

# Parity-Time coupled microresonators: Kramers-Kronig limitation

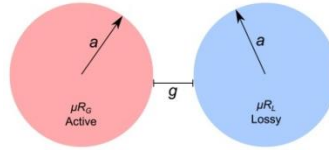
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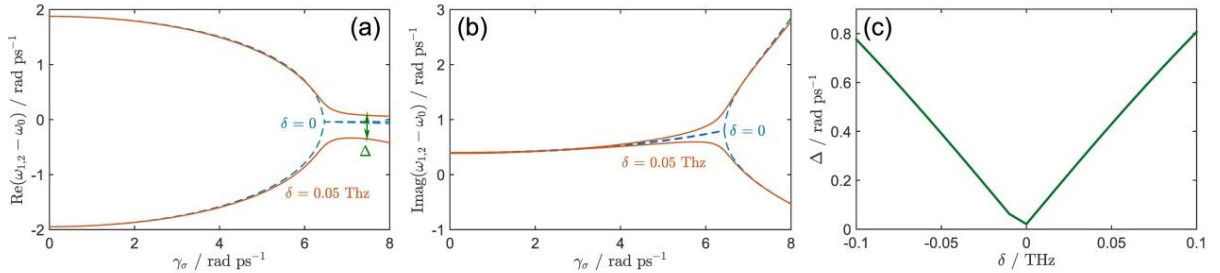
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New complex photonics structures, namely parity-time (PT)-symmetric structures, have inspired the innovative exploration of photonic structures with combined gain and loss. PT-symmetric structures are identified by their signature exceptional point (EP); operation below this point yields a completely real eigenmode/frequency, but above this point the eigen-frequencies are complex conjugates [1], [2]. Several PT-symmetric structures, such as coupled waveguides and Bragg gratings, have been studied theoretically and experimentally, and exhibit exciting behaviour and components such as unidirectional loss-induced transparency, a coherent perfect absorber laser, and a photonic analogue of a topological insulator.



**Fig. 1** Illustration of coupled micro-resonators, each with radius  $a$ , separated with a gap  $g$  and surrounded by air.  $\mu R_G$  acts as an active resonator and  $\mu R_L$  as a lossy resonator.

In passive structures, the relation between dispersion and losses is described by the Kramers-Kronig equations (KKE). Recently, the validity of those equations has been confirmed for PT-symmetric structures under a broadband excitation[3]. In this talk, we use a KKE argument to show that the presence of dispersion alters the eigen-frequencies of coupled PT coupled microresonators (Fig. 1), so that their eigen-frequencies are no longer complex conjugates after the EP. In a dispersive material the gain/loss spectra align to the resonant frequency only at a single frequency. Fig. 2(a) shows the real part of the eigen-frequencies of mismatched PT-coupled micro-resonators for material with gain/loss spectra centred at  $\omega_\sigma = 2\pi(f_0 + \delta)$ , for  $\delta = 0.05$  THz. It is noted that the coupled system no longer has an EP; the imaginary part of the eigen-frequencies splits early compared to the matched case,  $\delta = 0$ . The difference in the real part of eigen-frequency for a fixed gain/loss parameter  $\gamma_\sigma = 7.5$  rad/ps, is denoted by  $\Delta$  (see Fig. 2(a)) and is shown in Fig. 2(c). The figure shows that the operation of PT-coupled micro-resonators with practical parameters requires that the gain/loss spectra align with the resonant frequency of the resonator.



**Fig. 2(a,b)** Real and imaginary part of the eigen-frequencies of PT-coupled microresonators with mismatch between the material atomic transitional frequency and the resonant frequency where  $\omega_\sigma = 2\pi(f_0 + \delta)$ ; (c) the difference of real part of eigen-frequency  $\Delta$  (see Fig. 2(a)) for a fixed value of gain/loss parameter  $\gamma_\sigma = 7.5$  rad/ps and different mismatch parameter  $\delta$ . Operation in the (7,2) mode with resonant frequency  $f_0^{(7,2)} = 341.59$  THz.  $a = 0.24 \mu\text{m}$  and  $g = 0.24 \mu\text{m}$ . For material model refer to [4].

## References

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