Coercive Trade Policy

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Online Appendix: Reasonable Beliefs and Mixed Strategy Equilibria

A Reasonable Beliefs

As explained in the main text, there is a unique equilibrium in the trade-war continuation game, in which the type-γ Foreign government always chooses $\tilde{\tau}^*(\gamma)$ and the Home government always chooses $\tilde{\tau}$. Similarly, the Home government's decision of whether or not to concede when confronted with the IO's ruling τ^{io} is uniquely determined by sequential rationality.

However, multiplicity arises in the earlier stages of the model where, anticipating equilibrium moves in subsequent subgames, governments play a signaling game. In order to rule out PBEs supported by "unreasonable" beliefs off the equilibrium path, we concentrate on pure strategy equilibria that satisfy Cho' and Kreps' (1987) criterion D1.

Fix an equilibrium, and let $\widehat{W}^*(\gamma)$ be the payoff of the type-γ Foreign government in this equilibrium. According to criterion D1, what types of Foreign government can reasonably be thought to choose an off-the-equilibrium-path demand τ'? Let $MBR\,(F,\tau')$ be the Home government's set of mixed best responses to τ' when it has beliefs F about the Foreign government's type. Next, define $D_F\,(\gamma,\tau')$ be the set of mixed best responses $\alpha \in MBR\,(F,\tau')$ that make type γ strictly prefer τ' to its equilibrium strategy — that is, the type-γ Foreign government's expected payoff when the Home government adopts any strategy in $D_F\,(\gamma,\tau')$ is strictly greater than $\widehat{W}^*(\gamma)$. Thus, $D\,(\gamma,\tau') \equiv \bigcup_F D_F\,(\gamma,\tau')$ can be interpreted as the set of Home government's responses that make the type-γ Foreign government willing to deviate to τ'. The set $D^0\,(\gamma,\tau')$ of mixed best responses that make the type-γ Foreign government exactly indifferent is defined analogously. Accordingly, a type γ is deleted following demand τ' under criterion D1 if there is another type γ' such that $\left[D\,(\gamma,\tau') \cup D^0\,(\gamma,\tau')\right] \subset D\,(\gamma',\tau')$. In words, if the set of Home government's responses that make type γ willing to deviate to τ' is strictly smaller than the set of best responses that make type γ' willing to deviate, than the Home government should believe that type γ' is infinitely more likely to deviate to τ' than type γ is.

Now we establish a series of lemmata that are used in the proofs of our main results.

Coercion without the IO

LEMMA 1: *Consider an equilibrium in which some some subset of types of the form* $\left[\underline{\gamma}, \gamma^{\sup}\right]$*, with* $\gamma^{\sup} > \underline{\gamma}$*, obtain a concession* τ*. Reasonable beliefs assign zero probability to all types* $\gamma < \gamma^{\sup}$ *following any (off-the-equilibrium-path) demand* $\tau' \in \left(T\left(\gamma^{\sup}\right), \tau\right)$*.*

PROOF: Consider a deviation to demand $\tau' \in \left(T\left(\gamma^{\sup}\right), \tau\right)$. By definition of an equilibrium, all types $\gamma \in \left[\underline{\gamma}, \gamma^{\sup}\right]$ prefer successful demand τ to a trade war; that is: $W^*\left(\tau, \tau_0^*, \gamma\right) \geq \widetilde{W}^*(\gamma)$ for all $\gamma \in \left[\underline{\gamma}, \gamma^{\sup}\right]$. In addition, $\tau' < \tau$ implies that:

$$\widetilde{W}^*(\gamma) \leq W^*\left(\tau, \tau_0^*, \gamma\right) < W^*\left(\tau', \tau_0^*, \gamma\right) \text{ , for all } \gamma \in \Gamma_\tau \ . \tag{1}$$

Take an arbitrary type $\gamma \in \left[\underline{\gamma}, \gamma^{\sup}\right)$. The Home government's mixed best response α makes the type-γ foreign government prefer τ' to its equilibrium demand τ if and only if:

$$\alpha W^*\left(\tau', \tau_0^*, \gamma\right) + (1-\alpha)\widetilde{W}^*(\gamma) \geq W^*\left(\tau, \tau_0^*, \gamma\right) \ . \tag{2}$$

(Our restrictions on τ' ensure that any $\alpha \in [0,1]$ is a best response for some beliefs.) This inequality can be rewritten as

$$\begin{aligned}
\alpha \geq \bar{\alpha}(\gamma) \quad &\equiv \quad \frac{W^*\left(\tau, \tau_0^*, \gamma\right) - \widetilde{W}^*(\gamma)}{W^*\left(\tau', \tau_0^*, \gamma\right) - \widetilde{W}^*(\gamma)} \\
&= \quad \frac{w^*\left(\tau_0^*\right) - w^*\left(\tilde{\tau}^*(\gamma)\right) + \gamma\left[R_2^*\left(\tau_0^*\right) - R_2^*\left(\tilde{\tau}^*(\gamma)\right)\right] + R_1^*(\tau) - R_1^*\left(\tilde{\tau}\right)}{w^*\left(\tau_0^*\right) - w^*\left(\tilde{\tau}^*(\gamma)\right) + \gamma\left[R_2^*\left(\tau_0^*\right) - R_2^*\left(\tilde{\tau}^*(\gamma)\right)\right] + R_1^*\left(\tau'\right) - R_1^*\left(\tilde{\tau}\right)} \ , \qquad (3)
\end{aligned}$$

where $w(\tau^*) \equiv \mathbf{T}^*(\tau^*) + \Omega(\tau^*)$. The inequalities in (1) guarantee that $\bar{\alpha}(\gamma) \in [0,1)$ for all $\gamma \in \left[\underline{\gamma}, \gamma^{\sup}\right]$. Furthermore, as $R_1^*\left(\tau'\right) > R_1^*(\tau)$, the sign of the derivative of $\bar{\alpha}$ is the same as the sign of

$$\frac{d}{d\gamma}\left[w^*\left(\tau_0^*\right) - w^*\left(\tilde{\tau}^*(\gamma)\right) + \gamma\left[R_2^*\left(\tau_0^*\right) - R_2\left(\tilde{\tau}^*(\gamma)\right)\right]\right] = R_2^*\left(\tau_0^*\right) - R_2^*\left(\tilde{\tau}^*(\gamma)\right) < 0 \ . \tag{4}$$

(The equality follows from the Envelope Theorem: $\tilde{\tau}^*(\gamma)$ is the maximizer of $w^*(\cdot) + \gamma R_2^*(\cdot)$.) Hence, $\bar{\alpha}$ is strictly decreasing.

This implies that, for any $\gamma' \in \left[\underline{\gamma}, \gamma^{\sup}\right]$ such that $\gamma' > \gamma$, $D\left(\gamma, \tau'\right) \cup D^0\left(\gamma, \tau'\right) = [\bar{\alpha}(\gamma), 1] \subset (\bar{\alpha}(\gamma'), 1] = D\left(\gamma', \tau'\right)$. Criterion D1 then requires that, when confronted with demand τ', the Home government believes that the Foreign government is of type γ with probability 0. As γ was taken arbitrarily in $\left[\underline{\gamma}, \gamma^{\sup}\right)$, this establishes the lemma. □

LEMMA 2: *Consider an equilibrium in which a trade war ensues after every type's demand. Beliefs which assign a probability of 1 to type* $\underline{\gamma}$ *following any (off-the-equilibrium-path) unilateral demand* τ' *are reasonable.*

PROOF: If $\tau' < T(\overline{\gamma})$, then the lemma is trivial: the only best response for the Home government is to reject demand τ'. This implies that $D(\gamma, \tau') = \emptyset$ for all types $\gamma \in [\underline{\gamma}, \overline{\gamma}]$ and, therefore, that it is impossible to eliminate type $\underline{\gamma}$.

If $\tau' \in [T(\overline{\gamma}), T(\underline{\gamma})]$, then any $\alpha \in [0,1]$ may be a best response. As all types of foreign government make unsuccessful demands in equilibrium, we have

$$D(\gamma, \tau') = \begin{cases} (0,1] & \text{if } \tau' < T^*(\gamma) , \\ \emptyset & \text{otherwise,} \end{cases} \tag{5}$$

for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$. As T^* is strictly decreasing, this implies that $D(\underline{\gamma}, \tau') \supseteq D(\gamma, \tau')$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$. Therefore, beliefs which assign a probability of 1 to type $\underline{\gamma}$ following any (off-the-equilibrium-path) demand $\tau' \in [T(\overline{\gamma}), T(\underline{\gamma})]$ are reasonable.

Finally, if $\tau' > T(\underline{\gamma})$ then $D(\gamma; \tau', u) = \{1\}$ if $\tau' < T^*(\gamma)$, and $D(\gamma; \tau') = \emptyset$. By the same argument as above, it is impossible to eliminate type $\underline{\gamma}$ using criterion D1. □

COERCION WITH FULL COMMITMENT TO THE IO

LEMMA 3: *Consider an equilibrium in which a set of types of the form* $[\underline{\gamma}, \hat{\gamma}]$*, with* $\hat{\gamma} \in (\underline{\gamma}, \overline{\gamma}]$*, make a successful demand* $\tau > \tau^{io}$*. Reasonable beliefs must assign zero probability to all types* $\gamma < \hat{\gamma}$ *following any (off-the-equilibrium-path) demand* $\tau' \in (\tau^{io}, \tau)$*.*

PROOF: Consider a deviation $\tau' \in (\tau^{io}, \tau)$, and let F' be the Home government's beliefs following this demand. If F' makes the Home government indifferent between conceding and not conceding to τ', then $W(\tau^{io}, \tau_0^*) < W(\tau', \tau_0^*) = W(T(\gamma), \tau_0^*)$ — so that a trade war ensues when the Home government rejects τ'. In addition, all types in $[\underline{\gamma}, \hat{\gamma}]$ must prefer τ to a trade war; otherwise we would have $W^*(\tau, \tau_0^*, \gamma) < \min\left\{W^*(\tau', \tau_0^*, \gamma), \widetilde{W}^*(\gamma)\right\}$ for some type $\gamma \in [\underline{\gamma}, \hat{\gamma}]$ (which could the profitably deviate by making an unacceptable offer $\tau'' < \tau^{io}$). These observations imply that we can use the same argument as in the proof of Lemma 1 to obtain the result. □

LEMMA 4: *Suppose that, in equilibrium, all types of Foreign government successfully demand* τ^{io}*. Beliefs which assign a probability of 1 to type* $\overline{\gamma}$ *following any (off-the-equilibrium-path) multilateral demand* $\tau' \neq \tau^{io}$ *are reasonable.*

PROOF: Take an arbitrary (off-the-equilibrium-path) multilateral demand $\tau' \neq \tau^{io}$. Throughout this proof, the Home government's updated beliefs following demand τ' are denoted by F'. Suppose first that $\tau' < \tau^{io}$. In this case, it is never a best response for the Home government to concede to τ^{io}; so that $MBR(F', \tau') = \{0\}$. If its beliefs F' are such that $\tau^{io} \geq T(F')$ (i.e., it concedes to the IO ruling τ^{io}), then any type of Foreign government is indifferent between its successful equilibrium demand τ^{io} and the unsuccessful demand τ'; so that $D_{F'}(\gamma, \tau') = \emptyset$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$. If its beliefs F' are such that $\tau^{io} < T(F')$ (i.e., it does not concedes to the IO's ruling, thus triggering a trade war), then the type-γ Foreign government strictly prefers demanding τ' over demanding τ^{io} if and only if: $\widetilde{W}^*(\gamma) > W^*(\tau^{io}, \tau_0^*, \gamma)$ or, equivalently, $T^*(\gamma) < \tau^{io}$. This implies that

$$D_{F'}(\gamma, \tau') = \begin{cases} \{0\} & \text{if } T^*(\gamma) < \tau^{io} , \\ \emptyset & \text{otherwise,} \end{cases} \tag{6}$$

As $T^*(\gamma)$ is strictly decreasing in γ, this implies that $D_{F'}(\gamma, \tau') \subseteq D_{F'}(\overline{\gamma}, \tau')$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$. This in turn implies that $D(\gamma, \tau') \subseteq D(\overline{\gamma}, \tau')$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$, thus proving that beliefs which assign a probability of 1 to type $\overline{\gamma}$ following any (off-the-equilibrium-path) demand $\tau' < \tau^{io}$ are reasonable.

Suppose now that $\tau' > \tau^{io}$. If the Home government's beliefs F' are such that $\tau^{io} \geq T(F')$ (i.e., it concedes to the IO's ruling τ^{io}), then any type of Foreign government is always worse off making the unsuccessful demand τ'; so that $D_{F'}(\gamma, \tau') = \emptyset$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$. If its beliefs F' are such that $\tau^{io} < T(F')$ (i.e., it does not concede to the IO's ruling), then we must distinguish between three different cases:

(i) If F' is such that $T(F') < \tau'$, then the unique best response for the Home government is to accept τ' with a probability of 1: $MBR(F', \tau') = \{1\}$. As $\tau' > \tau$, any type of Foreign government is worse off. Hence, $D_{F'}(\gamma, \tau') = \emptyset$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$.

(ii) If F' is such that $T(F') = \tau'$, then the Home government is indifferent between conceding and not conceding to τ': $MBR(F', \tau') = [0, 1]$. An $\alpha \in [0, 1]$ makes the type-γ Foreign government (strictly) prefer τ' to its equilibrium demand τ^{io} if and only if:

$$\alpha W^*(\tau', \tau_0^*, \gamma) + (1 - \alpha)\widetilde{W}^*(\gamma) > W^*(\tau^{io}, \tau_0^*, \gamma) \ . \tag{7}$$

This implies that

$$D_{F'}(\gamma, \tau') = \begin{cases} [0, \bar{\alpha}(\gamma)) & \text{if } \widetilde{W}^*(\gamma) > W^*(\tau^{io}, \tau_0^*, \gamma) \ , \\ \emptyset & \text{otherwise,} \end{cases} \tag{8}$$

where
(9)

$$\bar{\alpha}(\gamma) \equiv \frac{w^*\left(\tau_0^*\right) - w^*\left(\tilde{\tau}^*(\gamma)\right) + \gamma\left[R_2^*\left(\tau_0^*\right) - R_2^*\left(\tilde{\tau}^*(\gamma)\right)\right] + R_1^*\left(\tau^{io}\right) - R_1^*\left(\tilde{\tau}\right)}{w^*\left(\tau_0^*\right) - w^*\left(\tilde{\tau}^*(\gamma)\right) + \gamma\left[R_2^*\left(\tau_0^*\right) - R_2^*\left(\tilde{\tau}^*(\gamma)\right)\right] + R_1^*\left(\tau'\right) - R_1^*\left(\tilde{\tau}\right)} .$$

As $R_1^*\left(\tau'\right) < R_1^*\left(\tau^{io}\right)$, the sign of the derivative of $\bar{\alpha}$ is the same as the sign of

$$-\frac{d}{d\gamma}\left[w^*\left(\tau_0^*\right) - w^*\left(\tilde{\tau}^*(\gamma)\right) + \gamma\left[R_2^*(\tau_0^*) - R_2\left(\tilde{\tau}^*(\gamma)\right)\right]\right] > 0 \tag{10}$$

(The argument is the same as in the proof of Lemma 1). Hence, $\bar{\alpha}$ is strictly increasing. This implies that $D_{F'}\left(\gamma, \tau'\right) \subseteq D_{F'}\left(\overline{\gamma}, \tau'\right)$ for all $\gamma \in \left[\underline{\gamma}, \overline{\gamma}\right]$.

(iii) If F' is such that $T(F') > \tau'$, then the unique best response for the Home government is to reject τ': $MBR\left(F', \tau'\right) = \{0\}$. As a consequence, the type-γ Foreign government is better-off demanding τ' rather than τ^{io} if and only of it prefers a trade war over agreement on τ^{io} or, equivalently, $T^*(\gamma) < \tau^{io}$. Hence,

$$D_{F'}\left(\gamma, \tau'\right) = \begin{cases} \{0\} & \text{if } T^*(\gamma) < \tau^{io} , \\ \emptyset & \text{otherwise.} \end{cases} \tag{11}$$

As $T^*(\gamma)$ is strictly decreasing in γ, this implies that, for any beliefs F', $D_{F'}\left(\gamma, \tau'\right) \subseteq D_{F'}\left(\overline{\gamma}, \tau'\right)$ for all $\gamma \in \Gamma$. We have thus proved that the latter relation is true for all possible beliefs and, therefore, that $D\left(\gamma, \tau'\right) \subseteq D\left(\overline{\gamma}, \tau'\right)$ for all $\gamma \in \Gamma$. As a result, beliefs which assign a probability of 1 to type $\overline{\gamma}$ following any (off-the-equilibrium-path) demand $\tau' > \tau^{io}$ are reasonable. This completes the proof of the lemma. □

LEMMA 5: *Consider an equilibrium in which all types of Foreign government unsuccessfully make demand τ^{io}, following which the domestic government does not comply the IO ruling. Beliefs that assign a probability of 1 to type $\underline{\gamma}$ following any (off-the-equilibrium-path) demand $\tau' \neq \tau^{io}$ are reasonable.*

PROOF: To prove the lemma, it suffices to show that $D\left(\gamma, \tau'\right) \subseteq D\left(\underline{\gamma}, \tau'\right)$ for all $\gamma \in \left[\underline{\gamma}, \overline{\gamma}\right]$ (so that $\underline{\gamma}$ cannot be eliminated).

Suppose first that $\tau' < \tau^{io}$. In this case, it is never a best response for the domestic government for any beliefs F it may have (it receives $\max\left\{W\left(\tau^{io}, \tau_0^*\right), \mathbb{E}_F\left[\widetilde{W}(\gamma)\right]\right\} \geq W\left(\tau^{io}, \tau_0^*\right) > W\left(\tau', \tau_0^*\right)$ by rejecting τ'); so that $MBR(F, \tau') = \{0\}$. If its beliefs F are such that $\tau^{io} \geq T(F)$ (i.e., it concedes to the IO ruling τ^{io}), then the type-γ foreign government strictly prefers unsuccessful demand τ' to the equilibrium trade war if and only if $\tau^{io} < T^*(\gamma)$; so that

$$D_F\left(\gamma, \tau'\right) = \begin{cases} \{0\} & \text{if } T^*(\gamma) > \tau^{io} , \\ \emptyset & \text{otherwise.} \end{cases} \tag{12}$$

As $T^*(\gamma)$ is strictly decreasing in γ, this implies that $D_F(\gamma, \tau') \subseteq D_F(\underline{\gamma}, \tau')$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$.

If its beliefs F are such that $\tau^{io} < T(F)$ (i.e., it does not concedes to the IO ruling, thus triggering a trade war), then any type of foreign government is indifferent between its unsuccessful equilibrium demand τ^{io} and the unsuccessful demand τ'; so that $D_F(\gamma, \tau') = \emptyset$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$.

Suppose now that $\tau' > \tau^{io}$. If the domestic government's beliefs F are such that $\tau^{io} \geq T(F)$ (i.e., it concedes to the IO ruling τ^{io}), then any type of foreign government is always worse off making the successful demand τ'; so that $D_F(\gamma, \tau') = \emptyset$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$. If its beliefs F are such that $\tau^{io} < T(F)$ (i.e., it does not concede to the IO ruling), then we must distinguish between three different cases:

(i) If F is such that $T(F) < \tau'$ — so that $\widetilde{W}(\gamma) < W(\tau', \tau_0^*)$ — then the unique best response for the domestic government is to accept τ' with a probability of 1: $MBR(F, \tau') = \{1\}$. Therefore, the type-γ foreign government is strictly better off demanding τ' if and only if $\tau' < T^*(\gamma)$. As $T^*(\gamma)$ is strictly decreasing in γ, this implies that $D_F(\gamma, \tau') \subseteq D_F(\underline{\gamma}, \tau')$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$.

(ii) If F is such that $T(F) = \tau'$ — so that $\widetilde{W}(\gamma) = W(\tau', \tau_0^*)$ — then the domestic government is indifferent between conceding and not conceding to τ': $MBR(F, \tau') = [0, 1]$. As, the type-γ foreign government strictly prefers successful demand τ' to the equilibrium trade war if and only if $\tau^{io} < T^*(\gamma)$, we have

$$D_F(\gamma, \tau') = \begin{cases} (0, 1] & \text{if } T^*(\gamma) > \tau' \ , \\ \emptyset & \text{otherwise.} \end{cases} \tag{13}$$

As $T^*(\gamma)$ is strictly decreasing in γ, this implies that $D_F(\gamma, \tau') \subseteq D_F(\underline{\gamma}, \tau')$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$.

(iii) If F is such that $T(F) > \tau'$ — so that $\widetilde{W}(\gamma) > W(\tau', \tau_0^*)$ — then the unique best response for the domestic government is to accept τ' with zero probability: $MBR(F, \tau') = \{0\}$. Therefore, all types of foreign government are indifferent between their equilibrium demand and τ'; so that $D_F(\gamma, \tau') = \emptyset$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$.

We have thus showed that the following is true for all domestic government's beliefs F: $D_F(\gamma, \tau') \subseteq D_F(\underline{\gamma}, \tau')$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$. This in turn implies that $D_F(\gamma, \tau') \subseteq D_F(\underline{\gamma}, \tau')$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$. It is therefore impossible to eliminate type $\underline{\gamma}$. □

Coercion with Partial Commitment to the IO

Observe that, in this version of the model, the Foreign government makes two choices: a coercion mode and a demand to the Home government. Therefore, a deviation is now of the form (τ', c) where $c \in \{u, m\}$ is the coercion mode adopted by the Foreign government when it deviates — u meaning "unilateral," and m "multilateral."

LEMMA 6: *Consider an equilibrium in which all types of Foreign government make unsuccessful unilateral demands. Beliefs that assign a probability of 1 to type $\underline{\gamma}$ following any off-the-equilibrium-path demand are reasonable.*

PROOF: We can apply the same argument as in Lemma 2 to show that beliefs assigning a probability of 1 to type $\underline{\gamma}$ following any deviation to a unilateral demand are reasonable. Now consider a deviation to a multilateral demand τ'. Suppose first that $\tau' > \tau^{io}$. Consider first a system of beliefs F such that the Home government complies with the IO ruling; that is, $\tau^{io} \geq T(F)$. In this case, the only best response for the Home government is to accept τ' with a probability of 1: $MBR(F, \tau', m) = \{1\}$. This implies that the type-γ Foreign government strictly prefers demanding τ' over its equilibrium demand if and only if $\tau' < T^*(\gamma)$. Hence,

$$D_F\left(\gamma, \tau', m\right) = \begin{cases} \{1\} & \text{if } T^*(\gamma) > \tau' \ , \\ \emptyset & \text{otherwise.} \end{cases} \tag{14}$$

As $T^*(\gamma)$ is strictly decreasing in γ, this implies that $D_F(\gamma, \tau', m) \subseteq D_F\left(\underline{\gamma}, \tau', m\right)$ for all $\gamma \in \left[\underline{\gamma}, \overline{\gamma}\right]$.

Consider now a system of beliefs F such that the Home government does not comply with the IO ruling; that is, $\tau^{io} < T(F)$. We must distinguish between three different situations:

(i) If $\tau' > T(F)$, then $MBR(F, \tau', m) = \{1\}$. We can use then use the same argument as above to obtain that $D_F(\gamma, \tau', m) \subseteq D_F\left(\underline{\gamma}, \tau', m\right)$ for all $\gamma \in \left[\underline{\gamma}, \overline{\gamma}\right]$.

(ii) If $\tau' < T(F)$, then $MBR(F, \tau', m) = \{0\}$. Therefore all types of Foreign government are perfectly indifferent between demanding τ' multilaterally and their equilibrium unilateral demand. Hence, $D_F(\gamma, \tau', m) = \emptyset$ for all $\gamma \in \left[\underline{\gamma}, \overline{\gamma}\right]$.

(iii) If $\tau' = T(F)$, then $MBR(F, \tau') = [0, 1]$. In this case,

$$D_F\left(\gamma, \tau', m\right) = \begin{cases} (0, 1] & \text{if } T^*(\gamma) > \tau' \ , \\ \emptyset & \text{otherwise.} \end{cases} \tag{15}$$

As $T^*(\gamma)$ is strictly decreasing in γ, this implies that $D_F(\gamma, \tau', m) \subseteq D_F\left(\underline{\gamma}, \tau', m\right)$ for all $\gamma \in \left[\underline{\gamma}, \overline{\gamma}\right]$.

Suppose now that $\tau' = \tau^{io}$. If the Home government's beliefs F are such that $\tau^{io} \geq T(F)$ — i.e. it complies with the IO ruling — then it is indifferent between accepting and rejecting demand τ': $MBR(F, \tau', m) = [0, 1]$. This implies that the type-γ Foreign government strictly prefers demanding τ' over its equilibrium demand if and only if $W^*(\tau', \tau_0^*, \gamma) = W^*\left(\tau^{io}, \tau_0^*, \gamma\right) > \widetilde{W}^*(\gamma)$ (or $\tau' < T^*(\gamma)$).

Hence,

$$D_F\left(\gamma, \tau', m\right) = \begin{cases} (0,1] & \text{if } T^*(\gamma) > \tau' \ , \\ \emptyset & \text{otherwise.} \end{cases} \tag{16}$$

As $T^*(\gamma)$ is strictly decreasing in γ, this implies that $D_F\left(\gamma, \tau', m\right) \subseteq D_F\left(\underline{\gamma}, \tau', m\right)$ for all $\gamma \in \left[\underline{\gamma}, \overline{\gamma}\right]$.

If the Home government's beliefs F are such that $\tau^{io} < T(F)$ — i.e. it does not comply with the IO ruling — then its only best response is to accept $\tau' = \tau^{io}$ with a zero probability: $MBR(F, \tau') = \{0\}$. Therefore all types of Foreign government are perfectly indifferent between demanding τ' multilaterally and their equilibrium unilateral demand. Hence, $D_F\left(\gamma, \tau', m\right) = \emptyset$ for all $\gamma \in \left[\underline{\gamma}, \overline{\gamma}\right]$.

Finally, suppose that $\tau' < \tau^{io}$. Then it is never a best response for the Home government, for any beliefs F (it receives $\max\left\{W\left(\tau^{io}, \tau_0^*\right), \mathbb{E}_F\left[\widetilde{W}(\gamma)\right]\right\} \geq W\left(\tau^{io}, \tau_0^*\right) > W\left(\tau', \tau_0^*\right)$ by rejecting τ'); so that $MBR(F, \tau', m) = \{0\}$. If its beliefs F are such that $\tau^{io} \geq T(F)$ (i.e., it concedes to the IO ruling τ^{io}), then the type-γ Foreign government strictly prefers unsuccessful demand τ' to the equilibrium trade war if and only if $\tau^{io} < T^*(\gamma)$; so that

$$D_F\left(\gamma, \tau', m\right) = \begin{cases} \{0\} & \text{if } T^*(\gamma) > \tau^{io} \ , \\ \emptyset & \text{otherwise.} \end{cases} \tag{17}$$

As $T^*(\gamma)$ is strictly decreasing in γ, this implies that $D_F\left(\gamma, \tau', m\right) \subseteq D_F\left(\underline{\gamma}, \tau', m\right)$ for all $\gamma \in \left[\underline{\gamma}, \overline{\gamma}\right]$.

If its beliefs F are such that $\tau^{io} < T(F)$ (i.e., it does not concedes to the IO ruling, thus triggering a trade war), then any type of Foreign government is indifferent between its unsuccessful equilibrium unilateral demand and the unsuccessful demand τ'; so that $D_F\left(\gamma, \tau', m\right) = \emptyset$ for all $\gamma \in \left[\underline{\gamma}, \overline{\gamma}\right]$.

We thus established that, for any belief system F, $D_F\left(\gamma, \tau', m\right) \subseteq D_F\left(\underline{\gamma}, \tau', m\right)$ for all $\gamma \in \left[\underline{\gamma}, \overline{\gamma}\right]$. Taking the union over all possible beliefs, we obtain $D\left(\gamma, \tau', m\right) \subseteq D\left(\underline{\gamma}, \tau', m\right)$ for all $\gamma \in \left[\underline{\gamma}, \overline{\gamma}\right]$. This proves that beliefs that assigns probability 1 to type $\underline{\gamma}$ are reasonable. □

LEMMA 7: *Consider an equilibrium in which all types of Foreign government coerce multilaterally, and all their demands are followed by the implementation of* $\tau^{io} \leq T^*\left(\overline{\gamma}\right)$. *Reasonable beliefs must assign a probability of 1 to type* $\overline{\gamma}$ *following any (off-the-equilibrium-path) unilateral demand* $\tau' < \tau^{io}$.

PROOF: Consider a deviation to a unilateral demand $\tau' < \tau^{io}$. If the Home government's beliefs, F, are such that its unique best response is to concede to τ' with a probability of 1, then all types of Foreign government are strictly better-off demanding τ' unilaterally: $D_F\left(\gamma, \tau', u\right) = \{1\}$ and $D_F^0\left(\gamma, \tau', u\right) = \emptyset$ for all $\gamma \in \left[\underline{\gamma}, \overline{\gamma}\right]$. If the Home government's beliefs are such that its unique best response is to concede to τ' with a zero probability, then all types $\gamma < \overline{\gamma}$ are strictly worse-off ($\tau' < \tau^{io} \leq T^*\left(\overline{\gamma}\right) < T^*(\gamma)$ for all $\gamma < \overline{\gamma}$). This implies that $D_F\left(\gamma, \tau', u\right) = D_F^0\left(\gamma, \tau', u\right) = \emptyset$ for all $\gamma < \overline{\gamma}$.

Finally, if the Home government's beliefs are such that it is indifferent between conceding and not conceding to τ'. In this case, a best response $\alpha \in MBR(F, \tau', u) = [0, 1]$ makes the type-γ Foreign government prefer to demand τ' unilaterally if and only if

$$\alpha W^*\left(\tau', \tau_0^*, \gamma\right) + (1-\alpha)\widetilde{W}^*(\gamma) \geq W^*\left(\tau^{io}, \tau_0^*, \gamma\right) \tag{18}$$

or, equivalently,

$$\alpha \geq \bar{\alpha}(\gamma) \equiv \frac{W^*\left(\tau^{io}, \tau_0^*, \gamma\right) - \widetilde{W}^*(\gamma)}{W^*\left(\tau', \tau_0^*, \gamma\right) - \widetilde{W}^*(\gamma)} \in [0, 1) \tag{19}$$

(with $\bar{\alpha}(\gamma) > 0$ for all $\gamma < \overline{\gamma}$). Therefore, $D_F\left(\gamma, \tau', u\right) = (\bar{\alpha}(\gamma), 1]$ and $D_F^0\left(\gamma, \tau', u\right) = \{\bar{\alpha}(\gamma)\}$ for all $\gamma \in \left[\underline{\gamma}, \overline{\gamma}\right]$. It is readily checked that $\bar{\alpha}$ is a strictly decreasing function (see the proof of Lemma 1). Hence, taking the union over all possible beliefs F, we obtain

$$\left[D\left(\gamma, \tau', u\right) \cup D^0\left(\gamma, \tau', u\right)\right] = [\bar{\alpha}(\gamma), 1] \subset \left(\bar{\alpha}\left(\overline{\gamma}\right), 1\right] = D\left(\overline{\gamma}, \tau', u\right) \; . \tag{20}$$

□

LEMMA 8: *Consider an equilibrium in which: all types in* $\left[\underline{\gamma}, \hat{\gamma}\right]$*, with* $\hat{\gamma} = \left(T^*\right)^{-1}\left(\tau^{io}\right)$*, make multilateral demands followed by the implementation of tariff* τ^{io}*; and all types in* $\left(\hat{\gamma}, \overline{\gamma}\right]$ *make unsuccessful unilateral demands. Reasonable beliefs must assign zero probability to all types* $\gamma \leq \hat{\gamma}$ *following any (off-the-equilibrium-path) unilateral demand* $\tau' \in \left(T\left(\hat{\gamma}\right), \tau^{io}\right)$.

PROOF: Observe that, in terms of equilibrium payoffs, this is similar to the case without IO in which all types in $\left[\underline{\gamma}, \hat{\gamma}\right]$ successfully demand τ^{io} and all types in $\left(\hat{\gamma}, \overline{\gamma}\right]$ fail to obtain a concession. We can therefore replicate the argument of Lemma 1 (replacing $\gamma^{\sup}$ by $\hat{\gamma}$) to prove that all types $\gamma \leq \hat{\gamma}$ must be eliminated according to the D1 criterion. □

B Mixed Strategy Equilibria

The proofs of Propositions 1 and 2 in the main text rely on the assumptions that: (i) the Home government cannot randomize between conceding and not conceding to the Foreign government's demand; and (ii) $T^*(\underline{\gamma}) < T(\underline{\gamma})$. These assumptions, however, are mainly made to ease the exposition and are not critical for our main conclusions. Indeed, we show in this section that the propositions carry over to the case where the Home and Foreign governments are allowed to randomize (and $T^*(\underline{\gamma}) < T(\underline{\gamma})$). (Note that we already allowed the Home government to randomize over actions when applying criterion D1.) In the next section, we will also establish that our main conclusions still hold if we assume that $T^*(\underline{\gamma}) > T(\underline{\gamma})$.

Proposition 1

Proposition 1, as stated in the main text, remains valid if we allow players to randomize over actions. Existence of a (degenerate) mixed strategy equilibrium in which the Home government never concedes to the Foreign government's demands follows immediately from the proof in the main text. To extend Proposition 1 to mixed strategy equilibria, therefore, it remains to show that in any *mixed strategy* equilibrium, the Home government never concedes to the Foreign government's demands.

Consider an arbitrary mixed-strategy equilibrium and suppose (toward a contradiction) that, in this equilibrium, some demands are made the Foreign government and accepted by the Home government with positive probability. Observe first that the type-γ Foreign government is better off making a demand τ_1 accepted with probability α_1 rather than making a demand τ_2 accepted with probability $\alpha_2 > \alpha_1$ if and only if

$$\alpha_1 W^*(\tau_1, \tau_0^*, \gamma) - \alpha_2 W^*(\tau_2, \tau_0^*, \gamma) + (\alpha_2 - \alpha_1)\widetilde{W}^*(\gamma) \geq 0 \ . \tag{21}$$

Differentiating the left-hand side of the above inequality with respect to γ (and applying the Envelope Theorem), we obtain $(\alpha_1 - \alpha_2)\big[R_2^*(\tau_0^*) - R_2^*\big(\tilde{\tau}^*(\gamma)\big)\big] > 0$. Hence, if a type γ_1 is better-off offering τ_1, then so is every type $\gamma > \gamma_1$; and, similarly, if a type γ_2 is better-off offering τ_2, then so is every type $\gamma < \gamma_2$. It follows that we can partition $[\underline{\gamma}, \overline{\gamma}]$ into (possibly degenerate) intervals $\{\Gamma_k\}_{k\in K}$, for some (possibly uncountable) index set K, such that: (i) each type in Γ_k makes the same demand τ_k, which is accepted by the Home government with some probability $\alpha_k \in [0, 1]$; and (ii) $\tau_k > \tau_{k'}$ and $\alpha_k \geq \alpha_{k'}$ for all $k' > k$.

As the contract curve is empty at $\underline{\gamma}$, the interval that contains $\underline{\gamma}$, say Γ_0, is not degenerate and its members make a demand that is accepted with a probability $\alpha_0 > 0$. Let τ be the demand made by the types in Γ_0. Confronted with demand τ, the Home government — whose updated beliefs F assign a probability of 1 to the event "$\gamma \in \Gamma_0$" — (weakly) prefers to concede in the equilibrium under consideration. As the distribution of types has full support on $\left[\underline{\gamma}, \overline{\gamma}\right]$, this implies that $\tau \geq T(F) > T\left(\gamma^{\sup}\right)$, where $\gamma^{\sup} \equiv \sup \Gamma_0$. If there exists another non-degenerate interval Γ_1 in which all types offer $\tau_1 \equiv \max\{\tau_k \colon \tau_k < \tau\}$, then all demands in the interval (τ_1, τ) are only made off the equilibrium path. If such an interval of types does not exist, then there is a small enough $\varepsilon > 0$ such that any equilibrium demand $\hat{\tau} \in (\tau - \varepsilon, \tau)$ is either successful with some probability $\hat{\alpha} \in (0, 1)$ or unsuccessful. In the former case, it is made by a single type $\hat{\gamma} \geq \gamma^{\text{sup}}$ and, therefore, $\hat{\tau} = T(\hat{\gamma}) \leq T(\gamma^{\text{sup}})$ (where the equality from the Home government indifference condition); in the latter case, $\hat{\tau} \leq T(\hat{F}) \leq T(\gamma^{\text{sup}})$, where $\hat{F}$ represents the Home government's updated beliefs, which assign a probability of 1 to the event "$\gamma \geq \gamma^{\text{sup}}$." This implies that there exists a tariff $\underline{\tau} \in \big(T(\gamma^{\text{sup}}, \tau\big)$ such that all demands in $(\underline{\tau}, \tau)$ are off the equilibrium path.

The desired contradiction follows from the following variant on Lemma 1:

Claim: Reasonable beliefs assign zero probability to all types $\gamma < \gamma^{\text{sup}}$ following any (off-the-equilibrium-path) demand $\tau' \in (\underline{\tau}, \tau)$.

PROOF: Suppose the Home government observes a deviation to a demand $\tau' \in (\underline{\tau}, \tau)$. For every $\gamma < \gamma^{\text{sup}}$, the Home government's mixed best response α makes the type-γ Foreign government prefer τ' to its equilibrium demand $\tau(\gamma)$ if and only if

$$\alpha W^*\big(\tau', \tau_0^*, \gamma\big) + (1 - \alpha)\widetilde{W}^*(\gamma) \geq \alpha_0 W^*\big(\tau, \tau_0^*, \gamma\big) + (1 - \alpha_0)\widetilde{W}^*(\gamma) \tag{22}$$

or, equivalently,

$$\alpha \geq \bar{\alpha}(\gamma) \equiv \alpha_0 \frac{W^*\big(\tau, \tau_0^*, \gamma\big) - \widetilde{W}^*(\gamma)}{W^*\big(\tau', \tau_0^*, \gamma\big) - \widetilde{W}^*(\gamma)} \ . \tag{23}$$

We can then apply the same argument as in the proof of Lemma 1 to show that $\bar{\alpha}$ is a strictly decreasing function, thus obtaining the result. □

Now take any tariff $\tau' \in (\underline{\tau}, \tau)$. When confronted with demand τ', the Home government believes that the Foreign government's type is lower than γ^{sup} with probability 0. As $\tau' > T(\gamma^{\text{sup}})$, the Home government concedes to demand τ' (off the equilibrium path). As $\tau' < T^*(\gamma^{\text{sup}})$, this implies that types in a neighborhood of γ^{sup} have a profitable deviation, giving the desired contradiction.

PROPOSITION 2

The original statement of Proposition 2 remains valid with mixed strategies, except for the knife-edge case where $\tau^{io} = T(F_0)$. (In the latter case, as we no longer break ties by assuming that the Home government chooses to comply when indifferent between τ^{io} and a trade war, there may also be equilibria where the indifferent Home government opts for a trade war with probability one.) Thus, the paper's main message — i.e., that an IO without enforcement power can be effective in preventing trade wars — is unaffected. This is established below.

Observe first that, in any equilibrium, if the Home government randomizes between accepting a demand τ and rejecting it to comply with the IO ruling with positive probability, then $\tau = \tau^{io}$ and the type-γ Foreign government receives $W^*(\tau^{io}, \tau_0^*, \gamma)$ with certainty. As any successful demand must be greater than or equal to τ^{io} (otherwise, the Home government could profitably deviate by rejecting it and complying with the IO ruling), the Foreign government always prefers τ^{io} with probability one over any demand $\tau \neq \tau^{io}$ accepted with probability one. Moreover, the type-γ Foreign government is better off obtaining τ^{io} with probability one than making a demand τ_1 accepted with probability $\alpha_1 < 1$ and followed by a trade war when rejected if and only if

$$W^*(\tau^{io}, \tau_0^*, \gamma) - \alpha_1 W^*(\tau_1, \tau_0^*, \gamma) - (1-\alpha_1)\widetilde{W}^*(\gamma) \geq 0 \ . \tag{24}$$

Differentiating the left-hand side of this inequality with respect to γ and rearranging terms, we obtain $-(1-\alpha_1)\big[R_2^*\big(\tilde{\tau}^*(\gamma)\big) - R_2^*(\tau_0^*)\big] < 0$. It follows that if type $\gamma > \underline{\gamma}$ obtains τ^{io} with probability one in equilibrium, then so do all types $\gamma' < \gamma$. In such an equilibrium, therefore, either all types of Foreign government obtain τ^{io} with certainty; or there is a threshold $\hat{\gamma}$ such that all types $\gamma < \hat{\gamma}$ obtain τ^{io} with certainty, and all types $\gamma > \hat{\gamma}$ make a demand rejected with positive probability — if they made successful demands with probability one, those demands would have to be greater than τ^{io} and, consequently, they could profitably deviate by mimicking types smaller than $\hat{\gamma}$. For the latter case to constitute an equilibrium, the Home government must prefer τ^{io} to a trade war for all types $\gamma < \hat{\gamma}$, but a trade war to τ^{io} for types $\gamma > \hat{\gamma}$, which is impossible (recall that $T(\cdot)$ is strictly decreasing). It follows that in any equilibrium, either all types of Foreign government obtain concession τ^{io} (possibly after an IO ruling) with probability one, or all types end up in a trade war, or some types make successful demands with positive probability, which are all strictly greater than τ^{io}. In the latter case, the same logic as in the proof of Proposition 1 above applies: such equilibria must involve pooling by the smaller types and are consequently eliminated by criterion D1.

We saw in the proof of Proposition 2 in the main text that a (pure strategy) equilibrium in which all types of Foreign government successfully demand τ^{io} exists if $\tau^{io} \geq T(F_0)$. As explained above, if the Home government rejects a demand τ and then complies with the IO ruling, then $\tau = \tau^{io}$. Therefore, in any equilibrium in which the Home government always implements τ^{io}, all types types of Foreign government demand τ^{io}. This in turn implies that the Home government's beliefs after observing demand τ^{io} must be $T(F_0)$. As it either accepts τ^{io} or complies wit the IO ruling, we must have $\tau^{io} \geq T(F_0)$.

Finally, as we saw in the proof of Proposition 2 in the main text, an equilibrium in which a trade war occurs with probability one for all types of Foreign government exists if $\tau^{io} < T(F_0)$.

C The $T^*(\underline{\gamma}) > T(\underline{\gamma})$ Case

As explained in the paper, we focused on cases where $T^*(\underline{\gamma}) < T(\underline{\gamma})$ because equilibrium existence problems arise when this condition does not hold: If $T^*(\underline{\gamma}) > T(\underline{\gamma})$,[1] then in the absence of an IO, pure strategy equilibria do not exist and existence of mixed strategy equilibria may require more restrictions on the parameters of the model. This section characterizes (mixed strategy) equilibria in the $T^*(\underline{\gamma}) > T(\underline{\gamma})$ case, and shows that our main conclusions remain valid in the specific cases where an equilibrium exists. Henceforth, we assume that $T^*(\underline{\gamma}) > T(\underline{\gamma})$, and we allow governments to use mixed strategies.

Consider first an equilibrium of the model without the IO. We saw in the proof of Proposition 1 above that, *irrespective of the relationship between $T^*(\underline{\gamma})$ and $T(\underline{\gamma})$*, we can partition $[\underline{\gamma}, \overline{\gamma}]$ into (possibly degenerate) intervals $\{\Gamma_k\}_{k\in K}$, for some (possibly uncountable) index set K, such that: (i) each type in the interior of Γ_k makes the same demand τ_k, which is accepted by the Home government with some probability $\alpha_k \in [0,1]$; and (ii) $\tau_k > \tau_{k'}$ and $\alpha_k \geq \alpha_{k'}$ for all $k' > k$. Moreover, the type-$\underline{\gamma}$ Foreign government must make a demand that is accepted with a positive probability; otherwise, it could profitably deviate by successfully demanding $\big(T^*(\underline{\gamma}) + T(\underline{\gamma})\big)/2 > T(\underline{\gamma})$. By the same logic as in the $T^*(\underline{\gamma}) < T(\underline{\gamma})$ case, there cannot be an equilibrium in which all types in a non-degenerate interval that includes $\underline{\gamma}$ pool. It follows that type $\underline{\gamma}$ must be the unique type making the highest demand. As this demand, denoted by $\underline{\tau}$, is accepted with positive probability by the Home government, it must be greater than or equal to $T(\underline{\gamma})$. If $\underline{\tau}$ is accepted with a probability $\alpha < 1$, then it must be equal to $T(\underline{\gamma})$ — otherwise the Home government would not be indifferent — and the type-$\underline{\gamma}$ Foreign government's payoff must be equal to $\alpha W^*\big(T(\underline{\gamma}), \tau_0^*, \underline{\gamma}\big) + (1-\alpha)\widetilde{W}^*(\underline{\gamma})$. This in turn implies that the latter can profitably deviate to $\underline{\tau} + \varepsilon > T(\underline{\gamma})$, for ε sufficiently small. Indeed, its payoff would then be $W^*\big(T(\underline{\gamma}) + \varepsilon, \tau_0^*, \underline{\gamma}\big) \approx W^*\big(T(\underline{\gamma}), \tau_0^*, \underline{\gamma}\big) > \alpha W^*\big(T(\underline{\gamma}), \tau_0^*, \underline{\gamma}\big) + (1-\alpha)\widetilde{W}^*(\underline{\gamma})$, where the strict inequality follows from $T^*(\underline{\gamma}) > T(\underline{\gamma})$ and $\alpha < 1$. Thus, the type-$\underline{\gamma}$ Foreign government's demand $\underline{\tau}$ must be accepted with probability one, which in turn implies that $\underline{\tau} = T(\underline{\gamma})$; otherwise it could profitably deviate by successfully demanding $\tau' = \big[\underline{\tau} + T(\underline{\gamma})\big]/2 < \underline{\tau}$, thereby obtaining a payoff of $W^*(\tau', \tau_0^*, \underline{\gamma}) > W^*(\underline{\tau}, \tau_0^*, \underline{\gamma})$.

We have established that in any equilibrium, the type-$\underline{\gamma}$ Foreign government reveals its type by demanding $T(\underline{\gamma})$. Our next step is to show that in any equilibrium, every type γ that obtains a concession with positive probability must

[1] We leave aside the knife-edge case where $T^*(\underline{\gamma}) = T(\underline{\gamma})$ because it only creates expositional complications without affecting the main results. For example, the statement of Proposition 1 would read "the Foreign government fails to obtain a concession with probability one" instead of "the Foreign government always fails to obtain a concession." There would indeed be an additional separating equilibrium in which the type-$\underline{\gamma}$ Foreign government, indifferent between successfully demanding $T^*(\underline{\gamma})$ and a trade war, would demand $T^*(\underline{\gamma})$, which would be accepted by the (indifferent) Home government; so that a trade war would be avoided in the measure-zero event $\{\gamma = \underline{\gamma}\}$.

separate from the other types and demand $T(\gamma)$. To do so, it suffices to prove that if all types γ in an interval $[\underline{\gamma}, \hat{\gamma}_0]$, $\hat{\gamma}_0 \geq \underline{\gamma}$, separate by demanding $T(\gamma)$, then there cannot be an interval of the form $(\hat{\gamma}_0, \hat{\gamma}_1]$ or $(\hat{\gamma}_0, \hat{\gamma}_1)$, in which all types make the same demand, accepted with positive probability.

Claim: Let $\underline{\gamma} \leq \hat{\gamma}_0 < \hat{\gamma}_1$. Consider an equilibrium in which every type γ in $[\underline{\gamma}, \hat{\gamma}_0]$ demands $T(\gamma)$, and all types in $(\hat{\gamma}_0, \hat{\gamma}_1)$ make the same demand, say τ, accepted with positive probability. Then, there exists $\varepsilon > 0$ such that reasonable beliefs assign zero probability to all types $\gamma < \hat{\gamma}_1$ following the (off-the-equilibrium-path) demand $\tau - \varepsilon$.

PROOF: Observe first that there exists a sufficiently small $\varepsilon > 0$ such that the demand $\tau - \varepsilon$ must be made off the path in any equilibrium satisfying the conditions of the claim. Indeed, it follows from the previous paragraphs that any demand $\tau' < \tau$ must be made by types $\gamma' \leq \hat{\gamma}_1$ on the path. Given that $\tau' < T(\underline{\gamma})$ must be accepted with a probability strictly less than one (and T is strictly decreasing), we must then have $\tau' \leq T(\hat{\gamma}_1) < T(F) = \tau$, where F represents the Home government's beliefs after observing demand τ. This in turn implies that any $\tau - \varepsilon \in \big(T(\hat{\gamma}_1), \tau\big)$ cannot be demanded on the equilibrium path.

Suppose the Home government observes a deviation to a demand $\tau - \varepsilon$. For every $\gamma \in (\hat{\gamma}_0, \hat{\gamma}_1)$, the Home government mixed best response α makes the type-γ Foreign government prefer $\tau - \varepsilon$ to its equilibrium demand τ if and only if

$$(25) \quad \alpha W^*(\tau - \varepsilon, \tau_0^*, \gamma) + (1 - \alpha)\widetilde{W}^*(\gamma) \geq \alpha_0 W^*(\tau, \tau_0^*, \gamma) + (1 - \alpha_0)\widetilde{W}^*(\gamma) \ ,$$

where $\alpha_0 \in (0, 1)$ is the equilibrium probability that the Home government concedes to τ. This inequality can be rewritten as

$$(26) \quad \alpha \geq \bar{\alpha}(\gamma) \equiv \alpha_0 \frac{W^*(\tau, \tau_0^*, \gamma) - \widetilde{W}^*(\gamma)}{W^*(\tau - \varepsilon, \tau_0^*, \gamma) - \widetilde{W}^*(\gamma)} \ , \text{for all } \gamma \in (\hat{\gamma}_0, \hat{\gamma}_1) \ .$$

By the same logic as in the proof of Proposition 1 in the main text, $\bar{\alpha}$ is a strictly decreasing and, consequently, $\left[D\left(\gamma, \tau - \varepsilon\right) \cup D^0\left(\gamma, \tau - \varepsilon\right)\right] \subset D\left(\gamma', \tau - \varepsilon\right)$, for all $\gamma, \gamma' \in (\hat{\gamma}_0, \hat{\gamma}_1)$ such that $\gamma < \gamma'$. Hence, all types $\gamma \in (\hat{\gamma}_0, \hat{\gamma}_1)$ must be deleted.

To complete the proof of the claim, it remains to establish that all types $\gamma \leq \hat{\gamma}_0$ must also be deleted. By continuity of W^* in γ, the type-$\hat{\gamma}_0$ Foreign government must be indifferent between demanding $T(\hat{\gamma}_0)$ and τ — otherwise some types in a neighborhood of $\hat{\gamma}_0$ could profitably deviate by demanding $T(\hat{\gamma}_0)$ instead of τ. We must then have

$$(27) \quad \alpha_0 W^*(\tau, \tau_0^*, \hat{\gamma}_0) + (1 - \alpha_0)\widetilde{W}^*(\hat{\gamma}_0) = \alpha(\hat{\gamma}_0) W^*\big(T(\hat{\gamma}_0), \tau_0^*, \hat{\gamma}_0\big) + \big[1 - \alpha(\hat{\gamma}_0)\big]\widetilde{W}^*(\hat{\gamma}_0)$$

or, equivalently,

$$\alpha_0 = \alpha(\hat{\gamma}_0)\frac{W^*\big(T(\hat{\gamma}_0), \tau_0^*, \hat{\gamma}_0\big) - \widetilde{W}^*(\hat{\gamma}_0)}{W^*(\tau, \tau_0^*, \hat{\gamma}_0) - \widetilde{W}^*(\hat{\gamma}_0)} \ , \tag{28}$$

where $\alpha(\hat{\gamma}_0)$ is the equilibrium probability that type $\hat{\gamma}_0$'s demand is accepted. Recall that, for all τ_1, τ_2 and $\alpha_1 < \alpha_2$, the difference $\alpha_1 W^*(\tau_1, \tau_0^*, \gamma) - \alpha_2 W^*(\tau_2, \tau_0^*, \gamma) + (\alpha_2 - \alpha_1)\widetilde{W}^*(\gamma)$ is strictly increasing in γ (see the proof of Proposition 1 for the $T^*(\underline{\gamma}) < T(\underline{\gamma})$ case above). Therefore, for all $\gamma \in [\underline{\gamma}, \hat{\gamma}_0]$, we have

$$\begin{aligned} \alpha(\gamma)\big[W^*\big(T(\gamma), \tau_0^*, \gamma\big) - \widetilde{W}^*(\gamma)\big] \quad &> \\ &= \quad \alpha_0\big[W^*(\tau, \tau_0^*, \gamma) - \widetilde{W}^*(\gamma)\big] \\ &= \quad \alpha(\hat{\gamma}_0)\frac{W^*\big(T(\hat{\gamma}_0), \tau_0^*, \hat{\gamma}_0\big) - \widetilde{W}^*(\hat{\gamma}_0)}{W^*(\tau, \tau_0^*, \hat{\gamma}_0) - \widetilde{W}^*(\hat{\gamma}_0)}\big[W^*(\tau, \tau_0^*, \gamma) - \widetilde{W}^*(\gamma)\big], \end{aligned} \tag{29}$$

where $\alpha(\gamma)$ is the equilibrium probability that type γ's demand is accepted. Rearranging terms, this yields

$$\alpha(\gamma)\frac{W^*\big(T(\gamma), \tau_0^*, \gamma\big) - \widetilde{W}^*(\gamma)}{W^*(\tau, \tau_0^*, \gamma) - \widetilde{W}^*(\gamma)} > \alpha(\hat{\gamma}_0)\frac{W^*\big(T(\hat{\gamma}_0), \tau_0^*, \hat{\gamma}_0\big) - \widetilde{W}^*(\hat{\gamma}_0)}{W^*(\tau, \tau_0^*, \hat{\gamma}_0) - \widetilde{W}^*(\hat{\gamma}_0)} = \alpha_0 \ . \tag{30}$$

Now take an arbitrary $\hat{\gamma} \in (\hat{\gamma}_0, \hat{\gamma}_1)$. As $\alpha_0 > \bar{\alpha}(\hat{\gamma})$, it follows from the above inequality that

$$\alpha(\gamma)\frac{W^*\big(T(\gamma), \tau_0^*, \gamma\big) - \widetilde{W}^*(\gamma)}{W^*(\tau, \tau_0^*, \gamma) - \widetilde{W}^*(\gamma)} > \bar{\alpha}(\hat{\gamma}) \ , \tag{31}$$

for all $\gamma \in [\underline{\gamma}, \hat{\gamma}_0]$. By continuity of W^* in τ, this in turn implies that there exists a sufficiently small $\varepsilon > 0$ such that

$$\alpha(\gamma)\frac{W^*\big(T(\gamma), \tau_0^*, \gamma\big) - \widetilde{W}^*(\gamma)}{W^*(\tau - \varepsilon, \tau_0^*, \gamma) - \widetilde{W}^*(\gamma)} > \bar{\alpha}(\hat{\gamma}) \ , \tag{32}$$

for all $\gamma \in [\underline{\gamma}, \hat{\gamma}_0]$.

Now, for every $\gamma \in [\underline{\gamma}, \hat{\gamma}_0]$, the Home government mixed best response α makes the type-γ Foreign government prefer $\tau - \varepsilon$ to its equilibrium demand $T(\gamma)$ if and only if

$$\alpha W^*(\tau - \varepsilon, \tau_0^*, \gamma) + (1 - \alpha)\widetilde{W}^*(\gamma) \geq \alpha(\gamma)W^*\big(T(\gamma), \tau_0^*, \gamma\big) + \big[1 - \alpha(\gamma)\big]\widetilde{W}^*(\gamma) \tag{33}$$

or, equivalently,

$$\alpha \geq \bar{\alpha}(\gamma) \equiv \alpha(\gamma)\frac{W^*\big(T(\gamma), \tau_0^*, \gamma\big) - \widetilde{W}^*(\gamma)}{W^*(\tau - \varepsilon, \tau_0^*, \gamma) - \widetilde{W}^*(\gamma)} > \bar{\alpha}(\hat{\alpha}) \ . \tag{34}$$

We conclude that $\left[D\left(\gamma, \tau - \varepsilon\right) \cup D^0\left(\gamma, \tau - \varepsilon\right)\right] \subset D\left(\hat{\gamma}, \tau - \varepsilon\right)$, for all $\gamma \in [\underline{\gamma}, \hat{\gamma}_0]$ and, consequently, that all types in $[\underline{\gamma}, \hat{\gamma}_0]$ must be deleted. □

It follows from the claim that types in $(\hat{\gamma}_0, \hat{\gamma}_1)$ that are sufficiently close to $\hat{\gamma}_1$ can profitably deviate by making demand $\tau - \varepsilon$, thus obtaining a better concession with probability one. This proves that in equilibrium, every type γ that obtains a concession with positive probability must separate from the other types and demand $T(\gamma) \leq T^*(\gamma)$. Now let γ^{sup} be the supremum of the set of types that obtain a concession with positive probability. By continuity of W^* in γ, either $\gamma^{\text{sup}} = \overline{\gamma}$, or type $\gamma^{\text{sup}} < \overline{\gamma}$ is indifferent between obtaining concession $T(\gamma^{\text{sup}})$ and a trade war (i.e., $T(\gamma^{\text{sup}}) = T^*(\gamma^{\text{sup}})$). In the former case, every type γ makes demand $T(\gamma) < T^*(\gamma)$ that is accepted with probability $\alpha(\gamma) > 0$ — except possibly type $\overline{\gamma}$ which is indifferent if $T(\overline{\gamma}) = T^*(\overline{\gamma})$. In the latter case, every type $\gamma < \gamma^{\text{sup}}$ makes demand $T(\gamma) < T^*(\gamma)$ that is accepted with probability $\alpha(\gamma) > 0$, and all the other types make demands that are rejected with probability one — except possibly type $\overline{\gamma}$ which is indifferent. The probability-of-acceptance function $\alpha\colon [\underline{\gamma}, \gamma^{\text{sup}}) \to [0, 1]$ must be selected in such a way that the following conditions hold: (i) $\alpha(\cdot)$ is strictly decreasing; (ii) $\alpha(\underline{\gamma}) = 1$; and (iii) every $\gamma \in [\underline{\gamma}, \gamma^{\text{sup}})$ is a solution to $\max_{\gamma'} \alpha(\gamma')W^*\big(T(\gamma'), \tau_0^*, \gamma\big) + \big[1 - \alpha(\gamma')\big]\widetilde{W}^*(\gamma)$. If such a function exists, then it is possible to construct a fully separating equilibrium, in which: the type-γ Foreign government demands $T(\gamma)$ if $\gamma \in [\underline{\gamma}, \gamma^{\text{sup}})$, and $T(\gamma) - \kappa$ for some $\kappa > 0$ otherwise; the Home government concedes to demand τ with probability one if $\tau \geq T(\underline{\gamma})$, with probability $\alpha(\gamma)$ if $\tau = T(\gamma)$ for all $\gamma \in [\underline{\gamma}, \gamma^{\text{sup}})$, and with probability zero for all $\tau \leq T(\gamma^{\text{sup}})$. For example, the function $\alpha\colon [\underline{\gamma}, \gamma^{\text{sup}}) \to [0, 1]$, defined by

$$\alpha(\gamma) \equiv \exp\left\{-\int_{\underline{\gamma}}^{\gamma} \frac{W_\tau^*\big(T(x), \tau_0^*, x\big)T'(x)}{W^*\big(T(x), \tau_0^*, x\big) - \widetilde{W}^*(x)} dx\right\} \ , \tag{35}$$

satisfies conditions (i) and (ii), as well as the first-order condition of the maximization problem in (iii). Additional delicate conditions on the curvatures of the functions W and W^* are required to guarantee that it also satisfies second-order conditions. In any case, the following variant on Proposition 1 follows from the discussion above.

PROPOSITION 1'. — *Suppose that there is no IO, and $T(\underline{\gamma}) < T^*(\underline{\gamma})$. There does not exist a pure strategy equilibrium. Furthermore, in any mixed strategy* *equilibrium, a trade war occurs with positive probability whenever the Foreign government's type exceeds $\underline{\gamma}$.*

Put differently, a trade war cannot be avoided with certainty unless the the Foreign government's type is exactly equal to $\underline{\gamma}$, which is a probability-zero event.

We now turn to the model with the IO. Suppose $\tau^{io} > T(F_0)$. (As in the $T^*(\underline{\gamma}) < T(\underline{\gamma})$ case, our conclusions may not hold for the knife-edge value $\tau^{io} = T(F_0)$ because we no longer break ties by assuming that the Home government chooses to comply when indifferent between τ^{io} and a trade war: there may also be equilibria in which the indifferent Home government opts for a trade war with probability one.) We showed in the proof of Proposition 2 for the $T^*(\underline{\gamma}) < T(\underline{\gamma})$ case (see above) that, irrespective of the relationship between $T^*(\underline{\gamma})$ and $T(\underline{\gamma})$, there can only be three types of equilibria in this model: either (i) all types of Foreign government obtain concession τ^{io} (possibly after an IO ruling) with probability one; or (ii) all types end up in a trade war; or (iii) some types make successful demands $\tau > \tau^{io}$ with positive probability, but no type obtains τ^{io} with certainty. As the equilibrium construction in the proof of Proposition 2 in the main text does not depend on the relationship between $T^*(\underline{\gamma})$ and $T(\underline{\gamma})$, there exists a pure strategy equilibrium of type (i). Moreover, there cannot be an equilibrium of type (ii): as $T^*(\underline{\gamma}) > T(\underline{\gamma})$, the type-$\underline{\gamma}$ could profitably deviate by making a successful demand $\tau \in \big(T(\underline{\gamma}), T^*(\underline{\gamma})\big)$. To obtain an equivalent to Proposition 2 for the $T^*(\underline{\gamma}) > T(\underline{\gamma})$ case, therefore, it remains to establish that there cannot be an equilibrium of type (iii).

Consider any equilibrium σ in which some types make successful demands $\tau > \tau^{io}$ with positive probability, but no type obtains τ^{io} with certainty. Observe that if the Home government randomizes between accepting a demand τ and rejecting it to comply with the IO ruling with positive probability, then $\tau = \tau^{io}$. Hence, rejection of any demand $\tau > \tau^{io}$ must be followed by a trade war with certainty. In equilibrium σ, τ^{io} cannot be the only demand made and conceded to with positive probability $\beta < 1$. Otherwise, either all types would demand τ^{io}, or only the types below some threshold $\hat{\gamma}$ would demand τ^{io} and the other types end up in a trade war. In the former case, as $\tau^{io} > T(F_0)$, the Home government would not be indifferent between complying to the ruling τ^{io} and a trade war, and would therefore deviate by complying with probability one. In the latter case, optimality of the Home government's response would require that $T(\hat{\gamma}) \leq \tau^{io} < T(\overline{\gamma})$, which is impossible since T is strictly decreasing. (Note that we must have $\hat{\gamma} < \overline{\gamma}$ in this case; otherwise, the Home government's beliefs would be $T(F_0) < \tau^{io}$ and types $\gamma < \hat{\gamma} = \overline{\gamma}$ would obtain τ^{io} with certainty).

It follows from the previous paragraph that, in equilibrium σ, some types of Foreign government make demands $\tau > \tau^{io}$ that are accepted with positive probability, and the other types either demand τ^{io} (and obtain it with positive probability) or end up in a trade war with certainty. Note that if the type-γ Foreign government demands τ^{io}, then she receives $\beta W^*(\tau^{io}, \tau_0^*, \gamma) + (1-\beta)\widetilde{W}^*(\gamma)$, where

$(1-\beta) \in (0,1)$ is the equilibrium probability that the Home government rejects the demand and does not comply with the IO ruling. From the point of view of the Foreign government's payoffs, this is thus equivalent to making demand τ^{io}, accepted with probability β and always followed by a trade war when rejected. By the same logic as in the model without the IO, the equilibrium must then have the following structure: there is a threshold $\gamma^{\sup}$ such that every type $\gamma < \gamma^{\sup}$ makes demand $T(\gamma) \geq \tau^{io}$ that is accepted with a decreasing probability $\alpha(\gamma)$, and all types $\gamma > \gamma^{\sup}$ (if any) end up in a trade war with probability one. As $T(\overline{\gamma}) < T(F_0) < \tau^{io}$, we must have $\gamma^{\sup} < \overline{\gamma}$. Let $\widehat{F}_0$ the distribution of types conditional on $\{\gamma \geq \gamma^{\sup}\}$. As F_0 has full support (and T is strictly decreasing), $T(\widehat{F}_0) < T(F_0)$. Among the equilibrium demands that are always rejected, there must then be at least one, say τ, such that the Home government's beliefs after observing τ, F_τ, satisfy $T(F_\tau) \leq T(\widehat{F}_0) < T(F_0)$. As $\tau^{io} > T(F_0)$, this in turn implies that the Home government can profitably deviate by complying with the IO ruling after rejecting τ. This proves that if $\tau^{io} > T(F_o)$, then only equilibria of type (i) can exist: tariff τ^{io} is always implemented and that a trade war never occurs in equilibrium.

If $\tau^{io} < T(F_0)$, then equilibria of types (i) and (ii) cannot exist: if all types obtain τ^{io}, then they must all demand $\tau^{io} < T(F_0)$ and, therefore, the Home government strictly prefers a trade war; if all types end up in a trade war, then type-$\underline{\gamma}$ can profitably deviate by making a successful demand $\tau \in \big(T(\underline{\gamma}), T^*(\underline{\gamma})\big)$. Equilibria of type (iii) may exist. If $\tau^{io} < T(\overline{\gamma})$, then we have the same equilibrium outcome as in the absence of an IO. If $\tau^{io} \in \big(T(\overline{\gamma}), T(F_0)\big)$, then it follows from the previous paragraph that an equilibrium can only exist if $\gamma^{\sup}$ (as defined above) satisfies the following conditions: (a) $T^*(\gamma) \geq T(\gamma)$ for all $\gamma < \gamma^{\sup}$ (otherwise, type γ would better off making an unacceptable demand to induce a trade war with certainty); and (b) we can partition $(\gamma^{\sup}, \overline{\gamma}]$ into subsets $\{\Gamma_\ell\}$ such that, for all ℓ, the distribution of types conditional on $\{\tilde{\gamma} \in \Gamma_\ell\}$, F_ℓ, satisfies $T(F_\ell) \geq \tau^{io}$ — i.e., given its updated beliefs, the Home government prefers a trade war to compliance with the IO ruling.

PROPOSITION 2'. — *Suppose the Foreign government is fully committed to the IO, and $T(\underline{\gamma}) < T^*(\underline{\gamma})$. Then:*

(i) If $\tau^{io} > T(F_0)$, then an equilibrium exists, and a trade war never occurs in equilibrium: Either the Foreign government obtains concession τ^{io}, or it makes an unsuccessful demand following which the Home government complies with the IO ruling.

(ii) If $\tau^{io} < T(F_0)$, then a trade war occurs with positive probability in any equilibrium.

Coupled, Propositions 1' and 2' show that the main conclusions of the paper carry over to the $T(\underline{\gamma}) < T^*(\underline{\gamma})$ case, explaining how IOs without enforcement power can be effective in preventing trade wars, and why sender governments are more likely to obtain concessions with multilateral coercion than with unilateral coercion.

The "negative" conclusion in the case where the Foreign government is only partially committed to the IO also carries over to the $T(\underline{\gamma}) < T^*(\underline{\gamma})$ case. By the same logic as in the no-commitment case above, existence of an equilibrium requires further conditions on the primitives of the model. When an equilibrium exists, however, we obtain again that a trade war can only be avoided with certainty in the probability-zero event where the Foreign government type is exactly $\underline{\gamma}$.

PROPOSITION 3'. — *Suppose the Foreign government is only partially committed to the IO, and $T(\underline{\gamma}) < T^*(\underline{\gamma})$. Then in any mixed strategy equilibrium, a trade war occurs with positive probability whenever the Foreign government's type exceeds $\underline{\gamma}$.*

To see this, suppose toward a contradiction that, in some equilibrium, there is a set of Foreign government types $\widehat{\Gamma} \neq \{\underline{\gamma}\}$ that never end up in a trade war. As established above, this implies either that $\widehat{\Gamma} = [\underline{\gamma}, \hat{\gamma}]$ or that $\widehat{\Gamma} = [\underline{\gamma}, \hat{\gamma})$, for some $\hat{\gamma} \in (\underline{\gamma}, \overline{\gamma}]$. Let $\hat{\tau}$ be the unique tariff that is implemented when the Foreign government's type is in $\widehat{\Gamma}$. Observe that types in $\widehat{\Gamma}$ may not pool: a successful demand $\hat{\tau}$ may be either made unilaterally or multilaterally; and, if $\hat{\tau} = \tau^{io}$, different unsuccessful multilateral demands may be followed by compliance with the IO ruling τ^{io}. Let $\mathcal{F}$ be the set of updated beliefs that the Home government may hold after observing the demands made in equilibrium by types in $\widehat{\Gamma}$. As these demands are never followed by a trade war, we must have $\hat{\tau} \geq \sup\{T(\widehat{F})\colon \widehat{F} \in \mathcal{F}\} > T(\hat{\gamma})$, where the second inequality follows from the fact that T is strictly decreasing (and, therefore, $T(\gamma) > T(\hat{\gamma})$ for all $\gamma \in [\underline{\gamma}, \hat{\gamma})$). By construction, any unilateral demands $\tau < \hat{\tau}$ made by types $\gamma \notin \widehat{\Gamma}$ (if any such type exists) must be unsuccessful with positive probability. As $\gamma \geq \hat{\gamma}$ for all $\gamma \notin \widehat{\Gamma}$, we thus have $\tau \leq T(\hat{\gamma}) < \hat{\tau}$. It follows that a unilateral demand $\tau' \in \big(T(\hat{\gamma}), \hat{\tau}\big)$ must be off the equilibrium path. We know from the analysis above that, following this demand, criterion D1 requires the Home government beliefs to assign zero probability to all types $\gamma < \hat{\gamma}$. The Home government's equilibrium strategy must therefore prescribe it to concede with probability one, thus making unilateral demand τ' a profitable deviation for Foreign government types in $\widehat{\Gamma}$ and yielding the desired contradiction.