papic, M.M., Mulligan, J.T. & Mitchelmore, M.C. (2011). Assessing the development of preschoolers' mathematical patterning. *Journal for Research in Mathematics Education*, 42(3), 237-269.

Pascal, C. & Bertram, T. (2012). Praxis, ethics and power: Developing praxeology as a participatory paradigm for early childhood research. *European Early Childhood Education Research Journal*, 20(4), 477-492.

Pasnak, R., Kidd, J.K., Gadzichowski, K.M., Gallington, D.A., Schmerold, K.L. & West, H. (2015). Abstracting sequences: Reasoning that is a key to academic achievement. *The Journal of Genetic Psychology*, 176(3), 171-193.

Perry, B. & Dockett, S. (2002). Young children's access to powerful mathematical ideas. *Handbook of International Research in Mathematics Education*, 2, 81-112.

Rittle-Johnson, B., Fyfe, E.R., Hofer, K.G. & Farran, D.C. (2017). Early math trajectories: Low-income children's mathematics knowledge from ages 4 to 11. *Child Development*, 88(5), 1727-1742.

Rittle-Johnson, B., Fyfe, E.R., McLean, L.E. & McEldoon, K.L. (2013). Emerging understanding of patterning in 4-year-olds. *Journal of Cognition and Development*, 14(3), 376-396.

Sarama, J. & Clements, D.H. (2009). *Early childhood mathematics education research: Learning trajectories for young children.* Oxon: Routledge.

Sarama, J., Clements, D.H., Starkey, P., Klein, A. & Wakeley, A. (2008). Scaling up the implementation of a pre-kindergarten mathematics curriculum: Teaching for understanding with trajectories and technologies. *Journal of research on educational effectiveness*, 1(2), 89-119.

Schmerold, K., Bock, A., Peterson, M., Leaf, B., Vennergrund, K. & Pasnak, R. (2017). The relations between patterning, executive function, and mathematics. *The Journal of Psychology*, 151(2), 207-228.

Stake, R.E. (1995). *The art of case study research*. London: Sage.

Steen, L.A. (1990). Pattern. In L.A. Steen (Ed.), *On the shoulders of giants: New approaches to numeracy*, 1-10. Washington DC: National Academy Press.

Strauss, L.I., Peterson, M.S., Kidd, J.K., Choe, J., Lauritzen, H.C., Patterson, A.B., Holmberg, C.A., Gallington, D.A. & Pasnak, R. (2020). Evaluation of patterning instruction for kindergartners. *The Journal of Educational Research,* 113(4), 292-302.

Threlfall, J. (2005). Repeating patterns in the early primary years. In A. Orton (Ed.), *Pattern in the teaching and learning of mathematics*, 18-30. London: Continuum.

Tirosh, D., Tsamir, P., Levenson, E.S., Barkai, R. & Tabach, M. (2019). Preschool teachers' knowledge of repeating patterns: focusing on structure and the unit of repeat. *Journal of Mathematics Teacher Education*, 22(3), 305-325.

Verschaffel, L., Torbeyns, J. & De Smedt, B. (2017). Young children's early mathematical competencies: Analysis and stimulation. *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education (CERME10),* 31-52. Dublin: CERME.

Wagner, J. (1997). The unavoidable intervention of educational research: A framework for reconsidering researcher-practitioner cooperation. *Educational Researcher*, 26(7), 13-22.

Warren, E.A. (2005). Patterns supporting the development of early algebraic thinking. *Building connections: Research, theory and practice*, 2, 759-766.

Warren, E. & Miller, J. (2013). Young Australian indigenous students' effective engagement in mathematics: The role of language, patterns, and structure. *Mathematics Education Research Journal,* 25(1), 151-171.

Waters, J. (2004). Mathematical patterning in early childhood settings. *Proceedings of MERGA 27: Mathematics Education for the Third Millennium, Towards 2010,* 565-572. Sydney: MERGA.

Wijns, N., De Smedt, B., Verschaffel, L. & Torbeyns, J. (2020). Are preschoolers who spontaneously create patterns better in mathematics? *British Journal of Educational Psychology*, 90(3), 753-769.

Wijns, N., Torbeyns, J., De Smedt, B. & Verschaffel, L. (2019a). Young children's patterning competencies and mathematical development: A review. In K.M. Robinson, H.P. Osana, & D. Kotsopoulos (Eds.), *Mathematical learning and cognition in early childhood: Integrating interdisciplinary research into practice,* 139-161. Cham: Springer International Publishing.

Wijns, N., Torbeyns, J., Bakker, M., De Smedt, B. & Verschaffel, L. (2019b). Four-year olds' understanding of repeating and growing patterns and its association with early numerical ability. *Early Childhood Research Quarterly*, 49, 152-163.

Wijns, N., Verschaffel, L., De Smedt, B. & Torbeyns, J. (2021). Associations between repeating patterning, growing patterning, and numerical ability: A longitudinal panel study in 4- to 6-year olds. *Child Development*, 92(4), 1354-1368.

Wilkie, K.J. (2012). Patterns and sequences in mathematics: teachers' knowledge and perceived practice in the upper primary years. *Proceedings of the International Conference of the Australian Association for Research in Education 2012*, 1-22. Sydney: Australian Association for Research in Education.

Zippert, E. L., Clayback, K. & Rittle-Johnson, B. (2019). Not just IQ: Patterning predicts preschoolers' math knowledge beyond fluid reasoning. *Journal of Cognition and Development*, 20(5), 752-771.