SYMMETRY DETECTION USING FREQUENCY DOMAIN MOTION ESTIMATION TECHNIQUES

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ABSTRACT

A frequency domain approach for the detection of symmetries in real images is presented. Our framework is based on recent state-of-the-art research where motion estimation techniques are employed to sequentially determine all the associated parameters. In particular, we introduce several modifications regarding the order of symmetry estimation and the detection of the axes of possible bilateral symmetry. Preliminary results demonstrate the efficiency of our approach.

***Index Terms*—** symmetry detection, correlation methods

1. INTRODUCTION

Symmetry is an important object visual attribute inferred by the human vision system which drives attention, facilitates perceptual grouping and helps in scene interpretation. Additionally, symmetry detection already finds applications in fundamental computer vision tasks such as image segmentation, recognition and pattern classification. Thus, a module designed for symmetry identification might be useful for machine vision systems which attempt to mimic successfully the way mammals sense and process visual information.

This works aims at detecting possible symmetries of $2D$ objects. For the class of similarity transforms, $2D$ patterns can be characterized by two types of symmetry: rotational and bilateral (or reflectional). A pattern is rotationally symmetric of order n, if it is invariant under $2\pi/n$ rotation about its center of mass, which is also called the center of symmetry. A pattern exhibits bilateral symmetry if it is invariant under reflection about one or more straight lines. A pattern characterized by n reflection symmetry lines is necessarily rotationally symmetric of order n.

Our approach follows recent research in symmetry detection [1], [2], [3], where techniques used in frequency domain motion estimation algorithms are employed to compute a number of parameters related to symmetry identification. In particular, we wish to robustly estimate the order of symmetry, recover the symmetry center, define its type (rotational or bilateral) and determine the axes of possible bilateral symmetry. A frequency domain formulation of symmetry detection has the advantage that the above parameters can be extracted sequentially, thus, reducing computational complexity and enhancing accuracy.

The approach in [1] detects symmetry by minimizing a functional obtained from the analytical Fourier-Mellin representation of a given image; however the scheme used to compute the Fourier-Mellin transform is based on the assumption that the center of symmetry is known. The method presented in [2] is fast, accurate and does not employ any conversion to the polar coordinate system. It is experimentally verified to correctly classify high-order symmetric patterns; nevertheless its applicability to real images is not demonstrated and maybe problematic. Finally, the method in [3] is capable of detecting symmetries under clutter/partial occlusion and is considered state-of-the-art in symmetry detection. The approach starts with obtaining a pseudo-polar Fourier representation of the given image which enables the estimation of the order of symmetry without any prior knowledge of the symmetry center. In particular, the magnitude of the Fourier transform is used to compute a translation-invariant representation of the symmetric pattern; then, the order detection is achieved by deriving and estimating the period of a $1D$ periodic function using MuSIC. Once the order has been identified, this information is used to recover the center of symmetry. Finally, the framework is also able to detect the axes of possible bilateral symmetry by examining the existence of a dominant global maximum of a second appropriately derived $1D$ function.

We step on this approach and based on the methodology suggested we propose a scheme which is briefly summarized in the following steps:

1. Extract image salient features by computing an edge map G which combines both the magnitude and the orientation of image gradients.

2. Feed G to a fast and accurate polar transform [4] and consider the magnitude M of the outcome solely. Estimate the order of symmetry by computing the $2D$ autocorrelation of M, followed by a singular value decomposition and periodicity detection schemes.

3. Exploit symmetry information to estimate the center of symmetry [3].

This work is sponsored by SELEX through the Systems Engineering for Autonomous Systems (SEAS) Defence Technology Centre.

4. Identify the axes of potential bilateral symmetry using gradient cross-correlation [5] combined with a local scheme which exploits symmetry to provide additional robustness to possible partial occlusion and clutter.

The contributions of this work are summarized in steps 1,2 and 4 where new schemes for the estimation of the order of symmetry and the recovery of the axes of bilateral symmetry are introduced.

2. METHODOLOGY

Our approach starts with computing a gray level edge map G of the given image I, which retains both magnitude and phase information, as follows:

$$G = G_x + jG_y \tag{1}$$

where $G_x = \nabla_x I$ and $G_y = \nabla_y I$ are the gradients along the horizontal and vertical direction respectively. This step provides the location, magnitude and orientation of the image high-activity structures which can be used as salient features to characterize symmetry. Assuming that the image mainly captures a large periodic pattern, this operation is very likely to reduce the effect of background clutter, emphasizing the features which vote for the existence of the symmetric structure. At the same time, low spatial frequency components inherent to the low pass nature of images will be filtered out as well. For real images, in most cases, the contribution of low spatial frequencies rather shadows the existence of periodicity than facilitates the symmetry detection process. Additionally, we note that a band-pass filtered version of the original image, as suggested by the use of practical differential operators, eliminates possible noise and aliasing effects [5]. The derived edge map is used throughout this work to estimate the order of symmetry, find the center of symmetry and detect the axes of possible bilateral symmetry as described in the remaining of the paper.

2.1. Estimating the order of symmetry

It is well known that the magnitude M of the Fourier transform of a symmetric image with unknown symmetry center is also symmetric around the origin of order n, if n is even, or $2n$, if n is odd. The main idea is to exploit this property to recover first the order of symmetry [3]. A polar representation $M(r, \theta)$ results in a repeated pattern of period n or $2n$ with respect to the angular parameter θ.

We start by computing the Fourier transform of G and considering its magnitude M solely. To obtain a polar representation of M, three options exist. The first is to evaluate the Fourier Transform over the Cartesian grid using the standard FFT and then interpolate the outcome over a polar grid. In general, such an approach is unstable and sensitive to interpolation errors even for good quality images [6]. The next option is to approximate the polar with a pseudo-polar grid for which a fast algorithm exists and its computation is algebraically stable. This approach is followed in [3]. Nevertheless, even in this case, the distortion caused by the uneven sampling of both polar parameters r and θ deteriorates the performance of the detection process. The last option is to use the fast and accurate polar Fourier transform recently proposed in [4]. From a computational point of view the algorithm's complexity is on the order of the standard Cartesian FFT, while experimental results report significant gain in performance compared to the pseudo-polar grid. This option is the approach adopted in this work.

Once M is computed, we need to measure its periodicity with respect to the angular parameter θ. The solution proposed in [3], starts with computing the expectation of $M(r, \theta)$ with respect to r and therefore reduces the problem of estimating the symmetry order to finding the dominant frequency of a $1D$ periodic function. For real images where the symmetric pattern is embedded in a complex background, noise is not evenly distributed over the whole spatial spectrum and averaging over r does not guarantee noise reduction. We take advantage of all information in M by computing the $2D$ autocorrelation function, as follows:

$$C(k, l) = F^{-1}\{M_F M_F^*\} \tag{2}$$

where M_F is the Fourier transform of M, $*$ denotes the complex conjugate operator and F^{-1} denotes the inverse Fourier transform. The indices k and l represent displacements with respect to r and θ respectively.

A straightforward solution to find the order of symmetry would be to estimate the periodicity of the $1D$ autocorrelation $C(0, l)$ using standard spectrum estimation techniques such as the Fourier transform. Alternatively, we may observe, that for any fixed k, $C(k, l)$ is a periodic pattern of period n. In practice, one would like to identify the range of k, for which $C(k, l)$ provides a reliable estimate of the periodicity and then combine this information to measure it. This idea can be implemented in one step by performing a singular value decomposition (SVD) of C and then finding the period of the dominant eigenvector of the subspace spanned by the rows of C:

$$C = USV^T \tag{3}$$

where the columns of U and V are the eigenvectors of the subspace spanned by the columns and rows of C respectively and S is a diagonal matrix containing the corresponding eigenvalues. The advantage of such an approach is that the estimation of the dominant eigenvector $\mathbf{v}_1$ is largely defined by the rows $C(k, l)$ which measure significant correlation. In contrary, the effect of the rows with low correlation values, is expected to be negligible. This directly comes from the fact that the SVD satisfies the minimum squared error criterion.

Which one of the two methods presented above should be chosen depends on the application. For clean symmetric patterns segmented from the background image, estimating the order of symmetry from $C(0, l)$ appears to be the most reasonable solution. For cluttered images which result in cluttered Fourier transforms, using the SVD approach might be beneficial. Two examples are shown in Fig. 1. It is evident that the $5-$order periodicity is better captured when the SVD approach is used.

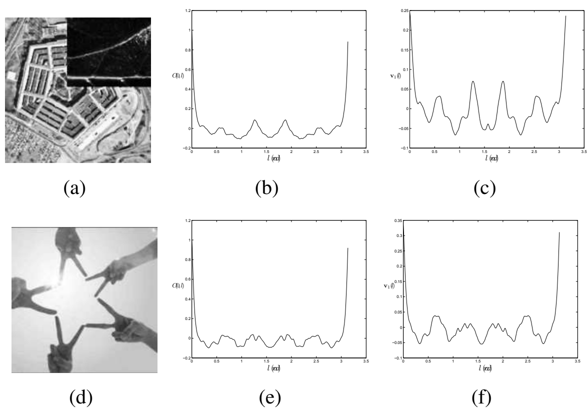

Fig. 1. (a)-(c): A corrupted symmetric image of order 5, the $1D$ autocorrelation $C(0, l)$ and the periodic pattern $\mathbf{v}_1(l)$ extracted using SVD.(d)-(f): Another example of a non-perfect symmetry and the obtained $C(0, l)$ and $\mathbf{v}_1(l)$ respectively.

To estimate the period of $C(0, l)$ or $\mathbf{v}_1$, we consider basic Fourier analysis. Ideally, the Fourier spectrum will exhibit peaks located at frequencies $\omega_k = kn, k = 1, \ldots$ multiples of the order of symmetry. From preliminary results with real images, we have observed that even for images with significant amount of clutter, using the scheme presented so far, the magnitude of the Fourier transform at $\omega = k$ is always among the one or two dominant frequency components. Therefore, considering for example the first two biggest peaks $\omega_i, i = 1, 2$ in the Fourier spectrum, we may prune the set of possible solutions. The real order of symmetry may be found by rotating the image by $2\pi/\omega_i$, computing the cross-correlation between the two images and choose $n = \omega_i$ for the solution which yields the biggest correlation peak.

2.2. Estimating the center of symmetry

Once the order of symmetry has been found, we estimate the center of symmetry using the approach presented in [3]. The method is based on the observation that symmetric images of order n, when rotated by $2\pi/n$, are related to each other by a pure translation. Therefore, we compute the gradient of the rotated image and we estimate the resulting translation using gradient cross-correlation. From this point, a simple geometric inspection reveals that both the distance and the direction of the center of symmetry with respect to the image center can be easily computed.

2.3. Estimating the axes of bilateral symmetry

The final step is to compute the axes of possible bilateral symmetry. If a_0 is the angle formed by the image horizontal axis and the first reflection symmetry axis, then all the lines of reflection symmetry are $a_i = a_0 + \pi i/n, \quad i = 0, \ldots, n-1$. To compute a_0 the image is flipped upside down about its center of symmetry. It can be easily seen that the original image I and its flipped version F are related by a rotation $2a_0$ [2].

To estimate a_0, we start by obtaining a polar representation $I_p(r, \theta)$ and $F_p(r, \theta)$ of the two images about the center of symmetry. It is evident that with respect to θ, I_p and F_p are repeated patterns of order n. We form:

$$I_p^i = I_p(r, \theta_i), \ \ \frac{2\pi i}{n} \leq \theta_i < \frac{2\pi(i+1)}{n}, \ \ i = 0, \ldots, n-1 \tag{4}$$

That is we segment the image I_p in n parts I_p^i according to the recovered so far symmetry information (order and center of symmetry). Then, we recover a_0 by matching any of I_p^i to F_p using gradient cross-correlation (that is, instead of using the original images we rather use image gradients). Now, for perfect bilateral symmetries, it is evident that correlating any I_p^i with F_p will yield n peaks in the derived correlation function located at $2a_0 + 2\pi i/n, \quad i = 0, \ldots, n-1$. For rotationally but not bilaterally symmetric objects, it is expected that no dominant peaks will appear. Similarly, for corrupted reflection symmetries, possibly due to partial occlusion, if I_p^i is partly or totally corrupted, peaks of small height or no peaks at all will appear respectively. Nevertheless, assuming that the corruption is partial, there will always exist a non-corrupted I_p^i which will yield $n_c < n$ dominant peaks in the resulting correlation.

From the above analysis, it is clear that the proposed scheme makes full use of the symmetry information to provide localization in the detection of the axes of bilateral symmetry, and therefore it is much more robust than the global approaches suggested in [2], [3]. A by-product of the above approach is the detection of possible occluded parts, which may be useful, for example, for segmentation purposes. Additionally, we emphasize that the detection process is based on gradient cross-correlation, a variation of phase correlation based on image gradients, which is expected to provide sharp peaks and better localization than standard cross-correlation techniques.

3. RESULTS

In this section we present preliminary results by applying the presented scheme to real images [1]. In Fig. 2a we show an example of an image which captures a non-perfect symmetry of order $n = 5$ embedded in background clutter. The recovered periodic pattern obtained by the singular value decomposition

[1]The pentagon images and the results related with the work in [3] are directly obtained from the paper.

of the $2D$ autocorrelation matrix and its Fourier spectrum are illustrated in Fig. 2b and 2c respectively. For the same image, the periodic pattern extracted by the approach proposed in [3] along with its Fourier spectrum are shown in Fig. 2d and 2e. For this example, it is evident that our scheme yields an easier estimation of the order of symmetry which can be identified correctly even with basic spectrum estimation techniques such as Fourier analysis.

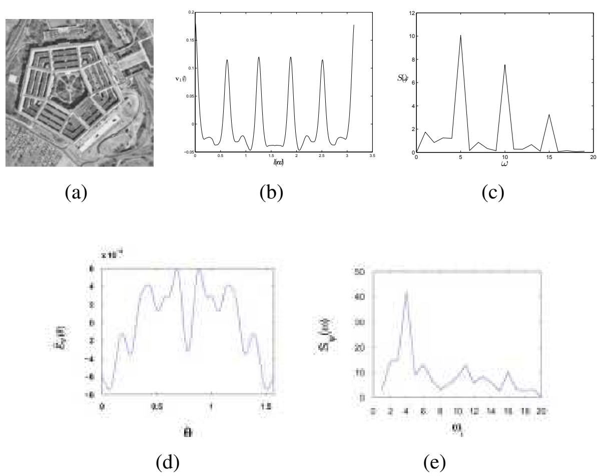

Fig. 2. (a): A a non-perfect symmetry of order 5. (b)-(c) the periodic pattern $\mathbf{v}_1(l)$ extracted using SVD and its Fourier spectrum.(d)-(e): The periodic pattern extracted using the method in [3] and its Fourier spectrum.

To illustrate the efficiency of our approach in estimating the axes of bilateral symmetry we consider the corrupted version of the same image shown in Fig. 1a, for which the scheme in [3] failed to identify correctly. Figures 3a and 3b show the $1D$ gradient cross-correlation functions obtained by considering the occluded and one non-occluded segment I_p^i respectively. We may observe that while no dominant peaks appear in the former case, spikes of large magnitude are evident in the latter case. The symmetry axes estimated from the biggest peak are sketched in Fig. 3c. The symmetry axes for the image in Fig. 1d are also shown in Fig. 3d. We conclude that the proposed scheme provides good accuracy.

4. CONCLUSIONS

We have presented a frequency domain framework for the detection of symmetries in real images. The approach is able to estimate the order and center of symmetry, define its type and identify the possible axes of bilateral symmetry. Preliminary results show promising performance.

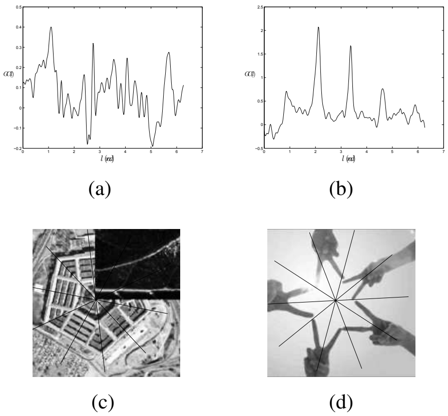

Fig. 3. (a)-(b): The $1D$ gradient cross-correlation functions for the occluded and one non-corrupted part of the symmetric pattern in Fig. 1a. (d)-(e): The estimated axes of bilateral symmetry.

5. ACKNOWLEDGEMENTS

We would like to thank Dr. Nikolaos Mitianoudis for useful discussions on this work and Dr. Michael Elad for providing the Matlab implementation of the polar FFT.

6. REFERENCES

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