Optimal Public Funding for Research: A Theoretical Analysis*

Gianni De Fraja†

University of Nottingham

Università di Roma “Tor Vergata”

and C.E.P.R.

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Abstract

This article studies how a government should distribute funds among research institutions and how it should allocate them to basic and applied research. Institutions differ in reputation and efficiency, and have an information advantage. The government should award funding for basic research to induce the most productive institutions to carry out more applied research than they would like. Institutions with better reputation do more research than otherwise identical ones, and applied research is inefficiently concentrated in the most efficient high reputation institutions. The article provides theoretical support for a dual channel funding mechanism, but not for full economic costing.

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†Nottingham School of Economics, Sir Clive Granger Building, University Park, Nottingham, NG7 2RD, UK, Università di Roma “Tor Vergata”, Dipartimento di Economia e Finanza, Via Columbia 2, I-00133 Rome, Italy, and C.E.P.R., 90-98 Goswell Street, London EC1V 7DB, UK; email: `gianni.defraja@nottingham.ac.uk`.

1 Introduction

Government expenditure on scientific research in the OECD countries amounts to around 0.8% of GDP, with peaks close or above 1% in the US, Sweden, Austria and Korea (OECD 2013, NSF 2012). These large sums of taxpayers' money are used to fund a wide variety of different institutions: public as well as private universities, much of whose research is publicly funded, but also dedicated research centres within the government and the armed forces, and firms, non-profit and possibly other organisations, which receive direct subsidies or tax incentives. This variety raises an immediate efficiency question. How should the total funding be shared among institutions, given that their reputation and potential for successful research may differ widely?

Also varied is the link between the funds provided and their use: in the UK, roughly 2/3 of the total government funding is distributed to institutions in consideration of past achievements, to spend as they see fit, the remaining 1/3 is grant funding, firmly linked to specific research projects. Is this proportion right, or would it be possible to re-allocate funding from one spending method to another and improve its impact on society? Moreover, grant funding is more concentrated: the top 25 UK universities received 85% of the aggregate research grant funding, and only 75% of the total quality related funding. Is this difference justified?

Along with the characteristics of the recipients and the mechanics of its funding, research also varies according to its nature, applied or basic. Some research benefits society in a concrete and tangible way; other research improves the "scientific climate" in society without an identifiable explicit benefit. Clearly there is a degree of arbitrariness in any binary attribution, and exceptions and special cases can always be found, but both the agencies – such as the US National Science Foundation – whose job it is to classify research into "applied" and "basic", and the existing academic literature see basic research (or fundamental, pure, curiosity-driven, upstream, unpredictable, Strandburg 2005) as driven by scientists' curiosity, its aim to acquire knowledge for knowledge's sake, unlike applied research, which is instead designed to solve well-defined practical problems.[1] Developed countries spend around one fifth of their R&D expenditure on basic research (Gersbach 2009). Is this a "good" ratio? More generally, should funding agencies be concerned with the nature, basic or applied, of the research carried out, or should they leave the choice to the institutions, which, after all, know more about research?

These are important questions, and yet the topic is barely touched in the literature. In this article, I aim to help fill this gap, and to provide a theoretical framework to address them: my intention here is to lay the foundations for a theory of the optimal public funding of research.

My approach is microeconomic: I leave the macroeconomic aspect of total spending in the background, and concentrate instead on the balance between basic and applied research and on the distribution of funding among different research institutions. I build a model based on two broad assumptions. Firstly, the *government's and the institutions' preferences regarding the type of research to be carried out are not perfectly aligned.* Institutions care about their prestige and reputation, and because both applied and basic research are recognised in the research community, lead to prestigious publications, and may win prizes and awards, institutions do not typically favour one kind over the other. Although the government is conscious that both basic and applied are necessary, at the margin it has a preference for applied, "useful"

[1] The National Science Foundation defines "basic research [...] as systematic study directed toward fuller knowledge or understanding of the fundamental aspects of phenomena and of observable facts without specific applications towards processes or products in mind." Conversely, "applied research is defined as systematic study to gain knowledge or understanding necessary to determine the means by which a recognized and specific need may be met" (NSB 2008).

research, which has a direct measurable impact on quality of life and national income.

Misalignment of preferences affects policy when there is also asymmetric information, and my second assumption is that *institutions have more precise information than the government as to their relative ability to carry out basic and applied research.* This better knowledge of their strengths and weaknesses gives an information advantage to the institutions that are relatively more productive in applied research, the kind of research preferred by the government. In line with the standard principal-agent set-up, this superior knowledge translates into informational rent and causes a distortion.

I show in Proposition 1 that if institutions' technology were observable, the government would allocate research funding in such a way that the marginal cost of applied research is the same for all institutions, irrespective of their efficiency and of their reputation. This is natural: were it not so, the government could reallocate funding and reduce the overall cost of a given aggregate amount of applied research.

However, if an institution's productivity is private information, the government is unable to enforce this policy, and, as shown in Proposition 3, the interplay between preference misalignment and the institutions' information advantage causes a distortion. At the optimal second best policy, which is implemented with the offer of funding contracts based on the amount of applied research to be carried out, the marginal cost of applied research is higher in the institutions which are more efficient at it. This is inefficient, these institutions do too much applied research, and welfare would be higher if applied research could be reallocated to less efficient institutions. This second best policy is implemented by offering incentive funding to institutions willing to carry out higher levels of applied research; institutions which accept this incentive funding spend it on basic research. Thus, in the second best, basic and applied research are positively correlated across institutions, even in the absence of any modelling assumption imposing this. This is in line with casual observation: for example, if one proxies the amount of *applied research* carried out with the number of patents granted to an institution, and the *total of basic and applied research* with the number of faculties who are members or fellows of learned institutions and academies,[2] then, with values of the parameters chosen to match data from the US patents office and the NSF, the correlation between basic and applied research carried out by the top 200 US universities from 1989 to 2011 ranges between 0.272 and 0.587. Also tallying with stylised empirical facts is the result that, at the second best research policy, publicly funded institutions have a minimum size. This is not due to economies to scale, which again are ruled out by assumption, but is a consequence of asymmetric information: at the first best some institutions receive a vanishing small amount to pay for applied research.

In addition to their efficiency, institutions differ according to their prestige and reputation. Reputation being by its very nature observable, funding can be made conditional on it, even though it does not *per se* affect research prowess. Proposition 4 shows that, with the plausible assumption that institutions which are good at research are more likely to be found among the prestigious ones, the optimal second best policy is such that funding is biased towards institutions with better reputation: given two *equally efficient* institutions, the one with better reputation receives more funding and does more research. This policy implication of my theoretical model matches the practice of some countries to skew research funding towards high reputation institutions. Taken together, Propositions 3 and 4 show that the information disadvantage of the government leads applied research to be inefficiently

[2]Such as the National Academy of Sciences or of Engineering or the Institute of Medicine, see Capaldi Phillips et al (2013), for details.

over-concentrated in the most efficient institutions and in those with a better reputation. To the extent that more research leads to better reputation, then reputation and prestige are self-perpetuating in the optimal second best funding mechanism.

The theoretical analysis of the article can also contribute to the design of policy in respect of funding mechanisms used in practice by showing how the optimal funding can be implemented. In the second best policy I derive, a dual funding system suggests itself naturally: all institutions receive an identical "block grant", as long as they carry out a threshold level of applied research. The least efficient institutions which receive this "block grant" spend it all on the applied research they must do to qualify for any funding, and so do no basic research. More efficient institutions, which can fund this minimum amount of applied research by spending less than the block grant, can therefore use this difference to engage in basic research, which, at their chosen point on the menu of contracts, they prefer to applied research. Additional financial resources are made available to institutions through a second funding channel, which, like research grant funding in practice, is linked to specific research projects. To become eligible to apply for these funds, an institution must meet a target of applied research funded with its own block grant. This target is lower for higher reputation institutions, which are therefore treated more favourably in the government's second best policy. Interestingly, the additional funding is lower than the cost of the additional applied research to be carried out: institutions need to "co-fund" the applied research grants they obtain. This is in contrast to the "cost-plus" approach, labelled "full economic costing", adopted by funding agencies in the UK and elsewhere, which typically award research grants to cover not only the full marginal cost but also a share of the institution's fixed costs.

The article is organised as follows. Section 2 presents the model, and

Section 3 the results. Section 4 shows how the policy can be implemented in practice; some additional remarks are in Section 5, and Section 6 concludes. Mathematical proofs are in the Appendix.

2 The model

Private benefits and cost of research

I model the publicly funded research sector of an economy. A continuum of research institutions compete to be the recipients of this funding. Institutions differ along two dimensions: their reputation and their ability to spend research funding efficiently. In practice, of course, the reputation and the research efficiency of a research institution change as time passes, but in this article I concentrate on the short term static problem of allocating research funding to institutions of given reputation and efficiency.

The capacity to carry out research efficiently is measured by a technology parameter $\theta \in \left[\underline{\theta}, \overline{\theta}\right] \subseteq \mathbb{R}_{++}$, ordered so that "better" institutions have lower θ. As an example, one can think of θ as the ratio of failed to successful research projects: an institution with low θ is efficient in the specific sense that, perhaps because of the ability of its scientists and the quality of its research environment, it is more capable of assessing *ex-ante* the chances of success of a project, and so relatively few of its projects turn out to be flops.

Reputation is naturally an ordinal concept: it does not have an objective measure, but we can say that an institution has a better or worse reputation than another.[3] To capture this idea formally, I posit that institutions are

[3] A good reputation may follow from the membership of formal clubs, the UK "Russell group", or the attribution of informal labels, the top twenty or the Ivy league universities, or the position achieved in the ubiquitous rankings for universities and research institutions, such as the SCImago Ranking, and those prepared by government agencies, such as the RAE/REF classification for the UK universities.

classified into N "reputation" groups, which are fixed within the time frame considered.

An institution's reputation does not affect directly the efficiency with which it carries out research, though it conveys information about this efficiency: an institution with a higher reputation is more likely to be efficient. Formally, I assume that, in each period, the value of θ for an institution with reputation $i \in \{1, \ldots, N\}$ is drawn from a differentiable distribution function $F_i(\theta)$, with density $f_i(\theta) = F_i'(\theta) > 0$, and monotonic hazard rate $\frac{d}{d\theta}\left(\frac{F_i(\theta)}{f_i(\theta)}\right) > 0$ for $\theta \in (\underline{\theta}, \bar{\theta})$, $i = 1, \ldots, N$,[4] and I capture the correlation between efficiency and reputation with the following.

Assumption 1 *For any $h, \ell \in \{1, \ldots, N\}$, with $h > \ell$, $F_\ell(\theta) < F_h(\theta)$ for $\theta \in (\underline{\theta}, \bar{\theta})$; that is, $F_\ell(\theta)$ strictly[5] first order stochastically dominates $F_h(\theta)$.*

Assumption 1 would follow from the natural idea that scientists like to have able scientists as colleagues, and that, when they consider job offers, they proxy the quality of the scientists an institution currently employs with its reputation. Thus a high reputation institution will find it easier to attract good scientists, and hence to draw a low θ, than one with a less established reputation.[6] A further channel would be at work if θ is affected by an institution's ability to supplement government funding with funds from private sources. To the extent that some of these, such as income from endowments, or, for universities, alumni donations and students' tuition fees, are larger

[4]Bagnoli and Bergstrom (2005) discuss at length the properties of these functions, and give numerous examples which show that most commonly used distribution functions do satisfy the monotonic hazard rate property.

[5]The assumption could be weakened by allowing strict first order stochastic dominance only over a range. This would add only slightly longer statements of results, with no additional insight.

[6]A similar externality is in Palomino and Sákovics's (2004) model, where institutions (sports leagues in their article) compete with each other, and their members (the individual teams) prefer to belong to leagues with better teams.

in prestigious institutions, this strengthens the correlation between i and θ. Other sources of private sector funding, like the commercial exploitation of research,[7] are probably less closely linked to reputation. Given the result of this article that, at the optimal second best policy, high reputation institutions do *ceteris paribus* more research, this creates a “multiplier” effect for current research investment: Assumption 1 would thus be naturally incorporated in a fully-fledged formal dynamic model where current research success enhances future reputation.

I assume that the payoff of research institutions is an increasing function of the amount of research they carry out in the current period. This again is a reduced form simplification for a richer dynamic set-up where institutions care about the present value of current and expected future research. If current research enhances reputation and, through this, future payoff, then success begets success and current and future research are complements, so there is no intertemporal trade-off, making this a plausible simplification. The exact link between current research and the present value of an institution’s payoff, which depends in general on the link between current research success and future reputation, and on the institution’s discount factor and risk tolerance, can be left in the background, captured by a monotonic function of the current research, which I normalise to the identity without further loss of generality.

An important decision institutions take is the type of research they carry out. The focus of the article is on the choice between applied and basic research. I assume that institutions are indifferent between them. This is because they care about repute, publications in prestigious journals and other

[7] Since the Bayh-Dole Act, in 1980, US institutions may patent federally funded inventions (Thursby and Thursby 2003 for a detailed analysis), and the income they earn can be substantial: at the upper end, in 2010 MIT earned over 80 million dollars through its licensing office. Recent work suggests that the effect is petering out (Leydesdorff and Meyer 2010). Other countries have systems in place similarly aiming to encourage patentable research in universities (Mowery and Sampat 2005).

signifiers of success, such as prizes and honours bestowed on its members, and these are all brought equally by basic and by applied successful research.

To sum up, an institution's payoff function can be written as

$$r_i(\theta) = a_i(\theta) + b_i(\theta), \qquad i = 1, \ldots, N, \tag{1}$$

where the link is made explicit between an institution's type, θ, its reputation i, and the amount of applied and basic research it carries out, $a_i(\theta)$ and $b_i(\theta)$ respectively, and their sum, $r_i(\theta)$. The latter is therefore both the rent[8] of institution of type θ and reputation i, and the total amount of research it carries out. An institution maximises (1) subject to the constraints imposed by the government agency. As well as being a static reduced form proxying a richer intertemporal payoff function, (1) is restrictive as it rules out both a preference, positive or negative, for variety, and differences in the marginal rate of substitution between basic and applied research which depend on an institution's type, θ, or reputation, i. My analysis should therefore be seen as the benchmark case where these assumptions hold.

Institutions use government funding to pay for their research.[9] Thus they maximise (1) subject to their current total cost not exceeding their current funding. Given my interpretation of θ, given above, any private funding received by an institution is included in θ.

I turn next to technology.

[8] $r_i(\theta)$ could thus be denoted by $U_i(\theta)$ as in some of the mechanism design literature.

[9] This being a static model, current funding pays for current research, as shown in Figure 1 below. In a fuller dynamic model, this would be the consequence of institutions not having access to capital markets, which would imply that they must spend all the government funding in the period they receive it.

Define B as the *total* amount of basic research carried out in the economy.

$$B = \sum_{i=1}^{N} q_i \int_{\underline{\theta}}^{\bar{\theta}} b_i(\theta) f_i(\theta) \, d\theta, \tag{2}$$

where $q_i \geqslant 0$ is the number of institutions in reputation group i, $i = 1, \ldots, n$.

I assume that B affects an institution's cost of doing applied research. The way I model this influence captures three of the characteristics attributed in the literature to basic research. First, the hierarchical link between basic and applied research, with the former preceding and providing the foundation to the latter (for example, Evenson and Kislev 1976, or more recently Aghion et al 2008). Next, the externality bestowed on society by basic research: basic research reduces the cost of applied research, and it is the latter that has direct beneficial effects, as posited below, in Section 2. And third, the unpredictable nature of the benefits arising from basic research. Each applied research project is, in expectation, helped equally by every basic research project: it is the very nature of basic research that makes it it is hard to pinpoint ex-ante what kind of benefits a given basic research project will bring if successful.

Nelson's early work (1959) already reports many examples of basic research projects which illustrate these characteristics. Among the cases studied more recently, Moody (1995) describes in detail the numerous strands of basic research which allowed the creation of the ubiquitous CD. A GPS navigation system would be far too inaccurate to be of any practical use without corrections of gravitational effects central to the theory of relativity (Haustein 2009). The abstract mathematical problem of covering a surface with tiles lies at the foundation of our understanding and exploitation of superconductors (Edelson 1992). Gauss's investigation into the distribution of prime numbers has led, with the contributions of some of the best mathematical minds over the course of two centuries, to the possibility of unbreakable cryptographic codes, without which e-commerce would not be possible (du Satoy 2003).[10]

The hierarchical structure, the externality and the unpredictability are captured by the assumption that only the total amount of basic research undertaken in society, determined by (2), affects an institution's cost of applied research.[11] A further justification for this is the scientists' incentives to make their discoveries known as quickly and as widely as possible and the limited appropriability of basic research, exemplified by the difficulty of patenting basic research discoveries. Thus the new knowledge embodied in basic research carried out in one institution is instantaneously diffused to the entire research community, and therefore all institutions benefit equally from it. In other words, an institution is unable to appropriate any of the externality bestowed by the basic research it carries out other than through its effect on the aggregate amount B. Mathematically, this can be captured by the assumption that the cost function is separable in the amounts of applied research, a, and basic research, b. In addition, I also assume that it is linear in b, which, as we see below, is convenient as it ensures that the model follows the standard mechanism design with monetary transfers.[12] There is no further loss of generality in normalising the coefficient of b to 1, and so an institution's total cost of

[10] Table 3 in Amon et al (2010) has a longer and more systematic list, and Stephan (2012) discusses further examples. The benefit of hindsight, of course, makes apparent the link between basic research and commercial applications of the applied research it generated (eg, Jensen and Thursby 2001).

[11] This is similar to Gersbach et al (2010), who posit that *aggregate* amount of basic research undertaken in society is a parameter of the function which gives the probability of a successful innovation in each of the continuum of industries where research is undertaken.

[12] The assumption that cost is linear in b is a notationally simple way of capturing the substantive assumption that, given that $c_{aa}(\cdot) > 0$, the marginal cost function for applied research is increasing at a faster rate than the marginal cost function for basic research. This implies that the optimal policy matches the stylised observation that funding agencies appear to have, at the margin, a stronger preference for applied research than the research institutions themselves.

carrying out a applied research and b basic research can be written as:

$$c\left(a,\theta,B\right)+b. \tag{3}$$

I impose the following restrictions on the cost function; here and in the rest of the article the partial derivatives of a function are denoted by subscripts.

Assumption 2 *An institution's total cost is given by (3). For every $a,B\geqslant 0$, for every $\theta\in\left[\underline{\theta},\bar{\theta}\right)$, the function $c\left(a,\theta,B\right)$ satisfies:*[13]

1. *(i)* $c_a\left(\cdot\right)>0$, *(ii)* $c_\theta\left(\cdot\right)>0$, *(iii)* $c_B\left(\cdot\right)<0$.
2. *(i)* $c_{aa}\left(\cdot\right)>0$, *(ii)* $c_{BB}\left(\cdot\right)>0$, *(iii)* $c_{a\theta}\left(\cdot\right)>0$, *(iv)* $c_{aB}\left(\cdot\right)\leqslant 0$.
3. *(i)* $c\left(0,\theta,B\right)=0$; *(ii)* $\lim_{B\to 0}c_B\left(a,\theta,B\right)=-\infty$.
4. $\lim_{\theta\to\bar{\theta}}c_a\left(0,\theta,B\right)=+\infty$.

Naturally, an institution's total cost (3) increases with a, b, and θ, and decreases with B, the last capturing the externality discussed above. Given θ, reputation does not affect cost: as explained above, it affects the likelihood of a good θ, which in turn reduces cost. Moreover, the marginal cost of applied research increases with θ, see Assumption 2.2.(iii). As the marginal cost of basic research is independent of θ, this implies that better (ie lower θ) institutions have a comparative advantage in applied research.[14] This is a natural consequence of my interpretation of θ as an institution's capacity

[13] An example of a functional form satisfying Assumption 2 is $\frac{a(a+1)\theta}{h(B)\left(\bar{\theta}-\theta\right)}$, where $h\left(B\right)$ satisfies $h\left(0\right)=0$, $h'\left(B\right)>0$, $h''\left(B\right)<0$, and $\lim_{B\to\infty}h\left(B\right)\geqslant\bar{\theta}$; one such $h\left(B\right)$ is $\frac{\bar{\theta}B}{1+B}$.

[14] In an earlier version of the article (De Fraja 2011), the marginal cost of basic research was also an increasing function of θ: this is the case where better institution have an *absolute* advantage in basic research, as well as a *comparative* advantage. All the results of the present version are maintained, provided that, loosely speaking, the marginal cost of applied research increases faster with θ than the marginal cost of basic research.

to tell good research projects from poor ones, and of the plausible fact that, given their less specific and more serendipitous nature, the expected benefits of basic research projects are harder to assess than those of applied ones. Several results in this article hinge on the assumption of comparative advantages in applied research for better institutions; in view of the fact that there are, as far as I am aware, no empirical results clarifying whether this is true or not, this article indicates as an avenue for further research the determination of comparative advantages in basic and applied research.

The sign of $c_{B\theta}(\cdot)$ is unrestricted: a positive sign indicates that efficient institutions can make more of the synergies between basic and applied research, and so benefit more from an increase in total basic research. In practice, the sign of this relationship is not obvious, but my results do not depend on this detail. As for the remaining restrictions, Assumption 2.2.(ii) posits, naturally, decreasing returns to scale for basic research; Assumption, 2.3.(i) rules out fixed costs, and 2.3.(ii) is an Inada condition which avoids unrewarding corner solutions by ensuring that if there is no basic research in society then a very small amount reduces the cost of research by more than it costs. Finally, Assumption, 2.4 is a "free entry from the bottom" condition: it ensures that there are potential institutions, whose efficiency parameter is just not adequate, but which would be willing to carry out applied research following a small improvement in their cost, for example following an increase in B. This is formalised in Definition 1 below.

Social benefits and cost of research

The aggregate amount of applied research is

$$A = \sum_{i=1}^{N} q_i \int_{\underline{\theta}}^{\bar{\theta}} a_i(\theta) f_i(\theta) \, d\theta. \tag{4}$$

Recall that $a_i(\theta)$ is the amount of applied research carried out in institutions with reputation i and efficiency parameter θ. A is assumed to affect *directly* national income, Y, and so I write:

$$Y(A), \qquad \text{with} \qquad Y'(A) > 0,\, Y''(A) \leqslant 0. \tag{5}$$

A is therefore the Solow residual at the cornerstone of the basic model of economic growth (Barro and Sala-i-Martin 1995). The idea conveyed by (4) and (5) is that applied research successfully carried out by institutions, translates, perhaps stochastically, into increases in total factor productivity, A. This formulation is general enough to allow for a feedback from applied to basic research, so that the cost of basic research is reduced if more applied research is carried out. Given the microeconomic focus of the article, this feedback can be left implicit in the functional shape of the Solow residual (5).

Unlike applied research, basic research has no direct effect on national income, only the indirect effect on the individual institutions' cost of carrying out applied research through the mechanism subsumed in (3). The externalities implied in (3) and (5) do not create the appropriability problems which beset R&D activities carried out in profit maximising firms, well-understood by the literature since at least Arrow (1962). This is both because all effects of research are internal to the government, which funds it,[15] and because individuals and institutions doing research are not concerned with its monetary appropriability: their reward is the *production* of knowledge, not its financial exploitation, as has long been recognised (see Stephan 1996 for a comprehensive review). In other words, from the decision makers' point of view, both

[15] In an international context, some of the benefits determined by the expenditure of one country's taxpayers' money do accrue to different countries. This can be captured by re-interpreting some of the parameters that measure the benefit of research or the shadow cost of public funds, λ and k in (6), to take this international spillover into account.

the government and the institutions, the value of a given project a is the same for a fully appropriable one, for example the development of a new therapy by a private profit-making pharmaceutical company receiving a government research subsidy, or a university selling a patent through a Technology Transfers Office,[16] or one with more diffuse benefits, such as an improvement in communication technology, which benefits all firms and consumers.

The government's objective function is the total national income (5), reduced by the cost of funding research, and increased by a direct benefit of research. In analogy with the static point of view taken with regard to institutions, I assume that the government maximises the current value of its payoff function. This separates the government intertemporal optimisation, whose nature is macroeconomic, from the analysis of the article, which I focus on the microeconomic aspects of the allocation of funds to different institutions.

The total taxes T necessary to fund the research sector, are simply equal to the total cost of research: $T = \sum_{i=1}^{N} q_i \int_{\underline{\theta}}^{\bar{\theta}} \left[c\left(a_i\left(\theta\right), \theta, B\right) + b_i\left(\theta\right)\right] f_i\left(\theta\right) d\theta$. The social cost of the tax collected must be increased by the distortionary and administrative costs they cause, assumed as standard to be proportional to $\lambda > 0$, the shadow cost of public funds, determined elsewhere in the economy.

Research has a direct benefit as well, measured by $k \in \mathbb{R}$. This may originate from several sources. Firstly, the government may include the institutions' current aggregate payoff in its own payoff function: given (1), (2), and (4) this is proportional to $A+B$. Secondly, k may capture the country's pride at the international prestige for conducting successful research, and similar less tangible benefits. In addition, k may be a reduced form way to describe the future benefits of any advancement in knowledge brought about by current research. It is worth stressing that all the results hold if $k = 0$, that

[16] The analysis of the role and effects of TTOs, outside the scope of this article, can be found for example, in Macho-Stadler et al (2007) and in the references reported there.

is if the government cares only about the current national income net of the cost of funding research. k could even be negative, indicating a "philistine" government: my results imply that such a government would tolerate, indeed *fund* research, both applied and basic.[17]

In sum, the government's payoff function is

$$Y(A) - (1+\lambda)T + k(A+B). \quad (6)$$

The shadow cost of taxation cannot be too high relative to the benefit of research, or else no research will be funded, and cannot be too low, otherwise it would be exceeded by the non-monetary benefit of research, pushing research to infinity. Formally, I make the following assumption.

Assumption 3 *For every $A \geqslant 0$, $k < 1 + \lambda < Y'(A) + k$.*

The viewpoint of this article is normative: the government views research funding as an investment and so it maximises its payoff function (6) by choice of its research funding policy. This is the offer, available to all potential research institutions, of contracts which link the amount of research, basic and applied, carried out by an institution and the funding provided by the government to that institution: reputation being observable, this link can be made dependent on it. In designing its policy, the government must of course obey its technological and informational constraints. After the government has announced the policy and committed to it, institutions choose their basic and applied research, and receive the corresponding funding. Figure 1 depicts schematically the sequence of the events described in detail in this section.

[17] The linearity of the non-monetary benefit of research could be relaxed, for example with a function satisfying $k'(\cdot) > 0$, $k''(\cdot) < 0$, and $\lim_{x\to\infty} k'(x) < 1+\lambda$ to ensure boundedness in the optimal amount of research. This would make expressions in the first order conditions for A and B in problem (18) slightly more complex, but, as will be apparent, no substantial change would occur.

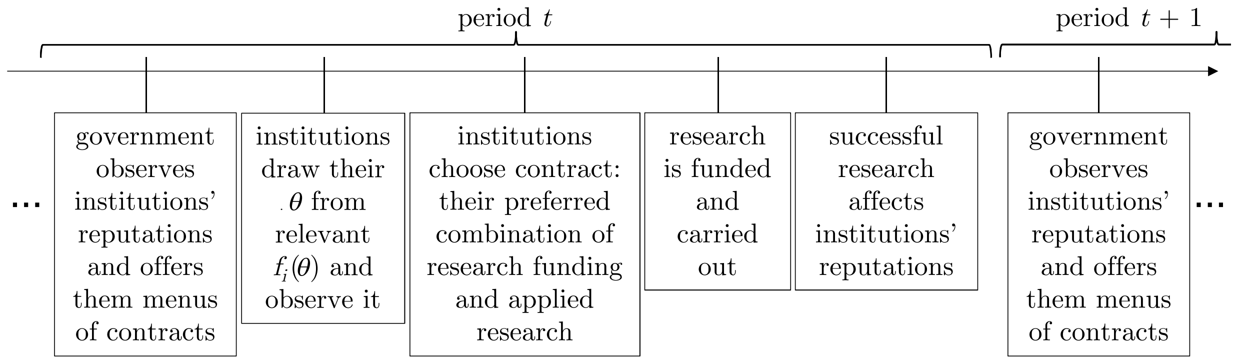

Figure 1: The sequence of events.

3 Results

Preliminaries

To present the results, it is convenient to define the amount of applied research which equates the marginal cost of applied and basic research.

Definition 1 *$a^0(\theta; B)$ is the value of a which solves*

$$c_a(a, \theta, B) = 1. \tag{7}$$

Also let $\theta^0(B)$ be the value of θ which solves

$$c_a(0, \theta, B) = 1. \tag{8}$$

That is, $a^0(\theta; B)$ is the amount of applied research which maximises a type θ institution's objective function when the aggregate amount of basic research is B, and $\theta^0(B)$ is the type of the least efficient institution which is willing to carry out applied research. Note that Assumptions 2.2.(iv) and 2.4 ensure that $\theta^0(B) < \bar{\theta}$. $a^0(\theta; B)$ can be defined as the individually efficient expenditure on applied research. This is because an institution spends an additional unit of funding on the less costly research type and this is applied research, up to level $a^0(\theta; B)$. Beyond that level, any additional funding is entirely spent on basic research, the marginal cost of which is lower. Note that $a^0(\theta; B)$ is independent of reputation: a low reputation institution, which has succeeded in acquiring the capacity and the personnel to carry out high quality research, has the same technology at its disposal as a better reputation institution.

Total differentiation of (7) gives $\frac{\partial a^0(\cdot)}{\partial \theta} = -\frac{c_{a\theta}(\cdot)}{c_{aa}(\cdot)} < 0$, efficient institutions have a higher individually efficient expenditure on applied research. In addition, an increase in the total level of basic research in society increases the individually efficient expenditure on applied research for all institutions, $\frac{\partial a^0(\cdot)}{\partial B} = -\frac{c_{aB}(\cdot)}{c_{aa}(\cdot)} \geqslant 0$; and new institutions are induced to enter, $\frac{d\theta^0(B)}{dB} = -\frac{c_{aB}(\cdot)}{c_{a\theta}(\cdot)} > 0$. This "crowding in" of basic research (Malla and Gray 2005) is natural.

Given (1), (2) can be replaced by:

$$B = \sum_{i=1}^{N} q_i \int_{\underline{\theta}}^{\bar{\theta}} \left[r_i(\theta) - a_i(\theta)\right] f_i(\theta)\, d\theta. \tag{9}$$

The government policy with perfect information

The first proposition gives the benchmark case in which the government has perfect information. Let $a^*(\theta; B^*)$, θ^*, A^*, and B^* be defined by:

$$c_a\left(a^*(\theta; B^*), \theta, B^*\right) = \frac{Y'(A^*) + k}{1 + \lambda}, \tag{10a}$$

$$c_a\left(0, \theta^*, B^*\right) = \frac{Y'(A^*) + k}{1 + \lambda}, \tag{10b}$$

$$A^* = \sum_{i=1}^{N} q_i \int_{\underline{\theta}}^{\theta^*} a^*(\theta; B^*) f_i(\theta)\, d\theta, \tag{10c}$$

$$\frac{k}{1+\lambda} = \sum_{i=1}^{N} q_i \int_{\underline{\theta}}^{\theta^*} c_B\left(a^*(\theta; B^*), \theta, B^*\right) f_i(\theta)\, d\theta + 1. \tag{10d}$$

By Assumption 3, $\frac{Y'(A^*)+k}{1+\lambda} > 1$, and so $a^*(\theta; B^*) > a^0(\theta; B^*)$ and $\theta^* > \theta^0(B)$.

Proposition 1 *If the government could observe perfectly the efficiency parameter of each institution, it would ask institutions of type $\theta \in [\underline{\theta}, \theta^*]$ and reputation i to carry out $a^*(\theta; B^*)$ applied research. It would ask institutions to carry out an aggregate amount of basic research given by B^*.*

The proofs of all the results are relegated to the Appendix.

Proposition 1 is a straightforward first best result: with perfect information, the government simply asks each institution to carry out the socially optimal amount of applied research: this is the level such that the marginal cost of applied research equals the social marginal benefit, and is the same in every institution. Institutions with type $\theta > \theta^*$ are unable to carry out applied research at a cost lower than this value, and so they do not do any. This is efficient; if it were not the case, the government could transfer research from one institution to another and reduce the overall cost of applied research. Because $\frac{Y'(A^*)+k}{1+\lambda} > 1$, the amount $a^*(\theta; B^*)$ imposed by the government exceeds $a^0(\theta; B^*)$, that is, it exceeds what each institution would choose if it were simply given a budget to spend as it pleases, and in particular, the marginal cost of applied research, which is equal across institutions, exceeds the marginal cost of basic research. This is because the government derives a larger benefit from applied research than individual institutions do. By the same token, $\theta^* > \theta^0(B)$: some institutions which would not carry out applied research with fixed funding, are instead instructed by the government to carry out some.

The distribution of the aggregate amount of basic research across institutions is made in such a way that all institutions have the same marginal cost of basic research; this is trivially so in this set-up, where the marginal cost of basic research is assumed constant, but would also hold in a more general model. As we see in Proposition 3, this is not the criterion that allocates basic research under asymmetric information, when instead basic research is used as a recompense to institutions that do more applied research. Note also that more efficient institutions do more applied research: $\frac{\partial a^*(\cdot)}{\partial \theta} = -\frac{c_{a\theta}(\cdot)}{c_{aa}(\cdot)} < 0$. They are better at it, so this is natural. Both an increase in k and a reduction in λ increase $a^*(\theta; B^*)$ for every $\theta \in [\underline{\theta}, \bar{\theta}]$, thus increasing A^* and B^*.

Reputation conveys no useful information, and so it is ignored: institutions with the same θ are treated identically, irrespective of their reputation. As I show below, this is no longer the case with imperfect information.

Information asymmetry

The assumption of symmetric information is implausible and out of kilter with the modern approach to government intervention in the economy. In what follows I therefore assume that the government cannot observe an institution's efficiency parameter θ, and so it knows only the probability distribution from which a given institution's θ is drawn.

The government commits to menus of contracts, which, reputation being observable, can depend on it. Subsequently, each research institution can apply for funds and choose any of the contracts available to institutions in its reputation group. A menu of contracts can be expressed as a non-linear price $T_i(a)$, where T_i is the total payment to an institution with reputation i that carries out amount a of applied research. I assume therefore that applied research is contractible. It does not matter whether or not basic research is contractible: this, as I discuss in the next subsection, is due to the direction of the misalignment of the government's and the institutions' incentives. The government funding policy is thus simply the vector of functions $\{T_i(a)\}_{i=1}^{N}$. The least efficient institution in each reputation group that applies for funding, denoted by $\hat{\theta}_i \in \left[\underline{\theta}, \overline{\theta}\right]$, can itself be treated as a choice variable for the government. If $T_i(a)$ is increasing, it determines immediately the amount of basic research carried out as $b_i(\theta) = T_i(a_i(\theta)) - c(a_i(\theta), \theta, B)$. Because $b_i(\theta) = r_i(\theta) - a_i(\theta)$, a contract can equivalently be written as a direct mechanism $\big(r_i(\theta), a_i(\theta)\big)$, and thus a policy is a vector of pairs of functions, which establish the amount of total research and applied research in an institution of type θ and reputation i:

$$\left\{\{r_i(\theta), a_i(\theta)\}_{\theta \in \left[\underline{\theta}, \hat{\theta}_i\right]}\right\}_{i=1}^{N}. \tag{11}$$

Contracts thus determine payments which depend on the amount of applied research carried out. If institutions are risk neutral, with many projects starting, the amount of applied research carried out, a, can be inferred from the number of research projects completed successfully in the period, which, plausibly, is observable.

Contracting basic research

One may think that contracts could also be made conditional on basic research, for example by specifying the number and quality of basic research publications. This would in theory compel institutions to reveal indirectly their type, and thus potentially allow direct implementation of the first best derived in Proposition 1. This policy, however, would be perverse and unlikely to be enforceable. The reason is that, as Proposition 1 shows, at the first best institutions are asked to carry out a combination of applied and basic research such that the marginal cost of applied research is higher than the marginal cost of basic research. Given this, if the government simply asked institutions to report their own θ, and offered them the corresponding funding, then a type θ institution, which is offered funding $T_i(a^*(\theta; B^*))$ and asked to carry out $a^*(\theta; B^*)$ applied research, would have an incentive to claim to have a higher θ than it has. If it did so, it would receive less funding, but nevertheless be able to increase the total amount of research it does with this lower funding, as it would be able to switch away from the more costly applied research and do more of the less expensive basic research. Formally, presented with a funding level menu $T_i(a^*(\theta; B^*))$, a type θ institution would claim to be of type $\max\left\{a^{*-1}(a^0(\theta; B^*)), \hat{\theta}_i\right\}$. If it did so, its marginal cost of doing applied research would be as near as possible to 1, its marginal cost of basic research. Thus, implementing the first best with a contract that conditions payments on the amount of basic research carried out would perversely require that institutions be punished if they have "too many" prestigious basic research publications.

To see this in an example, suppose that the applied research cost function is the one given in footnote 13: $\frac{a(a+1)\theta}{\frac{\theta B}{1+B}(\bar{\theta}-\theta)}$, with $\bar{\theta} = 1$ and $B = 100$, and that the optimal policy is such that the government asks an institution with $\theta = \frac{1}{10}$ to carry out 8 units of applied research and 2 units of basic research, for a total funding of \$10.08. This institution's payoff is 10, and its marginal cost of applied research is 1.9078, higher than its marginal cost of basic research. Suppose that the optimal policy asks slightly less efficient institutions, those with $\theta = \frac{11}{100}$, to do 7.9 unit of applied research and 1.3 units of basic research, for a funding of \$10.077. The efficient ($\theta = \frac{1}{10}$) institution's cost of carrying out 7.9 units of applied research is \$7.8903: therefore it has an incentive to report $\theta = \frac{11}{100}$, receive lower funding of \$10.077 instead of \$10.08, carry out 7.9 units of applied research, and use the residual \$2.1866 for basic research, improving its payoff from 10 to 10.087. A fine of \$0.1 for the 0.8866 "excess" units of basic research, were it imposed, would reduce the payoff to 9.9866 and thus eliminate the incentive to report a type $\theta = \frac{11}{100}$, but would be hard to justify in practice.[18] It would also be hard to enforce, as, quite apart from any possible difficulty to attribute precisely each publication to applied or basic research, an institution risking a penalty for having too many publications in basic research could always plausibly claim to have been "lucky", its expenditure on basic research churning out an unexpectedly high number of publications, or perhaps seek ways to elude penalties by "hiding" basic research in some way, such as delaying its publication.

To sum up, the government conditions the funding it gives an institution only on the amount of applied research this institution carries out. In case of dispute, an external adjudicator can verify whether the stipulated level of applied research a has been carried out, and hence confirm whether the conditions have been met for the agreed amount of funding to be paid out. The asymmetry between basic and applied research created by the fact that the government would like to induce institutions to do more of the latter than they would choose implies that contracts conditioning payments on basic research would be hard to enforce, because, beyond a certain level, they would dictate lower payments for increased success in basic research. Note that peer review based formal evaluation mechanisms, which are intended to assess the research effort of institutions, do distinguish between applied and basic research. For example, the version of the exercise carried out in 2014 in the UK, known as REF, has two measures of output, academic publications, judged solely on their academic merit, irrespective of their applied or basic nature, and *impact* of research on society: this needs to quantify "all kinds of social, economic and cultural benefits and impacts beyond academia" (HEFCE 2011), and, given both the short time in which impact must have measurable effects and the exacting standard of the required link from research to benefits, it applies in

[18] Lazear (1997) also points out the difficulty of providing explicit incentives for basic research.

practice only to applied research.

Incentive compatibility

In this subsection, I determine the constraints imposed by the information disadvantage of the government. I describe them via the standard revelation approach, that is supposing that the government asks each institution to report its own type, having committed to a policy as a function of the reported type. By the revelation principle, the government cannot improve on the payoff it can obtain by restricting its choices to policies that satisfy the incentive compatibility constraint. This is the property that no institution has an incentive to misreport its type.

Before deriving this constraint, I introduce an assumption which ensures that asymmetry of information is sufficiently important.

Assumption 4 *(i)* $c_\theta(\cdot) > \frac{c_{a\theta}(\cdot)}{c_{aa}(\cdot)}\left(1 + \frac{d}{d\theta}\left(\frac{F_i(\theta)}{f_i(\theta)}\right)\right)$, $i = 1, \ldots, N$; *(ii)* $c_\theta(\cdot) > \frac{c_{a\theta\theta}(\cdot)}{c_{aa\theta}(\cdot)}$, *(iii)* $c_{aa\theta}(\cdot) > 0$, $c_{a\theta\theta}(\cdot) \geqslant 0$.

Loosely speaking, (i) and (ii) require $c_\theta(\cdot)$ to be "large". Thus the first two statements in Assumption 4 require that an institution's cost of carrying out applied research varies enough with θ, making what is unobservable to government sufficiently important. This makes sense as it is this information disadvantage that renders the analysis relevant: if all research institutions had similar efficiency, the government's inability to observe their productivity would be obviously irrelevant. Of course, the considerable effort that funding agencies exert to ascertain the research potential of the research institutions they support financially does suggest strongly that these differences are indeed important in practice. The third statement in Assumption 4 is a regularity restriction.[19]

[19] As in the Laffont and Tirole (1993) contribution whose methodology is borrowed here,

Proposition 2 *A feasible and incentive compatible policy (11) satisfies, for* $\theta \in \left[\underline{\theta}, \hat{\theta}_i\right]$, $i = 1, \ldots, N$*:*

$$\dot{r}_i(\theta) = -c_\theta(a_i(\theta), \theta, B), \quad r_i(\underline{\theta}) \text{ free; } r_i\left(\hat{\theta}_i\right) = a^0\left(\hat{\theta}_i, B\right), \tag{12a}$$

$$\dot{a}_i(\theta) \leqslant 0, \tag{12b}$$

$$a_i(\theta) - a^0(\theta; B) \geqslant 0, \tag{12c}$$

$$r_i(\theta) - a_i(\theta) \geqslant 0. \tag{12d}$$

Here, as in some of the mechanism design literature, a dot over a variable denotes its derivative with respect to the unobserved parameter, θ in this case.

The derivation of this result, given in more detail in the Appendix, is standard. Faced with a contract $T_i(a)$, if a type θ institution carries out amount a of applied research, it can carry out an amount $T_i(a) - c(a, \theta, B)$ of basic research, and so its payoff is

$$r = a + T_i(a) - c(a, \theta, B). \tag{13}$$

The first order condition for maximisation of (13) by choice of a is given by

$$1 - T'(a) - c_a(a, \theta, B) = 0. \tag{14}$$

Because this institution's informational rent as a function of its type can be written as

$$r(\theta) = \max_a \{a + T(a) - c(a, \theta, B)\},$$

the link between an institution's type and its informational rent is determined

the restriction on the third derivative of the payoff function ensures the sufficiency of the first order conditions in the government second best problem and so it rules out stochastic mechanisms, but does not have a straightforward interpretation.

as

$$\dot{r}(\theta) = \left(1 - T'(a(\theta)) - c_a(a(\theta), \theta, B)\right) \dot{a}(\theta) - c_\theta(a(\theta), \theta, B). \tag{15}$$

In (15), the first term is 0 by the envelope theorem, (14), and this gives the differential equation in (12a). Condition (12b) is standard, (12c) follows from the requirement that total funding be decreasing in θ: if not, an institution could pretend to be of a worse type than it really is, receive more funding and also be required to do less applied research. (12d) is simply $b_i(\theta) \geqslant 0$.

Although the formal model is static, it is worth noting that the "ratchet effect" (Freixas et al 1985) would not be a problem in a fuller dynamic model based on the present analysis. According to this effect, current incentives are affected by the cost of reducing future informational rent, which occurs when revealing one's type provides information to the principal on the likelihood of future types. This does not apply here, because there is no intertemporal information trade-off, as current research enhances future reputation, and if, as Proposition 4 shows to be the case, institutions that are revealed to be of a better type today enjoy more informational rent in the future. Thus institutions would not want to alter their static preferred allocation of funds to basic and applied research in order to obtain future benefits.

One final assumption is needed before presenting the government maximisation problem. Define $\bar{B}$ as the value of B which solves the following equation:

$$B = \sum_{i=1}^{N} q_i \int_{\underline{\theta}}^{\theta^*} \int_{\theta}^{\theta^*} \left(c_\theta\left(a^*(x; B), x, B\right) dx - a^*(\theta; B)\right) f_i(\theta)\, d\theta. \tag{16}$$

Assumption 5

$$\frac{k}{1+\lambda} - \sum_{i=1}^{N} q_i \int_{\underline{\theta}}^{\theta^*} c_B\left(a^*\left(\theta;\bar{B}\right),\theta,\bar{B}\right) f_i\left(\theta\right) d\theta < 0. \qquad (17)$$

In words, as is shown below in the proof of Proposition 3, $\bar{B}$ is the value of aggregate research needed to provide every institution with incentives for self-selection, when all institutions are asked to carry out the first best level of applied research, derived in (10d) by Proposition 1. The LHS in (17) is the marginal social benefit of basic research, given by the sum of the direct benefit and the aggregate marginal reduction in the cost of applied research. Thus Assumption 5 states that when basic research is set to satisfy the incentive compatibility constraint for each institution, then the social marginal benefit of research is negative. This imposes a limit on the amount of basic research that the government is willing to fund: you can have too much basic research. If Assumption 5 did not hold, asymmetry of information would not prevent the government from obtaining the first best: this corresponds to the extreme case of zero cost of public funding.

The optimal funding policy

The government maximisation problem is the choice of a policy (11) which satisfies the constraints derived in Proposition 2 and maximises the government's objective function. As before, the aggregate amount of applied and basic research, A and B, are best treated as parameters of the problem, subject to their respective definition constraints, (4) and (9).

The government's problem is therefore the following.

$$\max_{\substack{\left\{\hat{\theta}_i,\{r_i(\theta),a_i(\theta)\}_{\theta\in[\underline{\theta},\hat{\theta}_i]}\right\}_{i=1}^N, \\ A,B}} \Bigg\{Y(A) + k(A+B) -$$

$$(1+\lambda)\sum_{i=1}^{N} q_i \int_{\underline{\theta}}^{\hat{\theta}_i} \left[c(a_i(\theta),\theta,B) + r_i(\theta) - a_i(\theta)\right] f_i(\theta)\, d\theta \Bigg\},$$

$$\text{s.t.: (4), (9), (12a), (12b), (12c), (12d).} \qquad (18)$$

Because institutions fund all their research from public sources, they all have zero reservation utility, and the participation constraint is implied by (12c): I have therefore omitted it.[20] The optimal funding policy is derived next.

Proposition 3 *Let Assumptions 2-5 hold. If problem (18) has a solution* $a_i(\theta)$, $b_i(\theta)$, *then there exist* $\tilde{\theta}_i, \theta_i^K, \hat{\theta} \in (\underline{\theta}, \overline{\theta})$ *with* $\underline{\theta} < \theta_i^K \leqslant \tilde{\theta}_i \leqslant \hat{\theta} < \theta^0(B)$, *for* $i = 1, \ldots, N$, *such that:*

if $\theta \in [\underline{\theta}, \theta_i^K)$ *then* $a_i(\theta) > a^0(\theta; B)$ *and* $b_i(\theta) > 0$;

if $\theta \in [\theta_i^K, \tilde{\theta}_i)$ *then* $a_i(\theta) = a^0(\theta; B)$ *and* $b_i(\theta) > 0$;

if $\theta \in [\tilde{\theta}_i, \hat{\theta})$ *then* $a_i(\theta) > a^0(\theta; B)$ *and* $b_i(\theta) = 0$;

if $\theta = \hat{\theta}$ *then* $a_i(\theta) = a^0(\theta; B)$ *and* $b_i(\theta) = 0$;

if $\theta \in (\hat{\theta}, \overline{\theta}]$ *then* $a_i(\theta) = b_i(\theta) = 0$.

Note that $\hat{\theta}$, the cut-off value of θ, is the same for all reputation groups. The best way to illustrate and discuss Proposition 3 is through the graphical

[20] If institutions were able to fund research independently, then the participation constraint would have to be considered explicitly, with the possible complication that the reservation utility itself would in general depend on θ. However, if institutions' own funds are non-increasing in θ, as is plausible, then the potential non-monotonicity studied by Jullien (2000) would not occur.

analysis in Figures 2 and 3, which plots the amount of research on the vertical axis as a function of an institution's efficiency parameter θ, and, in Figure 3, also of its reputation i. To this end, it is useful to restate the optimal policy described in Proposition 3 as Corollary 1, which is cast in terms of the function $a_i^K(\theta; B, \beta)$, defined as follows. For given $B > 0$ and $\beta \geqslant 0$, let $a_i^K(\theta; B, \beta)$ be the solution in a of

$$c_a(a, \theta, B) = \frac{Y'(A) + k}{1 + \lambda} + \beta - \frac{\beta F_i(\theta)}{f_i(\theta)} c_{\theta a}(a, \theta, B). \tag{19}$$

At the optimal policy, the parameter β is 1 minus the Lagrange multiplier of constraint (9); in economics terms, β measures the marginal net benefit of total basic research. At the first best, the government pushes the total amount of basic research to the level where its marginal benefit equals its marginal cost, and so $\beta = 0$, the last two terms in (19) are also 0, and the curve $a_i^K(\theta; B, \beta)$ coincides with $a^*(\theta; B)$, defined in (10a), the first best amount of applied research by a type θ institution. In the second best, if Assumptions 2 and 4 hold, there is a trade-off between research and distortionary costs of taxation, and $\beta > 0$, as shown in the proof of Proposition 3. Relatively to the position determined by (10a), curve (19) is rotated clockwise around a point with abscissa to the right of $\underline{\theta}$: its value at $\theta = \underline{\theta}$ increases, and, given Assumption 4, the slope increases in absolute value: this is the standard distortion due to asymmetric information (Laffont and Tirole 1993).

Corollary 1 *Let Assumptions 2-5 hold. Define*

$$\hat{a}_i(\theta; B, \beta) = \max\left\{a^0(\theta; B), a_i^K(\theta; B, \beta)\right\}, \tag{20}$$

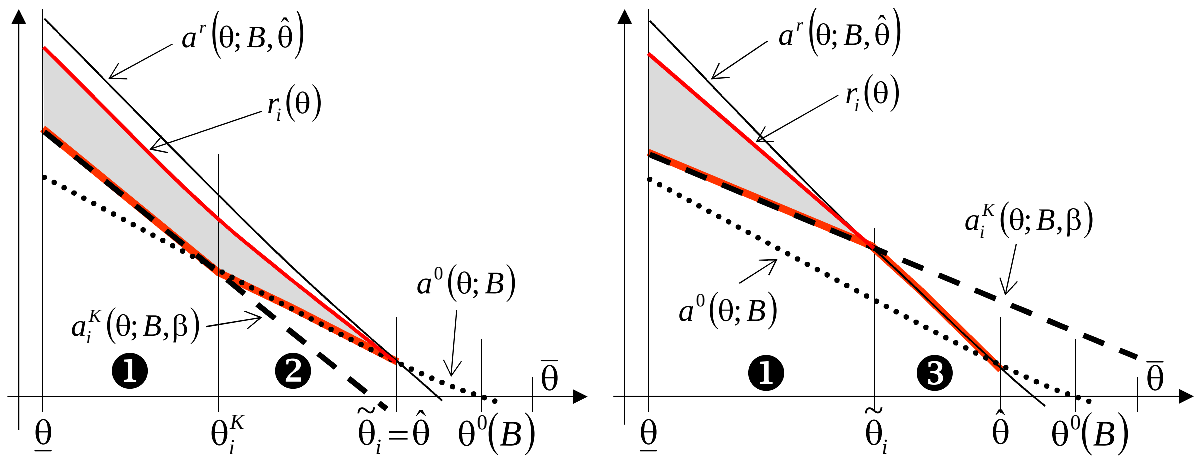

Figure 2: Applied and basic research. The second best policy.

and $a^r\left(\theta; B, \hat{\theta}\right)$ *as the solution to the following differential equation:*

$$\dot{x}(\theta) = -c_\theta(x(\theta), \theta, B), \qquad x\left(\hat{\theta}\right) = a^0\left(\hat{\theta}; B\right). \tag{21}$$

If problem (18) has a solution, then there exist $B > 0$, $\hat{\theta} \in \left[\underline{\theta}, \bar{\theta}\right]$, *and* $\beta > 0$, *such that:*

$$a_i(\theta) = \min\left\{\hat{a}_i(\theta; B, \beta), a^r\left(\theta; B, \hat{\theta}\right)\right\}, \tag{22}$$

$$b_i(\theta) = \max\left\{a^r\left(\theta; B, \hat{\theta}\right) - \hat{a}_i(\theta; B, \beta), 0\right\}, \tag{23}$$

for $\theta \in \left[\underline{\theta}, \hat{\theta}\right]$, *and* $a_i(\theta) = b_i(\theta) = 0$ *for* $\theta \in \left(\hat{\theta}, \bar{\theta}\right]$, $i = 1, \ldots, N$.

To illustrate the optimal policy, take Figure 2 first, which considers the institutions with a given reputation i. In each panel, the solid thin red line is the locus $a^r\left(\theta; B, \hat{\theta}\right)$, defined in (21); the dotted line is the locus $a^0(\theta; B)$, given in (7), and the dashed line is $a_i^K(\theta; B, \beta)$, defined in (19).

From Corollary 1, we can plot an institution's *applied* research as θ varies as the higher of the two curves $a^0(\theta; B)$ and $a_i^K(\theta; B, \beta)$, if it is below $a^r\left(\theta; B, \hat{\theta}\right)$, and otherwise as $a^r\left(\theta; B, \hat{\theta}\right)$ itself. This is the solid thick red curve; only institutions with θ below $\hat{\theta}$, which is, by construction, the intersection of $a^r\left(\theta; B, \hat{\theta}\right)$ and $a^0(\theta; B)$, apply for government funding. All three curves are strictly decreasing, and so $\dot{a}(\theta) < 0$. As explained above, $a_i^K(\cdot)$, is above $a^*(\theta; B)$ (not drawn, to avoid cluttering the diagram), determined in (10a), and therefore also above $a^0(\theta; B)$ in a right neighbourhood of $\underline{\theta}$: the most efficient institutions do more applied research than at the first best. The research carried out by a type θ reputation i institution is obtained from the incentive compatibility constraint, (12a):

$$r_i(\theta) = \int_{\theta}^{\hat{\theta}} c_\theta(a_i(x), x, B)\, dx + a^0\left(\hat{\theta}; B\right), \qquad i = 1, \ldots, N.$$

In the case drawn in the LHS panel, this is the higher thin red line in the diagram. Basic research therefore is the vertical distance between the two red curves, shaded in grey in the diagrams.[21] It is $a_i(\theta) \leqslant a^r\left(\theta; B, \hat{\theta}\right)$: thus $r_i(\theta) \leqslant a^r\left(\theta; B, \hat{\theta}\right)$, with strict inequality for $\theta \in \left[\underline{\theta}, \tilde{\theta}_i\right)$, and, as drawn, the thick red line is below the thin black line to the left of $\tilde{\theta}_i$.

The panels of Figure 2 differ in the position of the curve $a_i^K(\cdot)$. This can have three kinds of relationship with the two other relevant curves, indicated by the white numbers in a black disk, reflecting the three possible patterns of complementary slackness of the constraints in Problem (18). In region 1, both (12c) and (12d) are slack; in region 2, constraint (12c) is binding. In region 3, (12d) is binding. The conceptual difference between regions 1 and 2 is that research institutions in region 2 choose their "preferred" combination of applied and basic research, and those in region 1 are induced by the policy to do more than this amount. Institutions in region 3 do only applied research.

[21] If institutions varied also in their ability to carry out basic research, the analysis would be slightly modified to capture the trade-off between using basic research funding to provide incentives for applied research and allocating it to the more efficient institutions.

Because $\beta > 0$ at the second best policy, the marginal net benefit of aggregate basic research is higher than at the first best, and therefore the optimal value of B is lower: asymmetric information reduces the amount of basic research. This makes applied research more expensive, and hence reduces it, and, given that more efficient institutions do more applied research, some high θ institutions do less applied research than at the first best. Note also that, applied research being allocated less efficiently, its overall cost is not necessarily lower than at the first best even though its overall amount is smaller. Also note $\hat{\theta} < \theta^0(B)$, and therefore $\hat{\theta} < \theta^*$. Thus fewer institutions receive funding than at the first best: the funding agency achieves this by offering contracts that require institutions to carry out at least a threshold amount of applied research in order to qualify for any public funding.

Figure 2 can be used to carry out some simple comparative statics analysis. To this end, note that the position of the curve $a_i^K(\theta; B, \beta)$, defined in (19), is affected by four factors: the direct effect of applied research on national income, $Y'(A)$; the direct effect of research on the policy maker's payoff, k; the shadow cost of public funds λ; and finally, β, the endogenously determined net marginal benefit effect of basic research, its marginal reduction of institutions' cost of doing applied research, net of the cost of the raise in tax necessary to pay for it. The first three simply shift the dashed curve $a_i^K(\theta; B, \beta)$ up and down in a parallel fashion. Thus, other things equal, increases in k and in $Y'(A)$ and decreases in λ all increase the amount of applied research, and decrease the amount of basic research carried out by a type θ institution. Of course, changes in these parameters also affect β, the net benefit of basic research, so the above discussion is only a first approximation. The effect of changes in these parameters on the amount of basic research is in general ambiguous: when applied research becomes more valuable, there is a substitution effect, basic research becomes relatively less valuable and so

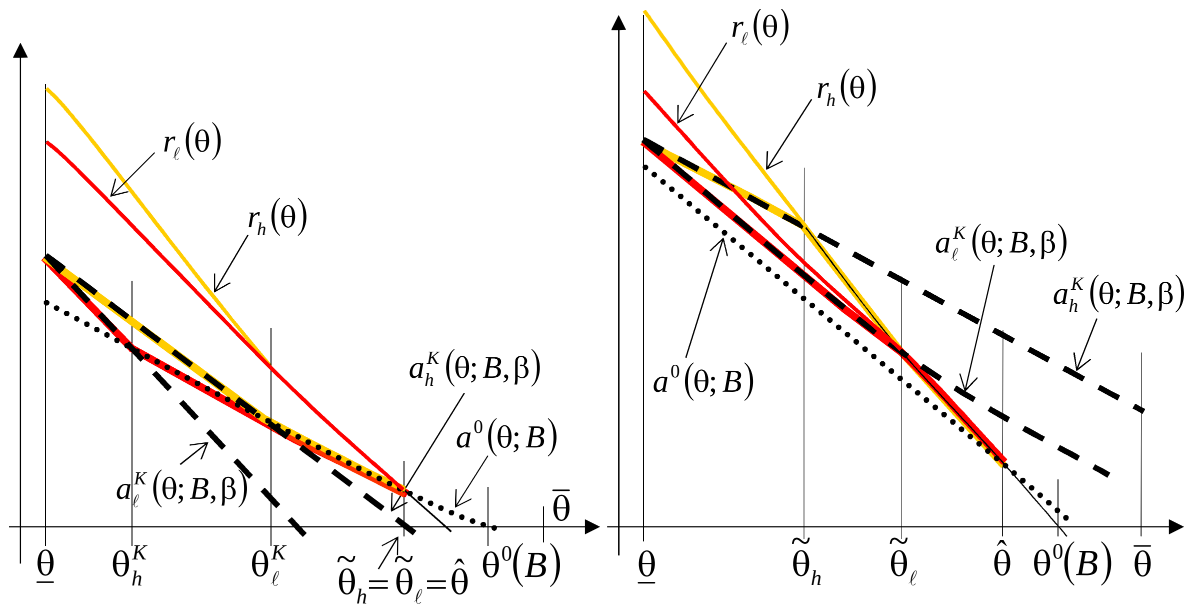

Figure 3: Institutions with different reputation.

less of it is carried out, and an "income" effect: as more applied research is done, the importance is increased of reducing its cost by increasing B.

The intuition for Proposition 3 is further illustrated when comparing institutions of different reputation. Note that, in the second best, where $\beta > 0$, the distribution $F_i(\theta)$ appears in the definition of $a_i^K(\theta; B, \beta)$ and therefore a type θ institution's amount of applied research does depend on its reputation. The next Proposition determines the direction of this dependence.

Proposition 4 *Let Assumptions 1-5 hold. Let $h > \ell$. Then $a_h^K(\theta; B, \beta) > a_\ell^K(\theta; B, \beta)$, for all $\theta \in \left(\underline{\theta}, \theta_\ell^K\right)$. Consequently $a_h(\theta) > a_\ell(\theta)$, and $r_h(\theta) > r_\ell(\theta)$ in a right neighbourhood of $\underline{\theta}$.*

To examine graphically how reputation affects research at the optimum, note that, in Figure 2, the following are independent of reputation i: the "starting point" of the dashed curve $a_i^K(\underline{\theta}; B, \beta)$, the dotted curve $a^0(\theta, B)$, and the type of the least efficient active institutions, $\hat{\theta}$. The rest of the diagram does vary with i. Thus, for example, if an institution has better reputation than another, then its curve $a_h^K(\theta; B, \beta)$ is pivoted anticlockwise around its value at $\theta = \underline{\theta}$, and hence point θ_h^K on the LHS panel and point $\tilde{\theta}_h$ on the RHS panel, both shift to the left of the corresponding points θ_ℓ^K and $\tilde{\theta}_\ell$, as shown by the gold curves in Figure 3, where $\theta_h^K < \theta_\ell^K$ in the LHS panel, and $\tilde{\theta}_h < \tilde{\theta}_\ell$ on the RHS panel. As Proposition 4 states, a higher reputation institution does more applied research than an institution with the same θ and a less established reputation.

Figure 3 also shows $r_h(\theta)$, the total research done by institutions with higher reputation h. Relative to $r_\ell(\theta)$, this is rotated *clockwise*, pivoting around θ_ℓ^K in the LHS panel, and around $\tilde{\theta}_\ell$ in the RHS panel: a higher reputation institution does more total research. It is necessarily the case that the most efficient institutions do more research and more basic research if they have high reputation. This can be reversed for higher θ; for example, in the RHS panel of Figure 3, institutions with efficiency parameter between $\tilde{\theta}_h$ and $\tilde{\theta}_\ell$ do no basic research if they have high reputation, and a strictly positive amount if they have low reputation.

To understand these results, consider first the intuition for the shape of the functions $a_i(\theta)$ and $r_i(\theta)$ for institutions in a given reputation group. With symmetric information, there is no incentive compatibility constraint, and so the last term in (19) is absent: when this term is included, the dashed curve in Figure 2 is rotated clockwise around the intersection with the vertical line $\theta = \underline{\theta}$. That is, the applied research schedule is steeper with asymmetric information. The reason is the standard one: the government needs to induce self-selection in an efficient institution, that is it needs to dissuade one which is contemplating choosing the combination of funding and applied research designed for an institution with slightly higher θ instead of that designed for its own type. The extra funding that comes with the extra applied research plays this role: it does serve as an incentive for an efficient institution, without at the same time tempting less efficient ones to pretend to be more efficient than they really are, because, given their higher θ, the extra applied research required to receive this extra funding would be too expensive for the latter.

Consider now the intuition for Proposition 4, according to which institutions with better reputation do, at the optimum, *more applied* and *more total* research, and so receive *more funding*, than equally efficient institutions with lower reputation.

At the optimal policy, the aggregate amount of applied research A exceeds the total amount that would result if institutions could choose their preferred level of applied research. Providing incentives for them to do more has a cost in terms of informational rent, and, to minimise the ensuing efficiency loss, given that $c_{a\theta}(\cdot) > 0$, and so $a^*(\theta; B^*) - a^0(\theta; B)$ decreases with θ, the more efficient institutions are "asked" for a greater increase over the individually efficient level $a^0(\theta; B)$. When Assumption 1 holds, there are more efficient institutions in higher reputation groups: to see this, take $\theta_2 > \underline{\theta}$ and consider a small interval $(\theta_2 - \varepsilon, \theta_2 + \varepsilon)$. In expectation, there are $[F_i(\theta_2 + \varepsilon) - F_i(\theta_2 - \varepsilon)] q_i \cong 2\varepsilon f_i(\theta_2) q_i$ institutions with reputation i in this interval, and $F_i(\theta_2) q_i$ with θ below θ_2. By Lemma A3 in the Appendix, $\frac{F_i(\theta)}{f_i(\theta)}$ increases with reputation, that is, there are proportionally more low θ institutions in a higher reputation group. In other words, high reputation institutions are more likely to have drawn a low θ and so be good at research. This implies that to be more likely to push more low θ institutions closer to their first best level of applied research, the government favours high reputation groups, where more low θ institutions are concentrated. It is of course more costly to reward high efficiency institutions, because they are asked to do more research, which is increasingly expensive as $c_{aa}(\cdot) > 0$; but the government does not mind this in the least, because it pays them with basic research, which it values itself, and so the cost incurred by the government is the cost of withdrawing basic research from less efficient institutions, more likely to be found in lower reputation groups. In the standard Laffont-Tirole procurement model, awarding an extra \$1 of information rent has a cost exceeding \$1 because of the shadow cost of public funding; here, for fixed B, the information rent is paid by lower reputation institutions, which do less basic research, and so it has a cost of exactly \$1.

In looser words, asymmetric information makes applied research more expensive, and the government prefers to ration it by allocating it to the institutions which are better at it, the low θ ones, and, in expectations, there are more such institutions among those with high reputation, and so the government biases funding towards high reputation institutions.

4 Implementation

This section investigates how a central funding agency can implement in practice the optimal policy described in Proposition 3 and Corollary 1. Recall that this agency offers all institutions contracts that stipulate a link between the amount of applied research carried out and the total amount of funding an institution of reputation i receives, that is a vector of functions $\{T_i(a)\}_{i=1}^N$. Because at the optimal policy there is a one-to-one relationship between θ and a, this is well defined.

To determine the shape of these functions, consider an institution of type θ and reputation i which, given the incentive compatible policy (11), chooses $r_i(\theta)$ and $a_i(\theta)$, and therefore receives total funding $T_i(a_i(\theta))$. If the amount of applied research identified in Proposition 1 is in region 1 in Figure 2, this total funding is

$$T\left(a_i\left(\theta\right)\right)=c\left(a_i^K\left(\theta;B,\beta\right),\theta,B\right)+\left[a^r\left(\theta;B,\hat{\theta}\right)-a_i^K\left(\theta;B,\beta\right)\right].$$

The first term is the cost of carrying out $a_i^K\left(\theta;B,\beta\right)$ applied research, and the sum in the square brackets the cost (and the amount) of basic research. For fixed B and β, let $\theta_i^K\left(a;B,\beta\right)$ be the inverse of the function $a_i^K\left(\theta;B,\beta\right)$: that is, $\theta_i^K\left(a;B,\beta\right)$ is the value of θ such that $a_i^K\left(\theta;B,\beta\right)=a$. Consider an i-reputation institution which, faced with a schedule $T_i\left(a\right)$, needs to choose the amount a of applied research to carry out; if the policy is incentive compatible, it has type $\theta_i^K\left(a;B,\beta\right)$, and the total funding it receives is given by:

$$T_i\left(a\right)=c\left(a,\theta_i^K\left(a;B,\beta\right),B\right)+a^r\left(\theta_i^K\left(a;B,\beta\right);B,\hat{\theta}\right)-a. \qquad (24)$$

Faced with (24), a reputation i type θ institution does indeed want to carry out precisely the amount $a=a_i^K\left(\theta;B,\beta\right)$ of applied research. To see this, note that, given (24), a type θ institution's optimisation problem is:

$$\max_{a\geqslant 0}\left\{a+\left[T_i\left(a\right)-c\left(a,\theta,B\right)\right]\right\}, \qquad (25)$$

where $T_i\left(a\right)$ is given by (24). The first order condition for (25) is

$$c_a\left(a,\theta_i^K\left(a;B,\beta\right),B\right)=c_a\left(a,\theta,B\right),$$

which gives $a=a_i^K\left(\theta;B,\beta\right)$ as required (provided it is at least $a^0\left(\theta;B\right)$, otherwise the institution does not apply for public funding). The following determines the shape of (24).

Corollary 2 *If $a_i\left(\theta\right)=a_i^K\left(\theta;B,\beta\right)$, then $T_i\left(a\right)$ is increasing and convex in a.*

The same procedure gives the shape of $T_i(a)$ in the other regions. Begin with region 2. Here, let $\theta^0(a;B)$ be the inverse function of $a^0(\theta;B)$, so that total funding is given by:

$$T_i(a) = c\left(a, \theta^0(a;B), B\right) + a^r\left(\theta^0(a,B); B, \hat{\theta}\right) - a. \tag{26}$$

Corollary 3 *If $a_i(\theta) = a^0(\theta;B)$, then $T_i(a)$ is constant in a.*

So in this region all institutions receive the same funding, irrespective of their efficiency and of their reputation. Finally region 3.

Corollary 4 *If $a_i(\theta) = a^r\left(\theta; B, \hat{\theta}\right)$, then $T_i(a)$ is increasing and convex in a.*

Moreover, at the boundary between regions 1 and 3 the slope of $T_i(a)$ is increasing in a. Having determined the shape of the function $T_i(a)$, I show next, by means of a graphical analysis, how the funding agency can implement it in practice.

A graphical analysis

Consider Figure 4. The axis pointing west measures θ, the south and east axes measure a, and the north axis measures total funding $T(a)$. I reproduce the RHS panel of Figure 3 in the diagram in the southwest quadrant, with the axes pointing in the opposite directions. The northwest diagram measures the cost of applied research, as a function of θ for a given value of a: this curve is drawn for three different values of a, namely $a^0\left(\hat{\theta}, B\right)$, $a_\ell^K\left(\theta_\ell^K; B, \beta\right)$, and $a_h^K\left(\theta_\ell^K; B, \beta\right)$. Note the subscripts in the latter two: these are the applied research by a type θ_ℓ^K institution with low and high reputation, respectively. Because $a_\ell^K(\cdot) > a_h^K(\cdot)$, the corresponding curve is higher. Because $c_{a\theta}(\cdot) > 0$, the curves fan out from the origin (and because $c_{aa\theta}(\cdot) > 0$, they are convex).

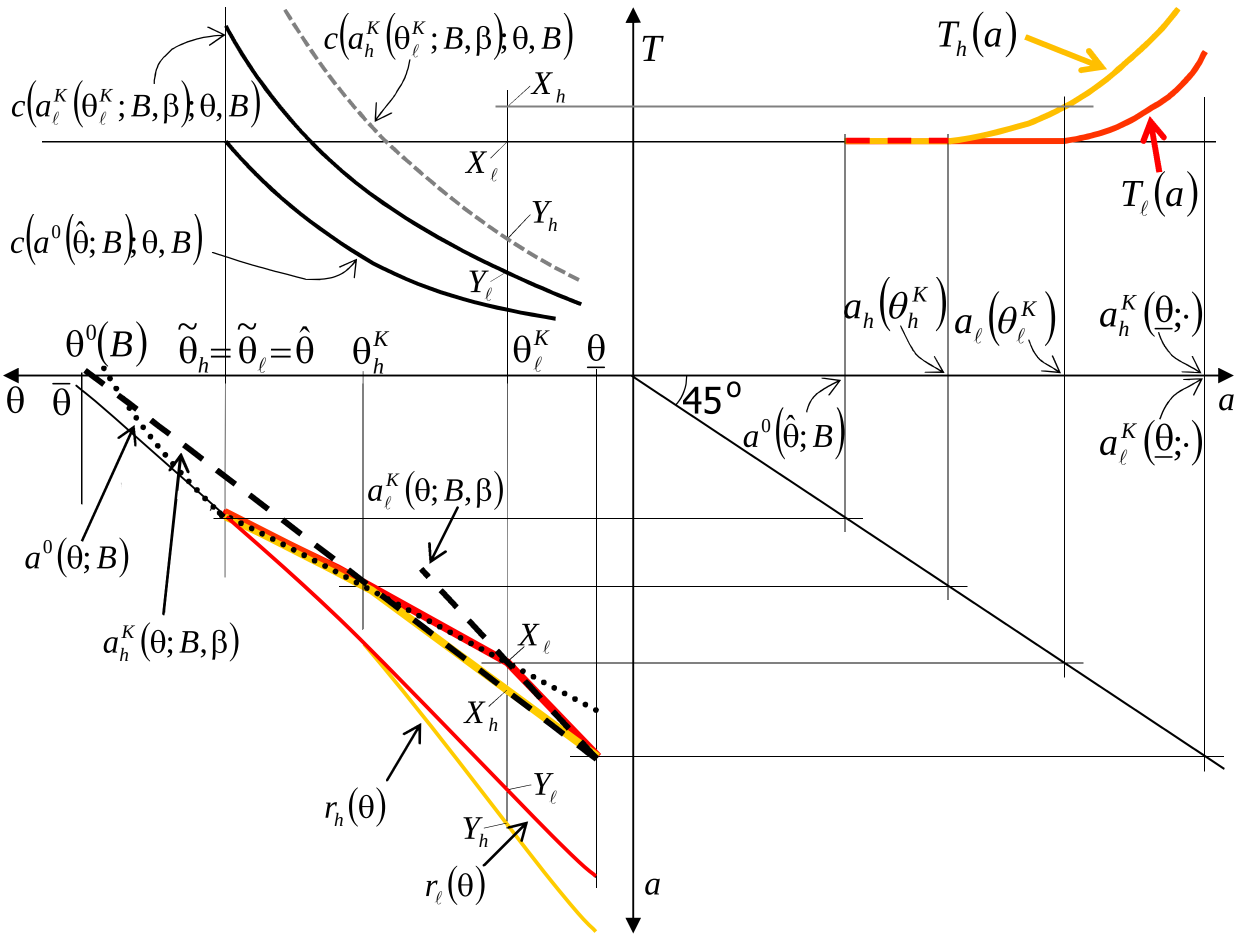

Figure 4: Building the funding menu from the optimal policy.

From the diagrams in the southwest and northwest quadrants, the northeast quadrant constructs, via the 45° line in the southeast quadrant, the funding schedules that implement the second best optimal policy. To see how, consider first the worst institution that receives funding. Its efficiency parameter is $\hat{\theta}$, and it needs to do $a^0\left(\hat{\theta}, B\right)$ applied research. The funding it receives, $T\left(a^0\left(\hat{\theta}, B\right)\right)$, just pays for this research. This amount is measured on the north axis. From the lower curve, it can be seen that the funding needed by institution $\hat{\theta}$ to do $a^0\left(\hat{\theta}, B\right)$ units of applied research is the ordinate of the lower curve on the north axis, $c\left(a^0\left(\hat{\theta}, B\right); \theta, B\right)$. An institution with slightly lower θ, say $\hat{\theta}-\varepsilon$, does, at the optimum, $a^0\left(\hat{\theta}-\varepsilon,B\right)$ applied research, and, by Corollary 3, receives the same amount of funding as a type $\hat{\theta}$ institution: therefore, the ordinate $T_i\left(a^0\left(\hat{\theta}-\varepsilon,B\right)\right)$ is the same as $T_i\left(a^0\left(\hat{\theta},B\right)\right)$, and so the function T_i in the northeast quadrant is flat between $\hat{\theta}-\varepsilon$ and $\hat{\theta}$, even though a $\left(\hat{\theta}-\varepsilon\right)$-type institution does more applied research with the same funding. This is true down to efficiency parameter θ_h^K: all institutions with θ higher than θ_h^K will choose a point in the interval $\left[\hat{\theta},\theta_h^K\right]$, with funding $T\left(a^0\left(\hat{\theta},B\right)\right)$ independent of their type and their reputation.

Consider now an institution with $\theta=\theta_\ell^K$ (or just below). If it has reputation ℓ, it needs to do $a_\ell^K\left(\theta_\ell^K;B,\beta\right)$ applied research, which has a cost $c\left(a_\ell^K\left(\theta_\ell^K;B,\beta\right);\theta,B\right)$, the ordinate of point Y_ℓ on the north axis. The funding it receives is instead $T\left(a^0\left(\hat{\theta}\right);B\right)$, which is higher. The difference between funding and cost of applied research is the vertical distance between points X_ℓ and Y_ℓ. This is the same in the northwest and in the southwest quadrant (a unit of basic research costs a unit of funding), and this amount is devoted by this institutions to basic research. An institution of the same efficiency θ_ℓ^K and higher reputation h does more applied research, so the relevant curve in the northwest is the higher solid gold line. Again the ordinate of point X_h is the total funding received by this institution, and again the distance between points X_h and Y_h is the same in the southwest and in the northwest diagram. Transporting the amount of funding to the northeast quadrant, the gold line is the funding that an h-reputation institution receives when it chooses to carry out the amount $a_h^K\left(\theta_\ell^K;B,\beta\right)$ of applied research. And so on for other values of θ.

In sum, the government funding agency commits to a set of funding schedules, each available to institutions of a given reputation level, as depicted in the northeast quadrant of Figure 4, and institutions choose the amount of

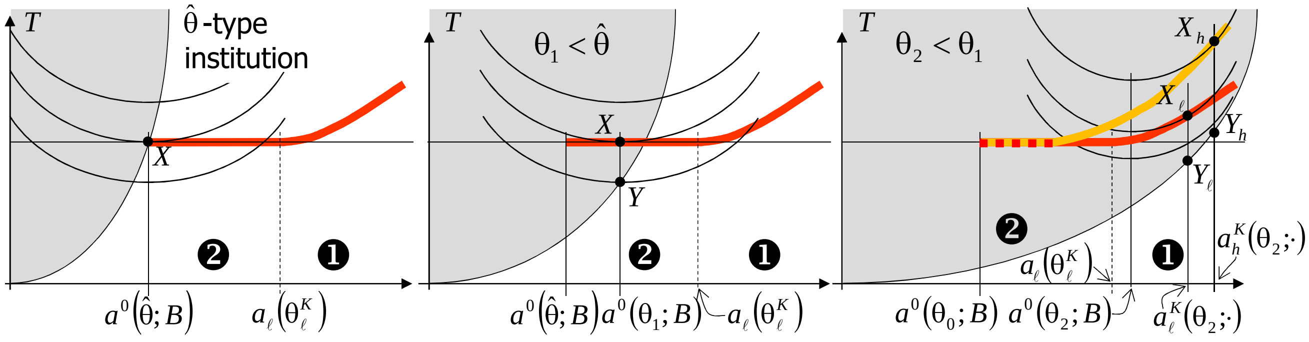

Figure 5: Implementation: The LHS of Figure 2.

applied research they do, and receive the corresponding level of funding, measured on the vertical axis, as the ordinate of the schedule corresponding to their reputation group.

Research and efficiency

Figure 5 shows in more detail how three institutions, with the same reputation ℓ and different efficiency, choose their preferred combination of applied research and funding when faced with the schedule $T_\ell(a)$, derived in the northeast quadrant of Figure 4. In each panel, the function $T_\ell(a)$, defined for $a \in \left[a^0\left(\hat{\theta}; B\right), a_\ell^K\left(\underline{\theta}; B, \beta\right)\right]$ is the thick red solid line; because the funding agency cannot tell types apart, it is the same in the three panels. Points on this locus represent combinations of funding and applied research which the funding agency allows research institutions with reputation ℓ to choose from (it is again drawn for the case depicted in the LHS panel of Figures 2 and 3). Each diagram shows, shaded, the "feasible set", the combinations of funding and the amount of applied research which a type θ institution is able to carry out with that funding. It also shows, as the solid thin lines, the indifference curves: these are the combinations of funding and applied research which allow the institutions to carry out a constant amount of research, basic plus applied.[22] The LHS, the middle and the RHS panel show these for the least efficient active institution, for a slightly more efficient one, and for a very efficient institution, respectively. Consider first a type $\hat{\theta}$ institution, shown on the LHS panel. Its feasible set, the grey shaded area, is the set $\left\{(a,T) \in \mathbb{R}_+^2 | c\left(a,\hat{\theta},B\right) \leqslant T\right\}$; its shape follows from $c_{aa}(\cdot) > 0$ postulated in Assumption 2.2.(i). This institution has effectively no choice: only the point $\left(a^0\left(\hat{\theta};B\right), T_\ell\left(a^0\left(\hat{\theta};B\right)\right)\right)$, marked by X in the LHS diagram, is both on the solid thick locus and in its "feasible set". Not so however for more efficient research institutions: take type $\theta_1 \in \left(\theta_\ell^K, \hat{\theta}\right)$, illustrated in the middle panel. Its feasible set is obviously bigger than a type $\hat{\theta}$'s. It therefore has a genuine choice among the points which are both in the grey area and on the thick solid red line. The best among such points is $(a^0(\theta_1;B), T_\ell(a^0(\theta_1;B)))$, point X in the diagram, the point of tangency between the highest indifference curve and the thick solid line. Notice that the required level of applied research, $a^0(\theta_1;B)$, will cost this institution only $c\left(a^0\left(\hat{\theta};B\right),\hat{\theta},B\right)$, the vertical height of point Y, which is less than $T_\ell(a^0(\theta_1;B))$ (which is equal to $T_\ell(a^0(\theta_1;B))$). After it has paid for its applied research, it will spend its "leftover" funding on basic research, which has marginal cost of 1, rather than on more applied research, which, if pushed above $a^0(\theta_1;B)$, would have a marginal cost exceeding 1. A type θ_1 institution, therefore, carries out an amount of basic research measured by the vertical distance between points Y and X in the middle panel of Figure 5.

Finally consider a very efficient institution, one with $\theta_2 < \theta_\ell^K$. Its efficient level of applied research is $a^0(\theta_2;B)$, the abscissa of the minima of the indifference curves in the RHS panel of Figure 5. This is the level it would choose

[22]Because of the separability in the cost function, the indifference curves all reach a minimum at $a = a^0(\theta;B)$, as can be seen by totally differentiating $a + t - c(a,\theta,B)$.

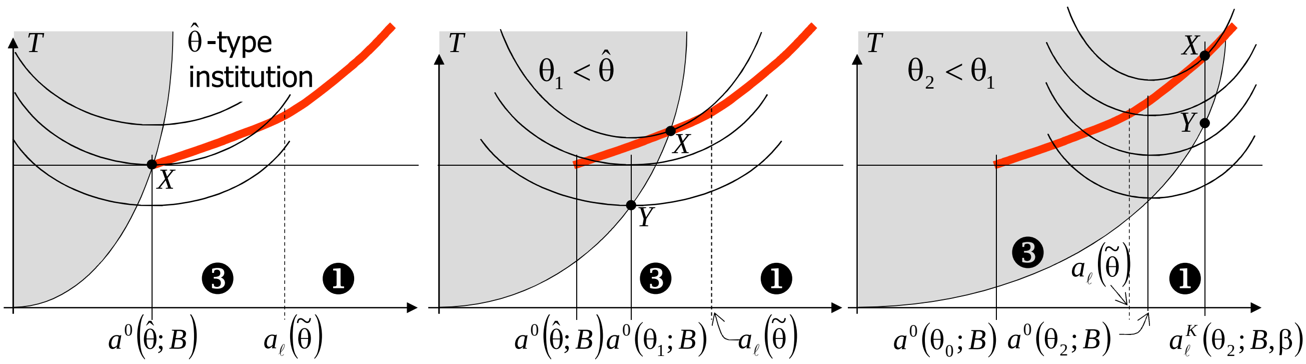

Figure 6: Implementation: The RHS of Figure 2.

if funding were constant. But the optimal policy is designed so that this institution does more than this amount: faced with the solid thick schedule, a type θ_2 research institution chooses the combination that allows it to be on the highest possible indifference curve, namely tangency point X_ℓ in the RHS panel of the diagram in Figure 5.[23] I explain in Section 5 below how research grant funding induces institutions to choose this point. This institution's cost of carrying out the amount of applied research $a_\ell^K(\theta_2; B, \beta)$ is the ordinate of point X_ℓ, and so a type θ_2 institution spends the rest, measured by the distance between X_ℓ and Y_ℓ, on basic research. I have also drawn the gold curve, which shows how an equally efficient institution with a better reputation would be offered a schedule such that it would choose a higher level of applied research, X_h, and spend the amount measured by the vertical distance between Y_h and X_h on basic research.

When the relative position of the curves $a^0\left(\hat{\theta}; B\right)$ and $a_\ell^K(\theta; B, \beta)$ is instead as shown in the RHS panel of Figure 2, the optimal funding can be implemented by the schedule illustrated in Figure 6. This differs from Figure 5 only in that the initial part of the schedule is also increasing. The RHS and

[23]When the curve $C_i(a)$ is convex, as in Figure 5, then the tangency point is a local, and hence a global, maximum; this is shown in Lemma A4. If the curve $C_i(a)$ were concave, then the tangency point would clearly be a maximum.

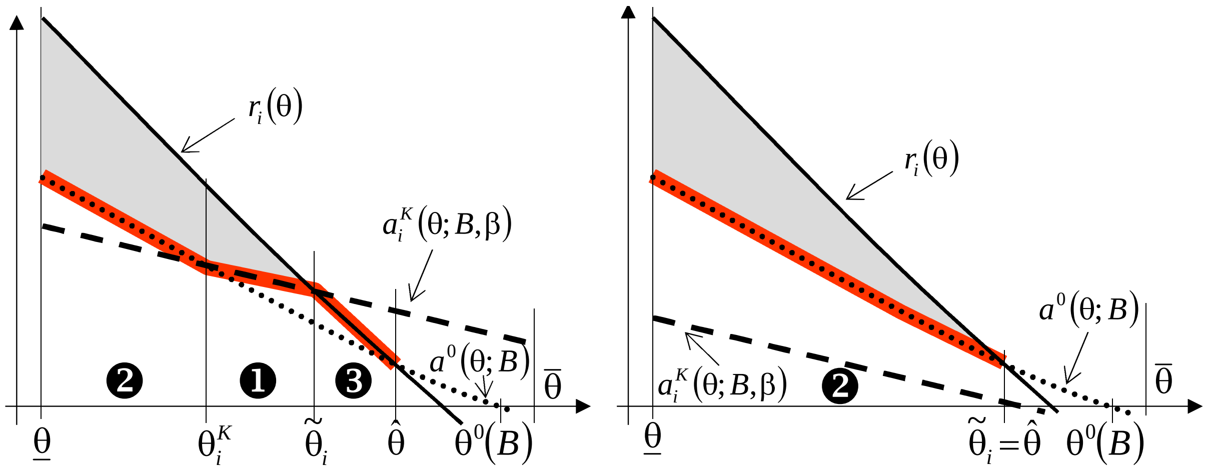

Figure 7: Applied and basic research. Low social value of applied research.

the LHS panels are conceptually identical in Figures 5 and 6: a type $\hat{\theta}$ institution has no choice (LHS) and efficient institutions do more applied research than they would like (RHS), and have enough funding to do basic research. In the middle panel, in contrast, an institution of an intermediate θ is seen to spend all of its budget on applied research, to do more than its efficient level of applied research, and to have no funding left for basic research: in the picture, the solid thick curve is steeper that the indifference curve at the boundary of the feasible set.

5 Remarks

Low social value of applied research

I end the article with three observations. The first sketches how the analysis changes when the second inequality in Assumption 3 is violated, that is when the social value of applied research is low. The curve $a_i^K(\cdot)$ is below $a^0(\cdot)$ at $\theta = \underline{\theta}$. This is illustrated in Figure 7. If a solution exists, the most productive research institutions carry out their preferred level of applied research. However, if there are institutions which are given an incentive to do more than this, as in the left hand side panel, they are the middle θ institutions: regions 1 and 2 in Figure 5 are "swapped". If the relationship between applied research and efficiency is reversed in this case, the relationship between reputation and applied research is not. Given that the schedule $a_i^K(\cdot)$ rotates anticlockwise around its leftmost point when reputation becomes higher (compare the gold and the red curve in Figure 3), the institutions whose applied research exceeds the individually rational level $a^0(\theta; B)$ do more research if their reputation is higher.

If the relative position of the various curves is as depicted in the RHS panel of Figure 7, then the optimal policy is implemented simply with constant funding: the rotation of the schedule $a_i^K(\cdot)$ due to the higher reputation has no effect and all research institutions that agree to carry out at least $a^0\left(\hat{\theta}; B\right)$ applied research, receive the funds necessary to pay for it, which they can then use in any way they choose. In this case the diagram of the funding schedule looks exactly the same as the flat segment, the initial portion of the thick solid line on Figure 5, from $a^0\left(\hat{\theta}; B\right)$ to $a_i\left(\theta_i^K\right)$.

Recall that the relative position of the two curves depends on the social value of research, which, as discussed, might be lowered by international spillovers. It seems plausible that the situation depicted in the RHS panel of Figure 7 applies to a small country, which would be less able to internalise the benefits of applied research. In this light, the discussion of this section would therefore loosely suggest that smaller countries should be more likely to adopt a constant funding scheme.

"Dual support system"

In many countries, research is funded through a dual channel funding mechanism: some funding is a lump-sum, and some is allocated on a project by project basis (for example, DBIS 2010). The optimal mechanism derived in Section 4 can be implemented in a way that resembles this principle: all institutions, regardless of their reputation, can apply for lump sum funding

$$c\left(a^0\left(\hat{\theta}; B\right), \hat{\theta}, B\right), \tag{27}$$

provided they carry out at least the "qualifying" level of applied research $a^0\left(\hat{\theta}; B\right)$. In addition, institutions can apply to have specific projects funded through a grant. However, these grants are not available to all institutions: to qualify to apply, an institution needs to carry out at least a threshold level $a^0\left(\theta_i^K; B\right)$ of applied research with the fixed sum (27). This higher threshold is set at a lower level for institutions with better reputation: this follows from $a^0\left(\theta_h^K; B\right) > a^0\left(\theta_\ell^K; B\right)$. The additional grant funding is governed by the formula

$$g_i(a) = T_i\left(a + a^0\left(\theta_i^K; B\right)\right) - c\left(a^0\left(\hat{\theta}; B\right), \hat{\theta}, B\right), \tag{28}$$

where $g_i(a)$ is the amount of grant awarded for agreeing to carry out a additional units of applied research, over and above to the qualifying level $a^0\left(\theta_i^K; B\right)$.

Intuitively, institutions with weaker reputations are set a higher hurdle before they are allowed to apply for a grant. An example that fits precisely this aspect of the optimal policy is the funding for UK doctoral centres in the social sciences, which is restricted to institutions which had a sufficiently high reputation, precisely defined as having obtained at least a target score in the previous research assessment exercise, regardless of the intrinsic merits

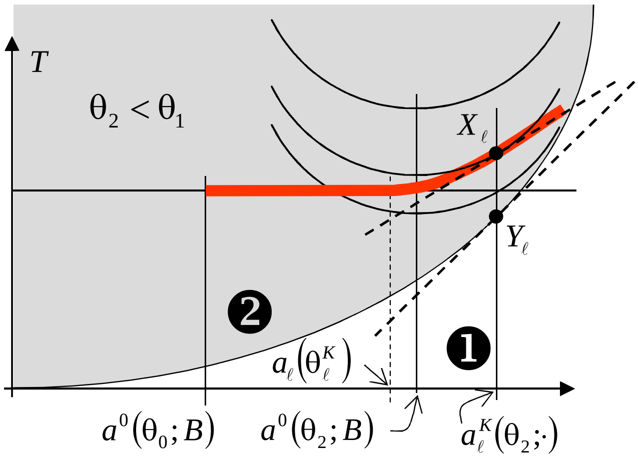

Figure 8: Marginal cost and marginal funding for research grants.

of the application.[24] Also, as again can be seen from Figure 4 above, the optimal policy is such that the amount of research funded by grants is higher in institutions with the same efficiency θ but better reputation.

Full economic costing

A consequence of (28) is that the amount awarded as a research grant for a specific project does not cover the additional cost of the project, except possibly for very high levels of funding. Formally.

Corollary 5 *Suppose* $\theta_i^K < \tilde{\theta} = \hat{\theta}$. *There exists* $\Delta > 0$ *such that there exists* $\theta_\Delta < \theta_i^K$ *such that* $g(\Delta) < c\left(a^0\left(\theta_i^K; B\right) + \Delta, \theta, B\right) - c\left(a^0\left(\theta_i^K; B\right), \theta, B\right)$ *for every* $\theta \in \left(\theta_\Delta, \theta_i^K\right)$.

Graphically, this is illustrated in Figure 8, which is the RHS panel of Figure 5 for an institution of reputation ℓ and efficiency parameter θ_2. Corollary 5 says that the slope of the solid thick red curve in a neighbourhood of

[24]More generally, even without a formal bar to apply for grant funding, low reputation institutions are often hampered by stringent requirements regarding, for example, research infrastructure and institutional support, and in practice they do receive as research grants a lower proportion of their funding than institutions carrying out more research.

$a_\ell^K(\theta_2; B, \beta)$, given by the dashed line through point X_ℓ, which represents the *additional funding* received by an institution of type θ_2 which exceeds by a small amount its level of applied research, $a_\ell^K\left(\theta_i^K; B, \beta\right)$ is less than the slope of the frontier at the same point, the dashed line through point Y_ℓ, which measures the *additional cost* incurred by such an institution for this increase in applied research.

In words, the additional funding does not cover the extra cost of grant funded applied research, which is therefore "co-funded" by the grant funding agency and the institution. This can be compared with the practice of "full economic costing", adopted, among others, by the research councils in the UK (RCUK/UUK 2010): the amount of funding for a research grant is calculated to exceed the cost to carry out the research it intends to fund. The rationale for this mechanism is that the additional funds cover the institution's fixed cost, thus avoiding cross-subsidisation among an institution's activities. My results here however do not lend support to this rationale. The optimal policy is more subtle and does entail cross-subsidisation from the block grant to co-funding specific research projects. This is arguably in line with the principle of designing incentives to delegate a decision to the economic agents possessing the private information relevant to that decision.

Finally, note that the argument underlying Corollary 5 does not apply if curve $T_i(a)$ is concave, and may moreover be reversed for higher values of applied research: very expensive applied research projects, which are carried out by very efficient institutions, may require funding that exceeds their cost.

6 Concluding remarks

The aim of this article is to lay a microeconomic foundation for the analysis of the public funding of research. Its building blocks are an information advantage of research institution vis-à-vis their funders, and the misalignment in objectives between funders and institutions, the latter, at the optimum, preferring basic research. I derive a number of theoretical conclusions, which help to assess the mechanisms used in practice to award government research funding: for example, institutions that are intrinsically better at applied research ought to receive more funding, not just in absolute terms, which is natural as they do more research, but per unit of research as well. This is both because they do more expensive research, and as an incentive payment: they are rewarded for taking on this more expensive applied research, and choose to spend this incentive payment on basic research, hence they do more *basic* research as well, even though they do not have an absolute advantage in this activity. They would of course do even more basic research if they also had an absolute advantage in this activity: modifying the model to allow the cost of basic research to increase with the idiosyncratic parameter θ would add algebraic complication but leave the results qualitatively unchanged, provided that the increase is less steep than for applied research, that is as long as better institutions have a *comparative* advantage in applied research.

The model is sufficiently precise to shed light on some of the mechanisms used in practice to allocate research funding. For example, government agencies typically award research grants on a "cost-plus" principle, whereas charitable bodies require co-funding of research activities. The latter can be justified on the basis of the analysis of the model, whereas the former cannot. The article also shows that distribution of government funds should depend on past success: the funding opportunities available to more prestigious institutions should be wider than those that less prestigious one can draw from.

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Appendix A

Proof of Proposition 1. Divide the government objective function (6) by $(1+\lambda)$, introduce the auxiliary variables A_i and B_i, $i = 1, \ldots, N$, defined in (A1b) and (A1c) as the amount of applied and basic research carried out in i-reputation institutions, and substitute (9) and the value of T to write the optimization problem as:

$$\max_{\substack{\left\{\{r_i(\theta), a_i(\theta)\}_{\theta\in[\underline{\theta},\hat{\theta}_i]}, A_i, B_i\right\}_{i=1}^{N} \\ A,B}} \left\{ \frac{Y(A) + k(A+B)}{1+\lambda} - \right. \tag{A1a}$$

$$\left. \sum_{i=1}^{N} q_i \int_{\underline{\theta}}^{\bar{\theta}} \Big[c(a_i(\theta), \theta, B) + r_i(\theta) - a_i(\theta) \Big] f_i(\theta)\, d\theta \right\},$$

$$\text{s.t.} \int_{\underline{\theta}}^{\bar{\theta}} a_i(\theta) f_i(\theta)\, d\theta = A_i, \qquad i = 1, \ldots, N, \tag{A1b}$$

$$\int_{\underline{\theta}}^{\bar{\theta}} [r_i(\theta) - a_i(\theta)] f_i(\theta)\, d\theta = B_i, \qquad i = 1, \ldots, N, \tag{A1c}$$

$$\sum_{i=1}^{N} q_i A_i = A, \qquad \sum_{i=1}^{N} q_i B_i = B, \tag{A1d}$$

$$r_i(\theta) - a_i(\theta) \geqslant 0, \;\; a_i(\theta) \geqslant 0, \qquad i = 1, \ldots, N. \tag{A1e}$$

Ignoring for the moment the constraints (A1e), the Lagrangean for (A1a) is:

$$\mathcal{L}(\cdot) = \sum_{i=1}^{N} \Big\{ -q_i \Big[c(a_i(\theta), \theta, B) + r_i(\theta) - a_i(\theta) \Big] + \sigma_i a_i(\theta) + (1-\beta_i)(r_i(\theta) - a_i(\theta)) \Big\} f_i(\theta)$$
$$+ (1-\beta) \left(\sum_{i=1}^{N} q_i B_i - B \right) + \sigma \left(\sum_{i=1}^{N} q_i A_i - A \right), \tag{A2}$$

where, following Leonard and van Long (1992), σ_i and $(1-\beta_i)$ are the (constant) Lagrange multipliers for constraints (A1b) and (A1c). Similarly, σ and $(1-\beta)$ are the Lagrange multipliers for the constraints in (A1d). I write the multipliers as $(1-\beta_i)$ and $(1-\beta)$ to lighten notation. The first order conditions give (see

Leonard and van Long, 1992, Theorem 7.11.1):

$$\frac{\partial \mathcal{L}}{\partial a_i(\theta)} = \left\{ q_i \left[-c_a\left(a_i(\theta), \theta, B\right) + 1 \right] + \sigma_i - (1-\beta_i) \right\} f_i(\theta) = 0, \tag{A3a}$$

$$\frac{\partial \mathcal{L}}{\partial r_i(\theta)} = \left(-q_i + (1-\beta_i)\right) f_i(\theta) = 0, \tag{A3b}$$

$$\sigma_i = \sigma q_i, \tag{A3c}$$

$$(1-\beta_i) = (1-\beta)\, q_i, \tag{A3d}$$

each for $i = 1, \ldots, N$, and

$$\sigma = \frac{k + Y'(A)}{1+\lambda}. \tag{A4}$$

(A3b) implies $1 - \beta_i = q_i$, $i = 1, \ldots, N$, and so $\beta = 0$ from (A3d). Next, substitute (A3c) into (A3a), note that the constraint which were ignored, $a_i(\theta) \geqslant 0$ and $r_i(\theta) - a_i(\theta) \geqslant 0$, are satisfied at this solution whenever $\theta \leqslant \theta^*$. This determines $\hat{\theta}_i = \theta^*$ for $i = 1, \ldots, n$. When $\theta > \theta^*$, then $a_i(\theta) = 0$. Using the same argument given below, at the end of Proposition 3, I can show that conditions (A3a)-(A3d) and (A4) are sufficient, and the result follows. ■

Proof of Proposition 2. The argument which derives (15) establishes (12a). Consider next the other statements. Clearly $b_i(\theta)$ must be non-negative, and so (12d) must hold. Now (12b): following Laffont and Tirole (1993), if

$$-c_{a\theta}\left(a_i(x), \theta, B\right) \dot{a}_i(x) \geqslant 0, \tag{A5}$$

then the first order conditions are sufficient for a maximum and the policy is incentive compatible. Given my assumption that $c_{a\theta}\left(a_i(x), \theta, B\right) > 0$, (A5) requires (12b) to hold.

Finally derive (12c). This follows from the observation that total funding must be decreasing in θ. If it were not the case, then an institution could simply claim to have a higher θ than it has, thus receiving more funding, which it could spend on basic research. Therefore

$$\frac{d\left(c\left(a_i\left(\theta\right),\theta,B\right)+r_i\left(\theta\right)-a_i\left(\theta\right)\right)}{d\theta}\leqslant 0. \tag{A6}$$

Expand (A6):

$$c_a\left(\cdot\right)\dot{a}_i\left(\theta\right)+c_\theta\left(\cdot\right)+\dot{r}_i\left(\theta\right)-\dot{a}_i\left(\theta\right)\leqslant 0,$$

which becomes, using (12a),

$$\left[c_a\left(a_i\left(\theta\right),\theta,B\right)-1\right]\dot{a}_i\left(\theta\right)\leqslant 0.$$

Because $\dot{a}_i\left(\theta\right)\leqslant 0$, $c_a\left(a_i\left(\theta\right),\theta,B\right)$ must be greater than or equal to 1, which is (12c). Finally, as $a^0\left(\theta,B\right)$ is lower than $\dot{r}_i\left(\theta\right)$ to the left of their intersection, by Assumption 4.(i), this also determines the boundary condition in (12a). ■

Proof of Proposition 3. The problem can be rewritten as Problem (A1a) with the additional constraints (12a), (12b), and (12c) the last replacing the constraints $a_i\left(\theta\right)\geqslant 0$. The Lagrangean for this problem is the same as (A2), with the following added terms:

$$\sum_{i=1}^{N}\Big\{\mu_i\left(\theta\right)c_\theta\left(a_i\left(\theta\right),\theta,B\right)+\gamma_i\left(\theta\right)\left(a_i\left(\theta\right)-a^0\left(\theta;B\right)\right)+\pi_i\left(\theta\right)\left(r_i\left(\theta\right)-a_i\left(\theta\right)\right)\Big\},$$

where $\mu_i\left(\theta\right)$, $\gamma_i\left(\theta\right)$, and $\pi_i\left(\theta\right)$ are the multipliers associated respectively to constraints (12a), (12c), and (12d). The first order conditions for $r_i\left(\theta\right)$ and $a_i\left(\theta\right)$ are given by:

$$-\frac{\partial\mathcal{L}}{\partial r_i\left(\theta\right)}=\dot{\mu}_i\left(\theta\right)=q_if_i\left(\theta\right)-\left(1-\beta_i\right)f_i\left(\theta\right)-\pi_i\left(\theta\right),\qquad \mu_i\left(\underline{\theta}\right)=0,\ \mu_i\left(\hat{\theta}_i\right)\ \text{free}; \tag{A7a}$$

$$\frac{\partial\mathcal{L}}{\partial a_i\left(\theta\right)}=\Big\{-q_i\left[c_a\left(a_i\left(\theta\right),\theta,B\right)-1\right]+\sigma_i+\left(1-\beta_i\right)\Big\}f_i\left(\theta\right)+\gamma_i\left(\theta\right)-\pi_i\left(\theta\right)$$
$$-\mu_i\left(\theta\right)c_{\theta a}\left(a_i\left(\theta\right),\theta,B\right)=0. \tag{A7b}$$

As in Problem (A1a), the first order conditions for A_i and B_i give (A3c) and (A3d), and so (A7a) can be written as

$$\dot{\mu}_i(\theta) = \beta q_i f_i(\theta) - \pi_i(\theta), \qquad \mu_i(\underline{\theta}) = 0, \qquad \mu_i\left(\hat{\theta}_i\right) \text{ free,}$$

where, as before, $(1-\beta) \geqslant 0$ and $\sigma \geqslant 0$ are the multipliers for the constraints in (A1d). The above has solution:

$$\mu_i(\theta) = \beta q_i F_i(\theta) - \Pi_i(\theta), \tag{A8}$$

having defined $\Pi_i(\theta) = \int_{\underline{\theta}}^{\theta} \pi_i(z)\, dz$.

The multipliers β and σ are obtained from the first order conditions for A and B. The one for A is identical to the one given in Proposition 1, giving again $\sigma = \frac{k+Y'(A)}{1+\lambda}$. The one for B is derived in the following Lemma.

Lemma A1

$$1-\beta = \frac{\frac{k}{1+\lambda} - \sum_i q_i c_B(\cdot) F_i\left(\hat{\theta}_i\right)}{1 - \sum_i \int_{\underline{\theta}}^{\hat{\theta}_i} F_i(\theta) c_{\theta B}(\cdot)\, d\theta} + \frac{\sum_i \int_{\underline{\theta}}^{\hat{\theta}_i} \left[\Pi_i(\theta) c_{\theta B}(\cdot) + \gamma_i(\theta) \frac{c_{aB}(\cdot)}{c_{aa}(\cdot)}\right] d\theta}{1 - \sum_i \int_{\underline{\theta}}^{\hat{\theta}_i} F_i(\theta) c_{\theta B}(\cdot)\, d\theta}.$$

Proof. Take the first order condition for B, use (A8) and the definition of $a^0(\theta; B)$, which implies $\frac{\partial a^0}{\partial B} = -\frac{c_{aB}(\cdot)}{c_{aa}(\cdot)}$, writing $(\cdot)$ for $\left(a_i\left(\hat{\theta}_i\right), \hat{\theta}_i, B\right)$:

$$1-\beta = \frac{k}{1+\lambda} + \sum_{i=1}^{N} q_i \int_{\underline{\theta}}^{\hat{\theta}_i} \left[-c_B(\cdot) f_i(\theta) - \left(\beta F_i(\theta) - \Pi_i(\theta)\right) c_{\theta B}(\cdot) + \gamma_i(\theta) \frac{c_{aB}(\cdot)}{c_{aa}(\cdot)}\right] d\theta. \tag{A9}$$

Integration by parts gives:

$$1-\beta = \frac{k}{1+\lambda} - \sum_{i=1}^{N} q_i \left\{ c_B(\cdot) F_i\left(\hat{\theta}_i\right) + (1-\beta) \int_{\underline{\theta}}^{\hat{\theta}_i} F_i(\theta) c_{\theta B}(\cdot)\, d\theta \right.$$
$$\left. + \int_{\underline{\theta}}^{\hat{\theta}_i} \left[\Pi_i(\theta) c_{\theta B}(\cdot) + \gamma_i(\theta) \frac{c_{aB}(\cdot)}{c_{aa}(\cdot)}\right] d\theta \right\},$$

which gives the Lemma. ■

Consider now the first order condition for B.

$$1-\beta=\frac{k}{1+\lambda}-\sum_{i=1}^{N}q_i\int_{\underline{\theta}}^{\bar{\theta}}c_B\left(a_i\left(\theta\right),\theta,B\right)f_i\left(\theta\right)d\theta. \tag{A10}$$

Notice first that $\beta\geqslant 0$: $(1-\beta)$ measures the benefit of relaxing the constraint $b_i(\theta)\geqslant 0$, which has a cost of 1, measured in the social value of monetary units. The funding agency can always increase $b_i(\theta)$ if it wants, because it can simply increase the funding to all research institutions, and, as at the optimum they all do at least $a^0(\theta;B)$, they all prefer to spend the additional funding on basic research. Therefore the benefit of increasing any of the $b_i(\theta)$'s cannot exceed the cost at the optimum: $(1-\beta)\leqslant 1$. Next I show that $\beta>0$. Suppose by contradiction that $\beta=0$. Then $\pi_i(\theta)=\Pi_i(\theta)=\gamma_i(\theta)=0$, and $\mu_i(\theta)=0$ (from (A8)), so that (A7b) and (A9) reduce to (10a) and (17). Therefore B is given by $\bar{B}$, determined in (17). However, the individual amount of basic research needs also to satisfy the incentive compatibility constraint (12a). By Assumption 5, $\bar{B}$ violates this requirement, and this is against the contradiction hypothesis $\beta=0$.

The final set of first order conditions are those for $\hat{\theta}_i$. They are given by (Leonard and van Long 1992):

$$c\left(a^0\left(\hat{\theta}_i;B\right),\hat{\theta}_i,B\right)-\sigma_i a^0\left(\hat{\theta}_i;B\right)=\frac{\mu_i\left(\theta\right)}{f_i\left(\theta\right)}c_\theta\left(a^0\left(\hat{\theta}_i;B\right),\hat{\theta}_i,B\right),$$

which, given $\mu_i(\theta)$, determines $\hat{\theta}_i$. Notice that because $a_i(\theta)$ is the same for $i=1,\ldots,N$ in a left neighbourhood of $\hat{\theta}_i$, then $\hat{\theta}_i$ must be the same for all $i=1,\ldots,N$.

To continue with the proof, return to the first order conditions for $a_i(\theta)$, and substitute (A8), (A3c), and (A3d) into (A7b) to rewrite it as:

$$c_a\left(a_i\left(\theta\right),\theta,B\right)=\frac{Y'\left(A\right)+k}{1+\lambda}+\beta+\frac{\gamma_i\left(\theta\right)-\pi_i\left(\theta\right)}{q_i f_i\left(\theta\right)}-\frac{\beta F_i\left(\theta\right)-\Pi_i\left(\theta\right)}{f_i\left(\theta\right)}c_{\theta a}\left(a_i\left(\theta\right),\theta,B\right). \tag{A11}$$

In what follows I consider a fixed i, and drop the subscript i to lighten notation.

Define the function $a^K_\Pi(\theta; B, \beta)$ as the solution in a of

$$c_a(a,\theta,B) = \frac{Y'(A)+k}{1+\lambda} + \beta - \frac{\beta F(\theta) - \Pi}{f(\theta)} c_{\theta a}(a,\theta,B). \tag{A12}$$

If $\Pi = 0$, then $a^K_\Pi(\theta; B, \beta) = a^K(\theta; B, \beta)$ and if $\Pi > 0$, then $a^K_\Pi(\theta; B, \beta) > a^K(\theta; B, \beta)$, because $c_{\theta a}(\cdot) > 0$.

Next notice that, depending on the combination of complementary slackness for constraints (12c) and (12d), a value of $a(\theta)$ belongs to one of four possible regions, defined by the pairs of inequality constraints which are satisfied as a strict inequality.

1. $a(\theta) - a^0(\theta; B) > 0$ and $r(\theta) - a(\theta) > 0$. Therefore, $\gamma(\theta) = \pi(\theta) = 0$, which means $r(\theta) > a(\theta) > a^0(\theta; B)$, and in this region, $r(\theta) = r^0\left(\theta; B, \hat{\theta}\right)$, $a(\theta) = a^K_{\Pi(\theta)}(\theta; B, \beta)$.

2. $r(\theta) - a(\theta) > 0$ and $\gamma(\theta) > 0$. Here, $a(\theta) - a^0(\theta; B) = 0$ and $\pi(\theta) = 0$, and so $r(\theta) = r^0\left(\theta; B, \hat{\theta}\right)$, $a(\theta) = a^0(\theta; B)$.

3. $a(\theta) - a^0(\theta; B) > 0$ and $\pi(\theta) > 0$. In this region $\gamma(\theta) = 0$ and $r(\theta) = a(\theta) = r^0\left(\theta; B, \hat{\theta}\right)$.

4. $\gamma(\theta) > 0$ and $\pi(\theta) > 0$. Here, $r(\theta) = r^0\left(\theta; B, \hat{\theta}\right) = a^0(\theta; B) = a(\theta)$, and therefore this region is just the single intersection point between $a^0(\theta; B)$ and $r^0\left(\theta; B, \hat{\theta}\right)$.

As a preliminary step, I show that

$$\begin{aligned} &\text{if } \theta \in \left[\underline{\theta}, \tilde{\theta}\right) \text{ then } a(\theta) > 0 \text{ and } b(\theta) > 0; \\ &\text{if } \theta \in \left[\tilde{\theta}, \hat{\theta}\right] \text{ then } a(\theta) > 0 \text{ and } b(\theta) = 0. \end{aligned}$$

Proposition 3 requires that $\underline{\theta}$ belongs to region 1, that is that $a(\underline{\theta}) < r^0\left(\underline{\theta}; B, \hat{\theta}\right)$. Suppose by contradiction that $a(\underline{\theta}) = r^0\left(\underline{\theta}; B, \hat{\theta}\right)$. Then $b(\theta) = 0$ in $\left[\underline{\theta}, \tilde{\theta}\right]$ for some $\tilde{\theta} \geqslant \underline{\theta}$. Notice next that it cannot be $\tilde{\theta} = \hat{\theta}$, otherwise $b(\theta) = 0$ in $\left[\underline{\theta}, \overline{\theta}\right]$ and so $B = 0$, against the Inada Condition, Assumption 2.3.(ii). That is, there is $\tilde{\theta} < \hat{\theta}$ such that $a(\theta) = a_{\Pi}^{K}(\theta; B, \beta) < r^0\left(\theta; B, \hat{\theta}\right)$ in a right neighbourhood of $\tilde{\theta}$, with of course $a\left(\tilde{\theta}\right) = r^0\left(\tilde{\theta}; B, \hat{\theta}\right) = a_{\Pi}^{K}\left(\tilde{\theta}; B, \beta\right)$. Now I show that at any intersection between $r^0\left(\theta; B, \hat{\theta}\right)$ and $a_{\Pi}^{K}(\theta; B, \beta)$, the latter is less steep than $r^0\left(\theta; B, \hat{\theta}\right)$, and thus we obtain a contradiction: if $a_{\Pi}^{K}(\theta; B, \beta)$ is less steep than $r^0\left(\theta; B, \hat{\theta}\right)$ then it must be above it in a right neighbourhood of $\tilde{\theta}$.

Lemma A2 $a_{\Pi}^{K}(\theta; B, \beta) > r^0\left(\theta; B, \hat{\theta}\right)$ *for* $\theta > \tilde{\theta}$.

Proof. To see this, compare $a_{\Pi}^{K}(\theta, B, \beta)$ and $r^0\left(\theta; B, \hat{\theta}\right)$ in a right neighbourhood of their intersection. Because $a(\theta)$ is above $a^0(\theta; B)$ in $\left[\underline{\theta}, \tilde{\theta}\right]$, it must be $\beta F(\theta) - \Pi(\theta) > 0$ in $\left[\underline{\theta}, \tilde{\theta}\right]$. Next totally differentiate (A12):

$$\left[c_{aa}(\cdot) + \frac{\beta F(\theta) - \Pi(\theta)}{f(\theta)} c_{\theta aa}(\cdot)\right] da +$$
$$\left[c_{a\theta}(\cdot) + \frac{\beta F(\theta) - \Pi(\theta)}{f(\theta)} c_{\theta\theta a}(\cdot) + c_{a\theta}(\cdot)\beta \frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right)\right] d\theta = 0.$$

Hence:

$$\frac{\partial a_{\Pi}^{K}(\theta; B, \beta)}{\partial\theta} = -\frac{c_{a\theta}(\cdot) + \frac{\beta F(\theta) - \Pi(\theta)}{f(\theta)} c_{\theta\theta a}(\cdot) + c_{a\theta}(\cdot)\beta \frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right)}{c_{aa}(\cdot) + \frac{\beta F(\theta) - \Pi(\theta)}{f(\theta)} c_{\theta aa}(\cdot)}.$$

I need to verify that the following holds:

$$-\frac{c_{a\theta}(\cdot) + \frac{\beta F(\theta) - \Pi(\theta)}{f(\theta)} c_{\theta\theta a}(\cdot) + c_{a\theta}(\cdot)\beta \frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right)}{c_{aa}(\cdot) + \frac{\beta F(\theta) - \Pi(\theta)}{f(\theta)} c_{\theta aa}(\cdot)} > -c_{\theta}(\cdot).$$

By Assumption 4, $c_{\theta aa}(\cdot) > 0$, and so I can multiply through and rearrange:

$$c_{\theta}(\cdot) c_{aa}(\cdot) - c_{a\theta}(\cdot)\left(1 + \frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right)\right) > \frac{\beta F(\theta) - \Pi(\theta)}{f(\theta)}\left(c_{\theta\theta a}(\cdot) - c_{\theta aa}(\cdot) c_{\theta}(\cdot)\right).$$

Again, by Assumption 4, the RHS is positive and the LHS is negative. Therefore, at their intersection, $\frac{\partial a_{\Pi}^{K}(\cdot)}{\partial\theta} > \frac{\partial r^0(\cdot)}{\partial\theta}$, that is $r^0(\cdot)$ is steeper, and so it is below $a_{\Pi}^{K}(\cdot)$ in a right neighbourhood of their intersection. ■

An implication of the Lemma is that if $a^K_{\Pi}(\cdot)$ defines the optimal schedule, then $a^K_{\Pi}(\cdot) = a^K(\cdot)$. This, by (A12), implies $c(a(\underline{\theta}), \underline{\theta}, B) > 1$, and so $a^K(\cdot) > a^0(\cdot)$, and $\theta^K > \underline{\theta}$.

The first order conditions (A7a) and (A7b) are also sufficient, for fixed threshold $\hat{\theta}$. This follows from Theorem 7.9.1 in Leonard and van Long (1992): the function in the first line of (A1a) is concave as $\frac{Y''(A)}{1+\lambda} < 0$, and the functions in (A1d) are linear and hence concave. The Lagrangen is concave, given that $\frac{\partial^2 \mathcal{L}}{\partial r_i(\theta)^2} = 0$ and

$$\frac{\partial^2 \mathcal{L}}{\partial a_i(\theta)^2} = -q_i c_{aa}(a_i(\theta), \theta, B) f_i(\theta) - \mu_i(\theta) c_{\theta aa}(a_i(\theta), \theta, B)$$

is negative, as $c_{aa}(\cdot) > 0$ by Assumption 2.2, $\mu(\theta) \geqslant 0$ by (A8), and $c_{\theta aa}(\cdot) > 0$ by Assumption 4.(iii).

The Proposition now follows immediately. Notice that constraint (12b) is satisfied, as all three curves $a^0(\theta; B)$, $a^K(\theta; B, \beta)$ and $r^0\left(\theta; B, \hat{\theta}\right)$ are decreasing in θ. ■

Proof of Corollary 1. Proposition 3 shows that $a_i(\theta)$ is one of $a^0(\theta; B)$, $a^K_i(\theta; B, \beta)$ or $a^r\left(\theta; B, \hat{\theta}\right)$. Moreover, as it must lie between $a^0(\theta; B)$ and $a^r\left(\theta; B, \hat{\theta}\right)$, it can only equal $a^K_i(\theta; B, \beta)$ – intersections excepted – between them. (23) follows from (22). ■

Proof of Proposition 4. Consider two reputation groups, h and ℓ, with $h > \ell$. Take the difference in the expressions in (19) for these reputation groups:

$$c_a(a_h, \cdot) - c_a(a_\ell, \cdot) = -\beta\left(\frac{F_h(\theta)}{f_h(\theta)} c_{\theta a}(a_h, \cdot) - \frac{F_\ell(\theta)}{f_\ell(\theta)} c_{\theta a}(a_\ell, \cdot)\right),$$

where, for the sake of brevity, the argument of a_i is omitted and "$\cdot$" stands for "θ, B". Add and subtract $\frac{F_h(\theta)}{f_h(\theta)} c_{\theta a}(a_\ell, \cdot)$ and rearrange:

$$\frac{c_a(a_h, \cdot) - c_a(a_\ell, \cdot)}{\beta} + \frac{F_h(\theta)}{f_h(\theta)}\left(c_{\theta a}(a_h, \cdot) - c_{\theta a}(a_\ell, \cdot)\right) = \left(\frac{F_h(\theta)}{f_h(\theta)} - \frac{F_\ell(\theta)}{f_\ell(\theta)}\right) c_{\theta a}(a_\ell, \cdot);$$

that is

$$\left(\frac{c_{aa}\left(\tilde{a},\cdot\right)}{\beta}+\frac{F_h\left(\theta\right)}{f_h\left(\theta\right)}c_{\theta aa}\left(\tilde{\tilde{a}},\cdot\right)\right)\left(a_h-a_\ell\right)=\left(\frac{F_h\left(\theta\right)}{f_h\left(\theta\right)}-\frac{F_\ell\left(\theta\right)}{f_\ell\left(\theta\right)}\right)c_{\theta a}\left(a_\ell,\cdot\right), \qquad \text{(A13)}$$

where $\tilde{a}$ and $\tilde{\tilde{a}}$ are appropriate intermediate value theorem values. In (A13), the coefficient of $(a_h - a_\ell)$ on the LHS is positive by Assumptions 2.2.(i) and 4.(iii); on the RHS, $c_{\theta a}(a_\ell, \cdot) > 0$, by Assumptions 2.2.(iii), and so the sign of $(a_h - a_\ell) > 0$ equals the sign of the coefficient of $c_{\theta a}(a_\ell, \cdot) > 0$, which is determined by the following Lemma.

Lemma A3 *Assumption 1 implies* $\frac{F_h(\theta)}{f_h(\theta)} > \frac{F_\ell(\theta)}{f_\ell(\theta)}$.

Proof. $F_h(\theta) > F_\ell(\theta)$ implies $-\ln F_h(\theta) < -\ln F_\ell(\theta)$, which can be written as:

$$\int_\theta^{\bar{\theta}} \frac{d\ln F_h\left(x\right)}{dx}dx < \int_\theta^{\bar{\theta}} \frac{d\ln F_\ell\left(x\right)}{dx}dx, \qquad \text{(A14)}$$

for every $\theta \in \left(\underline{\theta}, \bar{\theta}\right)$. Now use $\frac{d\ln F_i(x)}{dx} = \frac{f_i(x)}{F_i(x)}$ to write (A14) as

$$\int_\theta^{\bar{\theta}} \frac{f_h\left(x\right)}{F_h\left(x\right)}dx < \int_\theta^{\bar{\theta}} \frac{f_\ell\left(x\right)}{F_\ell\left(x\right)}dx \qquad \text{for every } \theta \in \left(\underline{\theta}, \bar{\theta}\right),$$

which implies $\frac{f_h(x)}{F_h(x)} < \frac{f_\ell(x)}{F_\ell(x)}$, or $\frac{F_h(x)}{f_h(x)} > \frac{F_\ell(x)}{f_\ell(x)}$, the statement in the Lemma. ■

This proves the first statement in Proposition 4. The same procedure establishes the second part, the relationship between reputation and total research: the difference $\dot{r}_h(\theta) - \dot{r}_\ell(\theta)$ is

$$\dot{r}_h\left(\theta\right)-\dot{r}_\ell\left(\theta\right) = -c_\theta\left(a_h\left(\theta\right),\cdot\right)+c_\theta\left(a_\ell\left(\theta\right),\cdot\right) = -c_{a\theta}\left(\tilde{a},\cdot\right)\left(a_h\left(\theta\right)-a_\ell\left(\theta\right)\right) < 0.$$

The research carried out by an institution of type θ and reputation i is

$$r_i\left(\theta\right) = \int_{\underline{\theta}}^{\hat{\theta}} \dot{r}_i\left(\theta\right)d\theta - \int_{\underline{\theta}}^{\theta} \dot{r}_i\left(\theta\right)d\theta, \qquad i = 1,\ldots,N,$$

which implies

$$r_h(\theta) - r_\ell(\theta) = -\int_\theta^{\theta_\ell^K} \left(\dot{r}_h(\theta) - \dot{r}_\ell(\theta)\right) > 0,$$

and establishes the second part of the statement. ■

Proof of Corollary 2. I continue to omit the subscript i, which does not generate possible confusion. Differentiate (24) with respect to a, using (12a):

$$T'(a) = c_a\left(a, \theta^K(a; B, \beta), B\right) - 1. \tag{A15}$$

The above is positive because $a_i^K(\theta; B, \beta)$ exceeds $a^0(\theta; B)$. T is therefore increasing. For the second part of the statement, expand $T''(a)$:

$$T''(a) = c_{aa}(\cdot) + c_{a\theta}(\cdot)\frac{\partial \theta^K(a; B, \beta)}{\partial a}.$$

This is positive if $-\frac{c_{a\theta}(\cdot)}{c_{aa}(\cdot)} = \frac{\partial a^0(\theta;B)}{\partial \theta} > \frac{\partial a^K(\theta;B,\beta)}{\partial \theta}$. ■

Proof of Corollary 3. The derivative of (26) is:

$$T'(a) = c_a(\cdot) + c_\theta(\cdot)\frac{\partial \theta^0(a;B)}{\partial a} + \frac{\partial a^r\left(\theta^0(a;B); B, \hat{\theta}\right)}{\partial \theta}\frac{\partial \theta^0(a;B)}{\partial a} - 1 = 0,$$

as $c_a(\cdot) = 1$ along $a^0(\theta; B)$. ■

Proof of Corollary 4. Let $\theta^r\left(a; B, \hat{\theta}\right)$ be the inverse function of $a^r\left(\theta; B, \hat{\theta}\right)$, and total funding is given by (recall that $b(\theta) = 0$ in this region):

$$T(a) = c\left(a, \theta^r\left(a, B, \hat{\theta}\right), B\right). \tag{A16}$$

Differentiation with respect to a yields:

$$T'(a) = c_a(\cdot) + \frac{c_\theta(\cdot)}{\frac{\partial a^r(\cdot)}{\partial \theta}} = c_a(\cdot) - 1.$$

This is because $a^r(\cdot)$ is the inverse of $\theta^r(\cdot)$; the second equality follows from the definition of $a^r\left(\theta;B,\hat{\theta}\right)$, given in (21), Corollary 1 in the text. Because $a^r\left(\theta;B,\hat{\theta}\right) > a^0(\theta;B)$ except at $\hat{\theta}$, the above is positive in $\left(\tilde{\theta},\hat{\theta}\right)$. To establish convexity, take $T''(a)$:

$$T''(a) = c_{aa}(\cdot) + c_{a\theta}(\cdot)\frac{\partial\theta^r\left(a;B,\hat{\theta}\right)}{\partial a},$$

which is positive as $-\frac{c_{a\theta}(\cdot)}{c_{aa}(\cdot)} > \frac{\partial a^r\left(\theta;B,\hat{\theta}\right)}{\partial\theta} = -c_\theta(\cdot)$.

For the second part of the statement, note that, in region 3 (that is to the right of their intersection), the slope of $T(a)$ is $c_a\left(a,\theta^r\left(a;B,\hat{\theta}\right),B\right) - 1$. In region 1, namely to the left of their intersection, the slope is $c_a\left(a,\theta^K(a;B,\beta),B\right) - 1$. Consider a right neighbourhood of their intersection: the difference in slope is

$$\begin{aligned} &c_a\left(a,\theta^r\left(a;B,\hat{\theta}\right),B\right) - c_a\left(a,\theta^K(a;B,\beta),B\right) \\ &\qquad = c_{a\theta}(a,\theta_3,B)\left(\theta^r\left(a;B,\hat{\theta}\right) - \theta^K(a;B,\beta)\right). \end{aligned} \tag{A17}$$

This is positive, as $\theta^r\left(a;B,\hat{\theta}\right) - \theta^K(a;B,\beta) > 0$, establishing the statement. ■

Lemma A4 *The tangency point $(a_2, T_i(a_2))$ is a local maximum of the indifference map in the feasible set.*

Proof. At the tangency point $(a_2, T_i(a_2))$, with $a_2 = a_i^K(\theta_2;B,\beta)$, the slope of the indifference curve is given by $c_a(a_2,\theta_2,B) - 1$. The slope of the funding schedule is given by (A15). In a neighbourhood of a_2, we have:

$$c_a(a_2+\varepsilon,\theta_2,B) - c_a\left(a_2+\varepsilon,\theta_i^K(a_2+\varepsilon;B,\beta),B\right) = c_{a\theta}(a_2+\varepsilon,\theta_3,B)\left(\theta_2 - \theta_i^K(a_2+\varepsilon;B,\beta)\right).$$

For some θ_3 in the interval with endpoints θ_2 and $\theta_i^K(a_2+\varepsilon;B,\beta)$. For $\varepsilon > 0$ (respectively $\varepsilon < 0$), the above is positive (respectively negative), as $a_i^K(\cdot)$ is decreasing and so $\theta^K(\cdot)$ is too. ■

Proof of Corollary 5. Omitted. ■